Urban and Regional Dynamics, towards an integrated approach

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Urban and regional dynamics—towards an integrated approach†

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Abstract. The generic structure of an integrated approach for urban and regional dynamics is presented. Interurban and intraurban interactions of agents are considered. The decision strategies of the different groups' agents on the microlevel, given certain constraints, constitute the fundamental starting point of the analysis. The master equation formalism provides the framework for the derivation of an appropriate set of nonlinear dynamic equations on the macrolevel, describing the state of the spatial system. Phenomena of self-organisation and of bifurcation may occur in such open systems when they are maintained under conditions of an active environment. It is also outlined how the trend parameters of the system can be estimated by using empirical data.

1 Introduction

Interdisciplinary research in recent years has encouraged new developments in model building and empirical testing, especially in the field of regional sciences.

The dynamics of city growth or decline influence the social and political structure of the society (Pumain et al, 1989). One aim in the designing of dynamic models is to understand and anticipate the evolution of sociospatial systems (Dendrinos and Mullaly, 1985; Weidlich and Haag, 1983; Wilson, 1971). New dynamic models mainly draw upon analogies from the physical sciences, for instance from chemical kinetics (Allen, 1978) or laser theory (Haag and Weidlich, 1984). Recently, the main broad approaches have been reviewed in order to assemble larger components to give an integrated urban model (for details, see Anas, 1990; Andersson et al, 1989; Bertuglia et al, 1990). Particular emphasis is given in these models to supply-side structures and to the way in which these patterns evolve and change. For a concise review of the literature on this issue see Griffith and Lea (1983), Griffith and Haining (1986), Batten et al (1985).

The evolution of urban and regional systems reflect the nested structure of the social and economic processes of a society. Conditions and problems related to the development of an integrated approach are discussed. The huge set of variables necessary to describe the economic and social situation of a spatial system demonstrates the difficult task of an appropriate way to model integrated sociospatial systems. Therefore, it seems to be appropriate, on the one hand, to introduce a subset of agglomerated macrovariables (for example, via the definition of an adequate basket) which in turn represents essential parts of the spatial system and is, on the other hand, accessible for modelling purposes.

The systems under study are described on at least two levels: system wide (macroscopic variables) and elementary (microscopic level). There is a very large

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number of elementary units (for example, households, workers, firms) in the system. Sometimes a third level of description is added, with subsystems comprising unfixed numbers of elementary units. Uncertainties (fluctuations) in the decision process on the microlevel play an important role.

The passage from the microscopic level to the macroscopic description of the system is the most arduous problem in such model building. It leads to stochastic equations when the interactions are not fully deterministic. Differential equations describing the evolution of the most probable state of the system—this is an appropriate macroscopic variable—are obtained via summation over the corresponding time-dependent probability distribution.

Phenomena of self-organisation and of bifurcation may occur in such open systems (Haken, 1974; Weidlich and Haag, 1983; Wilson, 1981) when they are maintained under the influence of an active environment (Sonis, 1981). Those systems may organise themselves in structures which are created or destroyed in the course of time. Phase transitions, for example, migratory phase transitions (Weidlich and Haag, 1987) may occur in periods of instability connected with a global change of the systems structure, by the amplification of small fluctuations (fluctuation enhancement). The equations of the model may therefore admit several stationary solutions or multiple equilibria, and it depends on the history and the evolution of the environment as to which of those solutions will be approached. The several possible trajectories described by the equations of motion corresponding to qualitatively different structures of the system may be driven towards one branch or another (that is, towards a given form of spatiotemporal organisation) depending on the current state of the system and its fluctuations.

Many sources of instability intervene in the evolution of such open systems when they are situated far from equilibrium. On the one hand, they continually undergo internal fluctuations, that is, variations in the level of their characteristic variables (which may result from changes in the microstate of the elementary units of the system); on the other hand, they are always subject to external perturbations stemming from the active environment. An open system is then continually adjusting the level (mean values) of its variables or the size of its subsystems. It maintains a relatively structural persistence only when this structure constitutes a quasi-stable state, at least for a certain period; in other words, a state towards which the system comes back after having gone slightly apart from it because of fluctuations. The structure is then viewed as an attractor on the trajectory of the system. A dynamic instability may induce a passage from one attractor to another, from one structure to another structure, that is, from one qualitative behaviour of the system to another, through a bifurcation or a series of bifurcations.

This kind of model structure provides our starting point towards an integrated approach for a dynamic system of cities comprising an agglomerated set of socioeconomic variables. We start from the microeconomic decision process of individual agents and add socioeconomic mechanisms to our probabilistic picture. It is a 'unified' model, as one algorithm is used to model all subsystems (Wegener, 1984).

2 The generic structure of model building

In this framework one has to define a geographical object as a set of elements which are either located elements or geographical zones. Location, at least relative location, must appear as a basic property of the elements of the system. The interactions between those elementary units must be at least partly spatial interactions, which are linked to the absolute (site) or relative (situation) location of the elements. The state of the system is defined as the geographical configuration of

its characteristic variables. A geographical structure is then given by a particular relative size and evolution for the state variables and/or located subsystems which are used for the description of the geographical system.

At the most disaggregate level, a spatial system can be formalised as a set of localised and interacting actors (individuals or groups such as persons, households, firms, associations, etc) who are using and continuously recreating geographical differences and spatial configurations. The interactions are different in nature and are responsible for competition for space, propensity to agglomerate, segregative tendencies, imitation effects, cooperation between different elementary units, and so on. Therefore, urban dynamics can be considered as the result of a broad stream of concurrent, unrelated or interrelated, individual or corporate, choices of human individuals (Wegener, 1984). Uncertainties in the decision process (fluctuations) disturb the evolution path of the system of cities and may trigger the system into another phase if it is near a critical point (Weidlich and Haag, 1987). Therefore, the study of spatial organisation is gifted with the theory of self-organising systems with all the consequences that implies (see Mela et al, 1987). Such a formalisation has many appealing features for geographers:

1 It allows stress to be placed upon the linkages between the individual behaviour of the actors and interactions where the realised configuration (macrolevel) of the system may have an impact on the individual decisions. Much research has been conducted and empirical regularities have been established for each scale of study separately, though a clear connection between the two levels of observation is not always established. Here, self-organisation phenomena, mesoscale or macroscale structures, are described as consequences of an interaction game between individuals, each animated by their own objectives. These consequences are not always intuitive to the observer, they are often not concerted, and most of the time are not perceived by the actors themselves.

2 This approach may also provide an interpretative framework for observed regularities in spatial systems. Temporal series appear then as possible sequences of complex dynamics, and very often the problem for social sciences, where experimentation is impossible, consists in identifying the dynamics which produce a particular series of observable structures; that is, a specific trajectory (Prigogine and Stengers, 1979).

3 Another interesting feature of this approach is that the historical dimension of social systems is taken into account via the concept of irreversibility. On the one hand, the explanation of the state of the system at a given date integrates its previous trajectory (its history), and the contemporary structure is the result of a sequence of previous bifurcations. On the other hand, the characteristic fluctuations of variables of dynamic systems imply that it is impossible to prepare initial conditions which would lead to identical features. The impossibility of exact predictions is then given as a theoretical 'a priori'. However, the analysis of the dynamic behaviour of the system, and of its sensitivity to variations of its parameters, allows the exploration of a limited number of possible futures, in accordance with the assumptions made about the evolution of the parameters (Allen and Sanglier, 1979; 1981).

These ideas may appear seductive to geographers (Pumain, 1989), but it is not because of their novelty—rather it is because they allow the relaxation of some of the oppressive restrictions imposed by the previous methods of model building, and because new 'experimental' tools related to these ideas are available. The present model is an extended application of some work described in Haag (1989b).

3 Problems and questions in urban and regional modelling

It has been shown that, when nonlinear equations and/or interdependencies between variables are used, even very simple mathematical models can produce very complicated dynamics (Dendrinos and Mullaly, 1981; May, 1976). Some of the models which have been proposed are too complex to be analytically tractable and must be solved and calibrated by means of simulation (Allen and Sanglier, 1979). Another potential method of classifying dynamic models of spatial structures is therefore to consider how they choose between the advantages of complexity (allowing possibly more realistic representations and also more complete descriptions of spatial features) and the gain in generality and soundness of results offered by the existence of more or less well-known analytical solutions.

Large urban models have suffered from many criticisms. Nowadays, however, stress is placed more upon qualitative features of spatial urban development than upon absolute prediction of growth. The ability of urban models to simulate such qualitative changes is one of their characteristics which give them more realistic chances of success. However, a valuable way to improve the design of future models is to consider the main difficulties encountered in their application. Problems are of a diverse nature, being either conceptual or practical.

3.1 *Conceptual problems*

The objections of such aplications are usually threefold: first, one must check that the model gives a good description of the kind of system under study; second, one needs to test that the model is able to replicate an observed evolution of a particular system; third, one must assess whether the model may under given assumptions lead to plausible predicted trajectories for the future of the system. The imprecision which may arise when borrowing concepts from other fields have then to be clarified. For instance, when taken theoretically, the use of the bifurcation concept looks rather simple: one slight change in the value of a parameter can drive the system on another trajectory representing a different kind of spatial configuration; that is, a structural change can occur. However, when using those concepts for practical purposes, we face the reverse problem. Observing a qualitative change in spatial configuration we need to assess if it corresponds to a bifurcation. Would one slight modification in one parameter value been sufficient to avoid it, or to alter it, in this region of instability, or is this change more deeply tied to the functioning of the system? In the second case, it would be naturally produced by the same trajectory defined by the nonlinear equations and the corresponding set of values, for the parameters would have to be estimated. So, observing the evolution of an urban structure, we face the question of discovering a plausible dynamic path which may have generated it. But to what extent is the model reliable enough to support predictions?

Data problems are not specific to this kind of model building but are of decisive importance in the quality of applications. The scarcity of localised temporal statistical series at a detailed geographical level is well-known, even for aggregate variables. For models needing information at a microlevel, the lack of data is even worse, as surveys establishing conditional probabilities in spatial behaviour are still far too rare.

3.2 *Interpretation problems*

In most of the applications discussed above (for instance, Pumain et al, 1989) it appears that the models were able to reproduce a large proportion of the wide variety of changes observed in most of the spatial units. The largest residuals between simulated and observed evolution are assumed to reflect specific local conditions which were not taken into account by the model. In other words,

the general mechanisms incorporated into the model seem to describe rather well the evolution of urban structure. However, a problem remains with the interpretation of the values calibrated for the parameters. The estimated values are not independent of the number of spatial divisions being considered, of the size of the zones, or of the measurement unit of the variables. As a consequence, it is almost impossible to arrive at a standardisation of the parameters. In addition, it is important to remember that the parameter values resulting from calibration reflect the joint effects of both the actual dynamics of the system and the way it has been disaggregated. When constructing or applying these types of dynamic models, special attention must therefore be given to the disaggregation problem: the design of spatial units, the selection of the state variables at the microlevel and macrolevel, according to the range of their spatial action, and also to the time scale of their intervention (that is, whether they are fast or slow dynamics).

4 A stochastic theory of urban and regional dynamics

We focus our interest now on the generic structure of dynamic models which are used to try to explain the evolution of spatial pattern at a mesolevel or macrolevel. The interactions between the elementary units of the system (the different agents) are introduced via appropriate transition rates which might depend on the distribution pattern of the elementary units of the system (Haag, 1989a). Therefore self-accelerating processes and saturation effects are included and the elementary decision process of an agent may depend on the psychological and social as well as on the economic situation.

If the individual decisions are stochastic, the evolution of the urban system composed of interacting individuals cannot be fully deterministic either. Instead the system must be described by an equation of motion for the evolution of a probability distribution over its possible states.

We introduce the dynamics in three steps: first, we define individual transition rates (decision rates), considering the whole complex distribution pattern of all individuals; second, we formulate the dynamic equations of motion for the probability to find a certain population distribution, third, we derive quasi-deterministic equations for the population numbers from the stochastic system.

We consider different groups of agents P_α ($\alpha = 1, 2, \ldots, A$) in the urban system. The individual agent is denoted by $A_\alpha^{(i)}$ ($i = 1, 2, \ldots, I_\alpha$), where I_α is the number of individual agents of subpopulation α.

The decision strategy of the individual agent $A_\alpha^{(i)}$ with respect to the choice set $x_\alpha^{(i)} = \{x_{\alpha 1}, x_{\alpha 2}, \ldots, x_{\alpha j}, \ldots, x_{\alpha L}\}^{(i)}$, taking into account certain constraints $N_\alpha^{(i)} = 0$, constitutes the fundamental starting point of our analysis. L denotes the number of mutually exclusive alternatives. Each individual agent tends to choose the option which is considered most desirable to him or her, given the attributes of each alternative as seen by the agent and his preferences. More precisely, it is assumed that an agent $A_\alpha^{(i)}$ measures the desirability of each alternative $x_\alpha^{(i)}$ by a function $Z_\alpha^{(i)}(x)$.

The corresponding Lagrangian function, $\mathscr{L}_\alpha^{(i)}$, or the derivation of the optimal solution, reads

$$\mathscr{L}_\alpha^{(i)} = Z_\alpha^{(i)}(x) + \lambda^{(i)} N_\alpha^{(i)}(x) , \tag{1}$$

where the first-order conditions

$$\frac{\partial \mathscr{L}_\alpha^{(i)}(x)}{\partial x_{\alpha j}} = 0 , \tag{2}$$

are necessary for an optimum. In other words, the agent $A_\alpha^{(i)}$ has reached her or

his optimal choice distribution if the *dynamic advantage function*,

$$E_{aj}^{(i)}(x) = \frac{\partial Z_a^{(i)}}{\partial x_{aj}^{(i)}} \Big/ \frac{\partial N_a^{(i)}}{\partial x_{aj}^{(i)}} , \tag{3}$$

is constant; that is, if

$$E_{aj}^{(i)}(x) = -\lambda_a^{(i)} = \text{constant} . \tag{4}$$

Assume that the transition rate $p_{kj}^{a}(x)$ of an individual i belonging to subpopulation a between two different alternatives, $x_{aj}^{(i)}$ and $x_{ak}^{(i)}$, depends on differences of the dynamic advantage functions (Haag, 1989b). The most reasonable form reads:

$$p_{kj}^{a(i)}(x) = \nu_{kj}^{a} \exp[E_{ak}^{(i)} - E_{aj}^{(i)}] , \tag{5}$$

where the symmetric time-scaling parameter $\nu_{kj}^{a} > 0$ denotes the flexibility of an individual to adopt different choices. In spite of the neoclassical optimisation procedures we admit transitions j to k, even if equation (4) is fulfilled. Obviously decisions towards an optimal choice distribution are reinforced against less optimal choices. In table 1 a few agents of the integrated urban system are mentioned.

The different agents in table 1 belong to the main subsystems underpinning an integrated model (Bertuglia et al, 1990); that is, housing market, services, land market, labour market, and transport, to mention a few.

In terms of dynamic modelling style, it is reasonable to neglect the dynamics of very fast processes (adiabatic elimination procedure; see Haken, 1974), which reach a steady state in a matter of a few days (like service demand choice) or even of a few hours (like route choice in a transport network) and to focus our attention on the medium-term time scale.

Actual computation can be done both on discrete and continuous time. It depends on the structure of the underlying decision process and the data base as to which of both procedures seem to be useful.

We shall now assume that the total urban population of individuals $N(t)$ consists of a $(= 1, \ldots, A)$ subpopulations, P_a, differing in their migratory (decision) behaviour. The urban system is divided into i $(= 1, 2, \ldots, L)$ cities and the hinterland, h. The *population configuration* is given by

$$n = \{n_{11}, \ldots, n_{ai}, \ldots, n_{AL}, n_{1h}, \ldots, n_{Ah}\} , \tag{6}$$

Table 1. Examples of the different agents of the urban system.

Agent	Decisions subject	Decision strategy	Controlled variable
$A_a^{(i)}$	$x_a^{(i)} = \{x_{aj}^{(i)}\}$	$\max_x Z_a^{(i)}(x)$, for $N_a^{(i)}(x) = 0$	Distribution x
Worker in industry	Place of work	Utility optimisation	Number of employees
Worker in tertiary sector	Place of work	Utility optimisation	Number of employees
Firms	Labour force	Profit optimisation	Stock
Firms	Production	Profit optimisation	Supply
Firms	Trade	Profit optimisation	Supply distribution
Retailers or firms	Prices of goods and services	Profit optimisation	Price level
Households	Place of residence	Utility optimisation	Housing demand
Household	Demand of goods and services	Utility optimisation	Demand
Landlords	Housing market	Profit optimisation	Housing stock
Landlords	Rents	Profit optimisation	Rent level

where $n_{\alpha i}$ is the number of individuals of subpopulation P_α living in city $i\,(=1,2,\ldots,L)$, $n_{\alpha h}$ is the subpopulation of the hinterland h. Hence the total number N_α of the urban subpopulation P_α is given by

$$\sum_{i=1}^{L} n_{\alpha i} = N_\alpha \,, \tag{7}$$

and the total number of individuals of the urban system is given by

$$\sum_{\alpha=1}^{A} N_\alpha = N \,. \tag{8}$$

However, there exists a set of 'material' variables, for example, housing stock, prices of certain commodities, commodities available. The available stock (or value, if it is an intensive variable) of the material variable $\gamma(\gamma = 1,2,\ldots,G)$ in city $i\,(=1,2,\ldots,L)$ is denoted by $m_{\gamma i}$. The *material configuration* is given by

$$m = \{m_{11},\ldots,m_{\gamma i},\ldots,m_{GL}\} \,. \tag{9}$$

In case of extensive variables the total stock M_γ of the (extensive) material variable is

$$\sum_{i=1}^{L} m_{\gamma i} = M_\gamma \,. \tag{10}$$

Of course, intensive variables must be treated in a different manner. The *socioconfiguration* (Weidlich and Haag, 1983) is given by

$$c = \{n, m\} = \{c_{\beta k}\} \,, \tag{11}$$

where $\beta = 1,2,\ldots,c\,(=A+G)$; $k\,(=1,2,\ldots,L)$ describes the spatial distribution of individuals of the different subpopulations and the different material variables of the system. We seek to understand the dynamics of $c(t)$.

4.1 *Interurban interaction of subpopulations*

According to equation (8), we introduce individual transition rates, $p_{ij}^{\alpha}(n, m)$, for the transition of one member of subpopulation α from city j to city i per time unit:

$$p_{ij}^{\alpha}(n, m) = \nu_{ij}^{\alpha}(t)\exp[u_i^{\alpha}(n, m) - u_j^{\alpha}(n, m)] \,, \tag{12}$$

composed of a symmetric mobility factor, ν_{ij}^{α} equal to ν_{ji}^{α}, and a push–pull factor depending on the utility of the origin region, $u_j^{\alpha}(n, m)$, and the destination region, $u_j^{\alpha}(n, m)$. Here it is assumed that the dynamic advantage function with respect to migration in the urban system can be represented by *dynamic regional utilities* (see Weidlich and Haag, 1988), or in other words we assume $E_{\alpha j}^{(i)} \to u_j^{\alpha}(n, m)$. A plausible form of the regional utilities reads

$$u_j^{\alpha}(n, m) = \delta_j^{\alpha}(m) + \sum_{\beta=1}^{A} \varkappa^{\alpha\beta}(m) n_{\beta j} + \sum_{\beta,\gamma=1}^{A} \varkappa^{\alpha\beta\gamma}(m) n_{\beta j} n_{\gamma j} + \mathrm{O}(n^3) \,, \tag{13}$$

where we have written the u_j^{α} as a truncated Taylor series [where $\mathrm{O}(n^3)$ is used for all contributions of higher order than 2]. Obviously, we have the following interpretation: the δ_j^{α} describe the size-independent preference of city i to members of subpopulation P_α (for example, income per capita, environment of the city, and so on). The intragroup interaction parameter, $\varkappa^{\alpha\alpha}$, is utilised to describe the agglomeration (clustering) trend of subpopulation P_α (for $\varkappa^{\alpha\alpha} > 0$) in the same city; whereas via the inequality $\varkappa^{\alpha\alpha\alpha} < 0$, saturation effects are taken into account. The intergroup interaction is represented by $\varkappa^{\alpha\beta}$ (for $\alpha \neq \beta$) and $\varkappa^{\alpha\beta\gamma}(\varkappa^{\alpha\beta\gamma} = \varkappa^{\alpha\gamma\beta})$. If $\varkappa^{\alpha\beta} > 0$, subpopulation P_α prefers the neighbourhood of subpopulation P_β, whereas $\varkappa^{\alpha\beta} < 0$ describes antagonistic effects. Self-reinforcing mechanisms on the

different subsystems are included in our dynamic picture via the cyclic coupling between causes and effects (Arthur, 1988).

We scale the utilities as follows:

$$\sum_{i=1}^{L} u_i^{\alpha}(n, m;\ t) = 0\ . \tag{14}$$

For further discussion we introduce the *regional variance* of the utilities:

$$\sigma_{\alpha}^2(t) = \frac{1}{L} \sum_{i=1}^{L} u_i^{\alpha}(n, m)^2\ , \tag{15}$$

which can be seen as a measure of the inhomogeneity of the urban system. Furthermore, it is possible to define a global mobility, $\nu_O^{\alpha}(t)$, characterising the mean mobility of the subpopulation α under consideration:

$$\nu_O^{\alpha}(t) = \frac{1}{L(L-1)} \sum_{i,j}^{L} \nu_{ij}^{\alpha}(t)\ . \tag{16}$$

However, a multitude of socioeconomic factors (for example, housing conditions, labour market, services available, income) merges into the utilities $u_j^{\alpha}(n)$ and the mobility matrix ν_{ij}^{α}. In particular, the effect of *distance* (in its most general meaning) will be reflected by the mobility matrix $\nu_{ij}^{\alpha}(t)$, which can be split into two factors (Weidlich and Haag, 1988):

$$\nu_{ij}^{\alpha}(t) = \nu_O^{\alpha}(t) f_{ij}^{\alpha}\ , \tag{17}$$

that is, into a time-independent *deterrence factor* f_{ij}^{α} $(= f_{ji}^{\alpha})$ describing the effects of space (distance) and a time-dependent global mobility $\nu_O^{\alpha}(t)$. It follows from equations (16) and (17) that the f_{ij}^{α} must fulfill a normalisation condition.

An *effective distance*, D_{ij}^{α} $(= D_{ji}^{\alpha} > 0)$ comprising all geographic, economic, and social distance effects may then be defined by:

$$\exp(-D_{ij}^{\alpha}) = \frac{1}{L(L-1)} f_{ij}^{\alpha}\ . \tag{18}$$

It can be shown that only for short-distance moves (less than 80 km) can a linear relation between the effective distance D_{ij}^{α} and the geographical distance d_{ij} be assumed. Long-distance moves, however, are characterised by a saturation of f_{ij}^{α}. The following hypothesis has proved its flexibility:

$$D_{ij}^{\alpha} \approx \frac{\beta^{\alpha} d_{ij}}{1 + \gamma^{\alpha} d_{ij}}\ . \tag{19}$$

Each of the $n_{\alpha j}$ members of subpopulation α of city j changes to city i with an individual transition rate $p_{ij}^{\alpha}(n, m)$ and thus gives rise to the population configuration transition rate $w_{ij}^{\alpha}(c)$ on the macrolevel as follows:

$$w_{ij}^{\alpha}(c) = n_{\alpha j} p_{ij}^{\alpha}(c)\ . \tag{20}$$

However, the $p_{ij}^{\alpha}(c)$ are here not individual transition rates in the sense that different transition rates must be attributed to different individuals. Instead we suppose that the different agents of the system (the individuals) belonging to the same ensemble P_{α} operate according to the same probability transition rate (representative agent). If panel data are available on the decision behaviour of a group of agents we are able to substitute the assumption (20) by direct computation of the configurational transition rates via (see Haag, 1989b)

$$w_{ij}^{\alpha}(c) = \sum_{i \in Q_j} p_{ij}^{\alpha(i)}(c)\ , \tag{21}$$

where we have to sum up over all individual contributions, $p_{ij}^{\alpha(i)}(c)$, of all agents $i(\in Q_j)$ belonging to subpopulation P_α and changing from state j to state i of behaviour per time unit. Because the transitions between all regions take place simultaneously, the total configurational transition rate is the sum of all contributions (20):

$$w_i^\alpha(c) = \sum_{\alpha=1}^{A} \sum_{i,j=1}^{L} w_{ij}^\alpha(c) . \tag{22}$$

4.2 *Intraurban interaction and birth–death processes*

Intraurban interaction of subpopulations can be treated as birth–death processes. When we assume (see Allen, 1983; Barentsen and Nijkamp, 1988) that limits of growth exist for each subpopulation P_α of each city j—or, in other words, if we assume there is a limited capacity to accommodate each agent in his or her new choice situation and that the ability to receive new members of subpopulation P_α depends on the number of agents having already opted for α—the following nonlinear configurational birth–death rates seem to be appropriate:

$$\begin{aligned} w_{i+}^\alpha(c) &= a_{i+}^\alpha c_{\alpha i} + b_{i+}^\alpha(c) c_{\alpha i}^2 , \\ w_{i-}^\alpha(n) &= a_{i-}^\alpha c_{\alpha i} + b_{i-}^\alpha(c) c_{\alpha i}^2 , \end{aligned} \tag{23}$$

with the inequalities $a_{i+}^\alpha > a_{i-}^\alpha > 0$ and $b_{i-}^\alpha > b_{i+}^\alpha > 0$ and the equalities $c_{\alpha i} = n_{\alpha i} = m_{\alpha i}$, respectively. (In order to assign the parameters to the birth and death rates, the symbols + and − are used. For example a_{i+}^α is the parameter in front of the linear term in the birth rate belonging to city i and subpopulation α.)

The city hinterland interaction, however, is rather complicated. The description of the hinterland by one level of attractivity is therefore questionable and must be empirically tested. We avoid this difficulty by considering the city hinterland interaction and the emigration and immigration rates to be birth–death events. The corresponding transition rates are denoted by W_{ih}^α and W_{hi}^α for transitions from h to i and from i to h, respectively.

4.3 *The stochastic level of description*

On a stochastic level, we consider the equations of motion for the probability $\wp(c, t) = \wp(c_{11}, c_{12}, \ldots, c_{CL}, t)$ to find a certain population configuration c realised at time t. This *configurational probability* has to fulfill the normalisation condition

$$\sum_c \wp(c, t) = 1 , \tag{24}$$

where the sum extends over all possible configurations. If migratory transitions (flows) as well as birth and death processes are taken into account, the configurational probability satisfies the following master equation:

$$\begin{aligned} \frac{\mathrm{d}\wp(c,t)}{\mathrm{d}t} = {} & \sum_\alpha^C \sum_{i,j}^L w_{ji}(c^{(ij)})\wp(c^{(ij)}, t) - \sum_\alpha^C \sum_{i,j}^L w_{ji}(c)\wp(c,t) + \sum_\alpha^C \sum_{i=1}^L w_{i+}(c^{(i+)})\wp(c^{(i+)}, t) \\ & - \sum_\alpha^C \sum_{i=1}^L w_{i+}(c)\wp(c,t) + \sum_\alpha^C \sum_{i=1}^L w_{i-}(c^{(i-)})\wp(c^{(i-)}, t) \\ & - \sum_\alpha^C \sum_{i=1}^L w_{i-}(c)\wp(c,t) + \sum_\alpha^C \sum_{i=1}^L W_{ih}\wp(c^{(i+)}, t) - \sum_\alpha^C \sum_{i=1}^L W_{ih}\wp(c,t) \\ & + \sum_\alpha^C \sum_{i=1}^L W_{hi}\wp(c^{(i-)}, t) - \sum_\alpha^C \sum_{i=1}^L W_{hi}\wp(c,t) . \end{aligned} \tag{25}$$

For details of the master equation formalism and the stationary solution $\wp_{st}(c)$ of the migratory part of the master equation (25), which corresponds to the urban system at equilibrium, see Haag (1989a).

The maxima and minima $\{\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_L\}$ of $\wp_{st}(c)$ describe the equilibrium configuration (configurations), or, in other words, the most probable stationary urban pattern.

If the distribution is unimodal the equilibrium is unique. However, if more than one solution exists, the distribution is multimodal with peaks corresponding to different possible equilibrium states of the urban system. The urban pattern would approach this virtual equilibrium solution if from this time on we were to keep the trend parameters constant.

5 Deterministic equations for interacting urban units

From equation (25), deterministic equations of motion can be derived. This nonlinear set of quasi-closed equations is sufficient for comparison with empiric data and can be used for simulation as well as for forecasting purposes. They are as follows:

$$\left.\begin{aligned}
\frac{\mathrm{d}\bar{n}_{\alpha k}}{\mathrm{d}t} &= \sum_{i=1}^{L} \bar{n}_{\alpha i}\, \nu_{ki}^{\alpha} \exp(u_k^{\alpha} - u_i^{\alpha}) - \sum_{i=1}^{L} \bar{n}_{\alpha k}\, \nu_{ik}^{\alpha} \exp(u_i^{\alpha} - u_k^{\alpha}) \\
&\quad + \overline{W}_{kh}^{(1)\alpha} - \overline{W}_{hk}^{(1)\alpha} + \varepsilon_k^{(1)\alpha}\, \bar{n}_{\alpha k}\, \frac{1 - \bar{n}_{\alpha k}}{C_{\alpha k}^{(1)}(\bar{n}, \bar{m})}, \\
\frac{\mathrm{d}\bar{m}_{\gamma k}}{\mathrm{d}t} &= \sum_{i=1}^{L} \bar{m}_{\gamma i}\, \nu_{ki}^{\gamma} \exp(E_k^{\gamma} - E_i^{\gamma}) - \sum_{i=1}^{L} \bar{m}_{\gamma k}\, \nu_{ik}^{\gamma} \exp(E_i^{\gamma} - E_k^{\gamma}) \\
&\quad + \overline{W}_{k+}^{(2)\gamma} - \overline{W}_{k-}^{(2)\gamma} + \varepsilon_k^{(2)\gamma}\, \bar{m}_{\gamma k}\, \frac{1 - \bar{m}_{\gamma k}}{C_{\gamma k}^{(2)}(\bar{n}, \bar{m})}.
\end{aligned}\right\} \tag{26}$$

where $k = 1, 2, \ldots, L$; $\alpha = 1, 2, \ldots, A$; and $\gamma = 1, 2, \ldots, G$; and where $\varepsilon_k^{(r)\alpha} = (a_{k+}^{(r)\alpha} - a_{k-}^{(r)\alpha}) > 0$ and $C_{\alpha k}^{(r)}(\bar{n}, \bar{m}) = (a_{k+}^{(r)\alpha} - a_{k-}^{(r)\alpha})/(b_{k-}^{(r)\alpha} - b_{k+}^{(r)\alpha})$ describe the rate of increase and the saturation level of population ($r = 1$), and material variables ($r = 2$), respectively [a and b are the parameters defined in equation (23)]. The $\overline{W}_{kh}^{(1)\alpha}$, $\overline{W}_{hk}^{(1)\alpha}$, denote city-hinterland interactions. The $\overline{W}_{k+}^{(2)}$ and $\overline{W}_{k-}^{(2)}$ describe export-import activities as well as the introduction of new products and the stopping of production. Spatial innovation diffusion can be treated within the same framework (Andersson, 1981; Fischer, 1987; Mensch, 1979; Sonis, 1981).

Evidently, equation (26) is a set of $(A+G)L$ coupled, nonlinear, first-order differential equations for the mean values of the spatial distribution pattern (subpopulations, material values). If saturation effects can be neglected it is easy to separate the birth-death processes from the migratory redistribution processes. Via computer simulation the arisal, competition, and self-organisation of urban structure can be simulated. However, this requires explicit values of the trend parameters in equation (26). In the next section, an appropriate estimation procedure will be briefly outlined.

6 Estimation of trend parameters

The trend parameters (ν_{kj}^{α} and $E_{\alpha k}$) in the transition rates $p_{kj}^{\alpha}(x)$ have to be estimated from empirical data. If the empirical (index e) data set $\{p_{kj}^{e\alpha}(t)\}$ is available for a sequence of t ($= 1, 2, \ldots, T$) years we can match the transition matrix p_{kj}^{α} to the empirical 'individual' flows $p_{kj}^{e\alpha} = w_{kj}^{e\alpha}(t)/c_{aj}^{e}$, by an optimal choice of the 'mobilities' $\nu_{kj}^{\alpha}(t)$ and of the 'dynamic advantage functions' $E_{\alpha k}$ ($k, j = 1, 2, \ldots, L$; $\alpha = 1, 2, \ldots, C$).

When using a log-linear estimation we obtain, by minimizing $F_\alpha(\nu, E)$:

$$F_\alpha(\nu, E) = \sum_{t=1}^{T} \sum_{k,l=1}^{L} \left\{ \ln p_{kl}^{e\alpha}(t) - \ln\{\nu_{kl}^{\alpha}(t) \exp[E_{\alpha k}(t) - E_{\alpha l}(t)]\} \right\}^2 , \tag{27}$$

for the decision functions

$$E_{\alpha i}(t) = \frac{1}{2L} \sum_{k=1}^{L} \ln \left[\frac{p_{ik}^{e\alpha}(t)}{p_{ki}^{e\alpha}(t)} \right], \qquad \text{for } i = 1, 2, \ldots, L, \text{ and } t = 1, 2, \ldots, T, \tag{28}$$

and for the flexibility matrix

$$\nu_{kj}^{\alpha}(t) = \nu_{jk}^{\alpha}(t) = [p_{kj}^{e\alpha}(t) p_{jk}^{e\alpha}(t)]^{1/2} > 0 . \tag{29}$$

The global 'mobility' reads

$$\nu_O^{\alpha}(t) = \prod_{\substack{k,l \\ k \neq l}}^{L} \left(\frac{\nu_{kl}^{\alpha}(t)}{f_{kl}^{\alpha}} \right)^{1/L(L-1)} , \tag{30}$$

with the deterrence factor

$$f_{kj}^{\alpha} = C^{\alpha} \prod_{t=1}^{T} \nu_{kj}^{\alpha}(t)^{1/2T} , \tag{31}$$

and where C is to be determined via definition (16).

It should be mentioned that instead of the log-linear estimation a nonlinear estimation can be implemented based on the minimisation of the expression

$$F_\alpha[\nu, E] = \sum_{t=1}^{T} \sum_{k,l=1}^{L} \{p_{kl}^{e\alpha}(t) - \nu_{kl}^{\alpha}(t) \exp[E_{\alpha k}(t) - E_{\alpha l}(t)]\}^2 . \tag{32}$$

The results of this optimisation are slightly better, but cannot be written in analytical form.

7 Conclusion

Via computer simulation, the arisal, competition, and self-organisation of urban structure can be simulated. Of course, this can lead only to a certain level of explanation. It can throw light on systems where the dynamics of elementary spatial interactions produces a morphogenesis; a spatial structure as observed at an upper level, the properties of which were not intended by the elements. For example, the intention of the individual actors are not from the outset to produce an urban system, with a given spatial organisation; but when such a system exists they use it and may try to improve some of its properties.

Various large metropolitan areas tend to lose their innovative potential in favour of medium-sized cities. A first step towards a disentangled consideration of the urban subsystem has been taken by Pumain and Saint-Julien (1984) via the introduction of compound macrovariables describing the stage of a city (*image de marque, modernité technique*). As a consequence, the number of 'material' variables in equation (26) can be considerably reduced. However, the testing and formulation of the interaction of those compound variables with the population dynamics as well as the derivation of appropriate equations of motion for those variables is still difficult and requires further empirical and theoretical work.

Urban economic theory is dealing with cities as centres of production of goods and services. In sociology and psychology, cities are treated as centres of cultural and social interaction inside and between the different subpopulations. The set of available empirical variables in both fields of consideration and the huge number of possible definitions of compound macrovariables require simplifying assumptions.

The definition of adequate variables, however, must be checked carefully in the spirit of empirical and theoretical needs.

References

Allen P M, 1978, "Dynamique des Centres Urbains" *Sciences et Techniques* **50** 15-19

Allen P M, 1983, "Self-organisation and evolution in urban systems", in *Cities and Regions as Nonlinear Decision Systems* Ed. R W Crosby (Westview Press, Boulder, CO) pp 29-60

Allen P M, Sanglier M, 1979, "Dynamic models and urban growth" *Journal of Social and Biological Structures* **1** 265-280, **2** 269-278

Allen P M, Sanglier M, 1979, "A dynamic model of growth in a central place system" *Geographical Analysis* **11** 256-272

Allen P M, Sanglier M, 1981, "Urban evolution, self-organisation, and decisionmaking" *Environment and Planning A* **13** 167-183

Anas A, 1990, "General economic principles for building comprehensive urban models", in *Urban Dynamics: Designing an Integrated Model* Eds C S Bertuglia, G Leonardi, A G Wilson (Routledge, Chapman and Hall, Andover, Hants) pp 7-44

Andersson A, 1981, "Structural change and technological development" *Regional Science and Urban Economics* **11** 267-268, 351-361

Andersson A, Batten D, Johansson B, Nijkamp P (Eds), 1989 *Advances in Spatial Theory and Dynamics* (North-Holland, Amsterdam)

Arthur W B, 1988, "Self-reinforcing mechanisms in economy", in *The Economy as an Evolving Complex System* Eds P W Anderson, J A Arrow (Addison-Wesley, Reading, MA) pp 1-21

Barentsen W, Nijkamp P, 1988, "Nonlinear dynamic modelling of spatial interaction" *Environment and Planning B: Planning and Design* **15** 433-446

Batten D, Casti J, Johansson B (Eds), 1985 *Economic Evolution and Structural Adjustment* (Springer, New York)

Bertuglia C S, Leonardi G, Wilson A G (Eds), 1990 *Urban Dynamics: Designing an Integrated Model* (Routledge, Chapman and Hall, Andover, Hants)

Dendrinos D S, Mullaly H, 1981, "Evolutionary patterns of urban populations" *Geographical Analysis* **13** 328-344

Dendrinos D S, Mullaly H, 1985 *Urban Evolution—Studies in the Mathematical* (Oxford, University Press, Oxford)

Fischer M, 1987, "Innovation, diffusion and regions", in *Proceedings of Research and Development, Industrial Change and Economic Policy* Symposium at the University of Karlstadt; available from Wirtschaftsuniversität Wien, A-1090 Wien

Griffith D A, Haining R P, 1986 *Transformations Through Space and Time* (Martinus Nijhoff, Dordrecht)

Griffith D A, Lea A C, 1983 *Evolving Geographical Structures* (Martinus Nijhoff, Dordrecht)

Haag G, 1989a *Dynamic Decision Theory: Applications to Urban and Regional Topics* (Kluwer, Dordrecht)

Haag G, 1989b, "Towards an integrated urban model", in *Spatial Analysis and Population Dynamics. Congress and Colloquia Number 6* Ed. D Pumain (John Libbey, London) chapter 14

Haag G, Weidlich W, 1984, "A stochastic theory of interregional migration" *Geographical Analysis* **16** 331-357

Haken H, 1974 *Synergetics, an Introduction. Springer Series Synergetics* **1** (Springer, Berlin)

May R M, 1976, "Simple mathematical models with very complicated dynamics" *Nature* **261** 459-467

Mela A, Preto G, Rabino G A, 1987, "Principles of spatial organization: a unifying model for regional systems", paper presented on the Fifth European Colloquium, Bardonecchia, 9-12 September; available from G A Rabino, Dipartimento di Ingegneria dei Sistemi Edilizi e Territoriali, Milano, 20133

Mensch G, 1979 *Stálemate in Technology* (Ballinger, Cambridge, MA)

Prigogine I, Stengers I, 1979 *La Nouvelle Alliance* (Gallimard, Paris)

Pumain D, 1989, "Spatial dynamics and urban models", in *Urban Dynamics and Spatial Choice Behaviour* Eds J Hauer, H Timmermans, N Wrigley (D Reidel, Dordrecht) pp 155-173

Pumain D, Saint-Julien Th, 1984, "Evolving structure of the French urban system" *Urban Geography* **5** 308-325

Pumain D, Sanders L, Saint-Julien Th, 1989 *Villes et Auto-organisation* (Economica, Paris)

Sonis M, 1981, "Flows, hierarchies, potentials" *Environment and Planning A* **13** 413-420

Wegener M, 1984, "Integrated forecasting models of urban and regional systems", paper prepared for the Fourth International Workshop on Strategic Planning, University of Liverpool, 4–6 April; copy from University of Dortmund, 4600 Dortmund-Eichlinghofen, August-Schmidt-Strasse, Germany

Weidlich W, Haag G, 1983 *Concepts and Models of a Quantitative Sociology. Springer Series Synergetics* **14** (Springer, Berlin)

Weidlich W, Haag G, 1987, "A dynamic phase transition model for spatial agglomeration processes" *Journal of Regional Science* **27** 529–569

Weidlich W, Haag G (Eds), 1988 *Interregional Migration, Dynamic Theory and Comparative Analysis* (Springer, Berlin)

Wilson A G, 1971, "A family of spatial interaction models, and associated developments" *Environment and Planning* **3** 1–32

Wilson A G, 1981 *Catastrophe Theory and Bifurcation: Applications to Urban and Regional Systems* (Croom Helm, Andover, Hants)