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ANALYSIS OF DIRICHLET-ROBIN INTERFACE CONDITION IN TRANSIENT CONJUGATE HEAT TRANSFER PROBLEMS.

APPLICATION TO A FLAT PLATE WITH CONVECTION.

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ABSTRACT

Growing optimization of gas turbines in terms of mass, energy efficiency and lifespan requires an accurate knowledge of solid temperature loads and resulting thermomechanical stress, especially during the sudden engine speed changes transient phases. This study deals with the development of a coupling strategy at fluid solid interface for transient heat transfer problems. A quasi-dynamic method is used between a finite-volume fluid code and a finite-element solid code. At the fluid-solid interface, Dirichlet-Robin conditions are employed. Various coupling relaxation parameters are tested on a test-case of flat plate with transient boundary conditions. It's shown that stability and computational cost increase when coupling relaxation parameter increase.

NOMENCLATURE

		<u>Subscripts and superscripts</u>
h	Convection coefficient ($W.m^{-2}.K^{-1}$)	f Fluid domain
n	Coupling cycle step	s Solid domain
q	Heat flux ($W.m^{-2}$)	ν Iteration step in coupling cycle
T	Temperature (K)	∞ Free stream
T_{ref}	Reference temperature at bottom side (K)	$\overline{(\quad)}$ Spatial mean quantity
T^*	Reference temperature at interface side (K)	
U	Velocity ($m.s^{-1}$)	
α	Coupling relaxation parameter ($W.m^{-2}.K^{-1}$)	

INTRODUCTION

In many industrial modern systems (combustors, turbine blades, heat exchangers, etc.), the high gas temperatures and high temperature gradients can result in significant thermal stresses in the solid structures which can lead to serious damage. As a result, a detailed knowledge of heat transfer characteristics is of prime importance in the design process to preserve the integrity of the components under extreme thermal conditions and therefore, an accurate representation of the temperature loading in the solid is essential.

Whenever there is a temperature difference between the fluid and the confining solid, heat is transferred and changes the flow properties in a non-trivial way. The term conjugate heat transfer (CHT) is used to describe those processes. They involve variations of temperature within solids and fluids, due to their mutual thermal interactions. A typical example is modern gas turbines that usually operate at temperatures higher than the melting temperature of the turbine blade materials. With advances in alloy technology, it is now possible to increase gas turbine operating temperatures and thus engine efficiency. But cooling techniques remain the essential factor for maintaining turbine blade integrity. CHT procedures are now commonly found in many real-world environments in which accurate heat transfer predictions are needed to design efficient cooling (or heating) systems.

In recent years many studies have been devoted to analyse the behavior of various CHT procedures, but these studies are most often limited to steady cases, i.e. when a fluid-solid thermal state is sought [1] [2] [3] [4] [5] [6] [7]. The simulation of the transient heat load in solid structures is much less common but is beginning to be employed in turbomachinery applications [8] [9] [10] or more generally to account for the time-dependent thermal response of a structures to ambient conditions, for instance in modeling heating, cooling and ventilating flows in building simulations [11] [12]. Transient CHT is costly in term of CPU time if inappropriate coupling strategies are used. That's the reason why its application remains limited to steady or simple transient simulations and this is especially true for coupled computations over a long period of time. Unsteady CHT may become increasingly used to assist and to improve the solid temperature prediction only if efficient procedures leading to accurate solutions at reasonable computational times are employed.

Recently, a new numerical coupling method to describe the transient temperature field in a solid via a conjugate heat transfer method was proposed [13] [14]. This method, called quasi-dynamic, presents advantages in terms of precision and computational time. It was shown that due to the significant discrepancies of time constants in the two media,- the fluid flow requires usually a much smaller temporal resolution than the structure -, the flow field can be considered as a sequence of steady states. Accordingly, this method is based on a two-way loose coupling of a dynamic thermal modeling in the solid and a sequence of steady states in the fluid. But this method requires appropriate choice of boundary conditions at the interface to ensure stability and a high convergence speed. Literature review points out that this choice remains to be clarified.

The paper focuses on unsteady CHT. The primary goal of this paper is to test various boundary conditions at the interface and to determine their impact on precision, stability and convergence, in order to choose optimal ones.

A QUASI-DYNAMIC COUPLING STRATEGY

Coupling algorithm

The quasi-dynamic algorithm was described recently in detail in a previous paper [13]. Thus, only the main topics will be presented here with emphasis put on the numerical coupling relaxation parameter used in the Dirichlet-Robin procedure.

The quasi-dynamic algorithm solves each subsystem by an individual solution scheme. This algorithm, illustrated in Figure 1 for a time cycle between t^n and t^{n+1} , is composed of 4 steps.

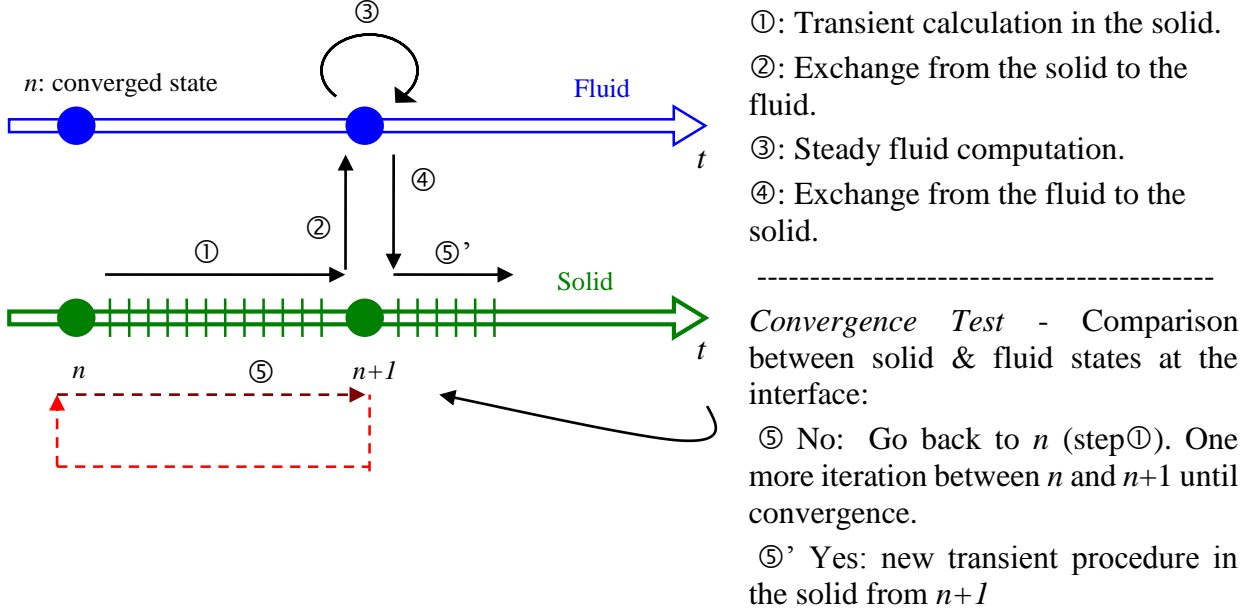


Figure 1: quasi-dynamic coupling algorithm [13]

At step 5, coupling cycle is repeated until convergence criterion is reached, defined at the v^{th} iteration as:

$$\frac{|q_s^{v+1} - q_s^v|}{|q_s^{v+1}| + |q_s^v|} < \varepsilon \quad (1)$$

Frequency of exchanges

Duration of a coupling cycle is the time between exchanges of boundary conditions at the interface between the fluid and solid domains. In this study frequency of exchanges is constant in the whole calculation.

Fluid-solid Interface conditions

Several boundary conditions at the interface can be chosen. In the more general case, Robin boundary conditions are imposed on fluid and solid sides [6]

$$\begin{cases} q_s^{v+1} = -q_f^v + \alpha_f^v (T_f^v - T_s^{v+1}) \\ q_f^{v+1} = -q_s^{v+1} + \alpha_s^{v+1} (T_s^{v+1} - T_f^{v+1}) \end{cases} \quad (2)$$

Where α_f^v and α_s^{v+1} are relaxation parameters.

Convection boundary condition can be imposed on fluid side:

$$q_s^{v+1} = h^v(T_{ref}^v - T_s^{v+1}) \quad (3)$$

With

$$h^v = \frac{q_f^v}{T_f^v - T_{ref}^v} \quad (4)$$

And T_{ref}^v a reference temperature.

Dirichlet boundary conditions (temperature imposed) and Neumann boundary condition (flux imposed) are particular cases of Robin boundary conditions.

If $\alpha_f^v \rightarrow \infty$:

$$T_s^{v+1} = T_f^v \quad (5)$$

If $\alpha_f^v = 0$:

$$q_s^{v+1} = -q_f^v \quad (6)$$

Flux and temperature can be relaxed between two successive iterations with a relaxation parameter β (different from relaxation boundary condition):

$$T_s^{v+1} = (1 - \beta)T_s^v + \beta T_s^{v+1} \quad (7)$$

$$q_s^{v+1} = (1 - \beta)q_s^v + \beta q_s^{v+1} \quad (8)$$

Influence of these boundary conditions on stability and convergence speed has been widely studied in steady-state. In [6], stability of a 1D diffusion model is analyzed by applying Godunov and Ryabenkii theory [15] on the discretized equations. In [16], another approach based on the physics of the problem, leads to choose appropriate boundary conditions at the interface based on adimensional numbers, like Biot number:

$$Bi = \frac{hL}{\lambda_s} \quad (9)$$

It's difficult to get a relevant reference temperature (equation 4). A possibility is to choose free stream temperature [13], but it's often difficult in industrial configurations. Another choice is to take the first fluid cell temperature [17]. However if this temperature is too close of the wall temperature it leads to divergent h values, which is not acceptable.

In equations 2, 7 and 8, increasing relaxation parameters improves stability but slows convergence speed [18] [19] [4].

Less work is devoted to influence of boundary conditions at the interface in transient phases. In [10], flux is imposed on the solid side, while a relaxed temperature is imposed on the fluid side with a constant relaxation parameter chosen empirically. Relaxation is over-estimated to ensure stability, leading to slow convergence speeds. Thus transient thermal analysis of a gas turbine internal air system with multiple cavities is computationally expensive.

This work is a contribution to find optimal boundary conditions at the interface in transient phases, in terms of stability, precision and convergence speed. In this paper temperature is imposed on fluid side, as done classically in literature [1]. Robin condition is imposed on solid side (equation 2) with a constant relaxation parameter α_f^v . This parameter is simply noted α in order to alleviate the text in the next. Thus boundary conditions at the interface are written:

$$\begin{cases} q_s^{v+1} = -q_f^v + \alpha(T_f^v - T_s^{v+1}) \\ T_f^{v+1} = T_s^{v+1} \end{cases} \quad (10)$$

CASE PRESENTATION

The test-case is a flat plate cooled by convection on its upper face and heated on its lower face. Convection boundary conditions applied on the underside are time-dependent. The fluid-solid coupling interface is the line $y=0$. The temporal evolution of temperature at several points of coupling interface will be studied.

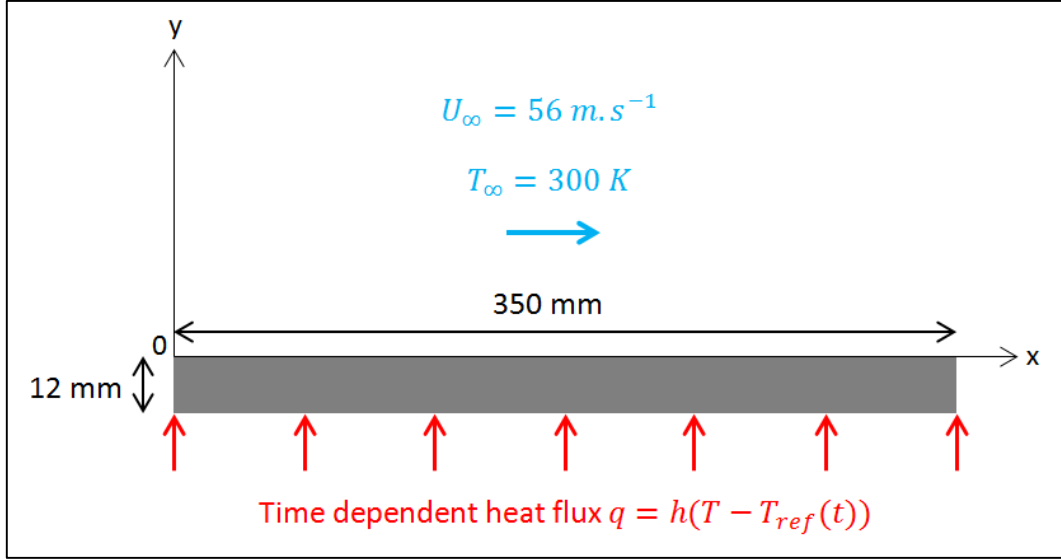


Figure 2: flat plate cooled by convection

Thermophysical properties of fluid (air) and solid (PVC) are detailed in Tables 1 and 2.

Notation	Name	Value
ρ_f	Fluid density	1,225 kg.m ⁻³
μ_f	Fluid dynamic viscosity	1,7894e ⁻⁵ kg.m ⁻¹ .s ⁻¹
λ_f	Fluid thermal conductivity	0,0242 W.m ⁻¹ .K ⁻¹
C_{pf}	Fluid heat capacity	1006,43 J.kg ⁻¹ .K ⁻¹

Table 1 : fluid thermophysical properties (air)

Notation	Name	Value
ρ_s	Solid properties	1200 kg.m ⁻³
λ_s	Solid thermal conductivity	0,16 W.m ⁻¹ .K ⁻¹
C_{ps}	Solid heat capacity	1400 J.kg ⁻¹ .K ⁻¹

Table 2 : solid thermophysical properties (PVC)

Temperature evolution is simulated over a long period of time ($t_{max} = 10800 \text{ s}$). Computation is divided into 18 coupling cycles of 600 s.

Convection conditions are applied at the bottom face ($y = -12 \text{ mm}$). Convection coefficient is constant ($h = 500 \text{ W.m}^{-2}.\text{K}^{-1}$). However, reference temperature (T_{ref}) is time-dependent, as shown on Figure 3.

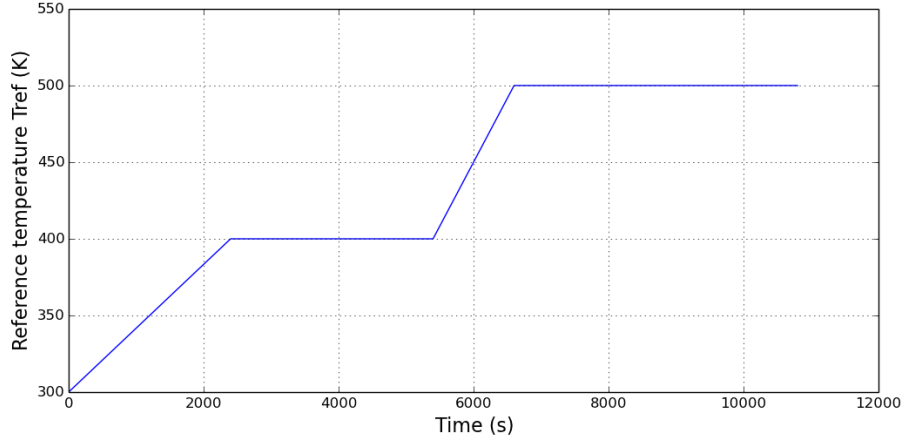


Figure 3: temporal evolution of reference temperature on bottom face ($y = -12$ mm)

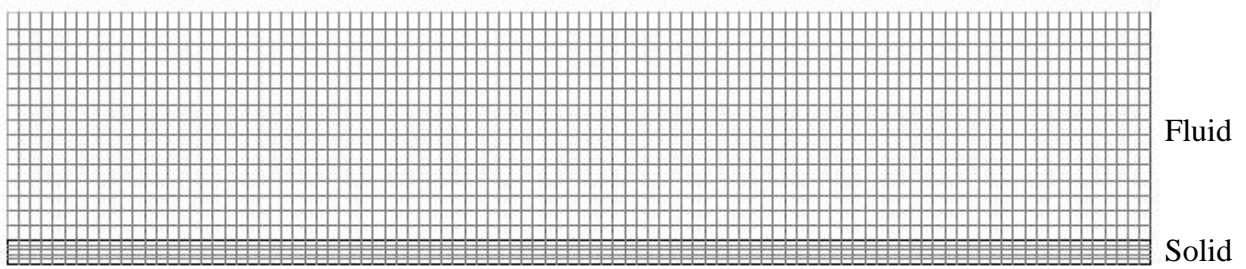


Figure 4: fluid and solid mesh

Fluid and solid meshes are coincident at the interface. However finite-volume Fluent solver defines the temperature in the cell center, while finite-element solve defines the temperature at the vertices of the element, requiring use of spatial linear interpolation.

Turbulence model is $k-\omega$ SST. A wall function is used for the near wall simulation, with $30 < y^+ < 60$ in the first fluid cell, in order to be in the logarithmic region of the boundary layer.

y^+ is defined as:

$$y^+ = \frac{\rho u^* y}{\mu} \quad (11)$$

With u^* the friction velocity defined by $u^* = \sqrt{\frac{\tau_w}{\rho}}$

And τ_w the wall shear stress defined by $\tau_w \approx \mu \frac{\partial U}{\partial y} \Big|_{y=0}$

NUMERICAL RESULTS

The influence of the coupling relaxation parameter α on stability and convergence is studied. Coupling relaxation parameters are arbitrarily chosen and set constant during the whole CHT calculation. Several calculations are performed to give qualitative tendencies. The temperature is analyzed at points $x=5$ cm (near leading edge) and $x=17$ cm (far from leading edge) on the interface. A partitioned approach is used for the quasi-dynamic procedure. Finite volume code FLUENT is used for fluid. Finite element code ANSYS is used for solid. Coupling is realized by ANSYS WORKBENCH multiphysics platform and by PYTHON scripts.

Results obtained with quasi-dynamic method are compared with monolithic ones, in which global fluid-solid system is solved with the single computational code. FLUENT is used for monolithic method. The monolithic method provides a reference solution in terms of precision because fluid and solid temperatures and heat fluxes are intrinsically equal at the interface. Gradients are weak to ensure stability. However computational cost of monolithic method in industrial configurations for transient problems is too much expensive, and contains many other drawbacks.

Influence on stability

It's observed from numerical computations that for $\alpha < 150$, CHT procedure diverges after some iterations and is unstable. For $\alpha \geq 150$, calculation is stable. $\alpha = 150$ is referred to as critical coupling relaxation parameter.

Influence on precision

Criterion for precision is relative error between quasi-dynamic and reference monolithic methods, defined as:

$$error(t) = \left| \frac{T_{quasi-dynamic}(t) - T_{monolithic}(t)}{\frac{1}{t_{max}} \int_0^{t_{max}} T_{monolithic}(t) dt - T_{\infty}} \right| \quad (12)$$

With $T_{\infty} = 300 \text{ K}$

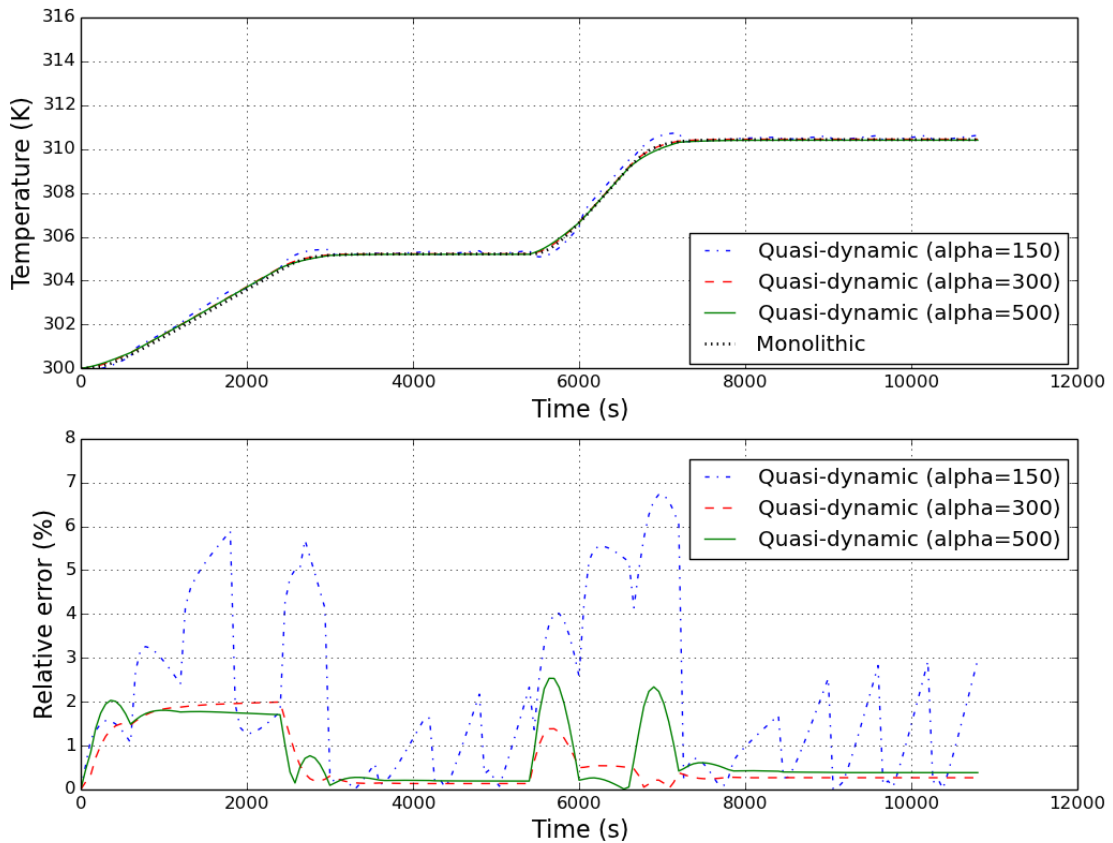


Figure 5: temporal evolution of temperature (above) and relative error (below) at point $x=5\text{cm}$ of upper face, for coupling relaxation parameters $\alpha = 150$, $\alpha = 300$ and $\alpha = 500$

Figure 5 shows the time evolutions of temperature and relative error near leading edge ($x=5\text{cm}$). With critical coupling relaxation parameter ($\alpha = 150$), evolution of temperature over time is strongly discontinuous from one coupling cycle to another. Relative error is quite important and can reach almost 7 % in transient phases. For higher parameters ($\alpha = 300$ and $\alpha = 500$), accuracy is quite good, with relative error around 2 % in transient phases. With all parameters, error is maximum in transient phases, and almost non-existent in steady-state phases.

Although precision is theoretically independent of coupling relaxation parameter α (Equation 4), quite important differences are observed for $\alpha = 150$ compared to other parameters. This can be explained by the fact that computation is not fully converged near critical relaxation parameter, because of strong heat flux and temperature oscillations during iterations.

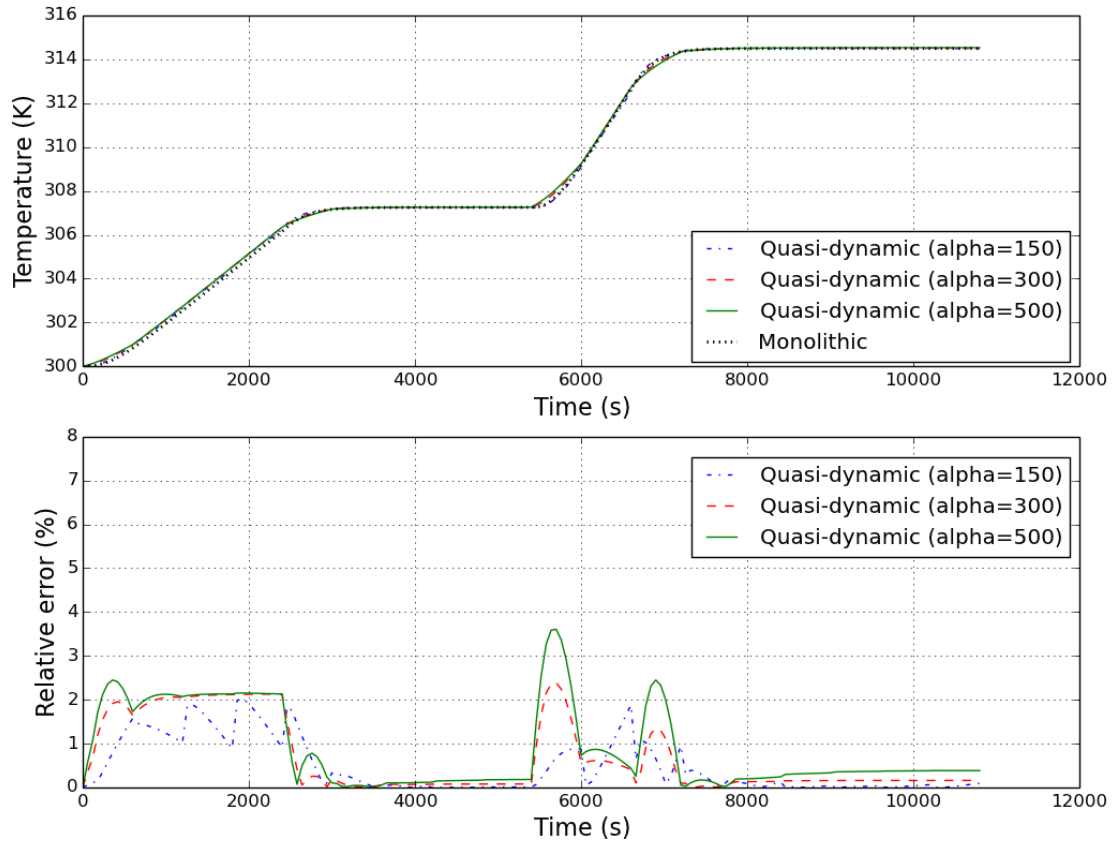


Figure 6: temporal evolution of temperature (above) and relative error (below) at point $x=17\text{cm}$ of upper face, for coupling relaxation parameters $\alpha = 150$, $\alpha = 300$ and $\alpha = 500$

Figure 6 shows the time evolutions of temperature and relative far from leading edge ($x=17\text{cm}$). It is also noticed that error is maximum in transient phases. Error slightly increases when coupling relaxation parameter increases, but remains very small. It has been verified that for all interface points far from leading edge, the error remains very small (on the same order than at point $x=17\text{cm}$).

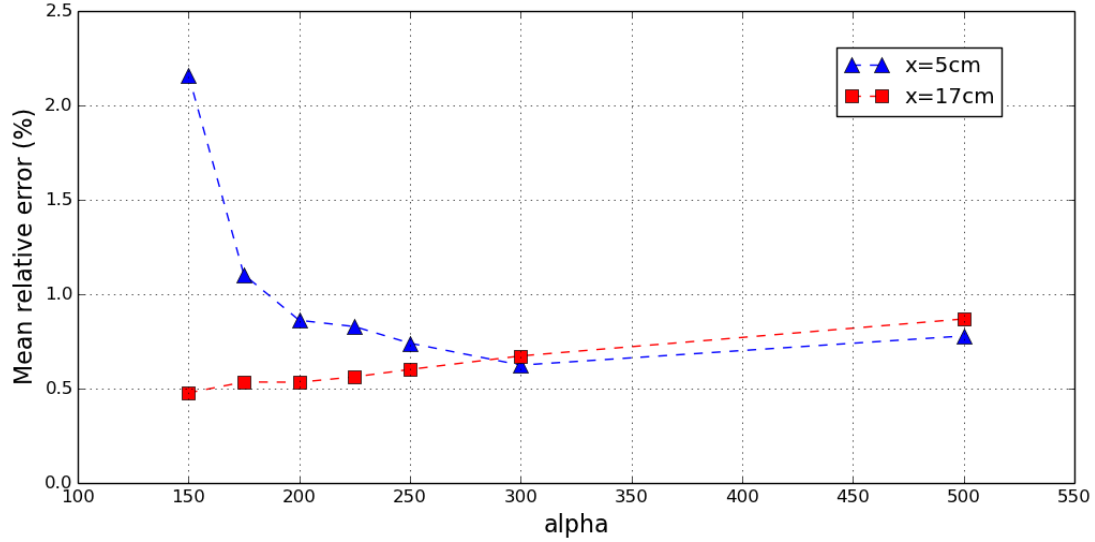


Figure 7: temporal mean of relative error at points $x=5\text{cm}$ and $x=17\text{cm}$ based on coupling relaxation parameter

Figure 7 shows mean temporal errors obtained for several relaxation parameters. Near leading edge ($x=5\text{cm}$), error increases when relaxation parameter decreases. Far from the leading edge ($x=17\text{cm}$), error increases when approaching critical relaxation parameter. But globally, if CHT calculation is stable, error is small and is little dependent on the relaxation parameter.

Near the leading edge, a coefficient near critical coefficient ($\alpha \approx 150$) makes the system almost unstable locally and explains the strong oscillations of temperature. This tends to show that a local and variable relaxation parameter would be better appropriate.

Influence on computational cost

Computational cost is measured by the total number of iterations required for the computation. Convergence criterion $\varepsilon = 10^{-4}$ is adopted.

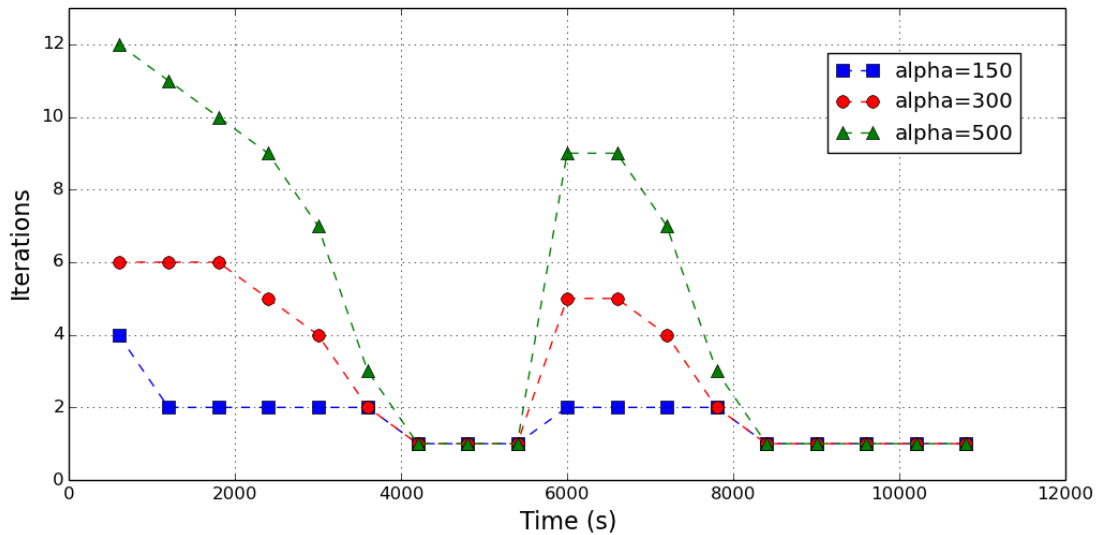


Figure 8: number of iterations for each coupling cycle for relaxation parameters $\alpha = 150$, $\alpha = 300$ and $\alpha = 500$

Figure 8 illustrates the number of iterations at each coupling cycle for several relaxation parameters. For each relaxation parameter, the number of iterations is higher in transient phases than in steady-state phases (Figure 3).

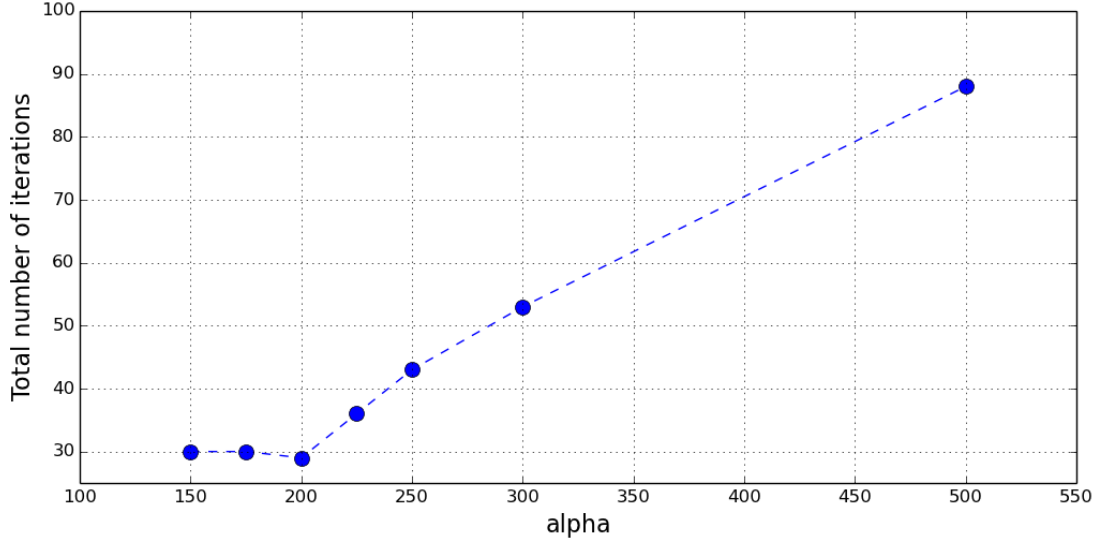


Figure 9: total number of iterations based on coupling relaxation parameter

Figure 8 and Figure 9 show that the number of iterations (i.e computational cost) increases when relaxation parameter increases. Thus it is possible to improve stability by increasing coupling relaxation parameter, but to the detriment of CPU time.

CONCLUSIONS

The relaxation parameter in Robin interface condition imposed on the solid side has a deep influence on stability, relative error and computational cost. Table 3 summarizes this influence.

Relaxation parameter	0	$\alpha_{critical}$	$\alpha_{optimal}$	∞
Stability	Unstable		Stable	
Relative error				
Computational cost				

Tableau 3: summary table of relaxation parameter influence

For parameters near critical relaxation parameter ($\alpha \approx 150$), oscillations at leading edge are quite important. The optimal coefficient is chosen when there is no more oscillations near the leading edge, around $\alpha \approx 250$.

The aim of this study was to give the qualitative influence of coupling relaxation parameter, and not quantitative values. We pointed out that an optimal relaxation parameter can be obtained. In this case, $\alpha_{critical} \approx 150$ (between stability and instability) and $\alpha_{optimal} \approx 250$ (for minimal error) were obtained, but these values depend on each configuration. Moreover, we showed that a local and variable relaxation parameters should be investigated in order to improve stability, precision and convergence speed.

Futur work will be devoted to research of analytical expressions of optimal coupling relaxation parameters depending on the main physical and numerical conditions involved in a coupled problem. Robin interface condition will be generalized, and imposed on the fluid side too. First results obtained on this simple flat plate test case are promising, and quasi-dynamic methods and optimal coupling relaxation parameters would be investigated on helicopter engine stators and rotors.

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