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**Advection-diffusion in porous media at low scale separation via  
higher-order homogenization**

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Asymptotic multiple scale homogenization allows to determine the effective behaviour of a porous medium by starting from the pore-scale description. The thus obtained continuum descriptions are valid when the given phenomenon is considered in a medium with a large number of heterogeneities. This hypothesis of separation of scales means that the macroscopic size  $L$ , is very large in comparison with the size  $l$  of the heterogeneities. It is therefore interesting to investigate how the described theories are modified when the hypothesis of scale separation is not perfectly respected. This happens when the porous medium is macroscopically heterogeneous or when large gradients are applied to macroscopically homogeneous media. The asymptotic multiple scale homogenization method is particularly well adapted to this type of analysis. The effect of low scale separation can be obtained by exploiting higher order equations in the asymptotic homogenization procedure and then by analyzing their role in the macroscopic description. Higher-order homogenization thus allows both the determination of the conditions under which the influence of the microstructure is negligible and also how the microstructure may modify the response of the medium. While higher-order homogenization has been widely applied in the field of mechanics of composite materials (e.g. (Gambin and Kröner, 1989), (Boutin, 1996)), porous media at low scale separation have received very little attention. However, an important work is presented in (Goyeau et al., 1997), where the method of volume averaging is used to obtain the correctors of Darcy's law. The same problem is tackled via higher-order homogenization and up the third order in (Auriault et al., 2005). The aim of the present study is to investigate higher-order terms of the advective-diffusive model to describe advection-diffusion in a macroscopically homogeneous porous medium at low scale separation. The advective-diffusive model is obtained by first-order homogenization when considering the convection-diffusion equations with a macroscopic Peclet number (i.e. the Peclet number measured by means of the macroscopic length  $L$ ) in the order of 1 on the pore scale (Auriault and Adler, 1995). The second and third order models are derived. The main result of the study is that the low separation of scales induces dispersion effects. In particular, for a macroscopically homogeneous medium, the second-order model is similar to the most currently used phenomenological model of dispersion: it is characterized by a dispersion tensor which can be decomposed into a purely diffusive component and a mechanical dispersion part, whilst this property is not verified in the homogenized dispersion model (obtained at higher Peclet numbers). The third-order description contains second and third concentration gradient terms, with a fourth order tensor of diffusion and with a third-order and a second-order tensors of dispersion.

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