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Louis de Mesnard. Infected persons by the fact of a public organism : is a fair compensation by the community realistic ? A graph-theory pproach . [Research Report] Laboratoire d'analyse et de techniques économiques(LATEC). 1999, 23 p., Illustration, ref. bib. : 12 ref. hal-01526506

HAL Id: hal-01526506

<https://hal.science/hal-01526506v1>

Submitted on 23 May 2017

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ISSN : 1260-8556

n° 9909

**Infected persons by the fact of a public organism :
is a fair compensation by the community realistic ?
A graph-theory approach**

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June 1999

**Infected persons by the fact of a public organism:
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Keywords. AIDS, hepatitis, infection, compensation, graph.

JEL Classification. I18.

Abstract. The paper focuses on indirectly infected persons (persons infected by a person, infected by a person, etc., itself infected by a public organism, for AIDS, hepatitis C, etc.). It is assumed that fairness implies the national community to indemnify them. Graph theory is used to explain what is indirect infection. The concept of chains of infection, and their length is central. Re-infections are considered also because even individuals that are initially infected by another cause could become later indirectly infected by the public agent. The number of persons to be indemnified is larger than for direct infection, so the cost of compensation is higher, unless if compensation per capita is lower. The required quantity of information is so large that this is unrealistic and not democratic. The precision of the determination of infecting causes, as well as the responsibility, decreases rapidly when the length of chains increases. With re-infection, potentially all infected persons could become relevant of a compensation. Compensation of indirectly infected persons is unrealistic, what is unfair when the directly infected persons are compensated.

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I. Introduction

A general tendency inside developed countries is the following: the national community have to compensate financially the disease generated by an infection contracted by the direct fact of a public organism ¹, as in the case of AIDS or hepatitis C contracted during a blood transfusion. This is fair: the community must support the consequences of its negative actions, even if they are involuntary ^{2 3} and even if the number of persons that are concerned by this

¹ The paper focuses on the case of public health organisms, even if in some countries, as France, both public and private health organisms can be sued, as private individuals, as hospital directors, doctors, etc. However, this is not the aim of this paper, even if some conclusions can be adopted. A first difference is that the number of victims of a same private individual is lower, even if the possibility of compensation that can be paid by a private agent is lower. A second difference is that insurance systems can be suitable more easily to pay the compensation of the injury.

Note that, the same principle can be discussed for other diseases contracted by the fact of a private organisms as cigarette producers, because it can be argued that the infected persons were knowing the risk: it is not the aim of this paper. In the paper, only infectious diseases will be considered.

² The question of a faulty behavior of the infecting organism is left out of this analysis; in the French law, the health organism must have done a fault to be condemned to pay a compensation but the victim does have to prove the fault of the health organism: this one has to prove its no-fault behavior; for Brazier and McLean (1993) and Harris (1997), compensation must be automatic because the distinction between two categories of victims

infection can be large (even if data are difficult to know) what is problematic for a compensation of their injury (perhaps this is why the French compensation law for post-transfusion hepatitis C infection that were planned in 1994 has never been achieved).

However, the case of *indirectly infected persons* can be examined. I call indirectly infected persons, the persons that have contracted an infectious disease as AIDS or hepatitis C from another person that have itself directly contracted it by the fact of a public organism, or that have contracted it from a person that have itself indirectly contracted it, etc. Usually, these persons are excluded from compensation but the fairness of such a decision can be discussed: do indirectly infected persons have to be compensated by the community? Generally, the programs of compensation by the community do not envisage to indemnify the individuals that are not directly infected by the public agent but that are only indirectly. Yet, the responsibility of the public agent is engaged, because if the initial infection could have been removed, the indirect infection could have been avoided itself: the initial cause is the behavior of the public organism ⁴.

(victims after a faulty behavior of the health organism and other victims) is unfair.

³ The question of how much has to be paid for a fair compensation does not concern this paper. About the theoretical aspects of the question, see (Johansson, 1995). About the measure of how high is the disability introduced by such a disease, see for example (Bowling, 1995, pp. 271-273). Note that a potential disease can be compensated even if the infection simply generates a fear (Dyer, 1998b).

⁴ Note that the responsibility of a public health organism cannot be reached for infections that are easy to contract in the ordinary life, even for nosocomial infections; for

So it is unfair not to indemnify the disease of indirectly infected persons when the responsibility of the public agent is indirectly engaged. Justice imposes a yes as answer to the question of the compensation of indirectly infected private individuals by the national community when infection has been caused by a public organism, even indirectly ⁵. However, the determination of who have been infected is difficult. In this paper, graph theory will be used to define precisely what means "indirectly infected", then to explore the consequences and the difficulties generated by the question of compensation of indirectly infected persons.

example, B hepatitis can be transmitted involuntarily by a simple oral contact when hepatitis C or AIDS can be transmitted only by blood transfusion or sexual contact.

Also, some public health organisms -- as in France -- have an obligation of result, not of mean: they have to guarantee the patients against all infectious agents, known or unknown.

⁵ This idea is not similar to the idea of compensation of the victim *by ricochet*. In the case of the neonatal infection both concepts, indirect infection and infection by ricochet, can be confused: the child is a victim by ricochet if its parents decease or cannot have a job. Another example: for the Creutzfeldt-Jakob disease (remember that this disease cannot be compared with AIDS or hepatitis C: it is not proved that it can be transmitted to another human), some families have obtain compensation for the infection and/or the death of their children (Dyer, 1997 and 1998a). About these juridical problems, see (Gromb, 1998).

II. Indirect infection by the public health organism

A. The idea of indirect infection

Denote i and j the indices for private individuals that have infected or that have been infected.

A binary relation is set up: considering a private individual j that was not infected before, i is in relation with j if i have directly infected j , what is denoted: $i R j \Leftrightarrow h_{ij} = 1$ and $h_{ij} = 0$ else: the term h_{ij} is a boolean variable that indicates that j have been infected by i ⁶. Naturally $h_{ii} = 0$. Re-infection is excluded at this step, so there are no circuits; for example, if $h_{ij} = 1$, then $h_{ji} = 0$. All terms h_{ij} are regrouped in a matrix \mathbf{H} . Matrix \mathbf{H} has only one 1 by column (even if it is possible to find more than one 1 in each row). For reasons that will be explained now, matrix \mathbf{H} has one additional row and column.

$$\mathbf{H} = \begin{bmatrix} 0 & \dots & h_{1i} & \dots & h_{1n} & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ h_{i1} & \dots & 0 & \dots & h_{in} & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ h_{n1} & \dots & h_{ni} & \dots & 0 & 0 \\ 0 & \dots & 0 & \dots & 0 & 0 \end{bmatrix}$$

A supplementary agent has to be introduced: it is any public organism, of which index is denoted s (for simplicity, it is assumed that the number of public organism is 1). Also, some private individuals can have initially contracted the disease without the help of the public agent and without the contact of any other persons infected by the help of a public organism. I call them "naturally infected agents". The vector \mathbf{e} denotes the initial infection. One has $e_s = 1$; the fact that i is a naturally infected agent is denoted $e_i = 1$. There can be many zeros in vector \mathbf{e}

⁶ At this step, the relation is assumed to be deterministic.

because all individuals, naturally infected or not, are in \mathbf{e} : for individuals i that are not naturally infected, the term i in \mathbf{e} is zero.

The public organism and naturally infected persons can be seen as exogenous entries of the system made up of private individuals. One have only two types of initially infected individuals: those that are initially infected by the public organism s -- compensation by the community is fair for them -- and those that are initially infected by an agent k -- compensation by the community does not seem fair (even if these persons are relevant of an health care system to pay the cost of their disease, as for everybody) --. The direct infection by the public organism or by initially infected agents is given by the vector $\mathbf{e}^{(1)} = \mathbf{H}' \mathbf{e}$.

Example.

$$\mathbf{H} = \begin{array}{c} \left[\begin{array}{cccccccc} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ s \end{array} \end{array} \text{ and } \mathbf{e} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

1 2 3 4 5 6 7 s

Agent 5 is a naturally infected individual. ■

Chains of indirect infection can appear. One say that an agent j is indirectly infected by an agent i if there is a chain between i and j . For an indirect infection implying only one intermediary individual (the chain is two arcs long):

$$h_{ij}^{(2)} = 1 \Leftrightarrow \text{there exists a person } j' \text{ such that } \{h_{ij'} = 1 \text{ and } h_{j'j} = 1\}$$

So, $h_{ij}^{(2)} = h_{ij'} * h_{j'j}$, where $*$ is the boolean product, obtained by multiplying \mathbf{H} with itself by the boolean product, denoted: $(\mathbf{H}')^2 = (\mathbf{H}') * (\mathbf{H}')$ because $\mathbf{e}^{(2)} = \mathbf{H}' \mathbf{e}^{(1)} = (\mathbf{H}')^2 \mathbf{e}$. This can be generalized for a t -arcs chain: $h_{ij}^{(t)}$ is the term of $(\mathbf{H}')^t$, where t is an index of iteration. As the length of the chain is $l_{ij} = t$, the higher t is, the more indirect is the infection between individuals. l_{sj} indicates the length of the infection chain from the public organism to the person j : $l_{sj} = 1 + l_{ij}$ if i was directly infected by s and if j was infected by i indirectly. In the above example, s has infected 1, that has infected 2, that has infected 3, etc. This corresponds to a chain in a tree (see the figure 1 that corresponds to the above example): $s \rightarrow 1 \rightarrow 2 \rightarrow 3$. Note that a same individual has only one backward chain, beginning either by s , either by a naturally infected person, the individual 5 in the figure.

Figure 1 about here

The boolean summation $\mathbf{e}^{(1)} + \mathbf{e}^{(2)} + \dots + \mathbf{e}^{(t)} + \dots = [\mathbf{H}' + (\mathbf{H}')^2 + \dots + (\mathbf{H}')^t + \dots] \mathbf{e}$ indicates all chains of indirect infection: if $\tilde{h}_{ij} = 1$ then i have directly or indirectly infected j by a chain of any length. The boolean summation matrix \mathbf{w} computed as,

$$\mathbf{w} = \mathbf{e} + \mathbf{e}^{(1)} + \mathbf{e}^{(2)} + \dots + \mathbf{e}^{(t)} + \dots = [\mathbf{I} + \mathbf{H}' + (\mathbf{H}')^2 + \dots + (\mathbf{H}')^t + \dots] \mathbf{e}$$

indicates who was infected by all chains of direct or indirect infection from the public organism s or from a "naturally infected agent": in \mathbf{W} can be found the "successors" of the public health organism or of naturally infected agents. This tends to $(\mathbf{I} - \mathbf{H}')^{-1} \mathbf{e}$.

Example (following):

$$\mathbf{e}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{ etc., and } \mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

As only infected individuals are taken into account, the vector \mathbf{w} is full of zeros. ■

To know who is infected by the public agent, it is simply sufficient to replace the vector \mathbf{e} by the vector \mathbf{e}^P such that $(\mathbf{e}^P)' = (0 \dots 0 1)$. The direct infection caused by the public agent is given by $\mathbf{e}^{P(1)} = \mathbf{e}^P \mathbf{H}'$. A vector \mathbf{w}^P is computed as $\mathbf{w}^P = \mathbf{e}^P [\mathbf{I} + \mathbf{H}' + (\mathbf{H}')^2 + \dots + (\mathbf{H}')' + \dots]$: it indicates all individuals to be compensated for their infection by the public agent.

Example (following).

$$\mathbf{e}^{P(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}^{P(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}^{P(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}^{P(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{ etc., and } \mathbf{w}^P = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Here, agents 1, 2, 3 and 6 are directly or indirectly infected under the direct or indirect responsibility of the public agent. ■

B. Compensation for indirect infections

One has two types of individuals.

- The individuals, j , that are not directly or indirectly infected by a public organism, along a chain of any length. For those individuals, there exists no chain of any length exists beginning by s . Thus, all chains, that can be set up to j , do not begin by s : $w_i^P = 0$ for all n . For these individuals, clearly a compensation by the community has not to be set.
- The other individuals, for whom a chain -- of any length -- can be set from s to them: $w_i^P = 1$. Many of the individuals are not directly infected by s , but only indirectly.

Unfortunately, there are more people with $w_i^P = 1$ than people with $e_i^{P(1)} = 1$, so the cost of compensation is higher when indirect infection is considered. This intuitive idea is easy to prove. First, the number of individuals directly or indirectly infected by the public organism can be globally estimated by $\frac{N}{1-p}$, where N is the number of persons initially infected by the public agent and p is the overall probability of transmission of any agent i to any other agent j (remember that it is the overall probability and not the probability at each period): you have N agents directly infected, Np indirectly infected at the first iteration, Np^2 at the second, etc.; this is decreasing but the total number $\frac{N}{1-p}$ is higher than n and can be much higher if p is strong. Second, in the boolean product $(\mathbf{H}')^2$, you have some "1" that are not in the same place than in \mathbf{H}' ; so, in the boolean sum $\mathbf{H}' + (\mathbf{H}')^2$, you have generally more "1" than in \mathbf{H}' ; a similar thing happens for $(\mathbf{H}')^3$ and $\mathbf{H}' + (\mathbf{H}')^2$, etc.

Compared to the number of persons indirectly infected by a naturally infected person, the number of persons indirectly infected by the public organism can be large or low: it depends

on the data ⁷, i.e., on the structure of \mathbf{H} and \mathbf{e} . The computation has to be done, so an information problem occurs. Only a precise study of the direct causes of contamination of **all** infected individuals of the society (even if their infection is not yet declared), followed by the computation of indirect infection chains, can help to answer to this question ⁸. This is not realistic, or this can be realized at a very high cost.

What is the information cost? There are two ways to evaluate who is infected. First, starting from an infected person, the question is: who has been infected by this person from the moment of its infection; it is the forward search. Potentially, there are n persons, minus the persons that are known to be yet infected. Second, starting from an infected person, the question is: by whom this person has been infected; this is the backward search. There is, by definition, only one person, and the complexity of the search is similar to the forward search but this will be not the case in an improved version of the model where some indecision may occur.

⁷ That is on the behavior of the people directly infected and on the disease: probably, these people have a less risky behavior than naturally infected persons regarding to the problem of AIDS infection, but for hepatitis C, the possibility of transmission does not depend on a risky behavior.

⁸ Do not confuse this question with the question of the estimation of HIV incidence over time from surveillance data, that is to say the estimation of the number of people that develop and AIDS from the number of persons simply infected by the HIV virus (Lessner, 1998a and b). I leave out the distinction between the fact to be infected -- by example by the HIV virus -- and the fact to be effectively ill -- by example to have effectively develop and AIDS syndrome.

Moreover, matrix \mathbf{H} and vector \mathbf{e} have a very large number of rows and column, even when the study is restricted to actually infected persons ⁹, so the cost of search is very high because it is proportional to the number of arcs. Fairness, that implies here perfect information, is impossible to reach in a democratic society unless to establish the inefficient Orwell's "big-brother" society.

C. Decreasing responsibility and probabilistic transmission

1. Decreasing responsibility

The hypothesis of a decreasing indirect responsibility of the public organism can be also introduced: in this case, the responsibility of the public organism decreases when the length of the infection chain increases. So the $\frac{1}{l_{sj}}$ can be an indicator of the decreasing responsibility of the public organism on the indirect infection of the person j . A less artificial view consists into considering that in the chain $s \rightarrow i \rightarrow j \rightarrow j' \rightarrow j'' \rightarrow \dots$, it is not acceptable that j could obtain a higher compensation than i has obtained itself, only by the fact that j has the possibility to sue both i and s (even i is a private individual) when i is able to sue only s . So, the responsibility of the infection of j must be shared between s and i : s is fully responsible of the infection of i , s and i become equally responsible of the infection of j , then s , i and j become equally responsible of the infection of j' , etc. So, naturally, the initial responsibility of the public organism becomes decreasing. The choice between one principle or the other depends on legal principles what goes outside the subject of this paper.

⁹ But I have said that an individual can be infected not initially but only later: potentially, **all** the population of the country could be considered.

2. Probabilistic transmission

It has been assumed that only one person can be infected by only one person but, when is considered the possibility of multiple partners and multiple acts per partner, some indecision remains about who have been really infected by whom: often, it is only possible to know with whom a person has had a potentially infectant contact (as a sexual relation) but not if this contact was really infectant (for example, X has been infected; he has had a contact with Y and Z , who are declared infected after a test, but Y and Z declare that they have had also a contact with other persons ¹⁰).

For these reasons, the transmission relation h becomes probabilistic; see: (Pinkerton, Holtgrave and Bloom, 1998, p. 1070). So, the precision of the search of infection causes can be considered as rapidly decreasing. Consider a probability p_{ij} of truth for the information: " i has infected j ". Naturally, considering the complementary event, " j has been infected by another cause than i " and considering its probability $p_{\bar{ij}}$, it follows that $p_{ij} + p_{\bar{ij}} = 1$. Along a chain $s \rightarrow i \rightarrow j \rightarrow j' \rightarrow \dots j^{(n-1)} \rightarrow j^{(n)}$, these probabilities of truth are multiplicative: $p_{sJ} = p_{si} p_{ij} p_{jj'} \dots p_{j^{(n-1)}j^{(n)}}$. As all probabilities are between 0 and 1, the precision of the determination of infecting causes decreases rapidly when the length of chains increases: the probability p_{sJ} tends toward zero when n tends to infinite. When a probability near zero is assumed to be equal to zero, the responsibility of a given infectious organism or person is

¹⁰ An additional difficulty occurs when a person is infected but the disease is not declared (or is hidden by the medical treatment (Zhang *et alii*, 1998): this person can be infectant without itself knowing). In this case, investigation can be difficult, or even impossible, because uncertainty is maximal.

limited forward: in practical terms, chains with only two up to four steps have to be considered instead of infinite chains what is more practicable.

On the other hand, when the transmission relation is probabilistic, if an indecision remains, one can admit that there are equally placed in the infection problem: one person can have been infected by multiple partners (the person can have had multiple potentially infectant contacts with many persons that are all infected, without the possibility to say that this is one or the other that has infected him). In this case, there can be more than one "1" in a same column in matrices \mathbf{H} . As a consequence, forward and backward searches have to be combined, the complexity of search is larger, the cost also.

III. Re-infection

1. Complications induced by the idea of re-infection

Depending on the considered disease, a re-infection may have no additional effect, or may have a serious effect. When re-infection does not add to the disease of the person, this re-infection can be neglected without consequences and the resulting graph remains a tree again (trees are more simple to study).

In the second case, if re-infection adds to the disease of the re-infected person, one can think about the case of the re-infection of any private individual i : a same individual has now many backward chains. As a general principle, a fair attitude could consist into compensate only the increasing of the disease introduced by the additional infection, when it can be calculated what

is no so easy: again detailed investigations are necessary but they can appear as incompatible with respect of of private life and democracy.

However, the most important case is those of naturally infected persons i that are re-infected by a person j that have itself the public agent s in its chain: even partially, these persons i leave the category of naturally infected persons and are able to receive compensation, as the other persons that will be infected by them later.

Loops in the chain are even possible but they are not true loops because time flows here: in a static ordinary graph, a loop is run through an infinity of times, so re-infection is repeated again and again, what is not realistic. A dynamic version of the model has to be introduced by taking time into account: loops become *pseudo-loops* and they will be run through only one time.

Remark. Short pseudo-loops, as $i \rightarrow j \rightarrow i$ are normally poorly re-infectious (the infectious agent is exactly the same and has a low additional effect). If they are long, these pseudo-loops can have an additional infectious effect because the infectious agent can have been modified between the first and the second infection (example: mutations of the virus) and even because a simple re-infection with the same infectious agent can reactivate the immunological process ¹¹. ■

In dynamics, things are more complicated than in statics. It can be assumed that there is an initial period, denoted $t = 0$, where the infection by the public organism (and by the naturally infected agents) occurs, and there are following periods t , where private individuals that have

¹¹ The case of multiple infections is left aside: it is known that an infection with a some virus as hepatitis C or AIDS can worsen another infection as rheumatoid polyarthritis.

been infected at preceding periods can infect other private individuals. Denoting by \mathbf{H}_t the matrix \mathbf{H} for period t then the product $(\mathbf{H}')^2$ has to be replaced by $\mathbf{H}'^{(2)} = \mathbf{H}'_1 * \mathbf{H}'_2$, then $(\mathbf{H}')^3$ by $\mathbf{H}'^{(3)} = \mathbf{H}'_1 * \mathbf{H}'_2 * \mathbf{H}'_3$, etc., up to $\mathbf{H}'^{(t)} = \mathbf{H}'_1 * \mathbf{H}'_2 * \dots * \mathbf{H}$ that replaces $(\mathbf{H}')^t$. Finally:

$$\begin{aligned} \mathbf{w} &= \mathbf{e} + \mathbf{e}^{(1)} + \mathbf{e}^{(2)} + \dots + \mathbf{e}^{(t)} + \dots \\ &= \mathbf{e} + \mathbf{H}'^{(0)} \mathbf{e} + \mathbf{H}'^{(1)} \mathbf{e}^{(1)} + \mathbf{H}'^{(2)} \mathbf{e}^{(2)} + \mathbf{H}'^{(3)} \mathbf{e}^{(3)} + \dots + \mathbf{H}'^{(t)} \mathbf{e}^{(t)} + \dots \end{aligned}$$

where $\mathbf{H}'^{(0)} \mathbf{e}$ corresponds to the initial infection by s and by the naturally infected agents, without indirect infections, at the beginning of the epidemic study ($t=0$ stands for the beginning date of the study).

Remark. With this dynamic model, even the decease of some persons can be taken into account. For the persons i that are deceased at a date t , the row i of $\mathbf{H}^{(t)}$ becomes entirely full of zeros. ■

Example.

$$\mathbf{H}^{(0)} = \begin{array}{c} \left[\begin{array}{cccccccccc} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ s \end{array} \\ \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & s \end{array} \end{array}$$

$$\mathbf{H}^{(1)} = \begin{array}{c} \left[\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ s \end{array} \\ \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & s \end{array} \end{array}$$

$$\mathbf{H}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ s \end{matrix}$$

$$\mathbf{H}^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ s \end{matrix}$$

$$\mathbf{H}^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ s \end{matrix} \text{ and } \mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Figure 2 about here

Figure 3 about here

In this scholarship example, where agent 1 is a naturally infected individual, you can find two pseudo-loops, $1 \rightarrow 2 \rightarrow 7 \rightarrow 9 \rightarrow 1$ and $1 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 1$ (see figures 2 and 3). In the first case, there is a loop but it is run through one time only. In the second case, the same problem occurs but, in addition, the temporal causality is not respected: this pseudo-loop is not run through even one time and it is only an artefact: the naturally infected agent 1 has been indirectly re-infected by s through 4 and 6 but 1 has no responsibility in the infection of 6

because the re-infection of 4 (through 3) is later. The same thing happens for $1 \rightarrow 8 \rightarrow 5 \rightarrow 2 \rightarrow 7 \rightarrow 9 \rightarrow 1$; for $8 \rightarrow 5 \rightarrow 2$ and $5 \rightarrow 2 \rightarrow 7$, temporal causality is not respected: even if 2 has been indirectly re-infected by s through 5, s has no responsibility in the infection of 7 and 9. ■

The idea of period itself can be discussed: is it the year, the month, the week, the day, or less? It is not only a question of time basis: the responsibility of agents depends on it. Inside a same period, all infections are assumed to be simultaneous. For example, if the organism s infects j , and at the following period, j infects j' , so s has infected j indirectly, when both periods are confused into only one, either s appears to have infected s and j' directly, either s appears to have infected only j and for j' the origin of infection appears to be unknown. The second possibility is more plausible. Again, the complexity of the problem, and its contestability, appears clearly.

2. Responsibility and compensation

For the individuals that are infected by a naturally infected individual l that have been indirectly re-infected by the public organism, before the re-infection, the public organism is not responsible, when after it is (again the static model does not work and the model must be dynamic). Moreover, when time passes, by the mechanism of re-infection of naturally infected persons, the ratio of the number of individuals that have s in their chains under the number of persons that fall under the first category will increase. So, if it is considered that re-infection has bad additional consequences, then when time passes the infected population, eligible for a compensation by the community, will increase even if the compensation concerns only the

additional consequences. So, the cost of compensation will be higher than in the case without re-infection, unless compensation per capita is reduced.

IV. Conclusion

It is not fair that the community compensates only individuals that are directly infected by the public organism. Persons that are indirectly infected by the public organism have to be compensated by the community. With the help of graph theory, it is demonstrated that this implies two consequences. First, the number of persons to be indemnified is larger than for direct infection, so the cost of compensation is higher, unless if compensation per capita is lower. Second, the quantity of information that is required is so large that it is unrealistic and contradictory with democracy. Third, the precision of the determination of the infecting causes, as well as the responsibility, decreases rapidly when the length of chains increases. In the case of persons that have been only indirectly re-infected by the public organism with bad consequences, conclusions are amplified: potentially all infected persons could be a matter for compensation. Finally, compensation of the indirectly infected persons is unrealistic, what is unfair if the directly infected persons are compensated.

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Fig i

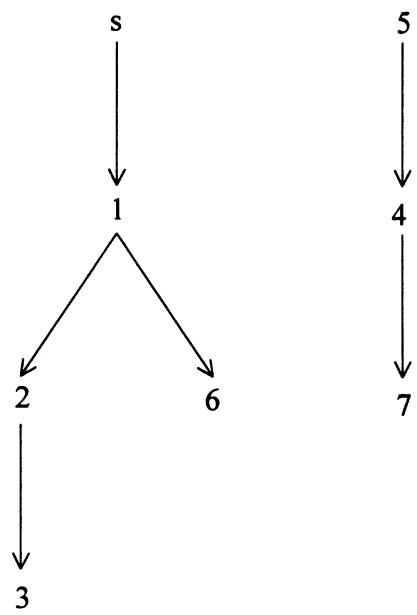


Figure 1. Tree of infections

Fig ii

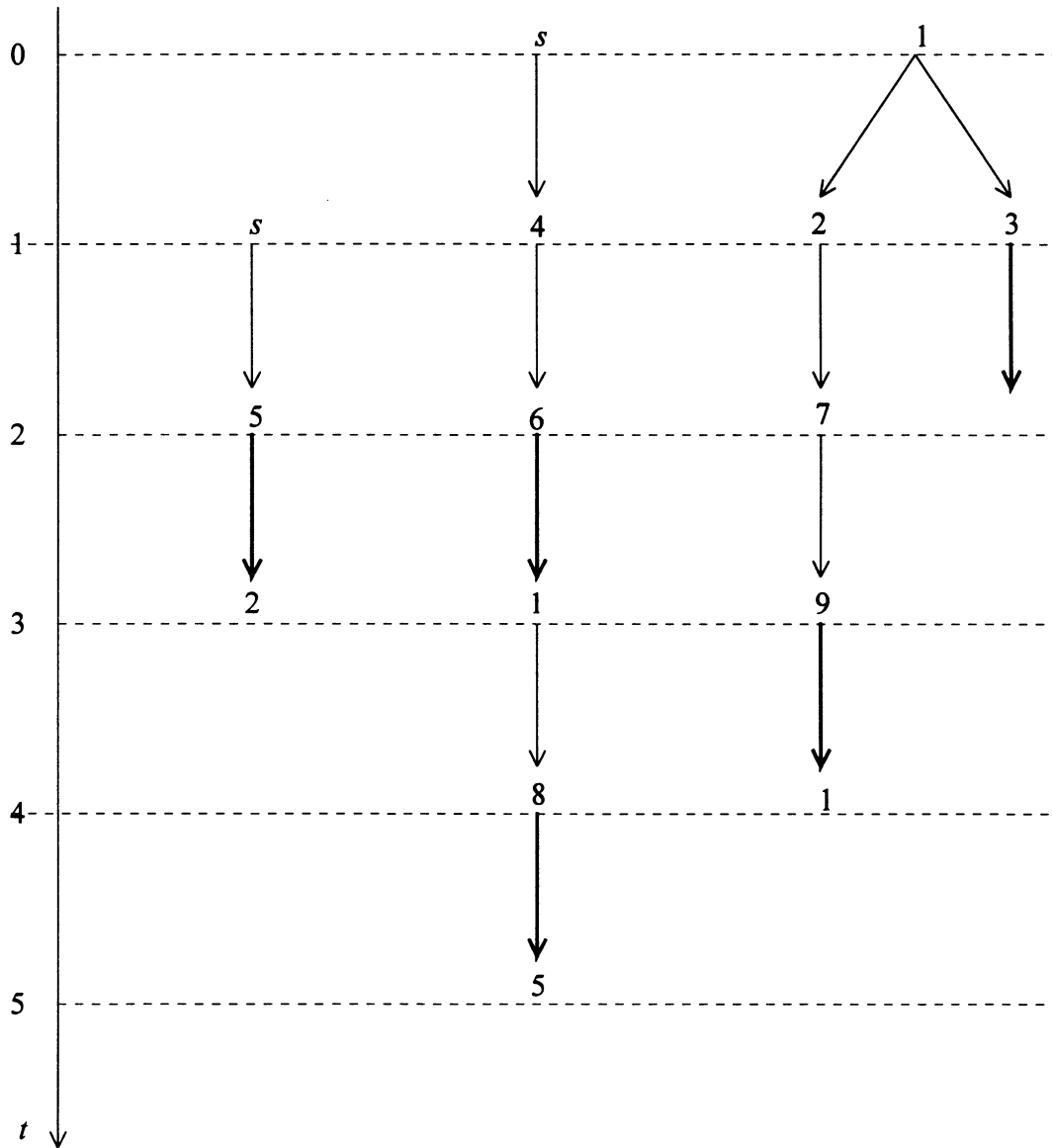


Figure 2. Example: graph with re-infections
(arcs in bold: re-infections; time flows from up to down)

Fig iii

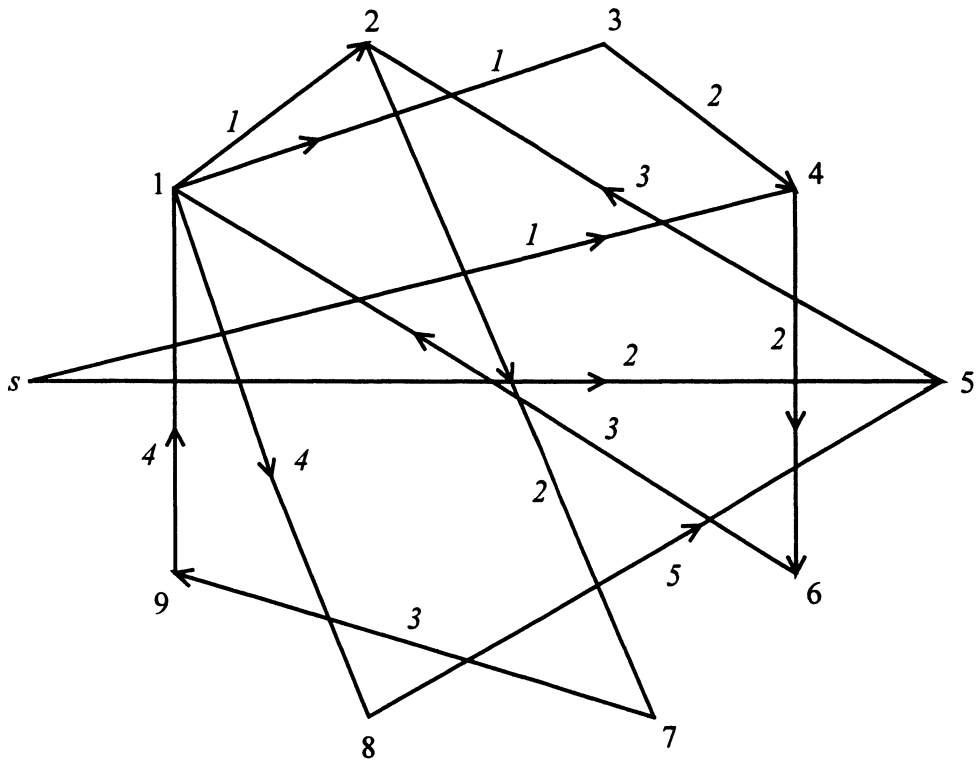


Figure 3. Example: synthetic graph of relations
(italic numbers over the arcs indicate the number of period;
remember that loops are only pseudo-loops)

