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## Litigation and settlement II: stability and other criteria

Urs Schweizer

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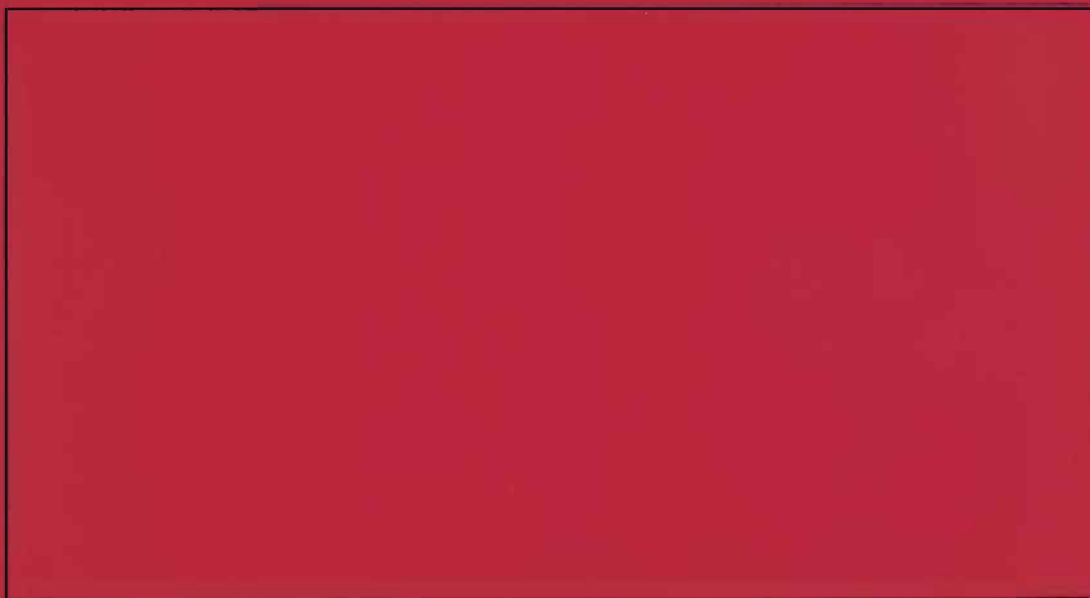


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**INSTITUT DE MATHEMATIQUES ECONOMIQUES**

UNIVERSITE DE DIJON

FACULTE DE SCIENCE ECONOMIQUE ET DE GESTION

4, BOULEVARD GABRIEL — 21000 DIJON

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LITIGATION AND SETTLEMENT II :  
STABILITY AND OTHER CRITERIA

Urs SCHWEIZER

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Litigation and Settlement II:  
Stability and Other Criteria

by Urs Schweizer<sup>1</sup>, University of Bonn

Abstract

The game of litigation and settlement introduced by Schweizer (1986) is taken as an example to explore the cutting power of stability and other tests. Stability and universal divinity which, in the present game, coincides with the test based on "never a weak best response" lead to some essentially unique outcome. The test based on dominance and the intuitive criterion, however, are shown to rule out none of the sequential equilibrium outcomes. Divinity, finally, has some cutting power but still allows a substantial variety of outcomes to pass.

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## Introduction

The following simple non-cooperative game of litigation and settlement with two-sided incomplete information was introduced by Schweizer (1986). The plaintiff (player A) claims damages of a certain amount from the defendant (player B). Each player holds private information on the merits of the case. The plaintiff has filed suit. Then, in a first move, player B proposes settlement terms which player A, in a second and final move, either accepts or rejects. If he accepts the case is settled out of court at the proposed terms, and litigation costs do not arise. If, however, player A rejects, the case will be litigated and litigation costs lead to a negative-sum situation. In the previous paper, the set of all sequential equilibria of the game has been fully characterized. Separating and pooling equilibria are shown to coexist. For many equilibria, the case is predicted to be litigated with positive probability, for others, settlement occurs with certainty. To make this variety of equilibria visible, we have chosen to describe the set of all payoffs for the first-moving player B which can be sustained in sequential equilibrium. Hereby, payoffs are listed from the interim view (i.e. right after nature provides parties with their private information) which means that as many expected payoffs have to be considered as player B obtains different signals from nature.

The defendant B as the first-moving player is, in an obvious sense, the leader in this game whereas the plaintiff A might be called the follower. As a leader, player B possibly has particular influence on the final outcome of the game. In the preceding paper, we hypothesized that the leadership position allows B to prevent any sequential equilibrium outcome which, from B's

interim view, is dominated by another equilibrium outcome. In other words, an equilibrium will be ruled out by the so-called interim rule if there exists another equilibrium which gives the leader a higher payoff for every possible signal and a strictly higher payoff for at least one of them. This rule can be criticized for valid reasons, not so much because it attributes strong influence to the leader, but rather because it confines his choice, somewhat artificially, to the set of sequential equilibria. In any case, the rule reduces the number of equilibria quite drastically. It does, however, not necessarily preserve two equilibria to coexist, one predicting positive probability of litigation the other certain settlement.

In the present paper, we take up the program as proposed by Cho and Kreps (1986) for signaling games in general and explore its significance in the context of our game of litigation and settlement. We shall discuss the notion of stability (Kohlberg and Mertens (1985)), the test based on dominance, the intuitive criterion as well as the tests of divinity and universal divinity (Banks and Sobel (1985)). Some of these tests turn out to rule out all but one equilibrium outcome, some others have no cutting power at all. The paper is organized as follows.

Section 1 contains a summary (without proofs) of those findings of Schweizer (1986) which are relevant for the present study. The presentation has been revised in order to, hopefully, improve readability. The only truly new result of this section deals with the ordering of moves. In particular, it is shown that being leader does not necessarily increase payoff as expected ex ante compared to the game with reversed moves. It is true however that, from the interim view, each player would prefer to be leader in the situation where nature sends

the signal which is good for him.

Section 2 deals with stability, a notion defined for games with only finitely many pure strategies. Finiteness is achieved by having the defendant select from a given finite sequence of settlement terms. A unique stable equilibrium outcome is shown to exist which depends, though inessentially, on the given sequence. As the sequence is refined, the stable outcome approaches, in the limit, a sequential equilibrium outcome which, however, is not stable itself in the strict sense. The unique stable outcome predicts litigation with positive probability for each parameter configuration. Somewhat surprisingly, the stable outcome could conceivably be ruled out by the interim rule.

In section 3, it is established that neither the test based on dominance nor the intuitive criterion have any substantial cutting power. In the first case, each sustaining sequential equilibrium profile passes the test, whereas, in the second case, each sequential equilibrium outcome can be sustained by at least some equilibrium profile which passes the intuitive criterion.

Section 4, finally, explores the tests of divinity and universal divinity which both are shown to have cutting power. While divinity reduces the number of equilibrium outcomes, it might still leave a substantial variety unaffected. In particular, outcomes might pass simultaneously which predict litigation and settlement, respectively. Universal divinity, however, which here coincides with the test based on "never a weak best response" leads, essentially, to a unique outcome. To be precise, the test applies for the finite as well as the infinite version of the game. In the (more natural)

infinite version, true uniqueness can be established. For the finite version, this is not quite correct. Both the stable outcome as well as the outcome to which the stable one converges as the sequence is refined pass the test of universal divinity. Strictly speaking, there exist two different universally divine equilibrium outcomes. Only one of them is stable. But, for both, the case will be litigated with positive probability.

The basic conclusion seems to be as follows. If we subscribe to the tests of stability or universal divinity then the principle according to which parties will voluntarily transact whenever some mutually beneficial transaction exists can not longer be considered to be valid, at least not in a situation of incomplete information.

### 1. Previous Results: Revised Presentation

Details of the game are as follows (c.f. Schweizer (1986)). In a preliminary move, nature sends independent signals on the merits of the case to both parties. Each of them receives one of two possible signals. For player A, let  $k = 1$  denote the bad and  $k = 2$  the good signal (bad and good for A) whereas, for player B, let  $l = 1$  denote the good signal (good for B, unfavourable for A) and  $l = 2$  the bad signal (bad for B, hence favourable for A). There exist four different states  $(k,l)$  of the world, accordingly, and state  $(k,l)$  is commonly believed to occur with probability  $p_k q_l$ . Hereby, signal  $k$  is sent with probability  $p_k$  ( $p_1 + p_2 = 1$ ) and signal  $l$  with probability  $q_l$  ( $q_1 + q_2 = 1$ ). In state  $(k,l)$ , the plaintiff's expected gain and the defendant's expected loss from litigation amount to  $E_{k,l}$  and  $F_{k,l}$ , respectively. To reflect legal costs, it is assumed that  $E_{k,l} < F_{k,l}$  and, in order to justify the terms good and bad as used above, it is assumed throughout that

$$E_{1,1} < E_{2,1}, E_{1,2} < E_{2,2} \text{ and } F_{1,1} < F_{1,2}, F_{2,1} < F_{2,2} .$$

Parties know their signal but not the full state of the world. From the interim view, player A's expected gain and B's expected loss amount to  $E_k = q_1 E_{k,1} + q_2 E_{k,2}$  and  $F_l = p_1 F_{1,l} + p_2 F_{2,l}$  if A receives signal  $k$  and B receives signal  $l$ , respectively.

After this preliminary stage, the first moving player B proposes settlement terms  $S$  which then the second moving player A accepts or rejects. Since the notions of sequential equilibrium (Kreps and Wilson (1982)) and of stability (Kohlberg and Mertens (1985)) are confined to games with finitely many pure strategies, player B is assumed to choose settlement terms from a given finite

sequence  $S_1 < \dots < S_m < \dots < S_M$  of values. The sequence includes, however, all critical values such as  $E_{k1}$  and  $E_k$  as well as intermediate terms. A mixed strategy for B is denoted by a matrix  $(s_{1m})$  where  $s_{1m}$  stands for the probability that B proposes settlement terms  $S_m$  if he obtains signal 1 ( $l=1,2$ ). The symbol  $s_1(S)$  defined by  $s_1(S) = s_{1m}$  if  $S = S_m$  will alternatively be used where convenient. In sequential equilibrium, player A is assumed to form beliefs about the other player's signal. Let  $q_{1m}$  denote the probability as assessed by A that B has obtained signal 1 (i.e.  $q_{1m} + q_{2m} = 1$ ). For out-of-equilibrium messages, beliefs are restricted in no way. If, however, message  $m$  is sent (i.e.  $s_{1m} + s_{2m} > 0$ ) then Bayes' rule

$$(q_1 s_{1m} + q_2 s_{2m}) q_{1m} = q_1 s_{1m}$$

applies. A mixed strategy for A is denoted by a matrix  $(r_{km})$  where  $r_{km}$  is the probability of A accepting terms  $S = S_m$  if he has obtained signal  $k$ . Under the above beliefs, player A's updated expected gain  $e_{km}$  from litigation amounts to  $e_{km} = q_{1m} E_{k1} + q_{2m} E_{k2}$  if A holds signal  $k$  and is offered terms  $S_m$ . The strategy is sequentially rational if  $r_{km} = 1$  whenever  $e_{km} < S_m$  and  $r_{km} = 0$  whenever  $S_m < e_{km}$ . Player A's interim expected payoff (after having received signal  $k$ ) is given as

$$u_k = E_k + \sum_m r_{km} (q_1 s_{1m} + q_2 s_{2m}) (S_m - e_{km}).$$

Since, for any beliefs,  $e_{1m} < e_{2m}$  sequential rationality requires that  $r_{1m} = 1$  whenever  $r_{2m} > 0$ . Therefore, player A's strategy can be fully recovered from his expected move  $r_m = p_1 r_{1m} + p_2 r_{2m}$ . If  $r_m \leq p_1$  then  $r_m = p_1 r_{1m}$  and  $r_{2m} = 0$  whereas if  $p_1 \leq r_m$  then  $r_{1m} = 1$  and  $r_m = p_1 + p_2 r_{2m}$ . In the following, player A's

strategy will be described simply by the M-vector  $(r_m)$  or, where convenient, by  $r(S)$  defined as  $r(S_m) = r_m$  if  $S = S_m$ . As for player B's payoff, the function

$$V_1(S, r) = \begin{cases} -F_{11} + r(F_{11} - S) & \text{if } r \leq p_1 \\ -F_{21} + r(F_{21} - S) & \text{if } r \geq p_1 \end{cases}$$

is introduced which expresses B's payoff if B holding signal 1 proposes terms  $S$  and if A reacts with  $r$ . Player B's interim expected payoff (after having received signal 1) amounts to

$$v_1 = \sum_m s_{1m} V_1(S_m, r_m).$$

For sequential equilibrium, it is required that  $v_1 \geq V_1(S_m, r_m)$  holds for all  $m$  with equality prevailing if message  $m$  is sent in equilibrium (i.e. if  $s_{1m} > 0$ ). A (sequential) equilibrium profile  $(s_{1m}, q_{1m}, r_{1m})$  consists of a pair of strategies  $(s_{1m})$  and  $(r_m)$  as well as of beliefs  $(q_{1m})$  which together form a sequential equilibrium. As a final piece of notation, "indifference curves"  $R^1 = R^1(S, v_1)$  are defined by  $V_1(S, R^1) = v_1$ . Useful properties of these curves are listed in the following lemmas which hold for any sequential equilibrium.

Lemma 1

For any message  $m$ , it holds that  $r_m \leq R^1(S_m, v_1)$  with equality prevailing if  $s_{1m} > 0$ .

Lemma 2

Associated with any sequential equilibrium, there exists a unique intersection  $(S^*, r^*)$  of indifference curves, i.e.  $R^1(S^*, v_1) = R^2(S^*, v_2) = r^*$ . It necessarily holds that

$$0 < r^* \leq 1 \text{ and } E_{11} \leq S^* \leq E_{22} . \quad (1)$$

Moreover,  $R^1(S, v_1) < (>) R^2(S, v_1)$  according to whether  $S < S^* (S > S^*, \text{ respectively})$ .

No proofs of these lemmas are given because they can easily be adapted from Schweizer (1986). The lemmas turn out to be useful for characterizing the set of all equilibria. According to the second one, an intersection  $(S^*, r^*)$  of indifference curves is associated with any equilibrium profile. Note that player B's interim payoffs  $(v_1, v_2)$  can be recovered from the intersection on its own by solving  $v_1 = V_1(S^*, r^*)$  and vice versa. Moreover, it is sufficient to know the equilibrium outcome (in the sense of Cho and Kreps) instead of the full profile to recover the intersection. Different profiles, however, and even different outcomes might lead to the same intersection of indifference curves. For a pair  $(v_1, v_2)$  of interim payoffs to arise from equilibrium, the condition

$$v_1 \geq v_1^0 = \text{Max}\{V_1(E_{12}, p_1), V_1(E_{22}, 1)\} \quad (2)$$

must necessarily be met as follows from lemma 1 and the fact that, in sequential equilibrium,  $r_m \geq p_1$  if  $S_m \geq E_{12}$  and  $r_m = 1$  if  $S_m \geq E_{22}$ . The following theorem describes the set of all equilibria.

#### Theorem 1

Consider any intersection  $(S^*, r^*)$  such that, for  $v_1 = V_1(S^*, r^*)$ , (1) and (2) hold. The intersection is consistent with equilibrium if and only if one of the conditions (i) - (iv) is met:

$$(i) \quad 0 < r^* \leq p_1 \text{ and } E_1 \leq S^* \leq E_{12}$$

- (ii)  $p_1 < r^* \leq 1$  and  $E_2 \leq S^* \leq E_{22}$
- (iii)  $v_2 = v_2^0$  and  $E_{21} \leq S^*$
- (iv)  $v_2 = v_2^0$  and  $R^1(E_{11}, v_1) \leq p_1$ .

For a proof of this proposition, the reader is referred to Schweizer (1986). The result can be visualized by plotting the set of intersections which are consistent with equilibrium (see fig. 1). Two cases must be distinguished according to whether  $R^2(E_{11}, v_2^0) > p_1$  (case a) or  $R^2(E_{11}, v_2^0) \leq p_1$  (case b). Intersections consistent with equilibrium are along  $Wcd$  and within  $cdX$  (figure 1a), along  $Yghcd$  and within  $ghZ$  or  $cdX$  (fig. 1b, respectively). Equilibria corresponding to  $W, X, Y$  and  $Z$  are of particular interest because they are the only candidates to survive the interim rule (see introduction). Hereby, an equilibrium leading to player B's interim payoffs  $(v_1, v_2)$  is eliminated by another equilibrium leading to  $(v_1', v_2')$  provided that  $v_1' \geq v_1$  holds for both signals  $l = 1$  and  $l = 2$  with strict inequality for at least one of them. In fig. 1, for example, equilibria along  $cd$  are eliminated by  $X$  because higher indifference curves correspond to higher payoffs. Other equilibria could be eliminated similarly and, eventually, equilibria  $W$  and  $X$  in case a whereas, in case b, equilibria  $Y, Z$  and  $X$  are the only ones possibly left. Notice, however, that  $W$  might still be eliminated by  $X$  and  $Y$  by  $Z$ . If, for instance, each player receives his bad signal with sufficiently low probability (probabilities  $p_1$  and  $q_2$  small enough) then not only case a can be shown to prevail but also  $X$  to eliminate  $W$ .

The equilibrium leading to  $W$  and  $Y$ , respectively, will play a distinguished role throughout the present paper. They can be sustained by the following separating equilibrium. Consider case a. Two subcases must be

distinguished according to whether

$$S^W = E_{21} \text{ and } V_1(E_{21}, r^W) > V_1(E_{11}, p_1) \quad (3)$$

or

$$S^W < E_{21} \text{ and } V_1(S^W, r^W) = V_1(E_{11}, p_1). \quad (4)$$

In subcase (3) the sustaining equilibrium satisfies  $s_1(E_{21}) = 1$ ,  $s_2(E_{22}) = 1$ ,  $r(E_{21}) = r^W$  and  $r(E_{22}) = 1$ . For out-of-equilibrium messages, beliefs and reactions can be defined with substantial degree of freedom as long as no conflict with lemma 1 arises. In subcase (4), the sustaining equilibrium is given by  $s_1(E_{11}) = 1$ ,  $s_2(E_{22}) = 1$ ,  $r(E_{11}) = p_1$  and  $r(E_{22}) = 1$  as far as equilibrium messages are concerned. Intersection Y can be sustained by a separating equilibrium in a similar way. For details, the reader might consult Schweizer (1986). Intersection X, finally, is to be sustained by the pooling equilibrium where B offers settlement terms  $S = E_2$  irrespective of his signal and where A accepts these terms with certainty. At X, the case is never litigated whereas at W and Y, respectively, litigation is predicted with positive probability.

To conclude this section, we investigate how much the leadership position is worth in terms of expected payoffs. To this end, the present game will be compared with the one where the ordering of moves is reversed. The question can only be asked for some particular equilibrium outcome. We choose W and Y, respectively, because they will be shown to have nice stability properties and the corresponding outcome in the reversed game. For the present, these outcomes might be characterized as the unique candidates to survive the interim rule and to be sustained by separating equilibria. In

the original game, B is leader and A is follower. Therefore, interim payoffs for A and B are denoted by  $(u_1^F, u_2^F)$  and  $(v_1^L, v_2^L)$ , respectively. In the reserved game, player A (plaintiff) as leader dictates settlement terms which then player B (defendant) as follower either accepts to pay or rejects in order to having the case litigated. Under the corresponding equilibrium outcome, interim payoffs in the reversed game are denoted by  $(u_1^L, u_2^L)$  for A and  $(v_1^F, v_2^F)$  for B. The following theorem can be established.

Theorem 2

For any state  $(k, l)$  of the world it holds that

$$u_k^L, u_k^F \geq E_k \text{ and } v_1^L, v_1^F \geq -F_1 . \quad (5)$$

Moreover,

$$u_2^L > E_2 = u_2^F \text{ and } v_1^L > -F_1 = v_1^F . \quad (6)$$

(Recall that  $k = 2$  and  $l = 1$  refer to signals which are good for the receiving party!).

Proof:

Each player can ensure, either by rejecting all terms or by proposing sufficiently low terms, that his interim expected payoffs are at least as high as under certain litigation. Therefore, (5) holds for any sequential equilibrium and not just for the ones under consideration in the above theorem. To establish (6), however, properties of the distinguished outcomes must be taken into account. For W (case a) it follows from (3) or (4) that  $v_1^L \geq V_1(E_{11}, p_1) = -F_1 + p_1(F_{11} - E_{11}) > -F_1$  whereas, for Y (case b), it holds that  $v_1^L = -F_1 + r^Y(F_{11} - E_{11}) > -F_1$ . In any case, we have  $v_1^L > -F_1$ .

Consider, next, interim payoffs for A in the original game where A is follower. We claim that  $u_2^F = E_2$ . Suppose this were not true. Then there must exist some message  $m$  which is sent by B with positive probability such that  $S_m > e_{2m}$ . Obviously, A accepts such terms, i.e.  $r_m = 1$ . For outcomes  $W$  and  $Y$ , it follows that  $S^* < S_m$  and, hence, that  $s_{1m} = 0$  and  $e_{2m} = E_{22}$ . This contradicts the fact that settlement terms exceeding  $E_{22}$  will never be offered and, hence,  $u_2^F = E_2$  is established.

The reversed game will not be discussed in detail. Obviously, the two games are quite symmetric and, therefore, have similar properties. In particular, it can be shown that the now leading A obtains strictly more than under certain litigation if he receives his good signal, i.e.  $u_2^L > E_2$ . Similarly, B as follower will not get more than under litigation if he receives his good signal, i.e.  $v_1^F = -F_1$ . By exploiting such symmetry, (6) is established readily.

Q.E.D.

We point out that, in the case where a party receives its bad signal, the interim payoff as follower could well be higher than the one achieved as leader. Examples can be constructed where, even from the ex ante view, the position of leader yields lower expected payoff than that of follower. For the plaintiff, this means that  $u^L = p_1 u_1^L + p_2 u_2^L$  could conceivably be lower than  $u^F = p_1 u_1^F + p_2 u_2^F$ , for the defendant that  $v^L = q_1 v_1^L + q_2 v_2^L < v^F = q_1 v_1^F + q_2 v_2^F$ . Obviously, such a possibility hinges on the incomplete information players have in the above games.

## 2. Stability

In the present section, the proper criterion of stability is shown to have substantial cutting power in the context of our game of litigation and settlement. A unique equilibrium outcome turns out to be stable. In general, a set of equilibrium profiles of a given game is called stable if any other game close enough has an equilibrium profile which is close to one of that set. Hereby, "close" refers to the following metrics for classes of games and their set of strategies. For any positive real number  $\delta$  and any strictly mixed strategy  $(s_{lm}^0)$  of player B (i.e.  $s_{lm}^0 > 0$  for all  $l$  and  $m$ ), the perturbed game which has the set of strategies  $(s_{lm})$  for player B restricted by

$$\delta s_{lm}^0 \leq s_{lm} \quad (l=1,2 \text{ and } m=1,\dots,M) \quad (7)$$

is said to be within  $\delta$  of the original game. Note that, for any such perturbation, the second moving player A's beliefs are fully determined by Bayes' rule. In particular, if message  $m$  is sent with minimum probability allowed by (7) then A's updated expected gain  $e_{km}^0$  from litigation amounts to

$$(q_1 s_{1m}^0 + q_2 s_{2m}^0) e_{km}^0 = q_1 s_{1m}^0 E_{k1} + q_2 s_{2m}^0 E_{k2} .$$

Strategy profiles  $(s_{1m}, s_{2m}, r_m)$  consist of a mixed strategy for each player and, hence, they can be represented as points of  $3M$ -dimensional Euclidean space. Two profiles are said to be within  $\epsilon$  if their Euclidean distance does not exceed  $\epsilon$ . A set  $N$  of Nash equilibria of the original game is called stable if, for any  $\epsilon > 0$ , there exists  $\delta > 0$  such that any perturbation within  $\delta$  of the original game has an equilibrium profile within  $\epsilon$  of the set  $N$ .

In essence, stability is a property of equilibrium outcomes (c.f. Cho and Kreps (1986)). Hereby, an equilibrium outcome is defined as a probability distribution over the set of endpoints of the game induced by some equilibrium profile. Associated with any such outcome is an intersection of indifference curves (see lemma 2). Different outcomes, however, might lead to the same intersection. Theorem 1 contains a full set of conditions for such an intersection to be consistent with equilibrium. In the following, we explore which of them would further be consistent with stability. Subcase (3) will be studied in detail. Other subcases are left to the reader.

To begin with, consider any equilibrium outcome leading to an intersection  $(S^*, r^*)$  with  $S^w = E_{21} < S^*$  (either along  $W \subset d$  or within  $c \subset d \subset X$  in fig. 1a). Such an intersection cannot be consistent with stability for the following reason. Take any message  $E_{21} < S_m < S^*$  and, for the perturbed game, define lower bounds such that  $e_{2m}^0 < S_m$  holds. Since  $S_m < S^*$  it follows that  $s_{2m} = \delta s_{2m}^0$  and, hence, that  $e_{2m} \leq e_{2m}^0 < S_m$ . For A's reaction to be sequentially rational it must be true that  $r_m = 1$  which, however, would not be consistent with equilibrium in the perturbed game because, as a consequence,  $r_m = 1 > R^2(S_m, v_2) > R^1(S_m, v_1)$ . In other words, if there exists some message  $m$  with  $E_{21} < S_m < S^*$  then, arbitrarily close to the original game, perturbations can be constructed for which the above intersection would not be consistent with equilibrium. It follows that  $E_{21} \leq S^* \leq E_{21}^+$  (where  $E_{21}^+ = S_{u+1}$  if  $E_{21} = S_u$  and, similarly, for other values of the sequence  $S_m$ ) are the only candidates left for stability. As a matter of fact, it can be shown that  $S^* < E_{21}^+$  is not consistent with stability either because, in this

case and for any perturbed game, player B holding signal  $l = 1$  would have no message to be sent in equilibrium. In this way, the strict condition  $S^* = E_{21}^+$  turns out to be necessary for stability. Now observe that, for  $S^* = E_{21}^+$ , sustaining equilibria in the original game are characterized by the following equilibrium path:  $s_1(E_{21}^+) = 1$ ,  $s_2(E_{21}^+)$  such that  $e_2(E_{21}^+) = E_{21}^+$ ,  $s_2(E_{22}) = 1 - s_2(E_{21}^+)$ ,  $r(E_{21}^+) = R^2(E_{21}^+, v_2^0)$  and  $r(E_{22}) = 1$ . The set of all equilibria with this path is easily shown to be stable (details left to the reader). To summarize subcase (3), while the outcome leading to  $W$  is not stable itself, the adjoining outcome  $W^+$  (defined by  $S^* = E_{21}^+$ ) is the unique stable outcome of the game. Obviously,  $W^+$  approaches  $W$  as the sequence  $S_m$  is refined.

The other subcases could be discussed similarly. Let, in general,  $W^+$  and  $Y^+$  denote the intersections adjoining  $W$  and  $Y$ , respectively. They are to be sustained in an obvious way. If the given sequence from which settlement terms have to be selected is fine enough then the following theorem holds.

### Theorem 3

The intersection  $(S^*, r^*)$  of indifference curves can be sustained by a stable equilibrium outcome if and only if it is associated with  $W^+$  and  $Y^+$ , respectively.

Stability has cutting power. A unique outcome survives the test. This outcome adjoins a separating equilibrium and has the case litigated with positive probability. It follows that, while sequential equilibria might exist where the case is settled with certainty, no such equilibrium leads to a stable outcome.

### 3. Dominance and the Intuitive Criterion

In this and the next section, the cutting power of the various selection criteria collected and reported by Cho and Kreps (1986) will be explored. We begin, in this section, by showing why neither the intuitive criterion nor that based on dominance has much cutting power in the context of our game of litigation and settlement. In order to avoid countless and tiresome subcases, the analysis is confined to parameter configurations where

$$E_{11} < E_{12} < E_{21} < E_{22} < F_{11} \quad (8)$$

and

$$V_1(E_{22}, 1) > V_1(E_{11}, p_1) \quad (9)$$

hold. Notice that this is a particular instance of subcase (3). Needless to say that other parameter configurations could be discussed similarly.

The criteria are as follows. In a first step, all message-response pairs  $(m, r)$  are deleted for which  $r$  is never a best response, no matter what A's beliefs are. The set of remaining responses is denoted by  $R(m)$ . Recall that, for all conceivable beliefs, the plaintiff's updated expected gain  $e_{km}$  from litigation necessarily satisfies  $E_{k1} \leq e_{km} \leq E_{k2}$ . It follows from (8) that  $R(m) = \{0\}$  if  $S_m < E_{11}$ ,  $R(m) = [0, p_1]$  if  $E_{11} \leq S_m \leq E_{12}$ ,  $R(m) = \{p_1\}$  if  $E_{12} < S_m < E_{21}$ ,  $R(m) = [p_1, 1]$  if  $E_{21} \leq S_m \leq E_{22}$  and  $R(m) = \{1\}$  if  $E_{22} < S_m$ .

According to the criterion involving dominance, player B holding signal 1 is believed not to send out-of-equilibrium message  $\mu$  if there exists some other message  $m$

with

$$\min_{R(m)} V_1(S_m, r) > \max_{R(\mu)} V_1(S_\mu, r) .$$

In words, if B would fare better, irrespective of A's reaction, by sending  $m$  instead of  $\mu$  while holding  $l$  then A if offered  $S_\mu$  is assumed to believe that B cannot having received signal  $l$  from nature. Let  $L^D(m)$  denote the set of B's remaining signals which A possibly attributes to B if offered terms  $S_m$ . Configuration (8) and (9) is chosen such that the following properties hold:

$$\min V_1(E_{22^+}, r) = \max V_1(E_{22^+}, r) = V_1(E_{22^+}, 1)$$

and, for  $S_\mu < E_{21}$  and  $E_{22^+} < S_\mu$ ,

$$V_1(E_{22^+}, 1) > \max_{R(\mu)} V_1(S_\mu, r) .$$

It follows that  $L^D(m)$  is empty for  $S_m < E_{21}$  and for  $E_{22^+} < S_m$  whereas  $L^D(m) = \{1, 2\}$  for  $E_{21} \leq S_m \leq E_{22^+}$ . A sequential equilibrium profile  $(s_{1m}, q_{1m}, r_m)$  passes the test based on dominance if, for any out-of-equilibrium message  $m$ , it holds that  $q_{1m} = 0$  for any signal  $l$  which does not belong to  $L^D(m)$ .

Every sequential equilibrium profile can be shown to pass the test for the following reason. Consider, first, any message  $m$  with  $S_m < E_{21}$ . It follows from (8) that  $r_m \leq p_1$  whenever  $r_m \in R(m)$  and, hence, from (9) that  $R^1(S_m, v_1) > r_m$ . Therefore, the test adds no restrictions as far as messages  $S_m < E_{21}$  are concerned. Second, for messages  $E_{21} \leq S_m \leq E_{22}$ , the test adds no restrictions because player A excludes none of player B's signals. For messages  $E_{22} < S_m$ , finally, the test adds no restriction because sequential rationality leads A to

accept the corresponding settlement terms which is consistent with A believing B to having received signal  $l = 2$ . The test based on dominance does not rule out a single equilibrium profile.

The intuitive criterion differs from the above test in that B's interim expected payoffs  $(v_1, v_2)$  of the profile to which the criterion is applied enters the restriction of A's beliefs. Player B holding signal  $l$  is assessed not to send out-of-equilibrium message  $\mu$  if the equilibrium payoff  $v_l$  exceeds what B could conceivably receive by offering terms  $S_\mu$ , i.e. if

$$v_l > \max_{R(\mu)} V_l(S_\mu, r)$$

holds. Let  $L^l(m, v_1, v_2)$  denote the set of signals which are not pruned if B offers terms  $S_m$ . A sequential equilibrium profile  $(s_{lm}, q_{lm}, r_m)$  with interim payoffs  $(v_1, v_2)$  passes the intuitive test if  $q_{lm} = 0$  for any signal  $l$  which does not belong to  $L^l(m, v_1, v_2)$ . There exist profiles which fail the test. But no substantial cutting power is involved because any equilibrium outcome can be sustained by at least some profile as is shown in the following way. Let  $S^l$  denote settlement terms with  $R^l(S^l, v_l) = 1$  for  $l=1$  and  $l=2$ . It follows that  $L = L^l(m, v_1, v_2)$  is empty for  $S_m < E_{21}$ ,  $L = \{1, 2\}$  for  $E_{21} \leq S_m \leq S^1$ ,  $L = \{2\}$  for  $S^1 < S_m \leq S^2$  and  $L$  empty again for  $S^2 < S_m$ . For out-of-equilibrium messages  $m$  with  $S^1 < S_m < S^2$ , the test restricts beliefs in that A assesses B to holding signal  $l=2$ . Therefore, A's updated expected gain from litigation amounts to  $e_{2m} = E_{22}$  and, since  $S^2 \leq E_{22}$ , A's best response is  $r_m = p_1$  for such messages. Any beliefs, however, leading to  $S_m \leq e_{2m}$  and any reaction  $r_m \leq R^2(S_m, v_2)$  would be consistent with sequential equilibrium. Note that condition (9) implies

$R^1(S_m, v_1) > p_1$  for both signals  $l$  and settlement terms in the above range. As a consequence, beliefs and reactions implied by the intuitive criterion are consistent with sequential equilibrium which means that not a single equilibrium outcome will be ruled out by the intuitive criterion.

To summarize the above findings, while every sequential equilibrium profile passes the test based on dominance, some profiles fail the test involving the intuitive criterion. At the more aggregate level of outcomes, however, neither test has cutting power in the context of our game.

#### 4. Divinity and Universal Divinity

In the final section, divinity tests as proposed by Banks and Sobel (1985) (c.f. Cho and Kreps (1986)) are investigated. Divinity is shown to rule out some equilibrium outcomes but, still, to leave a substantial number of them unaffected. The stricter test of universal divinity, however, leads to (almost) a unique outcome. Recall that outcome  $W^*$  ( $Y^*$ , respectively) is the only stable one (see Theorem 3) and, hence, must pass the test of universal divinity. But the fully separating outcome  $W$  ( $Y$ , respectively) itself passes the test as well. Therefore, as long as we insist on having  $B$  proposing terms out of a given finite sequence, exactly two outcomes pass the test. Only one of them is stable. The test can equally well be applied to the infinite version of the game where it then distinguishes some strictly unique outcome.

Divinity tests again include  $B$ 's interim expected payoffs  $(v_1, v_2)$  of the outcome to be tested. Let  $D_{1m}(v_1)$  and  $D_{1m}^0(v_1)$  denote the set of responses  $r$  with  $v_1 < V_1(S_m, r)$  and  $v_1 = V_1(S_m, r)$ , respectively, or what amounts to the same, with  $r > R^1(S_m, v_1)$  and  $r = R^1(S_m, v_1)$ . Suppose message  $m$  is such that  $B$  holding signal  $j$  strictly wishes to defect for all of  $A$ 's reactions for which  $B$  holding  $l$  wishes to defect, i.e.

$$D_{1m}(v_1) \cup D_{1m}^0(v_1) \subset D_{jm}(v_j) . \quad (10)$$

Then, (universal) divinity has  $A$  believing signal  $l$  to be (infinitely) less likely than signal  $j$ , i.e.  $q_{lm} \leq q_{jm}$  ( $q_{lm} = 0$ , respectively). Observe, by the way, that condition (10) is equivalent to

$$D_{1m}^0(v_1) \subset D_{jm}(v_j)$$

in the context of our game which makes the test based on "never a weak best response" coincide with that of universal divinity.

Condition (10) is straightforward to check. Let  $(S^*, r^*)$  be the intersection of indifference curves associated with the outcome under investigation (c.f. lemma 2). For messages  $m$  with  $S_m < S^*$ , condition (10) holds for signals  $l=2$  and  $j=1$  whereas, for  $S^* < S_m$ , the condition is met under signals  $l=1$  and  $j=2$ . An outcome passes the test of (universal) divinity if it can be sustained by a sequential equilibrium profile  $(s_{1m}, q_{1m}, r_m)$  such that  $q_{2m} \leq q_2$  ( $q_{2m} = 0$ ) for messages  $S_m < S^*$  and  $q_{1m} \leq q_1$  ( $q_{1m} = 0$ ) for messages  $S^* < S_m$ .

We, first, explore the cutting power of the divinity test which is shown to rule out equilibria such as (i) and (ii) of Theorem 1, at least if  $E_1^+ < S^*$  and  $E_2^+ < S^*$  holds, respectively. To establish this result, consider any equilibrium profile  $(s_{1m}, q_{1m}, r_m)$  leading to (i) (the case (ii) could be handled similarly). Since  $E_1^+ < S^*$  there exists some message  $m$  with  $E_1 < S_m < S^*$  and response  $r_m < r^* \leq p_1$ . For this to be sequentially rational, beliefs must be such that  $S_m \leq e_{1m}$ . Divinity, on the other hand, would require that  $q_{2m} \leq q_2$  and, hence, that  $e_{1m} \leq E_1$ . In this way, the outcome can be ruled out. Thus, the divinity test has cutting power. But a substantial number of outcomes might still survive. Reconsider, e.g., configuration (8) and (9) of section 3. We claim that any outcome leading to  $E_{21} \leq S^* \leq E_2$  (i.e. along  $Wc$  and  $cX$  in fig. 1a) passes the test which can be shown by constructing sustaining beliefs in the following way. For messages  $S_m \leq E_{21}$ , let A believe  $q_{2m} = 0$  and hence  $q_{1m} = 1$  and have A react with  $r_m \leq p_1$ . This is consistent both with sequential

equilibrium and divinity (even with universal divinity). For messages  $E_{21} < S_m < S^*$ , A is assumed to have beliefs such that  $e_{2m} = S^*$ . Since  $S^* \leq E_2$ , divinity allows such beliefs but note that universal divinity would not. Moreover, the induced sequentially rational response  $r_m = p_1$  is consistent with equilibrium as follows from (9). For messages  $S^* < S_m$ , finally, let A believe  $q_{1m} = 0$  and  $q_{2m} = 1$  as implied by universal divinity. The corresponding sequentially rational reaction is again consistent with equilibrium. It follows that any equilibrium outcome with  $E_{21} \leq S^* \leq E_2$  passes the test of divinity though not necessarily that of universal divinity. As a matter of fact, as long as there exists some message  $E_{21} < S_m < S^*$ , universal divinity imposes beliefs  $q_{2m} = 0$  and  $q_{1m} = 1$  which lead to updated expected gain  $e_{2m} = E_{21} < S_m$ . The only sequentially rational response  $r_m = 1$ , however, would not be consistent with equilibrium. These findings suggest that outcomes associated with  $S^* = E_{21}$  or with  $S^* = E_{21}^+$  are the only ones to pass the test of universal divinity. The result can be generalized beyond the particular configuration of the above example as the following theorem shows.

Theorem 4

Equilibrium outcomes associated with  $W$  and  $W^+$  ( $Y$  and  $Y^+$ , respectively) are the only ones passing the test of universal divinity.

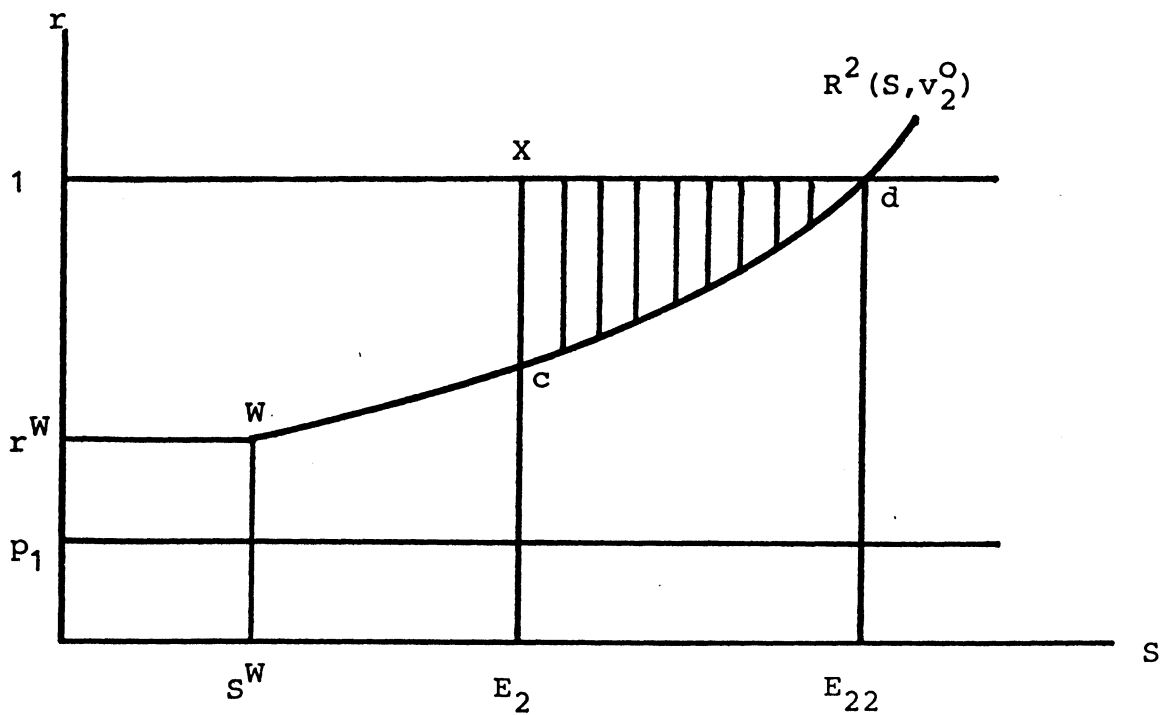
To prove the theorem, various subcases must be distinguished according to which conditions listed in Theorem 1 apply. A full proof would be rather tedious and lengthy. But, since the only arguments needed are of the same straightforward kind as in the above particular case, any details will be left to the reader.

Concluding Remarks

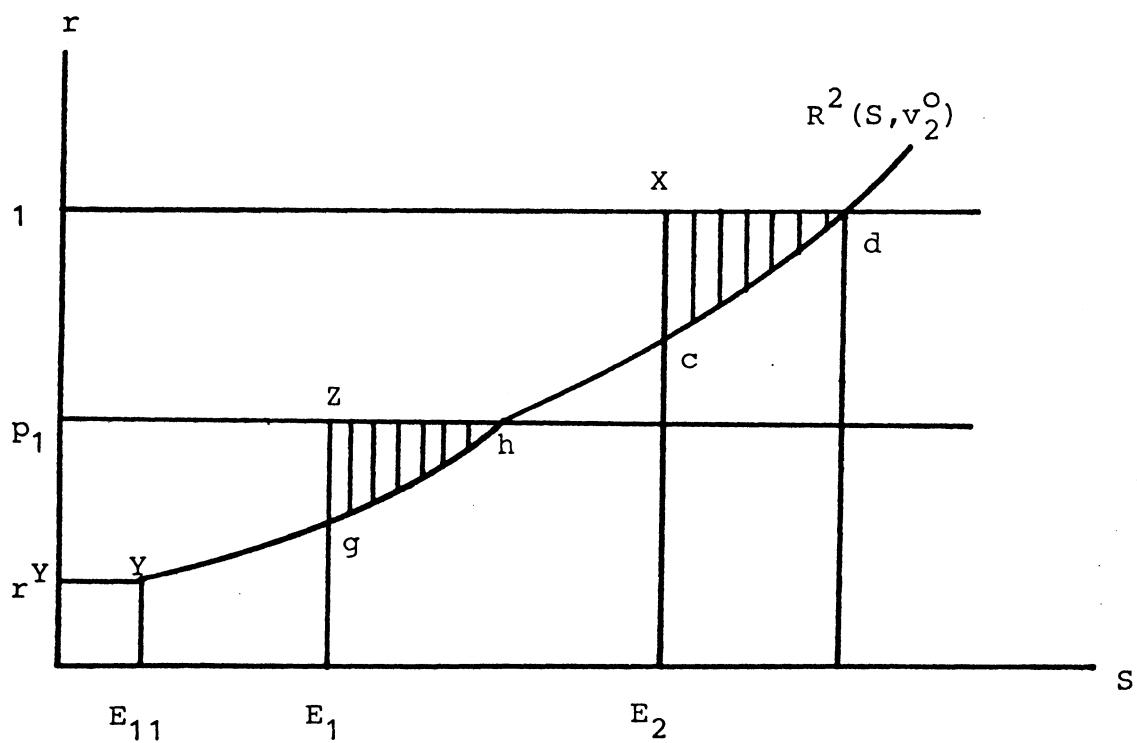
In this paper, the power of various tests for reducing the number of sequential equilibria has been investigated for a simple game of litigation and settlement. Most tests which have any substantial cutting power at all essentially turn out to allow a single outcome to pass which has some rather peculiar properties. First, the outcome is sustained by a separating equilibrium which fully reveals the first player's private information to the second one. Nevertheless, second, the negative-sum situation will not necessarily be avoided as the case is litigated with positive probability. In spite of the possibility to enhance efficiency, the required rearrangement of rights need not take place. We conclude that either the validity of the Coase Theorem must be questioned in a setup of incomplete information or, else, such incomplete information should explicitly be recognized as a source of transaction costs. Third, as far as comparative statics is concerned, it follows from section 3 of Schweizer (1986) that the single outcome surviving all tests might well be the one which has the first moving player's payoff (as expected ex ante) decrease with an improvement of his information quality (i.e. the accuracy of predicting his chances at trial). Hence, such properties are not only consistent with sequential equilibrium. They even survive the much stricter requirements as imposed by any of the tests reported by Cho and Kreps (1986).

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(a)



(b)

Figure 1 The set of intersections which are consistent with sequential equilibrium