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Stability of critical distance approach to predict fretting fatigue cracking: a "ℓ_{opt}-b_{opt}" concept

Camille Gandiolle, Siegfried Fouvry
LTDS, Ecole Centrale de Lyon, 36 Avenue Guy de Collongues, 69134 Ecully Cedex, France
*siegfried.fouvry@ec-lyon.fr, camille.gandiolle@ec-lyon.fr

Abstract

Fretting fatigue life prediction is a strategic issue for modern industry. Accurate prediction of total lifetime depends on crack nucleation time prediction. However, fretting fatigue is characterized by a high stress gradient which overestimates cracking risk at the hotspot. Hence non-local analyses are required. The present study focused on the critical distance method. However, no defined method is currently established to obtain this value, which has been shown to depend on microstructure or stress gradient. The present study investigated the influence of incipient crack length on critical distance crack nucleation prediction. An optimal ℓopt-bopt condition was introduced enabling accurate prediction of the crack nucleation condition whatever the loading condition. This approach was then applied to fretting endurance experiments, allowing to correlate fretting cracking endurance with shear fatigue data. Finally it was used on fretting fatigue lifetime experiments, showing good endurance prediction.

Keywords: critical distance; stress gradient; fretting fatigue; lifetime prediction.

Nomenclature

a	Half width of the contact area (mm)
b	Maximum projected crack length (µm)
b_0	Short crack to long crack transition (µm)
b_{opt}	Optimal crack nucleation length (µm)
b_{th}	Threshold crack nucleation length (µm)
d	Contact mesh size (µm)
E	Young's modulus (MPa)
K_I	Mode I stress intensity factor (SIF) (MPa.m$^{1/2}$)
K_{IC}	Critical mode I SIF
$\overline{K^*}$	Fatigue crack driving force parameter (MPa.m$^{1/2}$)
$\ell=\ell_D$	Critical distance (µm)
ℓ_{opt}	Optimal critical distance (µm)
ℓ_v	Representative material volume length scale (µm)
ℓ_∇	Gradient method length scale (µm)
N	Number of fretting cycles

N_{CN}	Number of fretting cycles to crack nucleation
N_f	Final number of fretting cycles from experiments
N_P	Number of propagating fretting cycles
N_T	Total number of fretting cycles from predictive model
P	Linear normal force (N/mm)
Q	Linear tangential fretting force (N/mm)
Q*	Linear tangential fretting force amplitude (N/mm)
Q^*_{CN}	Crack nucleation threshold (N/mm)
R	Radius of cylinder pad (mm)
R_F	Fatigue stress ratio
R_K	SIF stress ratio
R_{Q^*}	Fretting stress ratio (Q_{min}/Q_{max}=Q*+/Q*-)
S	Deviatoric part of Σ
x	Surface axis
z	Subsurface axis

<u>Greek letters</u>

$J_{2,a}$	Amplitude of the second invariant of the stress tensor deviator
$\nabla\sigma_H$	Hydrostatic stress gradient
ΔK	Nominal SIF (MPa.m$^{1/2}$)
ΔK_0	Long crack SIF range threshold (MPa.m$^{1/2}$)
$\mu=\mu_t$	Coefficient of friction at the sliding transition
σ	Stress (MPa)
σ_{11}	Stress in the 11 direction (MPa)
σ_F	Fatigue stress (MPa)
$\underline{\underline{\Sigma}}$	Stress tensor
δ	Fretting displacement (µm)
ν	Poisson's coefficient
σ_C	Equivalent stress of Crossland's criterion (MPa)
σ_d	traction-compression fatigue limit (R_F=-1) (MPa)
σ_H	Hydrostatic stress (MPa)
σ_y	Yield stress (MPa)
τ_d	Torsion fatigue limit (R_F=-1) (MPa)

<u>Subscripts</u>

FF	Fretting fatigue
max	Maximum
mean	mean
min	minimum
PF	Plain fretting

1. Introduction

Fretting is a small-amplitude oscillatory movement between two surfaces in contact [1,2]. Fretting involves two sliding conditions depending on the displacement amplitude: partial slip, which involves an inner stick zone, and larger amplitude gross slip, inducing a full sliding response in the interface. We focus on the partial slip condition which leads primarily to cracking damage [3,4]. Fretting cracking is often found in industrial assemblies and combined with fatigue; it can induce premature failure of the assembly. Accurate prediction of fretting fatigue crack initiation is essential for estimating the lifetime of assemblies.

Fretting fatigue is characterized by a high stress gradient, which may be one order greater than observed in notch fatigue experiments. Stress gradients lead to over-estimation of cracking risk at the hot spot (i.e., the top surface contact border in fretting fatigue). Hence, local analysis at the hot spot is not suitable, as it leads to an over-conservative estimation of crack nucleation risk.

Non-local fatigue stress analyses are required for severe contact stress gradient. These methods are not new, having been introduced for notch fatigue [5,6], and are widely used because they can easily be combined with finite element analysis, allowing accurate prediction of fracture and fatigue behavior in practical engineering problems. Various strategies can be applied. The most common one used in industry consists in computing a mean stress loading path averaged over a representative process volume domain surrounding the fatigue hot spot. A key aspect of this approach is the determination of the representative material volume, usually defined by a cubic length scale (ℓ_V). Using this mean stress path, crack nucleation risk is computed applying a multi-axial fatigue analysis [7].

A second strategy consists in using a stress gradient weight function strategy [8]. Fatigue is still estimated at the hot spot, but the equivalent stress obtained is multiplied by a coefficient w. This coefficient, defined as a linearly decreasing function of the hydrostatic stress gradient state operating near the hotspot ($|\nabla\sigma_H|$) $w=1-k|\nabla\sigma_H|$ with $k>0$ (note that the weight function is equal to 1 in the absence of a stress gradient, which corresponds to the common fatigue test condition). This strategy was successfully applied by Amargier et al. [9] to predict fretting crack nucleation risk in a nickel-based alloy. However, prediction still depends on the representative volume, usually defined by a cubic length ℓ_∇ over which the stress gradient is computed. Another strategy, extensively applied in academic research, is based on the critical distance method developed by Taylor and co-authors [10]. Equivalent in principle to the process volume method, this approach consists in considering the equivalent fatigue stress at a critical distance (ℓ) from the hot spot stress. Hence, whichever the method, prediction will depend on the length scale variable (i.e., ℓ_V, ℓ_∇ or ℓ, respectively) used to compute the representative stress states to be compared to the material fatigue data to predict the cracking risk. Various approaches consider microstructural grain size as a relevant parameter [7,11–13]. An alternative strategy,

developed by Taylor, considering the Kitagawa-Takahashi formalism [14], assumes that the optimal critical distance is equal to half the value of the transition from short to long crack propagation regime (b_0). The latter can be approximated as a function of the alternative fatigue limit (σ_d) and the long crack stress intensity factor (SIF) range threshold (ΔK_0):

$$\ell_D = \ell_T = \frac{b_0}{2} = \frac{1}{2\pi}\left(\frac{\Delta K_0}{1.12\sigma_d}\right)^2 \quad (1)$$

This approach was initially applied to notch fatigue [10,15]; but Araújo and co-authors achieved reasonable prediction of fretting fatigue endurance [16]. However, more recent research, investigating small crack nucleation conditions (b=10µm) in a large spectrum of cylinder radii (i.e., stress gradient conditions), showed that optimal critical distance evolves with the stress gradient condition [17] so that a single length scale dimension cannot provide a global description of contact stress gradient effects. In this research, very short crack lengths were investigated, and it was suggested that prediction would depend on the crack nucleation length used to establish the crack nucleation condition. Indeed depending on the crack length used to define the crack nucleation condition, different contact loading thresholds are determined (Q^*_{CN}). This evolution can influence the prediction of critical distance. Hence, an optimal crack nucleation length b_{opt} and its corresponding optimal critical distance ℓ_{opt} are expected to optimize the crack nucleation prediction provided by the critical distance method.

This study was conducted for industrial purposes thus applying corresponding severely plastic solicitations. We aim to predict lifetime of fretting fatigue experiments by decoupling crack initiation life and crack propagation life. The latter will be classically addressed with Paris law [18–22]. This paper focuses on the former by clarifying the crack length influence and (ℓ_{opt}-b_{opt}) strategy on the optimization of crack nucleation prediction, exploring crack nucleation responses obtained for various plain fretting and fretting fatigue conditions. This (ℓ_{opt}-b_{opt}) crack nucleation prediction strategy will then be validated through lifetime prediction of fretting fatigue experiments.

2. Material and tests

The study material was a 32Cr1 industrial steel (E=200GPa, ν=0.3) which shows low yield stress (σ_{y_flat}=400MPa). Its atomic composition is: 0.32%C, 0.347%Cr, 0.743%Mn, 0.201%Si, 0.046%Cu, 0.016%Mo, 0.047%Ni, 0.013%Al, 0.007%P, 0.002%S [23]. The material was tested under fretting fatigue conditions, using a cylinder/plane contact with a cylinder of R=4.6 mm radius applied with a normal force P on the flat material. The cylinder was also made of industrial steel (E=200GPa, ν=0.3), but with higher yield stress ($\sigma_{y_cylinder}$=600MPa >> σ_{y_flat}), to investigate cracking on the plane specimen only.

Their low yield stresses implied to consider elastic-plastic descriptions for both material behaviors. Because this study was conducted for industrial purposes, the industrial laws were used to describe the hardening of the materials (Fig. 1). Monotonic laws with isotropic hardenings were thus implemented to describe the cyclic fretting fatigue loading. This paper will show that despite this hypothesis, really good prediction of crack nucleation and crack propagation may be achieved.

These laws were obtained by tensile test and are plotted in Fig. 1. All stress values were normalized by the alternating shear fatigue limit at 10^7 cycles τ_d. This limit was determined experimentally by a staircase method.

The contact was tested under fretting and fretting fatigue. The plain fretting test set-up is shown Fig. 2a. The normal force P was applied to the contact by a mechanical spring system. Then a hydraulic actuator imposed a purely alternating sinusoidal cyclic displacement δ(t) on the plane, generating an alternating cyclic tangential load Q(t) on the contact surface. During the test, displacement δ, normal force P and tangential force Q were recorded, enabling the Q-δ fretting loop to be plotted and thus the fretting regime to be identified. As cracking, and more specifically crack nucleation, was to be investigated, the displacements were kept small enough to maintain partial slip conditions.

Fretting fatigue test allows to apply a bulk fatigue loading which helps propagate the cracks compared to plain fretting. Fretting fatigue experiments may lead to failure while plain fretting cracks will always stop propagating. This configuration is often more representative of industrial problems.

The fretting fatigue test machine is composed of three actuators respectively for the normal force P, the fretting force Q and the fatigue force, as shown in Fig. 2b. This multiple actuator set-up allows independant application of the fatigue force and the fretting force on the flat sample. In-phase sinusoidal cyclic tangential and fatigue forces were applied so that the maximum fretting load Q^*_{max} was applied at the same time as the maximum fatigue stress $\sigma_{F,max}$.

All tests were performed up to 10^6cycles. The frequency was fixed at 12 Hertz, high enough to investigate a long test condition and low enough to guarantee test control stability. The fretting stress ratio was kept constant at $R_{Q^*} = Q^*_{min}/Q^*_{max} = -Q^*/+Q^* = -1$.

Crack analysis was restricted to the flat specimens. The chosen dimensions allowed plane strain conditions along the central axis of the fretting scar. All the fretting scars were analyzed using a classic destructive method. First, the sample was cut in the middle, and then imbedded in epoxy to be polished to a mirror-like surface state. Next, cracks were observed by optical microscope. The polishing and observation steps were repeated two more times so that the crack measurement was performed on 6 different planes. From these measurements, the maximum projected crack length **b** was determined. Fig. 3 shows an example of a fretting crack and the measure of the projected crack length b. Chemical attack was performed to observe the microstructure. Cracks were found to be transgranular. The microstructure was also identified: the steel was found to be martensitique, with grain size about 15µm near the surface.

3. Experimental results

For each experiment, maximum projected crack length b was considered and plotted as a function of tangential force amplitude Q* (Fig. 4). The incipient crack nucleation condition, related to a projected crack length b=0µm, was defined by extrapolating the evolution of b to the x-axis: $Q^*_{CN}(b \rightarrow 0)$. Using this strategy, various crack nucleation thresholds Q^*_{CN} could be considered, depending on the chosen crack length threshold b_{th}.

A first set of tests was run to establish the evolution of b as a function of fretting tangential amplitude Q*(N/mm) for plain fretting conditions (Fig. 4). Then the fretting fatigue experiments were performed. Maximum fatigue stress $\sigma_{F,max}=1.86\times\tau_d$ was held constant while the fatigue stress ratio $R_F= \sigma_{F,min}/\sigma_{F,max}$ was varied from 0.6 to 1 (i.e., prestressed test). Only high fatigue stress ratios were tested, in line with the industrial application of the studied materials. For each fatigue condition, fretting loading was adjusted to establish the evolution of b as a function of Q* like for plain fretting. Fig. 5 presents the various crack nucleation and extension results. Application of fatigue loading compared to plain fretting did not affect the crack nucleation threshold Q^*_{CN} at 0 µm, but did increase crack extension. Considering the fretting fatigue results, it was interesting to note that, whatever the contact, for b_{th}=0, crack nucleation was independent of the fatigue stress ratio R_F. This suggests that activation of an incipient fretting fatigue crack is governed only by cyclic fretting contact stress.

Usual fretting crack nucleation analyses consider longer cracks, around 10µm, for crack nucleation so as to avoid any confusion with surface defects [19,24]. For longer cracks, the

fatigue stress ratio effect was more effective: an increase in fatigue stress amplitude (decrease in R_F) induced extension of crack propagation for a similar Q* fretting loading. On this experimental basis, it was possible to identify the stability of the critical distance approach in relation to the initial defect.

4. Test modeling

Finite Element Analysis (FEA) was carried out using Abaqus 6.10 software. A 2D plain strain model of the fretting fatigue test was generated (Fig. 6a). The dimensions and boundary conditions matched the parameters of the physical experiment. To model the plain fretting test, the ball-bearing, which is underneath on Fig. 6a was removed, and the fatigue part was blocked on both right and left sides and underneath.

The model was meshed with CPE3-type linear triangular elements, except in the contact zone where CPE4R-type linear quadrilateral elements were used; this zone was also meshed more densely than the other regions (5µm squares). The depth of the refined zone was adjusted from 200 µm for crack nucleation condition up to 6 mm for failure conditions (case of Fig. 6a).

To evaluate Finite Element Modeling (FEM) stress convergence, a similar fretting fatigue condition was simulated varying the surface mesh size from d = 1 µm to d = 10 µm. For this particular invastigation, elastic assumption was considered to compare FEM results with an exact analytical profile [4,25]. Fig. 6b compares z-profiles of σ_{11} stress at the trailing contact border x=-a. It shows strong mesh size dependence on the contacting surface. With a mesh d = 5 µm, the first surface node, at the surface hot spot, displays an error of 20%. The larger the mesh size, the larger is the underestimation of the surface stress provided by FEM computation.

Fig. 6b also suggests that a rather good estimation is achieved at the 2nd and 3rd node. The error falls below 5% on the 2nd node and is nearly null on the 3rd. It can be concluded that cautions must be taken regarding the surface FEM stress estimation. However, reliable estimations are achieved below the surface which implies that consistent crack nucleation predictions can be obtained if the fatigue stress analysis exclusively involves subsurface stress components defined for instance at the second and third nodes below the surface. Thus the choice of a 5µm mesh size in the contact allows saving considerable computation time, and is small enough compared to the critical distances that will be used.

Surface-to-surface discretization with small sliding was adopted for contact accommodation. The Lagrange multiplier was selected as the contact algorithm. The friction coefficient of each contact was determined experimentally using the variable displacement technique described by Voisin et al. [26]. The friction coefficient was µ=1.0. The cylinder and fatigue plane sample behaviors were described by the monotonic plastic laws introduced in Fig. 1. As mentioned above, whatever the contact, the normal force with which the cylinder was applied to the plane

was high enough to generate plasticity. The added fatigue loading contributed to extend the plastic state.

Using elastic-plastic laws to simulate the plane response to fretting fatigue, the level of activated plasticity decreased after each cycle, due partly to material hardening but mostly to plastic accommodation of the contact geometry. For each simulation, the contact border integration point was monitored. Fig 7a&b plots for a given condition the cumulative plastic strain and σ_{11} stress evolutions. The cumulative plastic strain increased until reaching an asymptotic evolution after around 50 cycles, i.e. a stable state corresponding to elastic shakedown. This stabilization of the strain corresponds to a stabilization of the stresses. Fig. 7c plots the corresponding stress-strain loops.
Numerical analysis showed that elastic shakedown was achieved after around 20 loading cycles for plain fretting. Higher loading conditions such as fretting-fatigue needed at least 50 cycles to accommodate. Fatigue analysis was therefore performed on the stable elastic shakedown state.

5. Crack nucleation criterion

Being well adapted for formalizing the multiaxial infinite endurance of the studied steel, Crossland's multiaxial fatigue criterion was used to identify the crack nucleation condition. It expresses crack risk as a linear combination of the square root of the maximum amplitude of the second invariant of the stress deviator defined by $\sqrt{J_{2,a}}$, and the maximum hydrostatic pressure $\sigma_{H,max}$ [27]:
The non-cracking condition is expressed by:

$$\sqrt{J_{2,a}} + \alpha.\sigma_{h\,max} < \tau_d \tag{2}$$

where

$$\sigma_{h\,max} = \max_{t \in T}\left(\frac{1}{3}\mathrm{trace}\left(\underline{\underline{\Sigma}}(t)\right)\right) \tag{3}$$

$$\sqrt{J_{2,a}} = \frac{1}{2}\max_{t_0 \in T}\left\{\max_{t \in T}\left[\frac{1}{2}\left(\underline{\underline{S}}(t) - \underline{\underline{S}}(t_0)\right):\left(\underline{\underline{S}}(t) - \underline{\underline{S}}(t_0)\right)\right]^{1/2}\right\} \tag{4}$$

$$\alpha = \frac{\tau_d - \dfrac{\sigma_d}{\sqrt{3}}}{\dfrac{\sigma_d}{3}} \tag{5}$$

Cracking risk can then be estimated by comparing the equivalent Crossland stress:

$$\sigma_C = \sqrt{J_{2,a}} + \alpha.\sigma_{h\max} \quad (6)$$

with the torsion fatigue limit. When $\sigma_C \geq \tau_d$, there is a risk of cracking.

Fig. 8 plots the subsurface distribution of the Crossland equivalent stress related to the crack nucleation condition defined for the plain fretting crack nucleation condition (b_{th}=0µm). The distribution was characterized by severe stress gradients. As previously underlined, the maximum cracking risk (i.e., hot spot stress) was located on the surface at the contact borders [17]. The computed value was significantly larger than the expected $\sigma_C = \tau_d$ crack nucleation value despite the mesh size surface issue. The real hot spot stresses shoul be even higher.

Using the tools presented above, the various experimental fretting fatigue crack nucleation conditions (b=10µm) extracted at hot spot were compared versus the material boundary in the so called $\sqrt{J_{2,a}} - \sigma_{H,\max}$ diagram (Fig. 9).

The material boundary was expressed for 10^6 cycles through the corresponding fatigue limit σ_d and τ_d at 10^6 cycles. The $\sqrt{J_{2,a}}$ and $\sigma_{H,\max}$ values were extracted at the hot spot at the trailing contact border, which is the zone of maximum cracking risk (Fig. 8). All crack nucleation conditions were well over the material boundary despite the underestimation of surfaces stresses in the simulation. The predictions were over-conservative.
One possible explanation for this was that the crack length used to describe the crack nucleation was not representative. To determine a possible effect of crack length on prediction, the transition from short to long crack was considered as the maximum crack nucleation length. The transition value was calculated with equation 1; for the plane component, this value was b_0=50µm. Fretting fatigue crack nucleation conditions for b_{th}=50µm are also plotted in Fig. 9. They showed a very large discrepancy with respect to the material boundary. Hence, whatever the crack length, hot spot stress was not representative of crack nucleation. This is a well-known result that comes from the stress gradient effect generated by the fretting loading (Fig. 8) [7,16]. In addition there was scattering between the 10µm and 50µm conditions, and thus prediction vary depending on the crack nucleation length considered.

This representation shows that there are two main parameters for accurate prediction of fretting fatigue crack nucleation. The first is the stress gradient effect. To reduce misestimating of hot spot stress, the non-local critical distance approach was considered. It is equivalent to point stress analysis but, instead of considering the surface hot-spot stress at the surface trailing contact border, fatigue analysis was performed at a critical distance below the surface, vertical to the hot spot stress contact border (x=-a, z=ℓ). The second parameter is the crack nucleation

length considered. The scatter shown in Fig. 9 suggests that the size of the crack length defining the crack nucleation condition influences prediction. The question was which initial size b and which critical length ℓ have to be considered? In order to identify which (ℓ_{opt}-b_{opt}) combination needed to be applied, the stability of the critical distance was investigated as a function of crack nucleation length.

6. Stability of crack nucleation prediction : a (ℓ_{opt}-b_{opt}) approach

The first question was to extrapolate a relevant value for ℓ. Previous analysis [28] showed that this value could, for a given $Q^*_{CN}(b_x)$ crack nucleation threshold, be extrapolated from reverse analysis of plain fretting cracking experiments by solving the following expression:

$$\sigma_C(\ell_x) = \tau_d \tag{7}$$

Fig. 10 plots the identification of the critical distance ℓ by reverse analysis of the experimental plain fretting crack nucleation condition ($Q^*_{CN}/(\mu.P)=0.15 \leftrightarrow b_{th}=10\mu m$).

For each loading condition, that is plain fretting and fretting fatigue at R_F={0.6,0.8,0.9,1}, and for each crack length from 0 µm to 100µm, the critical distance was calibrated with the previous method. Table 1 presents the values obtained. It is important to note that all identified critical distance values are bigger than 15µm. So, with a 5µm mesh size, the stresses are extracted at least on the third node and are unaffected by the mesh size as demonstrated in section 4 and Fig. 6b.

Table 1: Critical distance values in microns for each loading and each crack nucleation length calibrated with reverse analysis (10^6 cycles, f=12Hz)

Crack length b	Plain fretting	Fretting fatigue, $\sigma_{F,max}=1.76\times\tau_d$, R_F : 0.6	0.8	0.9	1
0	15.0	26.9	20.5	17.0	16.1
10	17.0	27.1	21.7	20.4	20.7
20	20.0	27.6	22.2	21.4	22.6
30	22.0	28.1	22.8	22.9	23.9
40	25.0	28.0	25.7	26.6	27.4
50	28.0	28.4	26.2	27.3	28.6
60	31.0	28.6	26.4	28.2	29.6
70	33.2	29.2	26.9	28.9	31.2
80	36.0	29.2	27.7	33.1	33.7
100	41.0	30.0	32.3	34.0	35.0

Then, for each possible crack nucleation length b, the mean value of the computed critical distance $\overline{\ell}$ and the related standard deviation $\mathrm{SD}(\ell)$ were over the 5 studied cases, determined using the following expressions:

$$\overline{\ell} = \frac{\sum \ell}{n} \quad (8)$$

$$\mathrm{SD}(\ell) = \sqrt{\frac{\sum(\ell-\overline{\ell})^2}{n}} \quad (9)$$

With n=5 for the following cases: plain fretting ; Fretting Fatigue (FF) with R_F=0.6 ; FF with R_F=0.8 ; FF with R_F=0.9 ; FF with R_F=1 (prestressed).

The evolution of the mean value and the standard deviation are plotted as a function of crack initiation length in Fig. 11a.

The standard deviation showed a remarkable bell shape, with a clear minimum for b=50µm. This suggests that crack nucleation has to be calibrated with respect to a representative incipient crack length for which prediction error is minimal. If the chosen crack length is too small (i.e., b<50µm), the crack length under study is probably not representative of a damaging crack, since such very small cracks may never propagate and therefore are not representative of the conventional fatigue limit. By contrast, if the chosen crack length is too long (i.e., b>50µm), it is probably partly governed by crack propagation rather than pure crack nucleation. Hence there is an optimal (crack initiation length / critical distance) couple (ℓ_{opt}-b_{opt}) for which crack nucleation prediction will be optimal. It is noteworthy that the optimal crack nucleation condition b_{opt} corresponds to the short-to-long crack transition $b_{opt}=b_0$ as expressed by equation 1. In addition, the ratio ℓ_{opt}/b_{opt} equaled 0.56. This result confirms Taylor's theory, but also completes it, suggesting that the optimal crack length used to identify the crack nucleation condition must be equivalent to the b_0 short-to-long crack transition and implying that the ratio with the optimal critical distance is slightly larger than the 0.5 value derived from the model:

$$\ell_{opt}=28\mu m=0.56\times b_0 \quad (10)$$

To confirm this hypothesis, the various experimental fretting fatigue crack nucleation conditions at (ℓ_{opt}=28µm - b_{opt}=50µm) were compared versus the material boundary in the $\sqrt{J_{2,a}} - \sigma_{H,max}$ diagram (Fig. 11b). They showed very good correlation with the material crack nucleation boundary. Moreover, this (ℓ_{opt}-b_{opt}) proposal has many advantages:

- For a given contact configuration (R, P), it is possible to achieve excellent predictions as long as a representative incipient crack length b_{opt} and its corresponding critical distance ℓ_{opt} are considered for crack nucleation prediction.
- There is no need for an extensive experimental campaign to determine the (ℓ_{opt}-b_{opt}) couple, as $b_{opt}=b_0$ and $\ell_{opt}=0.56\times b_{opt}$.

- Since the crack nucleation length corresponds to the short-to-long crack transition, propagation prediction is made easier by not having to consider the short crack propagation regime to predict fretting fatigue endurance.

To examine the reliability of this methodology, it was applied in fretting fatigue endurance experiments.

7. Fretting fatigue prediction

7.1. Identification of crack nucleation endurance

A second set of plain fretting experiments was performed, using the same contact configuration (R=4.6mm, P) to identify the tangential force loading conditions inducing a crack nucleation length of $b_{th}=b_0=50\mu m$ for different test durations (i.e., $Q^*_{CN}(b_0, N)$). The methodology is sketched in Fig. 12. From this analysis, the crack nucleation endurance such that $b_{th}=b_0$ could be expressed using a power law formulation:

$$N_{CN,Q}=A_1\left(\frac{Q^*-Q_\infty}{\mu P}\right)^{-B_1} \text{ with } A_1=6000,\ B_1=0.73 \text{ and } Q_\infty=0.23\times\mu.P \quad (11)$$

Each interface was simulated to extract the corresponding Crossland equivalent stress at the optimum critical distance value $\sigma_{C,PF}(\ell_{opt}=0.56\times b_0,N)$. The obtained values are compiled in Table 2.

Table 2 : Crossland equivalent stress related to tangential loading nucleation condition with crack nucleation life

N_{CN}	$Q^*_{CN}(b_0)/(\mu.P)$	$\sigma_{C,PF}(\ell_{opt})/\tau_d$ (MPa)
20,000	0.423	1.5
50,000	0.285	1.14
100,000	0.251	1.03
500,000	0.232	0.96
1,000,000	0.231	0.95

Fig. 13 plots the equivalent Crossland stresses determined at the critical distance $\sigma_C(\ell_{opt})$ versus the corresponding crack nucleation endurance. Again, a power law formulation can be considered:

$$N_{CN,\sigma}=A_2\left(\frac{\sigma_{C,PF}(\ell_{opt})}{\tau_d}-1\right)^{-B_2} \text{ with } A_2=8.96\times10^3 \text{ and } B_2=0.96 \quad (12)$$

These data are compared to the alternative shear fatigue endurance of the studied steel (Fig. 13).

Although the critical distance approach was calibrated using the on the 10^6 cycles fatigue limits, a very good correlation is observed with the shear endurance curve, at least in the high cycle fatigue domain ($N>5.10^4$ cycles). This result suggests that the (b_{opt}, ℓ_{opt}) approach, considering a constant value $b_{opt}=b_0$ and $\ell_{opt}=0.56\times b_0$, is well adapted for describing the high cycle fatigue fretting crack nucleation response using shear fatigue data, as it satisfies equation 7. There was, however, significant divergence in the low cycle fatigue domain, where the $\sigma_C(\ell_{opt})$ stress ($N<10^4$cycles) increased drastically excluding application of equation 7. This is consistent with the fact that Crossland criterion was developed for high cycle fatigue conditions and is probably not suited to describe low cycle fatigue endurance. Other fatigue criterion, such as the Manson-Coffin, could be considered. In the meantime, a more relevant strategy is to consider the Crossland equivalent stress identified by reverse identification on plain fretting experiments. Hence the following conditions ($b=b_0=50\mu m$) can be considered to formalize the crack nucleation endurance under fretting fatigue loading:

- for $N_{CN} \ll 5.10^4$: $\sigma_C(\ell_{opt},N) = \sigma_{C,PF}(\ell_{opt},N)$
- for $N_{CN} > 5.10^4$: $\sigma_C(\ell_{opt},N) = \tau(N)$ (13)

7.2. Application of the (ℓ_{opt}-b_{opt}) approach to predict lifetime of fretting fatigue test

To validate the above proposal, several fretting fatigue tests were performed until failure. Two test strategies were followed: 4 tests were carried out at constant fretting force Q*/(μ.P)=0.45 but varying fatigue loading, and 5 tests at constant fatigue loading $\sigma_{F,mean}=1.74\times\tau_d$, $R_F=0.6$ but varying tangential fretting loading. The fretting fatigue endurance curves are plotted in Fig. 14 and the related data are reported in Table 3.

Table 3: Fretting fatigue lifetime experiments (finite endurance results)

	Q*/(μ.P)	$\sigma_{F,mean}/\tau_d$	R_F	N_f (cycles)	N_{CN} (cycles)	N_P (cycles)	N_A/N_P	$N_T=N_{CN}+N_P$ (cycles)
Constant fretting force, variable fatigue amplitude	0.45	1.04	0.1	65,500	14,200	83,500	0.17	97,700
	0.45	1.48	0.6	140,600	18,100	130,000	0.14	148,100
	0.45	1.22	0.6	503,100	49,800	234,400	0.21	184,200
	0.45	1.57	0.7	277,700	18,500	169,700	0.11	188,200
Constant fatigue force, variable fretting loading	0.52	1.74	0.6	77,500	19,200	74,100	0.16	93,300
	0.45	1.74	0.6	156,300	20,000	81,800	0.24	101,800
	0.30	1.74	0.6	143,400	25,900	161,500	0.16	187,400
	0.22	1.74	0.6	238,600	33,200	268,800	0.12	302,000
	0.15	1.74	0.6	277,000	62,600	349,500	0.18	412,100

The total lifetime was defined as the sum of fretting fatigue cycles related to the crack nucleation stage $N_{CN}(b=0\rightarrow b_0)$, and the fretting fatigue loading cycles related to the propagation stage $N_P(b=b_0\rightarrow \text{failure})$ so that:

$$N_T=N_{CN}+N_P \tag{14}$$

The number of cycles to crack nucleation N_{CN} was obtained by defining the Crossland equivalent stress at the optimal critical distance $\ell_{opt}=0.56\times b_0$ for each fretting fatigue loading, and then considering the proposal developed in equation 13. The loading cycles related to the propagation stage were computed using Kujawski's $\overline{K}^*$ fatigue crack driving force parameter, integrated from b_0 up to failure [29]. The Paris' propagation law parameter (C,m) cannot be disclosed here, for reasons of confidentiality (equations 15&16).

$$N_P = \int_{b=b_0}^{b_{failure}} \frac{db}{C\overline{K}^{*m}} \tag{15}$$

$$\overline{K}^* = (\Delta K^+)^{\alpha} K_{max}{}^{1-\alpha} \tag{16}$$

with α=0.5, $\Delta K^{+}=\Delta K$ for $R_K \geq 0$ and $\Delta K^{+}=K_{max}$ for $R_K<0$, and $b_{failure}$ for $\overline{K}^{*}=K_{max}=K_{IC}$

The computed N_{CN} and N_P and related N_{CN}/N_P ratio and N_T are gathered in Table 3. Fig. 15 plots predicted lifetime as function of experimental lifetime. The results fell within a scatter band shown by a dashed line of value 2xN. Reasonable predictions were achieved with our method despite the different fretting fatigue loading conditions investigated. This suggests that the proposed method is relevant.

From the N_A/N_P ratios given in Table 3 it can be concluded that total lifetime was mainly governed by propagation stage, as less than 25% of the endurance was related to the nucleation process. This tendency can be explained by the following 2 points:

- The R_F ratios were rather high, leading to slow propagation rates and increased propagation lifetime;
- The loadings were chosen to focus on the low part of high cycle fatigue endurance domain corresponding to rather severe loadings for the studied material. In this way, a fast nucleation process was induced.

Following this critical analysis of the results, the usefulness of optimizing crack nucleation lifetime prediction may be questioned, as lifetime is essentially governed by propagation $N_T \approx N_P$. However, future studies will apply lower stress loading, to increase crack nucleation endurance N_{CN} and clarify the relevance of the proposed crack nucleation model (Fig. 16). A second objective will be to study infinite endurance fretting fatigue conditions (in a dedicated test campaign with unbroken specimens) to establish whether they are governed by the non-nucleating condition or by the crack arrest situation.

8. Conclusion

A combined experimental-modeling analysis was carried out to clarify the influence of incipient crack length on critical distance crack nucleation prediction. A cylinder/plane contact was subjected to plain fretting and fretting fatigue at various high fatigue ratios R_F. This analysis was performed under severe plastic fretting fatigue loadings that were taken into account in the simulations by using an elastic-plastic law. Considering all loading conditions, an optimal (ℓ_{opt}-b_{opt}) condition was identified for which cracking risk prediction error was minimal: b_{opt}=50µm optimal incipient crack length, with a corresponding optimal critical distance $\ell_{opt}=0.56\times b_{opt}$. Too small cracks are not representative of a damaging crack, since it may never propagate, whereas too long cracks are probably partly governed by crack propagation rather than pure crack

nucleation. Under this optimal (ℓ_{opt}-b_{opt}) condition, very good prediction of crack nucleation conditions was achieved.

In addition, b_{opt} was found to be equal to the short-to-long crack transition as defined by the Kitagawa-Takahashi diagram [14].. The optimal critical distance $\ell_{opt}=0.56b_{opt}$ was found to be very close to the estimation of critical distance made by Taylor and co-author that is $\ell=0.5\times b_0$ [10]. Thus our analysis confirmed Taylor's theory but also highlighted that such an approach is satisfied if crack nucleation is defined for a b_0 crack nucleation length.

This observation is consistent with engineering design, as the crack nucleation condition corresponds to the knee short crack arrest condition as defined by the Kitagawa-Takahashi model [14]. Propagation prediction was further simplified by eliminating the need to consider a short crack propagation regime to predict fretting fatigue endurance. The robustness of the proposal was studied via fretting fatigue endurance prediction. Reverse analysis of plain fretting crack nucleation conditions ($Q^*_{CN}(b_{opt},N)$) showed a good correlation with plain fatigue experiments for numbers of cycles greater than 5.10^4. However, divergence appeared for smaller initiation lifetimes. Hence, a specific strategy was proposed. For high cycle fatigue endurance ($N_{CN}>5.10^4$cycles), plain shear fatigue endurance can be considered, while for low cycle fatigue endurance ($N_{CN}>5.10^4$cycles), crack nucleation endurance derived from reverse analysis of plain fretting experiments must be considered to predict crack nucleation time accurately. Finally, this strategy was applied to fretting fatigue lifetime experiments. Good predictions were achieved, validating the approach. In addition, it was shown that using fretting experiments to calibrate data, such as the (ℓ_{opt}-b_{opt}) and crack nucleation conditions, enabled successful prediction of fatigue lifetime. However, the studied conditions were representative only of low cycle fatigue crack nucleation. Future work needs to be done in the high cycle fatigue crack nucleation domain to clarify the relevance of the present proposal. Besides, the analysis was carried out with one geometry of fretting pad, keeping constant the normal loading. Current research comparing various geometries and contact sizes is underway to evaluate the stability of the proposed (ℓ_{opt}-b_{opt}) approach in predicting fretting crack nucleation.

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