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Industrial Organization with profit rate maximizing firms

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Industrial Organization with Profit Rate Maximizing Firms

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KEYWORDS.

Coordination, Rate of Profit, Oligopoly, Behavior.

ABSTRACT.

We study the impact on industrial organization of the switching of objective function, from pure profit to profit rate maximization. The output level of firm is lower at optimum. This lead to a new conception of efficiency. Cases of no coordination are considered. In perfect competition, price signal disappears; factors remain paid at their marginal productivity, but modified. In imperfect competition, reaction functions may vanish even if collusion remains possible; limit of oligopoly remains perfect competition of profit rate; the paradox of Bertrand may remain; a new concept is studied: mixed duopoly, where firms can choose and change their objective.

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I. Introduction

Coordination among firms is one of the major themes of economics. It is the central idea of both signal theory and the theory of games.

- In perfect competition, each firm equalizes the price and their marginal cost to maximize its pure profit. The price signal allows coordination between the firm and the market.
- In imperfect competition with a small number of firms, as in duopoly-oligopoly, firms construct reaction functions in which conjectural variations play the central role, indicating the reaction of the firm(s) other than i with respect to the action of firm i (terms $v_i = \frac{\partial \tilde{Q}_i}{\partial Q_i}$ with Q_i as the output level of firm i and $\tilde{Q}_i$ as the output level of other firms). The intersect of the reaction functions determines the optimal output level of all firms. It is one of the starting points of the theory of games.

We will explore the consequences on coordination among firms of relinquishing the neoclassical objective function - the maximization of the pure profit - for another objective function, the maximization of the rate of profit, which is considered closer to actual business practice.

The variables that we will introduce are:

- $p(Q)$, the inverse of the demand curve, decreasing with Q.
- $R(Q) = p(Q)\,Q$, the income of the firm.
- $C_g(Q)$, the *cost of production* (subscript g for "gross"). It includes all the costs (direct costs and opportunity costs) of variable factors, like labor, plus renting costs and writing off costs of fixed capital. It excludes interest on debts, remuneration and opportunity cost of the equity capital.
- $K(Q)$, the positive *capital* (monetary, not physical) used in production.
- t , the constant and positive interest rate in the financial market, valid for both borrower and moneylender: as the financial market is assumed to be perfect, the same rate t is used for the debt of the firm and for the opportunity cost of the funds.
- $K_e(Q)$, the positive *equity capital* used in production.

  Even if remuneration is not necessarily paid, equity has an opportunity cost $t\,K_e(Q)$ sustained over each period at the fixed interest rate t.
- $K_d(Q)$, the *borrowed capital* (the debts) used in production.

  Interest is paid by the firm for each period: $t\,K_d(Q)$.

We obtain by hypothesis: $K(Q) = K_e(Q) + K_d(Q)$.

- $\Pi_g(Q) = R(Q) - C_g(Q)$, the *production profit* or *gross profit*.
- $\Pi(Q) = \Pi_g(Q) - t\,K_d(Q) - t\,K_e(Q)$, the *pure profit*.
- $\pi(Q) = \dfrac{\Pi_g(Q)}{K_d(Q) + K_e(Q)} = \dfrac{\Pi_g(Q)}{K(Q)}$, the *economic rate of profit* denoted in the following the *rate of profit*.

In the following, we will neglect to write the argument Q in the writing of functions when there is no confusion.

The neoclassical microeconomic firm maximizes pure profit, subtracting the debts and the opportunity cost of the capital to the gross profit: max Π. We shall examine another objective for firms: the maximization of the rate of profit, max π_e. As $\Pi = \Pi_g - t\,K$, these two objectives are independent to the financing decision i.e. to the division in equity capital and debts.

These two objectives are exactly equivalent concerning the condition of entry into the sector, i.e. when deciding whether the firm should or should not produce. A set of conditions of entry into the sector for the firm is as follows :

$$\Pi \geq 0 \Leftrightarrow \Pi_g - t\,K \geq 0 \qquad (1)$$

and $$\pi \geq t \Leftrightarrow \frac{\Pi_g}{K} \geq t \qquad (2)$$

However, classical microeconomic theory chooses condition (1) and not condition (2). In fact, there are two possibilities for a firm : *to maximize absolute satisfaction* or *to maximize relative satisfaction*. If the entry conditions are equivalent, the question is: why has the subtractive form of the profit function, Π, prevailed over the "divisive" form π? Why did firms historically choose to maximize pure profit? We may conjecture as follows .

In the pre-industrial age and from the beginning of the industrial revolution, the modern legal corporate form of firms was not widespread and as the banking system and the financial market were non-existent, capitalists disregarded debts and opportunity costs. Capitalists (or landowners) often had no alternative opportunity for investment [1] than their own business (or property): either they could leave their funds idle, or they could produce. In either case, they could count on their own funds only. Under the circumstances, maximization of profit was a valid objective even if the profit was only one cent for one million in gold coin. This is why the pre-classical authors, such as Bacon or Hobbes, naturally adopted the principle that best complied with the observed reality of the day, maximization of satisfaction, i.e. maximization of earnings, later modernized as maximization of profit.

Even if the classical authors emphasized the rate of profit (Smith, Ricardo, etc.), they considered the social rate of profit and the rate of profit of the sectors as Marx did later (see the law of

[1] Imagine that a landowner finds a casket with one million in gold coin inside.

equalization of the sector rate of profit), more than the rate of profit of the firm, and they did not think in terms of maximization of the rate of profit.

Later, the neoclassical authors retained the principle of profit maximization, taking into account debts and opportunity costs to create the concept of pure profit [2]. As in General Equilibrium the profit of each firm is the income of the productive agent, it seemed natural to maximize the income of the agent. The maximization of pure profit did not change ways of thinking too much.

Now, clearly, in the real world, a firm tries to maximize its return on capital, i.e. its rate of profit. This proves that it is necessary to explore an alternative theory of the rate of profit maximization. The microeconomic consequences of this have not been studied, although it is an old idea in the business world. We deliberately leave aside the improvements made to the neoclassical theory of the firm [3], such as incomplete information, the theory of bounded rationality [Simon, 1962], the theory of agency [Grossman and Hart, 1983], the theory of contracts and incentive [Hart 1983], managerial [Baumol,] and behaviorist theory of the firm [Cyert and March, 1963] and separation of ownership and control [Scherer, 1980], the theory of risk aversion [Tirole 1989, p. 35] etc., to focus only on the question. of the consequences of profit rate maximization. Clearly these arguments may be applied also to this alternative theory and we agree with Tirole when he said [Tirole 1989, p. 35],

> It will then be argued that even if managerial slack invalidates the profit-maximization hypothesis, the implications of this hypothesis for industrial organization need not to be erroneous.

applying this quotation to profit rate maximization.

However, it must be noted that even if many authors have well thought of the consequences of these theoretical improvements, not anyone has explored the consequences of profit rate maximization, as if pure profit maximization goes without saying.

Remarks.

1) The concept of profit remains unchanged from pure profit maximization to rate of profit maximization [4]

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[2] Even calculating pure profit is difficult because the opportunity costs of the capital are hard to know (this argument was presented historically by K.E. Boulding). Indeed, the alternative investment of the funds must be well defined, unique and known, in order to calculate the opportunity cost. Some authors treat the remuneration of equity as an ordinary cost [WU 1989, p. 250]: it is excessive.

[3] Remaining in what Simon calls the Theory of the Firm (F-theory), in contrast to Organization Theory (O-theory) [Simon, 1952-53].

[4] Remember that profit is due to an abstinence, as in Senior, or to an expectation, as in Marshall, or to a preference for the present as in Böhm-Bawerk or to an impatience, as in Fisher.

2) Many theories are based on profitability: the theory of investment choice (with the rate of actualization), the theory of capital demand (Keynes, among others, with the marginal efficiency of capital), etc.

3) Obviously, in the short-term, classical microeconomic theory assumes that capital is fixed in which case maximizing pure profit is equivalent to maximizing rate of profit.

Classical theory argues that: in the short-run, capital is invariable while varying in the long-term. This is incorrect, because, in the short-term, if capital is fixed, the scale of production is given; thus it is necessary to introduce this constraint into the classical calculation of short-run equilibrium: the program is $\underset{Q}{Max}\ \Pi$ under $Q \in \left\lfloor \check{Q}, \hat{Q} \right\rfloor$, in which $\check{Q}$ and $\hat{Q}$ are the bounds of Q due to the physical capital invested, which generates the fixed costs. This is why, in what follows, we will not consider a long-term in which capital is variable as in [de Mesnard, 1992] (and the number of firms is also variable) because it is confusing, but a short term with an *ex ante* calculation, in which the amount of capital is free (while *ex post*, it is fixed) and the number of firms remains invariable [5]. Note that both profit rate maximization and pure profit maximization are concerned with this remark because both include capital in their formula.

4) We may incidentally define many other types of profit ratios, such as [6]

$$\pi_f = \frac{\Pi_g - t\,K_d}{K_e},$$

which is the ratio of accounting profit over equity, or return on equity, which we term *financial rate of profit*. This type of ratio is familiar to shareholders who use it to measure profitability. It is obvious that this ratio is maximized when $K_e(Q)$ is minimized. However, it fails to discriminate between the production decision to the financing decision, because, with it, when trying to calculate the optimal output level, the manager also decides about financing, assuming an *ex ante* link between production and financing: clearly it is a function. That this link may be complicated is not the problem: the problem is that the link is exogenously determined and it may be unstable [7].

With the rate of profit, we may separate the production decision and the financial decision, as we usually do with pure profit maximization: that is why we shall adopt this function in the rest of the paper even if it is far removed from the objective function of shareholders.

In the first part of the story, we shall see that results diverge when we seek to maximize pure profit or the rate of profit: "max Π" diverges from "max π". In a second part of the paper, we will

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5 Even if it is possible to maximize the profit rate under constraints of capital.

6 As in [de Mesnard 1992].

7 Even if Modigliani and Miller theorem proves that the value of the firm is independent of the interest rate [Modigliani and Miller 1958]and even if Jensen and Meckling prove that there is an optimal financial structure in the financial theory because of agency cost [Jensen and Meckling 1976]: agency cost spent by the moneylender to control the firm may prevent it to reduce the part of shareholders; also bankrupt cost imply an optimal financial structure.

prove that coordination among firms may fail when firms maximize a profit ratio instead of the pure profit [8] [9].

II. The optimal output level maximizing the rate of profit

We shall study a single firm in a market as a general case for the firm (i.e. a monopoly).

Proposition 1. Consider the monopoly with function of capital. The firm produces up to the point where marginal profit is equal to average profit multiplied by the relative elasticity of capital to output level.

Proof. Consider the rate of profit $\pi = \frac{\Pi_g}{K}$. Its maximization gives:

$$\Pi'_g = \Pi_g \frac{K'}{K} \Leftrightarrow \Pi'_g = e_{K/Q}\, \bar{\Pi}_g$$

in which $e_{K/Q}$ is the relative elasticity of the capital to the output level

and $\bar{\Pi}_g = \frac{\Pi_g}{Q}$ is the average gross profit. ■

We have also:

$$\Pi'_g = \pi\, K'$$

As the optimum of the pure profit maximizing monopoly is given by:

$$\Pi'_g = t\, K'$$

Thus π has the role that t plays in the condition of maximization of pure profit. In a certain sense, the difference between the classical theory and the new theory is weak: where in the classical case, cost of capital is calculated with the exogenous fixed rate t, in the profit rate maximization, the cost of capital is calculated with the endogenous variable rate π: there is no evaluation of the cost of capital in the formula $\pi = \frac{\Pi_g}{K}$ and everything is as if the evaluation was made at the intern rate π while in the formula $\Pi = \Pi_g - t\, k$, the evaluation of the capital is made directly at the extern rate t. It is logical: if $\pi > t$ at the optimum, then the firm uses less capital and produces less because it evaluates the cost of its capital at a higher amount [10].

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[8] However, most part of the following results may be retrieved with the financial rate of profit.

[9] The exposé is volountary similar to first degree manuals, to focus on the specificty of the aaproach.

[10] With the financial rate of profit, only cost of equity is calculated at the rate π . It could seem more satisfying, (independently of the difficulties posed by the ratio) but we must remember that the opportunity cost of each dollar of equity is equal to the cost of each dollar of

Proposition 1'. The monopoly with a fixed coefficient of capital k produces up to the point where marginal profit is equal to average profit (or to the point where average profit is at a maximum).

Proof. We have : $K = k\,Q$ with $k > 0$, and then $e_{K/Q} = 1$. Hence, $\Pi'_g = \bar{\Pi}_g$. ■

As claimed in [de Mesnard 1992], this corresponds to the observed behavior of firms: a firm produces and invests so that the profit contributed by an extra unit is greater than or at least equal to the average profit yielded by other units.

Proposition 1". If the cost per unit is a constant $\bar{c}$ and if there is a fixed coefficient of capital then the optimal output level of the profit rate maximizing monopoly is indeterminate.

Remember that in this case the optimum for a pure profit maximizing monopoly does not exist excepted in the case $p = \bar{c} + t\,k$.

Proof. The proof is obvious because $\pi = \dfrac{(p - \bar{c})}{k}$ is a constant.■

Proposition 2. The optimum is independent of the level of fixed costs and of the constant term in the demand equation. Thus, if demand increases uniformly, or if fixed costs increase, a monopoly maintains an unchanged optimum when it maximizes the rate of profit.

Proof. Suppose $C_g = C_g^F$ + f(Q) in which C_g^F are fixed costs and $D = D_0$ + g(Q) . Then Π'_g does not include the terms C_g^F and D_0.■

Remark.

1) This equilibrium is not the point of long-term equilibrium of the traditional monopoly (i.e. $R' = C_g^{LT\prime} + t\,K^{LT\prime}$) and does not correspond to the point where average cost is minimum, even if capital is variable.

2) The criteria of **efficiency is changed**: an efficient system is no more a system where "marginal cost equals marginal revenue", it becomes a system where "maximum average profit maximum" (or more generally "marginal profit equals average profit multiplied by elasticity of capital"). For example, in health economics, if we use the new criteria, less treatments will be provided. In education economics, less teaching will be provided. Etc. In all sectors of economic life, the new criteria of efficiency leads to a limited service but also less expenditures. This consequence of the new criteria of the rate of profit may seem strange or revolting. However, the new criteria (the profit rate maximization) is not more doubtful than the old one (the pure profit maximization). ■

Denoting Q^{11} as the classical pure profit equilibrium and Q^{π} as the rate of profit equilibrium, we obtain the following graph:

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debts: in both cases, it is t.

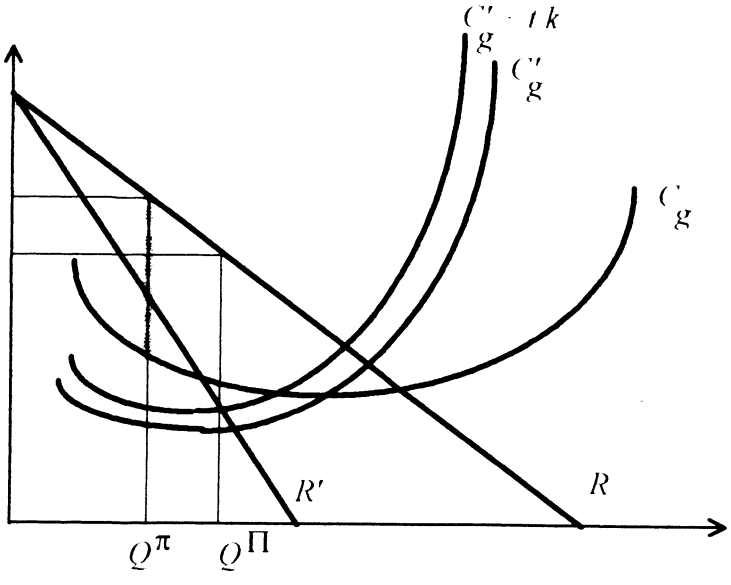

Figure 1. Short-term monopoly with fixed coefficient of capital

We may compare in the general case the optimum of the rate of profit with the classical microeconomic optimum (pure profit); we obtain the following fundamental theorem.

Theorem 1. Consider the more probable case $K'(Q^{\pi}) > 0$, in which Q^{π} is the optimal quantity that maximizes the rate of profit π . Compared with the maximization of pure profit, maximization of the rate of profit gives **a lower optimal output level** (respectively higher) **when the rate of profit is higher** (respectively lower) **than the interest rate at the optimum of the rate of profit**.

- $\pi(Q^{\pi}) > t \Rightarrow$ (max π) occurs before (max Π)
- $\pi(Q^{\pi}) < t \Rightarrow$ (max π) occurs after (max Π): it is less probable because in this case the firm leaves the sector.

The results are reversed if $K'(Q^{\pi}) < 0$.

Proof. The first order condition of optimality [11] for the maximization of the pure profit Π is $\Pi'_g - t\,K' = 0$. The first order condition of optimality for the maximization of the rate of profit π is $\Pi'_g - \pi\,K' = 0$.

Consider the curve $f_{\pi} = \Pi'_g - \pi\,K'$. Assume that Π' is decreasing and write Π' as a function of f_{π}: $\Pi' = f_{\pi} + [\pi - t]\,K'$. Denote Q^{Π} as the point of maximization of pure profit, such as

[11] It would be necessary to study second order conditions. However, the mathematical aspects (convexity, etc.) are similar in pure profit and in profit rate maximization.

$\Pi'(Q^{\Pi}) = 0$ and denote Q^{π} as the solution of the maximization of the rate of profit, such that $f_{\pi}(Q^{\pi}) = 0$.

1) Suppose that $K' > 0$ and $\pi(Q^{\pi}) > t$. Then, $\Pi'(Q^{\pi}) > f_{\pi}(Q^{\pi}) = 0$. Near Q^{Π} , curve f_{π} is under curve Π'. Thus, Q^{Π} must be greater than Q^{π}. At the limit, if Π' is vertical near Q^{Π}, Π' will cut the x-axis at Q^{π} and then $Q^{\Pi} = Q^{\pi}$. Remark that it in fact sufficient that $K'(Q^{\pi}) > 0$.

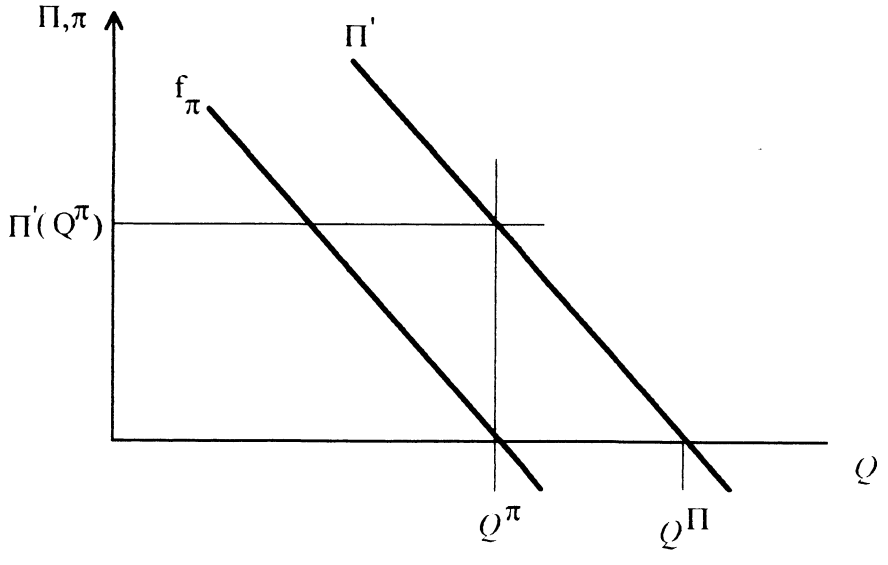

Figure 2. Mathematical comparison of optima

2) Suppose that $K' > 0$ and $\pi(Q^{\pi}) < t$. Then, $\Pi'(Q^{\pi}) < f_{\pi} = 0$. Then, near to Q^{π} , curve f_{π} is over curve Π' . Thus, Q^{Π} must be smaller than Q^{π} .■

Following this theorem, the monopolistic firm produces less, so at a higher price, when it maximizes the profit rate. Thus the monopoly is harsher with profit rate maximization.

Now, we will prove that coordination among firms could fail when firms maximize the rate of profit instead of pure profit.

III. Perfect competition

A. Price signal

Consider a single firm in a market in short-run perfect competition (the number of firms is invariable). We apply here theorem 1 above, assuming that the price is independent of Q , i.e. $p = \bar{p}$.

Proposition 3. In perfect competition at the optimum, we have $\bar{p} - C'_g = e_{K/Q}\left(\bar{p} - \bar{C}_g\right)$.

Proposition 3'. In perfect competition with a fixed coefficient of capital or with a relative elasticity of capital over output equal to 1, the firm produces up to the point where the average gross cost $\bar{C}_g$ is minimum.

Proof. We have : $e_{K/Q} = 1$. Thus $C'_g = \bar{C}_g$ at optimum. ■

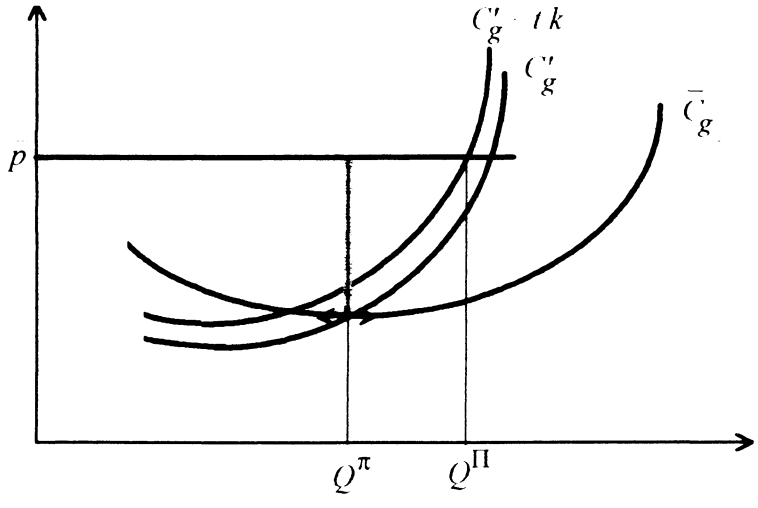

Figure 3. Short-term equilibrium of the firm in perfect competition with coefficient of capital

In this figure, the optimum of pure profit is given by $\bar{p} = C'_g + t\,K'$.

Remarks.

1) The optimum does not coincide with the classical point of long-term equilibrium of the firm which is situated at the minimum of $\bar{C}_g + t\,\bar{K}$. However, let's not get confused: we are not in long-term, but only *ex ante*, as said before. The number of firms remains fixed in our calculation, whereas the number of firms is variable in the long-run.

On the other hand, in the long term, $\bar{p}$ decreases in both cases because news firms enter the sector. The movement stops at the same point when individual pure profit is nil for a pure profit maximizing firm and when the profit rate is equal to t for a profit rate maximizing firm. This point is the entry point in the sector, corresponding to the minimum of $\bar{C}_g^{LT} + t\,\bar{K}^{LT}$.

2) The figure 3 suggests that if $t\,k > \bar{p}$ then Q^{Π} is before Q^{π}. It is right, but impossible, because in this case $\Pi < 0$.

We deduce from proposition 3 the following conclusions:

1) **The optimum does not depend on price**; thus, **there is not coordination in the market**. Each firm is alone, doing nothing with the price. Price is no more a signal. We need no more

additional theories to explain that the output is independent to the price (like *cost plus pricing*, U-curves of cost, etc.)

2) As a corollary, **the output level does not vary if price varies** (as long as the profit remains positive: when it becomes negative, the firm leaves the sector as usual). This corresponds to observed behavior. Consequently, **output level does not matter with taxation or other manipulations of prices**.

3) **The individual supply curve is vertical**, even in perfect competition. The global supply curve is completely inelastic and the market is in global equilibrium, following a classical pattern [12] :

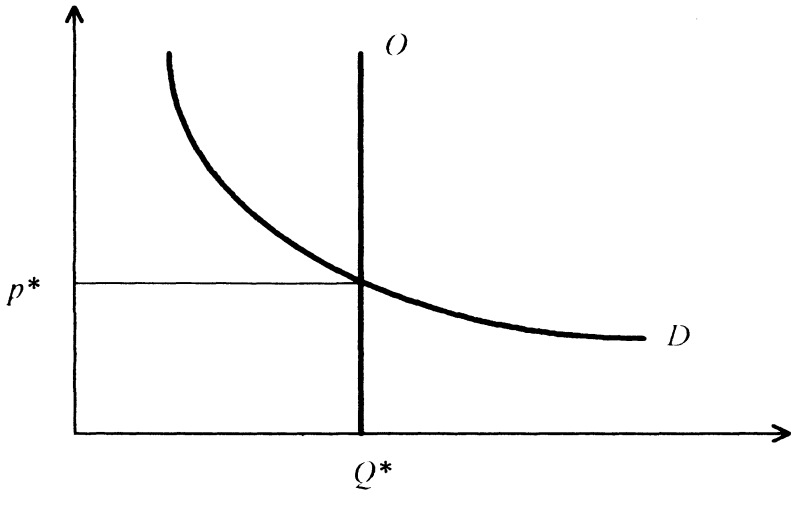

Figure 4. Global equilibrium in short-run perfect competition

This proves that a global equilibrium is compatible with no coordination by price, unlike good sense tends to believe. It must be noted that, unlike in the classical pure profit maximization (where price is determined by the intercept of the global curve of demand with the global curve of supply), it is not the global equilibrium which determines individual equilibrium of the firm, but it is the individual equilibrium which determines global equilibrium by adding up: price is only a consequence of the global equilibrium without impact on individual level (except for the amount of earnings of the firm but not for its optimal output).

Moreover, if demand curve is also completely inelastic, there may be no equilibrium; if it is only very inelastic, price may be very variable to slipping of demand (without any effect on the optimal output of firm, obviously). Note that if price decrease, some firms may go bankrupt and leave the sector: in this case global supply decreases [13].

[12] In [de Mesnard 1992], it is claimed that, as price is no longer a signal for the output level of firms, the optimum does not automatically clear the market in the short-term (as assumed in the neoclassical theory). This is false: the market is cleared in the sense that price is fully adaptive, as the quantity remains fixed.

[13] Individual supply is equal to Q^* if firm remains, or to zero if it leaves.

4) As firms are **smaller**, there could be **more firms**; thus **interaction among firms could be lower and competition greater** (even if the concept of competition becomes particular).

5) Accordingly with proposition 1", if the cost per unit is a constant $\bar{c}$ with a fixed coefficient of capital, $\pi = \frac{(\bar{p}-\bar{c})}{k}$ is a constant then the level of output is undetermined whereas pure profit has no optimum except if $\bar{p} = \bar{c} + t\,k$.

B. Optimal input combination and distribution

Consider n factors, used in quantity q_i, at a price $\bar{p}_i$. The function of production is $Q(q_1,...,q_n)$. The costs of production are $C(q_1,...,q_n) = \sum_{i=1}^{n} \bar{p}_i\, q_i$. The capital $K(q_1,...,q_n)$ is not considered as a factor , even if the cost of the capital (interest of debts and opportunity cost of equity) is a part of the total cost (at the interest rate t).

Recall. A pure profit maximizing firm remunerates it at its marginal productivity minus (in value) the marginal amount of capital required by this factor evaluated at the rate t.

$$\Pi(q_1,...,q_n) = \bar{p}\, Q(q_1,...,q_n) - \sum_{i=1}^{n} \bar{p}_i\, q_i - t\, K(q_1,...,q_n) = \bar{p}\, \tilde{Q}(q_1,...,q_n) - \sum_{i=1}^{n} \bar{p}_i\, q_i$$

denoting $\tilde{Q}(q_1,...,q_n) = Q(q_1,...,q_n) - t\, K(q_1,...,q_n)$ as the net output.

Then, $\frac{\partial \Pi(q_1,...,q_n)}{\partial q_i} = 0 \Leftrightarrow \bar{p}_i = \bar{p}\, Q'_i - t\, K'_i \Leftrightarrow \bar{p}_i = \tilde{Q}'_i$

Q'_i denoting the marginal productivity $\frac{\partial Q(q_1,...,q_n)}{\partial q_i}$ of factor i

K'_i denoting the marginal amount of capital $\frac{\partial K(q_1,...,q_n)}{\partial q_i}$ required by factor i

and $\tilde{Q}'_i = Q'_i - \frac{t}{\bar{p}} K'_i$ denoting the *net marginal productivity* of the factor i.

The classical distribution rule for homogeneous production function with degree 1 is the following:

$$\sum_{i=1}^{n} q_i\, \tilde{Q}'_i = \tilde{Q} \Rightarrow \sum_{i=1}^{n} \bar{p}_i\, q_i = \bar{p}\, \tilde{Q}.$$

Proposition 4. A profit rate maximizing firm remunerates each factor at its marginal productivity (in value) minus the marginal amount of capital required by this factor evaluated at the rate π.

Proof. It is very simple.

$$\pi(q_1,\ldots,q_n) = \frac{\bar{p}\,Q(q_1,\ldots,q_n) - \sum_{i=1}^{n} \bar{p}_i\, q_i}{K(q_1,\ldots,q_n)}$$

$\pi(q_1,\ldots,q_n)$ will be noted π further.

Then,

$$\frac{\partial \pi}{\partial q_i} = 0 \Leftrightarrow \bar{p}_i = \bar{p}\, Q_i' - \pi\, K_i'$$

Remark. The variable rate π plays the role of the fix interest rate t. ■

The last condition can be noted:

$$\bar{p}_i = \bar{p}\, \hat{Q}_i',$$

denoting $\hat{Q}_i' = Q_i' - \frac{\pi}{\bar{p}} K_i'$ as the *modified net marginal productivity* of the factor i. Thus, we can say that each factor is remunerated at its modified net marginal productivity (in value) with profit rate maximization: things are not so different... And as said before, in the profit rate maximization the cost of capital is calculated at the rate π where it is calculated at the rate t in the pure profit maximization.

Proposition 4'. If $\pi > t$, which is normal situation, then the remuneration of each factor is lower with the profit rate maximization than with the pure profit maximization: each factor is no more remunerated at its net marginal productivity. Consequently, if the price of each factor is exogenous, a lower amount of each factor is used when the firm maximizes its profit rate (remember that in this case the output is lower: $Q^{\pi} < Q^{11}$).

Proof. If $\pi > t$, we have the following graph:

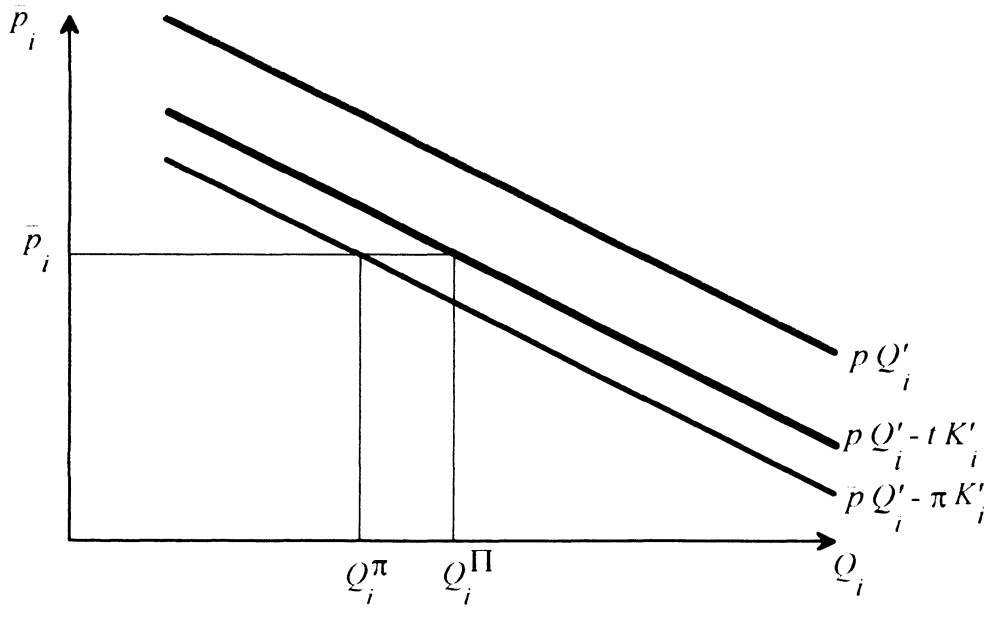

Figure 5. Marginal remuneration of factors

Q_i^{π} denotes the quantity of factor i used by firm when it maximizes the profit rate

Q_i^{Π} denotes the quantity of factor i used by firm when it maximizes the pure profit.

Corollary. If $\hat{Q}'_i < \tilde{Q}'_i$ for all i then the net output is not exhausted:

$$\sum_{i=1}^{n} q_i\,\hat{Q}'_i < \tilde{Q} \Rightarrow \sum_{i=1}^{n} \bar{p}_i\,q_i < \bar{p}\,\tilde{Q}.$$

In fact denoting $\hat{Q}(q_1,\ldots,q_n) = Q(q_1,\ldots,q_n) - \pi\,K(q_1,\ldots,q_n)$ as the *modified net product*, it is the modified net product that is exhausted:

$$\sum_{i=1}^{n} q_i\,\hat{Q}'_i = \hat{Q} \Rightarrow \sum_{i=1}^{n} \bar{p}_i\,q_i = \bar{p}\,\hat{Q}.$$

IV. Imperfect competition

A. *Duopoly and vanishing of reaction functions*

Let's look at the case of duopoly (competition in quantities). The result will be generalized to oligopoly later. The demand for the sector with two firms 1 and 2 is $D(Q^1+Q^2)$. The inverse demand is $p=D^{-1}(Q^1+Q^2)$ assumed to be decreasing. The total revenues of each firm are $R^i(Q^1+Q^2)=Q^i\,D^{-1}(Q^1+Q^2)$. The production costs of each firm are $C_g^i(Q^i)$. The total capital (equity plus debt) of each firm is $K^i(Q)$. The pure profits of each firm are $\Pi^i(Q^1,Q^2)=R^i(Q^1,Q^2)-C_g^i(Q^i)-t\,K^i(Q^i)$. The profit rates are:

$$\pi^i(Q^1,Q^2)=\frac{R^i(Q^1,Q^2)-C_g^i(Q^i)}{K^i(Q^i)}.$$

For the classical maximization of pure profit, we solve the equations $\dfrac{\partial\,\Pi^i(Q^1,Q^2)}{\partial\,Q^i}=0$. We obtain the following system:

$$\left\{\begin{array}{l}\dfrac{\partial\,D^{-1}(Q^1+Q^2)}{\partial\,Q^1}\,Q^1+D^{-1}(Q^1+Q^2)-C_g^{1\prime}(Q^1)-t\,K^{1\prime}(Q^1)=0\\ \dfrac{\partial\,D^{-1}(Q^1+Q^2)}{\partial\,Q^2}\,Q^2+D^{-1}(Q^1+Q^2)-C_g^{2\prime}(Q^2)-t\,K^{2\prime}(Q^2)=0\end{array}\right\}.$$

Generally, except special cases [14], the system has two reaction functions: the solution is found by the intersect of two curves (see the example later for a very simple linear demand function). These reaction functions allow coordination between firms, by way of conjectural variations [15] denoted $v^i=\dfrac{\partial\,Q^j}{\partial\,Q^i}$.

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[14] There is the question of uniqueness [Gaudet and Salant, 1991], existence or convergence of equilibrium [Novshek, 1985], and for example the question of discontinuity in reaction functions involving a non-existent equilibrium [Roberts and Sonnenchein, 1977].

[15] We do not enter in the debate about the interest of reaction functions and the fact that they are essentially concerned with competition in price (with an auctioneer). From Edgeworth to Kreps and Scheinkman, it is known that competition in price and competition in quantities are linked [Edgeworth, 1897] [Kreps and Scheinkman, 1983] [Benoit and Krishna, 1987] (see [Tirole, 1989, Chap 5] for a complete discussion). However, it must be noticed that profit rate maximization is compatible obviously with a model including capacity constraints (recalling note 1 on the remark in introduction, it is possible to maximize profit rate with bounds on the

Now, assume that each firm maximizes its rate of profit. The necessary conditions of optimality are:

$$\frac{\partial \pi^i(Q^1, Q^2)}{\partial Q^i} = 0 \text{ for all } i .$$

$$\Rightarrow \left\{ \begin{array}{l} \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1+Q^2)\,(1 - e_{K^1/Q^1}) = C_g^{1\prime}(Q^1) - \bar{C}_g^1(Q^1)\, e_{K^1/Q^1} \\ \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1+Q^2)\,(1 - e_{K^2/Q^2}) = C_g^{2\prime}(Q^2) - \bar{C}_g^2(Q^2)\, e_{K^2/Q^2} \end{array} \right\}$$

in which $\bar{C}_g^i(Q^i) = \dfrac{C_g^i(Q^i)}{Q^i}$, and $C_g^{i\prime}(Q^i) = \dfrac{d\,C_g^i(Q^i)}{d\,Q^i}$, and $e_{K^i/Q^i} = \dfrac{\dfrac{d\,K^i(Q^i)}{K^i(Q^i)}}{\dfrac{d\,Q^i}{Q^i}}$.

In a dynamic perspective [16], the classical iterative process is: firm 1 announces that it will produce quantity $Q^1_{(t-1)}$. Firm 2 calculates its own output level $Q^2_{(t)}$ that maximizes its profit, taking the output level of firm 1 as fixed, i.e., $d\,Q^1_{(t-1)} = 0$. Then, firm 1 recalculates $Q^1_{(t+1)}$ maximizing its profit, with $Q^2_{(t)}$ fixed, i.e., $d\,Q^2_{(t)} = 0$. The following system to be solved is the expression of the two reaction functions :

$$\frac{\partial \pi^i\left(Q^1_{(t+1)}, Q^2_{(t)}\right)}{\partial Q^i_{(t+1)}} = 0 \text{ for all } i .$$

It is easy to find counter-examples in which the reaction functions disappear with maximization of the rate of profit, while they occur with the maximization of pure profit. Generally, reaction functions disappear if the first equation excludes Q^2, and the second equation excludes Q^1 .

Proposition 5. Whether reaction functions can be canceled depends only on the parameters of the demand function and on the elasticity of equity capital with respect to the output level, not on the cost function.

Proof. In the above system of reaction functions, the right-hand side of the equation of equilibrium of firm i does not depend on the level of output Q^j of the other firm j , but the left-hand side does.■

This is an interesting result because there is a reaction function in all other models of duopoly, even though it is known that duopoly does not always have a stable solution (and even if

capital).

[16] As pointed out by Tirole, the dynamic perspective is the only valid approach for Nash game equilibrium, because in a static perspective, firm choose its action before observing the other and there is no reaction [Tirole 1989, p. 208]. However, it is not here a repeated game.

dynamics may be complicated or chaotic [Rand 1978] [Piatecki 1994]) or even this solution can be non-unique or cannot exist

Here are some counter-examples of non-existence when maximizing profit rate.

Proposition 6. In duopoly, the reaction functions vanish if the inverse function of demand is linear: $D^{-1}(Q^1, Q^2) = d - a\,Q^1 - b\,Q^2$, and if the elasticity of equity capital with respect to the output level is equal to 1 for each firm: $e_{K^i/Q^i} = 1$ for all i (this occurs when there is a coefficient of capital : $K^i(Q^i) = k\,Q^i$ for any i) .

Proof. With these conditions, the system of reaction functions becomes:

$$\left\{ \begin{array}{l} -a\,Q^1 - b\,v^1\,Q^1 = C'^{\,1}(Q^1) - \bar{C}^1(Q^1)\,e_{K^1/Q^1} \\ -a\,Q^2 - b\,v^2\,Q^2 = C'^{\,2}(Q^2) - \bar{C}^2(Q^2)\,e_{K^2/Q^2} \end{array} \right\} .$$

As each equation is independent to the other, there are no more reaction functions. Thus, Q^1 does not depend on Q^2 and reciprocally. ■

When reaction functions are canceled, that is when the solution is not found by the intersect of two functions, each firm acts independently of the other to find the optimal solution (even it has an opinion about the other firm). In a dynamic perspective, **each firm never observes the other firm** and there is **no iterative adjustment** ("immediate" solution). Finally, there is **no coordination** among firms and **no game**. We can say that duopoly is **degenerated**. Nash equilibrium exists anyway but it is special. Note that, as there is no reaction function, their slope does not matter in this case thus it does not matter to know if firms are strategic substitutes or strategic complements to the mind of [Bulow, Geanakoplos and Klemperer, 1985].

Example. The above proposition corresponds to a very common case, as in Milleron's example [1979, pp. 117-122], adapted to introduce pure profit. Data are:

$$p = d - (Q^1 + Q^2),$$

$$R^i(Q^1 + Q^2) = d\,Q^i - (Q^i)^2 - Q^i\,Q^j \text{ for all } i$$

$$C^i_g(Q^i) = c^i + (Q^i)^2 \text{ for all } i\ ,$$

$$K^i(Q^i) = k\,Q^i \text{ for all } i\ .$$

Thus, if both firms maximize the rate of profit in a general duopoly, the system to be solved is not a Cramer system:

$$\left\{ \begin{array}{c} -Q^1_{(t+1)}\,(1 + v^1) = Q^1_{(t+1)} - \dfrac{c^1}{Q^1_{(t+1)}} \\ -Q^2_{(t)}\,(1 + v^2) = Q^2_{(t)} - \dfrac{c^2}{Q^2_{(t)}} \end{array} \right\} \Rightarrow Q^i_* = \sqrt{\frac{c^i}{2 + v^i}} \text{ for all } i$$

Here, each firm does not consider demand or capital parameters (nor interest rate) but only its fixed costs and conjectural variations when determining its output level at equilibrium (moreover, decisions are not identical because costs are not equal).

This solution could be compared to the equivalent of the general duopoly maximizing pure profit:

$$\left\{ \begin{array}{l} (4+v^1)\, Q^1_{(t+1)} + Q^2_{(t)} = d - t\,k \\ Q^1_{(t-1)} + (4+v^2)\, Q^2_{(t)} = d - t\,k \end{array} \right\}$$

The solution of this Cramer system is:

$$Q^i_* = \frac{(d-t\,k)\,(3+v^j)}{15+4v^1+4v^2+v^1v^2} \text{ for all } i$$

Note that i is assumed to know the conjectural variation of j : it is not realistic.

When $v^1 = v^2 = v$ we obtain $Q^i_* = \dfrac{(d-t\,k)}{5+v}$ for all i .

For the original Cournot duopoly [Cournot, 1838], the equation is solved assuming that when either one firm modifies its output level, the other firm would not react and change its own output level ($v^1 = v^2 = 0$) :

$Q^i_* = \sqrt{\dfrac{c^i}{2}}$ for all i, for the rate of profit: it behaves than a profit rate maximizing monopoly.

It could be compared to the corresponding duopoly of Cournot-Nash maximizing pure profit: $Q^i_* = \dfrac{d-t\,k}{5}$ for all i (which is Milleron's result assuming that $k = 0$).

These results prove that, unlike in monopoly or perfect competition, a profit rate maximizing firm in duopoly may have a greater output than a pure profit maximizing firm. ■

Where demand is not linear, reaction functions may also disappear:

<u>Proposition 7</u>. Reaction functions may be canceled in duopoly if the inverse demand function is a simple quadratic function : $D^{-1}(Q^1,Q^2) = d - a\,Q^1\,Q^2$ and if the elasticity of capital versus output level is equal to 2 for each firm : $e_{K^i/Q^i} = 2$ for all i .

<u>Proof</u>. The left hand-side of the equation for firm 1 becomes:

$$-2\,a\,Q^1\,Q^2 - a\,v^1\,(Q^1)^2 + d - d\,e_{K^1/Q^1} + e_{K^1/Q^1}\,a\,Q^1\,Q^2 \,.$$

The terms Q^2 vanish if $e_{K^1/Q^1} = 2$. ■

However, it is false to say that the elasticity condition seems to be closely tied to the degree of the demand function. As a counter-example, see the following proposition:

Proposition 8. Reaction functions are not canceled if the quadratic inverse demand function includes a linear term: $D^{-1}(Q^1, Q^2) = d - a\,Q^1 - b\,Q^2 - f\,Q^1\,Q^2$.

Proof. The terms including Q^2 on the left hand-side of the equation for firm 1 are $-f\,Q^1\,Q^2\,(2 - e_{K^1/Q^1}) - b\,Q^2\,(1 - e_{K^1/Q^1})$. This does not vanish whatever the level of output. ■

B. Collusion

One question remains. Does collusion is possible with profit rate maximization? If yes, a certain form of coordination remains possible. Classically, the collusion equilibrium is found by the maximization of the sum of the pure profit functions: max $\Pi(Q^1 + Q^2)$,

where $\Pi(Q^1 + Q^2) = \Pi^1(Q^1) + \Pi^2(Q^2)$

$$= D^{-1}(Q^1 + Q^2)\,(Q^1 + Q^2) - C_g^1(Q^1) - C_g^2(Q^2) - t\,[K^1(Q^1) + K^2(Q^2)]$$

solving $\dfrac{\partial \Pi(Q^1 + Q^2)}{\partial Q^i} = 0$ for all i.

With the profit rate maximization, we calculate:

max $\pi(Q^1 + Q^2)$ where $\pi(Q^1 + Q^2) = \dfrac{D^{-1}(Q^1 + Q^2)\,(Q^1 + Q^2) - C_g^1(Q^1) - C_g^2(Q^2)}{K^1(Q^1) + K^2(Q^2)}$

and we solve $\dfrac{\partial \pi(Q^1 + Q^2)}{\partial Q^i} = 0$ for all i.

To simplify calculations, we choose to work on the above example.

$$\Pi(Q^1 + Q^2) = (d - Q^1 - Q^2)\,(Q^1 + Q^2) - \left(c^1 + (Q^1)^2\right) - \left(c^2 + (Q^2)^2\right) - t\,k\,(Q^1 + Q^2)$$

Thus, for the pure profit maximization, we solve:

$$\left\{ \begin{array}{l} (4 + 2\,v^1)\,Q^1 + (2 + 4\,v^1)\,Q^2 = (1 + v^1)\,(d - t\,k) \\ (2 + 4\,v^2)\,Q^1 + (4 + 2\,v^2)\,Q^2 = (1 + v^2)\,(d - t\,k) \end{array} \right\}$$

Classically, it is assumed that $v^1 = v^2 = v$ when there is collusion. Thus, the collusion equilibrium is: $Q^1 = Q^2 = \dfrac{(d - t\,k)}{6}$.

For the profit rate maximization, we solve:

$$\left\{\begin{array}{l} (Q^1)^2 + 2\,(1+v^1)\,Q^1\,Q^2 + v^1\,(Q^2)^2 = (1+v^1)\,\dfrac{c^1+c^2}{2} \\ v^2\,(Q^1)^2 + 2\,(1+v^2)\,Q^1\,Q^2 + (Q^2)^2 = (1+v^2)\,\dfrac{c^1+c^2}{2} \end{array}\right\}.$$

With $v^1 = v^2 = v$, the solution is $Q^1 = Q^2 = \sqrt{\dfrac{c^1+c^2}{6}}$ and collusion remains possible. Again, demand and cost parameters play no role (and naturally conjectural variations also).

C. Oligopoly

In the above example, we shall study the case of oligopoly.

Proposition 9. The limit of a profit rate maximizing oligopoly remains perfect competition, but perfect competition with profit rate.

Proof. For a general oligopoly with n firms, it is easy to verify that the solution of the profit rate maximizing oligopoly is $Q^i_* = \sqrt{\dfrac{c^i}{2+\sum_{j=2}^n v^i_j}}$ for all i, where $v^i_j = \dfrac{\partial Q^j}{\partial Q^i}$,

i.e. $$Q^i_* = \sqrt{\frac{c^i}{2+(n-1)\,v^i}} \text{ for all } i, \text{ if } v^i_j = v^i \text{ for all } j.$$

If $n \to \infty$ then $Q_* = \sum_i Q^i_* \to \sqrt{\dfrac{n\,c}{v}} \to \infty$ (assuming that $c^i = c$ and $v^i = v$ for all i) and $p \to d - \sqrt{\dfrac{n\,c}{v}} \to 0$: global quantity increases continuously, even if $Q^i_* \to 0$ for all i (with $n = \dfrac{v\,d^2}{c}$: the number of firms is limited).

Perfect competition with n profit rate maximizing firms is identical, assuming that $v = 0$:

$Q^i_* = \sqrt{\dfrac{c^i}{2}}$ for all i, $Q_* = \sum_i Q^i_* \to n\sqrt{\dfrac{c}{2}}$ and $p = d - n\sqrt{\dfrac{c}{2}}$; when $n \to \infty$, $Q_* \to \infty$ and $p \to 0$ (with $n = \sqrt{\dfrac{2\,d^2}{c}}$).

Recall. For the pure profit maximizing oligopoly with n firms,

$$Q^i_* = \frac{(d - t\,k)\,\prod_{i=2}^n (3+v^i)}{\begin{vmatrix} 4+v^1 & 1 & \dots & 1 \\ 1 & 4+v^2 & \dots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \dots & 4+v^n \end{vmatrix}} \text{ for all } i$$

i.e. $Q_*^i = \dfrac{(d - t\,k)}{n+3+v}$ for all i if $v^i = v$ for all i and if $n \to \infty$, global quantity becomes stationary (even if $Q_*^i \to 0$), $Q_* \to \sum_i Q_*^i \to d - t\,k$, with $p \to t\,k$ (the number of firms may increase indefinitely) : it is perfect competition.■

D. Competition in prices

The solution of Bertrand's model [Bertrand, 1883] adapted to a profit rate maximizing duopoly may differ strongly to the solution of the Bertrand's pure profit maximizing duopoly in prices. We will take up the data classically used for the Bertrand's model.

Proposition 10. With the same hypothesis than classical duopoly of Bertrand (fixed cost per unit of output, discontinuous demand), if there is a coefficient of capital then the solution of the symmetric duopoly in price does not correspond to perfect competition: price is not equal to marginal cost.

Proof. Consider the demand to firm i : $D^i(p^1,p^2) = \left\{ \begin{array}{c} D(p^i) \text{ if } p^i < p^j \\ \frac{1}{2} D(p^i) \text{ if } p^i = p^j \\ 0 \text{ if } p^i > p^j \end{array} \right\}$

and consider that the cost is c by unity of output. Thus, the pure profit function of firm i is $\Pi^i(p^1,p^2) = (p^i - \bar{c})\, D^i(p^1,p^2) - t\ k\, D^i(p^1,p^2)$ and its profit rate function is:

$\pi^i(p^1,p^2) = \dfrac{(p^i - \bar{c})\, D^i(p^1,p^2)}{k\, D^i(p^1,p^2)} = \dfrac{(p^i - \bar{c})}{k}$ whatever are the relative values of p^1 and p^2.

The classical Bertrand solution for a pure profit maximizing duopoly is perfect competition because each firm has interest to lower its price in order to take all the market; thus $p^1 = p^2 = \bar{c} + t\,k$ with $\Pi^i(p^1,p^2) = 0$ for both firms. The corresponding solution for a profit rate maximizing duopoly would be $\pi^i(p^1,p^2) = t$ with $p^1 = p^2 = \bar{c} + t\,k$. However, the problem is complicated. First, the solution $p^1 = p^2$ for this duopoly is obviously infinite with a profit rate also infinite because profit rate has no maximum (it is a straight increasing line) and the situation of long run perfect competition $\pi^i(p^1,p^2) = t$ is never realized. Second, starting from $p^1 = p^2$, the same reasoning than Bertrand does with the pure profit maximizing duopoly is applicable here: one firm may lower its price just a little to take all the market but, as this does not increase its rate of profit, the firm does not adopt this behavior, prices do not fall down to $\bar{c} + t\,k$ and there is not perfect competition.■

The most important conclusion of the Bertrand's model, which is named the paradox of Bertrand, is called into question. For Bertrand, the behavior of two firms which compete with prices is necessarily competitive; this is seen as the forerunner of Baumol-Panzar-Willig approach [Baumol, Panzar and Willig, 1986]. With profit rate maximization, both firms may earn a profit rate superior to interest rate by increasing price indefinitely.

A disturbing aspect of this conclusion is that the problematic of coordination is reversed. The paradox of Bertrand may be interpreted as a default of coordination among firms which causes (as in the Prisoner Paradox and in non-cooperative games) a less favorable solution compared to the cooperative one, while with profit rate, the solution is always favorable (even if the technical cause is the same than in case of perfect competition and duopoly in quantities: demand plays a minor or nil role in the solution).

Yet, the hypothesis of coefficient of capital seems improper regarding to the form of demand function: it may be more suitable to suppose that capital is proportional to the global demand because firm expects to cover entire demand.

Proposition 11. With the same hypothesis than classical duopoly of Bertrand (fixed cost per unit of output, discontinuous demand), if the amount of capital required by a firm, when both prices are equal, is always equal to the amount of capital required by this firm when its price is the lower (i.e. $K^i(Q) = k\,D(p^i)$ whatever p^i), then the equilibrium is competitive in pure profit maximizing and in profit rate maximizing: $p^1 = p^2 = \bar{c}$.

Proof. When $p^1 = p^2$, the profit rate is $\pi^i(p^1,p^2) = \dfrac{(p^i - \bar{c})\,D^i(p^1,p^2)}{k\,D_i(p^i)} = \dfrac{(p^i - \bar{c})}{2\,k}$ and the optimal profit rate for firm 1 is also $\dfrac{(p^1 - \bar{c})}{2\,k}$: it is lower than the optimal profit rate when $p^1 < p^2$ which is $\dfrac{(p^1 - \bar{c})}{k}$ (remind that if $p^1 > p^2$ then $\pi^1(p^1,p^2) = \dfrac{(p^1 - \bar{c})\,0}{k\,D_1(p^1)} = 0$).

Thus each firm may want to lower its price, and there is a movement toward $p^1 = p^2 = \bar{c} + 2\,t\,k$ which is the competitive solution ($\pi^i = t$). In the other hand, pure profit maximization gives also perfect competition, $p^i = \bar{c} + 2\,t\,k$, with a nil pure profit. ■

The paradox of Bertrand remains in this case.

E. Mixed duopoly

Both firms may maximize either pure profit or profit rate. Denote ${}^{\pi}Q^i_*$ the optimal output for firm 1 when both firms maximize profit rate (denoted π^i_* for firm i) and ${}^{\Pi}Q^i_*$ the optimal output for firm i when both firms maximize pure profit (denoted Π^i_* for firm i). Denote ${}^{\pi M}Q^i_*$ the optimal output for firm i when it maximizes its profit rate (denoted ${}^{M}\pi^i_*$) whereas simultaneously firm j maximizes its pure profit (denoted ${}^{M}\Pi^j_*$) with an optimal output of ${}^{\Pi M}Q^j_*$. We have four cases, that is for optimal outputs:

Optimal output	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	${}^{\pi}Q_*^1$ and ${}^{\pi}Q_*^2$	${}^{\pi M}Q_*^1$ and ${}^{\Pi M}Q_*^2$
1 maximizes its pure profit	${}^{\Pi M}Q_*^1$ and ${}^{\pi M}Q_*^2$	${}^{\Pi}Q_*^1$ and ${}^{\Pi}Q_*^2$

and for optimal objective function:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	1 π_*^1 and π_*^2	2 ${}^{M}\pi_*^1$ and ${}^{M}\Pi_*^2$
1 maximizes its pure profit	3 ${}^{M}\Pi_*^1$ and ${}^{M}\pi_*^2$	4 Π_*^1 and Π_*^2

The second table looks like a game matrix but it is not a game because we have not comparable earnings in it [17]. In an ordinary game, to find Nash equilibria, the reasoning is the following: if firm 2 chooses strategy *a*, what is the best reply for firm 1, *a* or *b*, and then, if firm 2 chooses strategy *b*, what is the best reply for firm 1, *a* or *b*? The evaluation of each strategy is made looking at the value of a fixed objective function. There is a Nash equilibria if both players choose the same case of the matrix. If the reply is the same in both cases, for example firm 1 must chose strategy *a* in both cases whatever the choice of firm 2, then this strategy is always played by firm 1.

However, in our table, the objective function changes [18]! We cannot compute a maximin equilibrium as usual, even if some computations are possible. Cases 1 and 4 are *pure*, cases 2 and 3 are *mixed* [19]. A firm cannot compare its profit rate with its pure profit. Cases 1 and 4 corresponding to similar choices of objective functions for both firms are not at all comparable. Cases 1 and 2 and cases 3 and 4 are not comparable for firm 2, while cases 1 and 3 and cases 2 and 4 are not comparable for firm 1: firm 1 can compare case 1 with case 2, and case 3 with case 4 and firm 2 can compare case 1 with case 3 and case 2 with case 4; that is to say, firm *i* can compare Π_*^i with ${}^{M}\Pi_*^i$ or π_*^i with ${}^{M}\pi_*^i$. It can only compare the values of its objective function when the other firm changes its objective function (even it cannot force it to change) [20] ; for example, when firm 2 chooses to maximize its pure profit, firm 1 cannot compare ${}^{M}\pi_*^1$ and Π_*^1.

[17] The choice between "be a profit rate maximizing firm" or "be a pure profit maximizing firm" may not be seen as an ordinary game. Only in evolutionary games players may chose a strategy in order to satisfy a *meta-objective*: for example, survive [Maynard Smith 1982]. Here, if we suppose that players may chose their objective function, we must have a meta-objective, like survive. And it is not sure that choosing the objective function is realistic for a firm. We see here the interest of econometric studies to determine what is the objective function of real firms.

[18] The term "multiobjective duopoly" is not suitable because each firm has only one objective at the same time. That is why we prefer "mixed duopoly".

[19] Do not confuse with pure strategies and mixed (i.e. probabilistic) strategies.

[20] As a small comparison, it is false to say that rugby is better than soccer because players

Even if a firm maximizes its profit rate, it can calculates the corresponding pure profit, thus a firm can compare its old pure profit Π^i_* with the pure profit, denoted $\Pi({}^M\pi^i_*) = {}^{\pi M} K^i_* \, {}^M\pi^i_*$, that it obtains when it maximizes its rate of profit ${}^M\pi^i_*$ (${}^{\pi M}K^i_*$ is the optimal amount of capital of firm i when this firm maximizes its profit rate and the other firm maximizes the pure profit). This argument is perfectly right, but $\Pi({}^M\pi^i_*) < \Pi^i_*$ (equality is the particular case where firm i just enters in the sector: see theorem 1) because Π^i_* is yet a maximum! The same reasoning can be made with π^i_* and $\pi({}^M\Pi^i_*) = \frac{{}^M\Pi^i_*}{\Pi M K^i_*}$, and for other configurations. Thus, the firm which abandon one objective to another always looses in terms of the first objective (in other words: there is never pareto-optimality between the two objectives). The real question is: does it freezes the movement? The answer is not, because we think here in terms of objective, not simply in terms of decisions: the problem is not only to take a decision or another decision, it is to switch the objective function. This theme is not much studied in economic theory: when an agent thinks about switching, it do not calculates the value of both objective functions. Example: if a man thinks about the possibility to work hard to do a great career or to be happy, he knows that he will be less happy with a great career (to much work) or he will have a poor career if he his happy (to much tranquillity); anyway, he chooses or switches.

We will say that a mixed case is dominated for a firm if, starting from a pure case to go on this mixed case, this firm is loosing (when the comparison is possible). Suppose that starting from case 1, firm 2 changes its objective function (choosing pure profit instead of profit rate) and suppose that ${}^M\pi^1_* < \pi^1_*$ and suppose that starting from case 4, firm 1 changes its objective function (choosing profit rate instead of pure profit) and suppose that ${}^M\Pi^2_* < \Pi^2_*$. Then case 2 is dominated [21].

For example, to see if case 2 is dominated [22], we compute ${}^M\pi^1_*$ and ${}^M\Pi^2_*$, and we solve the system:

$$\left\{ \begin{array}{l} \dfrac{\partial \pi^1(Q^1,Q^2)}{\partial Q^1} = 0 \\ \dfrac{\partial \Pi^2(Q^1,Q^2)}{\partial Q^2} = 0 \end{array} \right\}$$

may earn more points in rugby than in soccer: it is another game and if a player leave out from rugby to go playing soccer, he may have regrets but not for the quantity of points, even if a rugby team may notice that its results have change in good or in bad when this player leave rugby to go playing soccer.

[21] The same analysis can be made with a more familiar objective function, Baumol's maximization of turnover; for example, with three firms, each one with a different objective.

[22] Reversing indexes gives the conditions to know if case 3 is dominated.

$$\Rightarrow \left\{ \begin{array}{l} \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1+Q^2)\,(1-e_{K^1/Q^1}) = C_g^{1\prime}(Q^1) - \bar{C}_g^1(Q^1)\, e_{K^1/Q^1} \\ \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1+Q^2) - C_g^{2\prime}(Q^2) - t\, K^{2\prime}(Q^2) = 0 \end{array} \right\}$$

In the case of the above example, we solve:

$$\left\{ \begin{array}{l} -Q^1_{(t+1)}\,(1+v^1) = Q^1_{(t+1)} - \dfrac{c^1}{Q^1_{(t+1)}} \\ Q^1_{(t-1)} + (4+v^2)\, Q^2_{(t)} = d - t\,k \end{array} \right\}$$

thus ${}^{\pi M}Q^1_* = \sqrt{\dfrac{c^1}{2+v^1}}$ and ${}^{\Pi M}Q^2_* = \dfrac{d - t\,k - {}^{\pi M}Q^1_*}{4+v^2}$.

We see that the profit rate maximizing firm determines its output independently to the other firm, when it is not the case of the second firm; there is a unilateral link between firm 1 and firm 2: firm 1 is dominant in a certain sense.

Remember that ${}^{\pi}Q^i_* = \sqrt{\dfrac{c^i}{2+v^i}}$ for both firms and ${}^{\Pi}Q^i_* = \dfrac{(d - t\,k)}{5+v}$ for both firms.

- Suppose that both firms are initially profit rate maximizing (case 1) and then firm 2 becomes pure profit maximizing (case 2): in this case, only firm 2 is affected for its output level. If ${}^{\pi}Q^1_* \leq {}^{\Pi}Q^1_*$ (i.e. if firm 1 has the same behavior than in monopoly or perfect competition), as ${}^{\pi M}Q^1_* = {}^{\pi}Q^1_*$, we have:

  $${}^{\Pi M}Q^2_* = \frac{d - t\,k - {}^{\pi}Q^1_*}{4+v^2} \geq {}^{\Pi}Q^2_*.$$

  If ${}^{M}\pi^1_* > \pi^1_*$ then firm 1 my want that firm 2 becomes pure profit maximizing. Does firm 2 have lost anything in this operation? Not, because it is assumed to be equally satisfied with its new objective function as it was with the old one: firm 2 plays another game, not the same than before.

- Suppose that both firms are initially pure profit maximizing (case 4) and then firm 1 becomes profit rate maximizing (case 2): in this case, both firms are affected. Moreover, if ${}^{M}\Pi^2_* > \Pi^2_*$ then firm 2 may want that firm 1 becomes profit rate maximizing.

As we said, there is no game strictly speaking. However, there are cases that are wished by firms.

If a mixed case, 2 or 4, is dominated for both firms, starting from pure cases, no firm wants that the other firm changes its objective function: the mixed cases are never wished. In this situation, there is a potential conflict because, starting from a mixed case, each firm wants that the other

firm changes its objective function, or there is a potential jamming because each firm waits the change of the other firm [23]. For example:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	1 $\pi_*^1 = 0.15$ and $\pi_*^2 = 0.20$	2 ${}^M\pi_*^1 = 0.13$ and ${}^M\Pi_*^2 = 130$
1 maximizes its pure profit	3 ${}^M\Pi_*^1 = 80$ and ${}^M\pi_*^2 = 0.17$	4 $\Pi_*^1 = 100$ and $\Pi_*^2 = 150$

Conversely, pure cases 1 and 4 can be dominated for both firms: starting from mixed case, no firm wants a change in the objective function of the other firm. For example:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	1 $\pi_*^1 = 0.15$ and $\pi_*^2 = 0.20$	2 ${}^M\pi_*^1 = 0.18$ and ${}^M\Pi_*^2 = 170$
1 maximizes its pure profit	3 ${}^M\Pi_*^1 = 110$ and ${}^M\pi_*^2 = 0.22$	4 $\Pi_*^1 = 100$ and $\Pi_*^2 = 150$

Some **complex movements** may arise. Suppose that case 2 is dominated for firm 1 but not for firm 2. For example:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	1 $\pi_*^1 = 0.15$ and $\pi_*^2 = 0.20$	2 ${}^M\pi_*^1 = 0.13$ and ${}^M\Pi_*^2 = 170$
1 maximizes its pure profit	3 ${}^M\Pi_*^1 = 110$ and ${}^M\pi_*^2 = 0.17$	4 $\Pi_*^1 = 100$ and $\Pi_*^2 = 150$

A complex movement is wished: starting from case 4, firm 2 wishes that firm 1 changes its objective function going in case 2, and there, assuming that firm 1 executes the movement (remember that firm 1 cannot compare case 4 and case 2), firm 1 wishes that firm 2 changes its objective function going in case 1; if we assume that firm 2 executes also the movement (firm 2 cannot compare case 2 and case 1), then both firms abandon the pure profit maximization to adopt profit rate maximization. Conversely, the opposite movement is not wished because, starting from case 1, firm 1 wishes that firm 2 keep on profit rate maximization (i.e. wishes that firm 2 do not go on case 2), and from case 2, firm 2 wishes that firm 1 remains profit rate maximizing.

Does a hoped movement is realized? This is the question of **incentives**. We may imagine that a firm, *i*, which wishes that the other firm *j* do a movement pay for it (for example, firm *i* pays to firm *j*, following Nash's rule, the half of what it earns by this movement): logically, firm *j* must accept the deal [24].

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[23] It is known that in a classical framework, waiting strategies may lead to war of attrition [Tirole 1989, p. 380].

[24] Remember that even if the pure profit is lower when a firm maximizes its profit rate, the

Clearly, the complex wished movement can start from a mixed case toward a mixed case, for instance from case 3 to case 2 by way of case 4. For example:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	[1] $\pi_*^1 = 0.12$ and $\pi_*^2 = 0.20$	[2] ${}^M\pi_*^1 = 0.13$ and ${}^M\Pi_*^2 = 170$
1 maximizes its pure profit	[3] ${}^M\Pi_*^1 = 90$ and ${}^M\pi_*^2 = 0.22$	[4] $\Pi_*^1 = 100$ and $\Pi_*^2 = 150$

There can be a **cyclical movement**, like: case 4 → case 2 → case 1 → case 3 → case 4. This type of cycle in wished movement has no end, but we must remember that we said above about the incentives required to obtain a real movement. For example:

Optimal objective function	2 maximizes its profit rate	2 maximizes its pure profit
1 maximizes its profit rate	[1] $\pi_*^1 = 0.15$ and $\pi_*^2 = 0.20$	[2] ${}^M\pi_*^1 = 0.13$ and ${}^M\Pi_*^2 = 170$
1 maximizes its pure profit	[3] ${}^M\Pi_*^1 = 90$ and ${}^M\pi_*^2 = 0.22$	[4] $\Pi_*^1 = 100$ and $\Pi_*^2 = 150$

V. Conclusion

We have examined an alternative objective for the firm, the maximization of the rate of profit over capital instead of the maximization of pure profit. Both objectives are independent of financing decisions. Even if the conditions of entry into the sector are similar for both objectives, the optima differ. We have proved that the optimal output level of firms is lower when they adopt this objective. Thus, we have a conception of efficiency different to the classical one.

We have proved that coordination among firms may fail when the objective of firms is changed from maximizing pure profit to maximizing rate of profit. We have found some major instances in which each firm acts independently of the other. In competition with a fixed coefficient of capital, the price signal plays no role: firms are not coordinated even if they are smaller than in the classical model but there is a global equilibrium with no inelastic supply curve.If we adopt a special definition of marginal productivity (replacing interest rate by profit rate), factors may remain paid at their modified marginal productivity. In imperfect competition, in duopoly in quantities, reaction functions may disappear in some common cases, such as with a linear inverse demand function with a coefficient of capital: there is no more game among firms; however, collusion remains possible; so, firms do not consider other firms to determine their

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firm remains indifferent between both objective function: when the firm is profit rate maximizing, it engages less capital in production, the rest of capital may be engaged elsewhere out of this duopoly.

optimal output. In oligopoly in quantities equilibrium may not correspond to perfect competition. For competition in prices, things are more complex.

As a general (even surprising) rule, demand plays no role in the determination of equilibrium in perfect competition in any case and in imperfect competition in some cases. Does it mean that there is nothing at all in this economy? Not, because the classical concept of equilibrium remains. Remark that these results about non-coordination are independent of the rate of profit chosen: if the financial rate of profit replaces the rate of profit, they remain.

These simple cases may be seen as counter-examples of non-coordination because in the standard classical model there is always coordination [25]. Even if these cases are not absolutely general, there are sufficiently eloquent to prove the fragility of the behavior classically claimed for firms. Conversely, coordination appears to be an artifact, that is a consequence of the choice made about the objective function. On the other hand, it could be said that pure profit maximization is the best objective because results are "well behaved" while it is not the case with profit rate maximization; this argument is not suitable because a result is a result even it is not attractive and we cannot push back profit rate maximization only because its results are embarrassing. We must test these theories, and it could be suited to do an econometric test to see how many firms choose to maximize the rate of profit instead of the pure profit. One question again: what could be a general competitive equilibrium for profit rate maximizing producers [26]?

[25] Even if some cases of coordination failures are studied in the literature; see for example [Heller, 1986] [Bagwell and Ramey, 1994].

[26] The starting point would be the section above about factors. In this context, we must define the counterpart of the profit rate for consumers and other non-productive agents, defining the effort to buy the good and then making a ratio satisfaction / effort.

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