



**HAL**  
open science

## Some Irreducible Laman Graphs

Adel Moussaoui, Samy Ait-Aoudia

► **To cite this version:**

Adel Moussaoui, Samy Ait-Aoudia. Some Irreducible Laman Graphs. [Research Report] ESI - Ecole nationale Supérieure en Informatique - Alger. 2016. hal-01521779v2

**HAL Id: hal-01521779**

**<https://hal.science/hal-01521779v2>**

Submitted on 11 Jul 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Some Irreducible Laman Graphs

Adel Moussaoui, Samy Ait-Aoudia

ESI – Ecole nationale Supérieure en Informatique (ex INI), Oued-Smar, Algiers, Algeria

[a\\_moussaoui@esi.dz](mailto:a_moussaoui@esi.dz), [s\\_ait\\_aoudia@esi.dz](mailto:s_ait_aoudia@esi.dz)

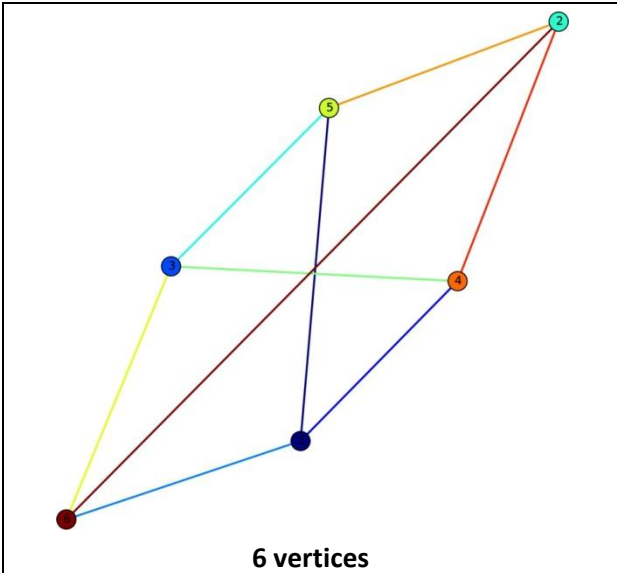
In graph theory, Laman graphs [1] describe the minimally rigid plane structures composed of bars and joints. Removing a bar leads to the non-rigidity of the structure. In the domain of geometric constraints solving, this property is commonly referred to as well-constrained graphs or structures. Formally, a graph  $G=(V, E)$  where  $|V|=n$  and  $|E|=m$  is a Laman graph if and only if  $m=2*n-3$  and  $m' \leq 2*n'-3$  for any induced sub-graph  $G'=(V', E')$  where  $|V'|=n'$  and  $|E'|=m'$ . Another characterization of rigid structures was given in 1911 by Henneberg [2] for which any minimal rigid plane structure is obtained starting from an edge joining two vertices and adding one vertex at a time using one of the two following operations. *Operation HI*: add a new vertex  $v$  to  $G$ , then connect  $v$  to two chosen vertices  $u$  and  $w$  from  $G$  via two new edges  $(v, u)$  and  $(v, w)$ . *Operation HII*: add a new vertex  $v$  to  $G$ , chose an edge  $(u, w)$  and another vertex  $z$  from  $G$ , then add three edges  $(v, u)$ ,  $(v, w)$  and  $(v, z)$  to  $G$ , finally delete the edge  $(u, w)$ .

An irreducible Laman graph is a graph  $G=(V, E)$  where  $|V|=n$ ,  $|E|=m$ ,  $m=2*n-3$  and  $m' < 2*n'-3$  for any induced sub-graph  $G'=(V', E')$  where  $|V'|=n'$  and  $|E'|=m'$ . An irreducible Laman graph is a graph that contains no Laman subgraphs. Based on the constraint system reduction algorithm [3], Laman irreducible graphs are obtained using the generator of 2D Geometric Constraint Graphs described in [4]. Laman irreducible graphs with up to 100 vertices are given in the appendix.

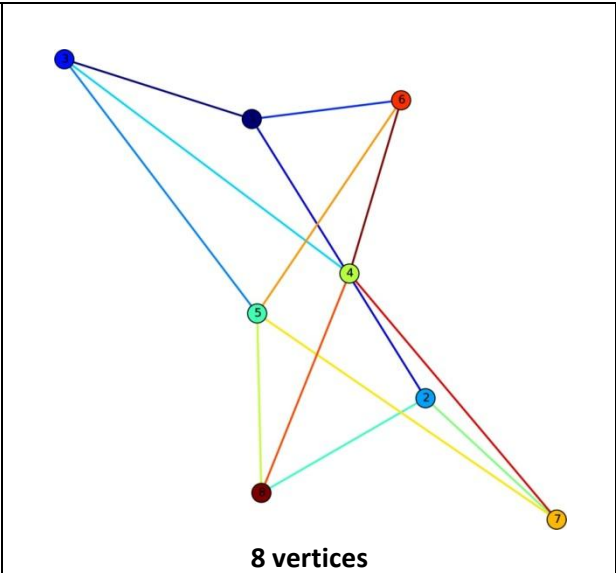
## REFERENCES

- [1] Laman, G. (1970). On graphs and rigidity of plane skeletal structures *Journal of Engineering mathematics* 4.4 (1970): 331-340. *Journal of Engineering mathematics*, 4 (4), 331-340.
- [2] Henneberg, L. (1911). *Die graphische Statik der starren Systeme* (Vol. 31). BG Teubner.
- [3] Ait-Aoudia, S., Jegou, R., & Michelucci, D. (1993). Reduction of constraint systems. *Compugraphics*. Algarve, Portugal.
- [4] Moussaoui, A., & Ait-Aoudia, S. (2016). Generator of 2D geometric constraint graphs. *Computer-Aided Design and Applications*, 13 (3), 271-280.

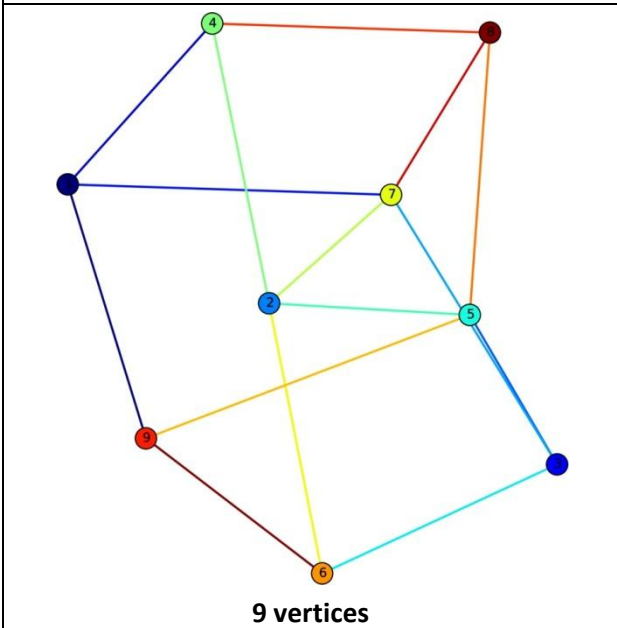
**APPENDIX**



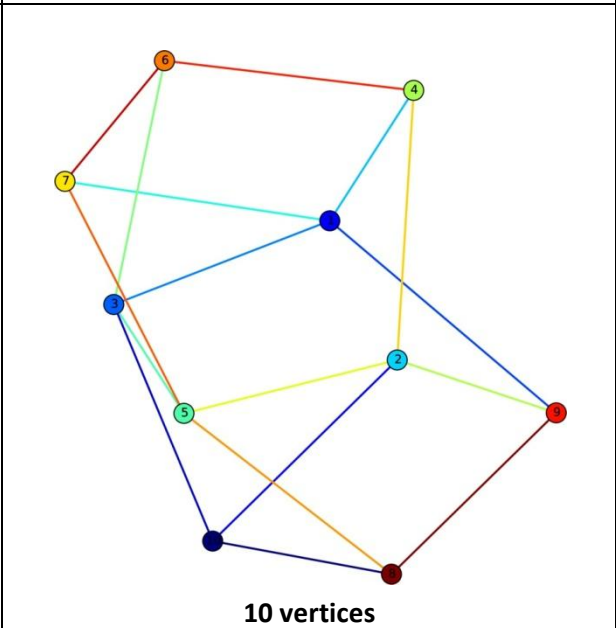
**6 vertices**



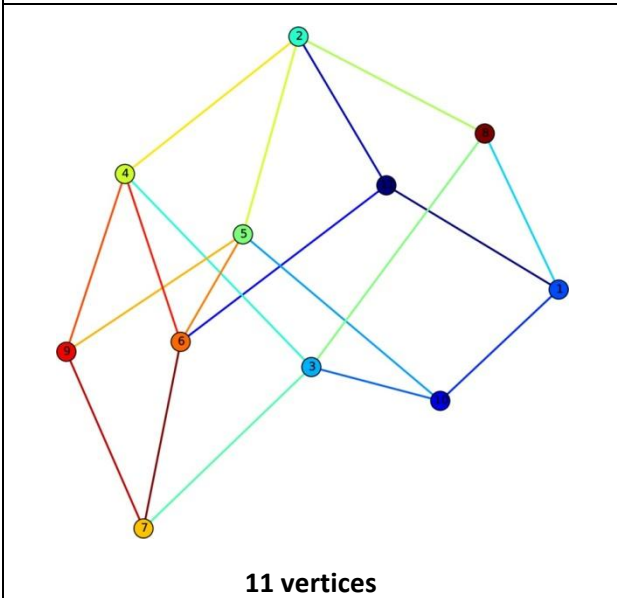
**8 vertices**



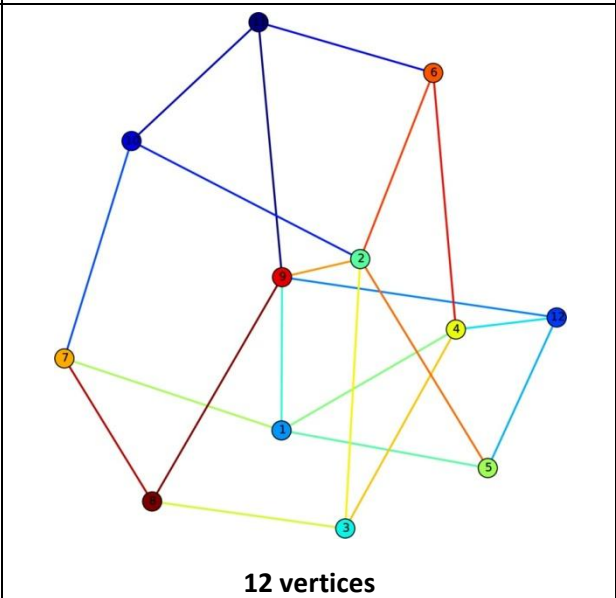
**9 vertices**



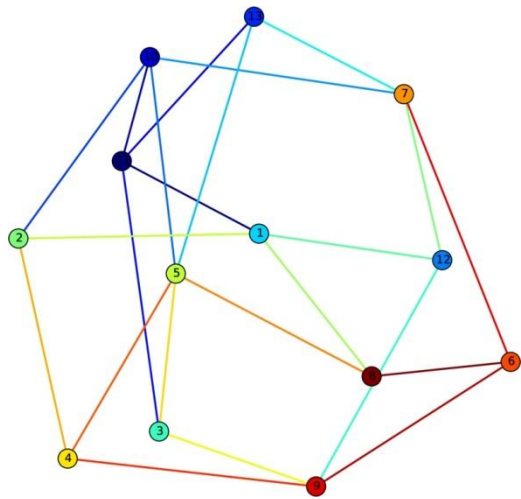
**10 vertices**



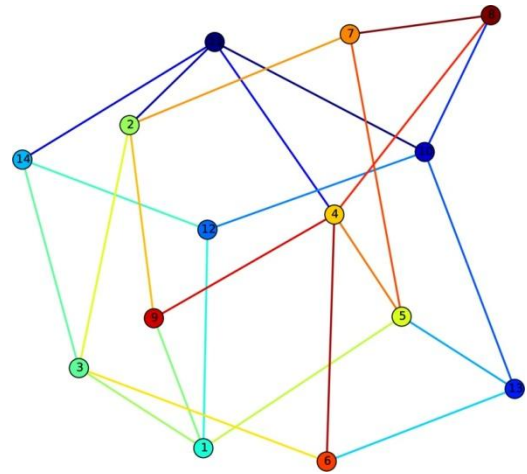
**11 vertices**



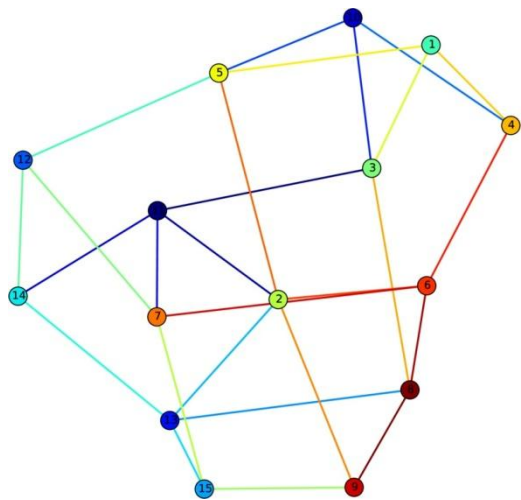
**12 vertices**



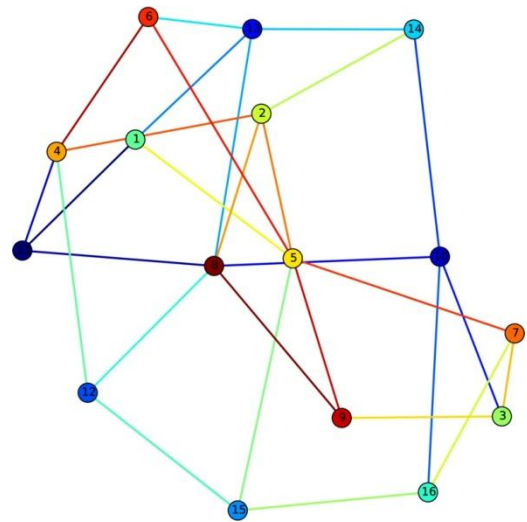
13 vertices



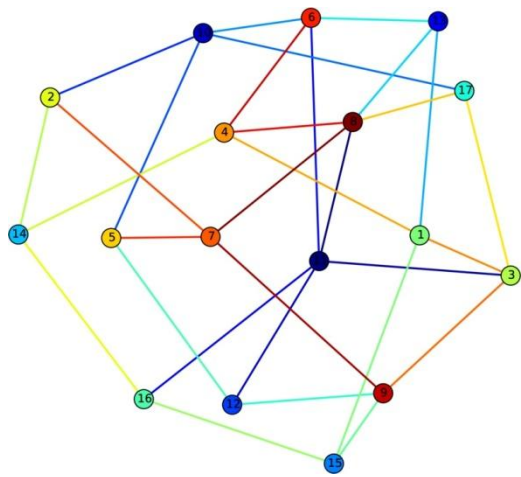
14 vertices



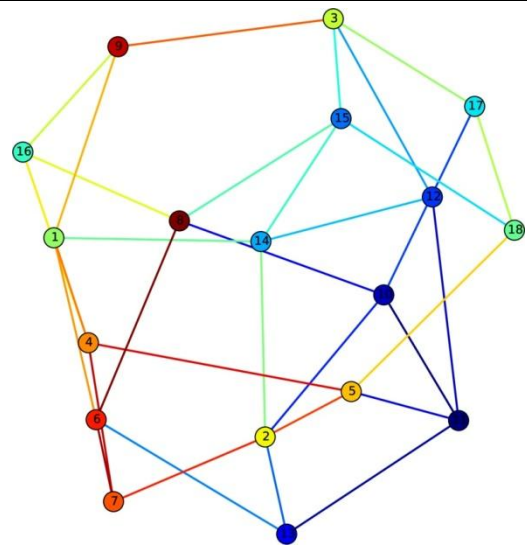
15 vertices



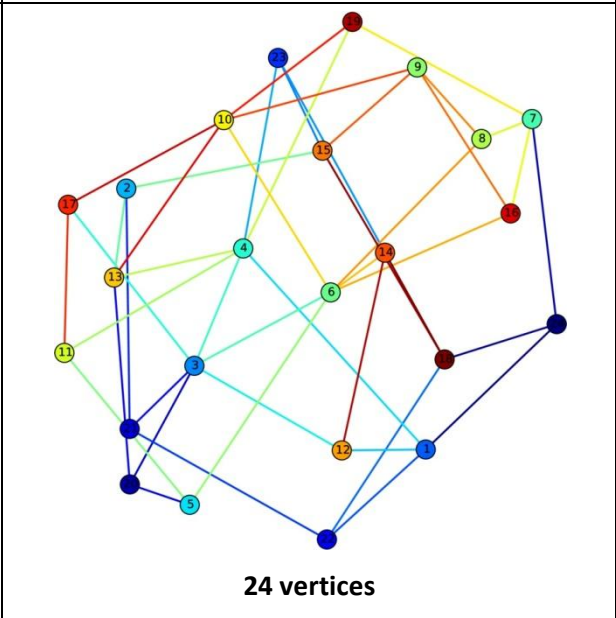
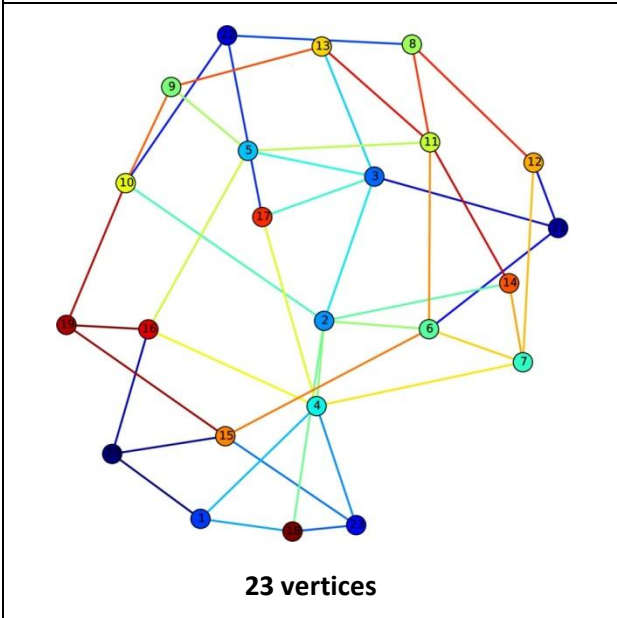
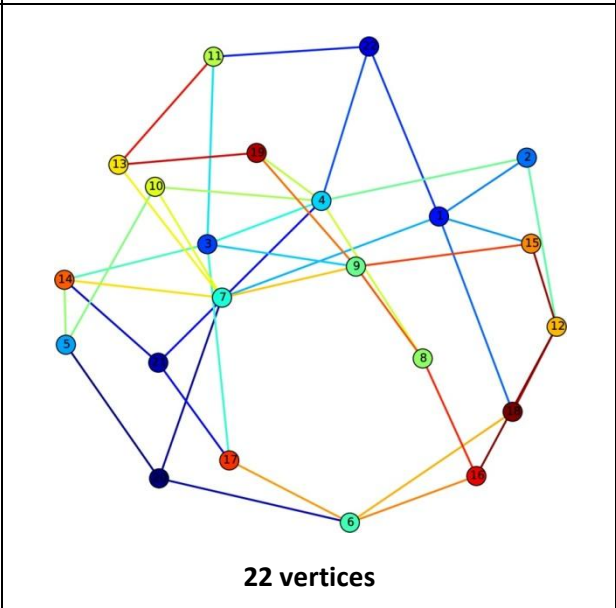
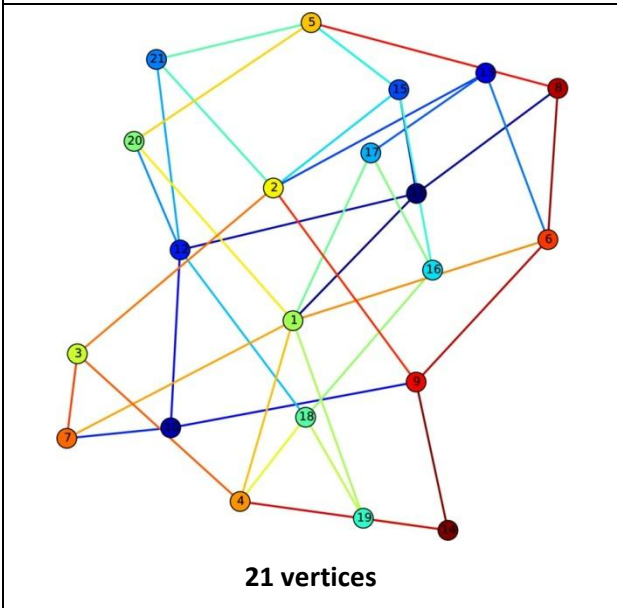
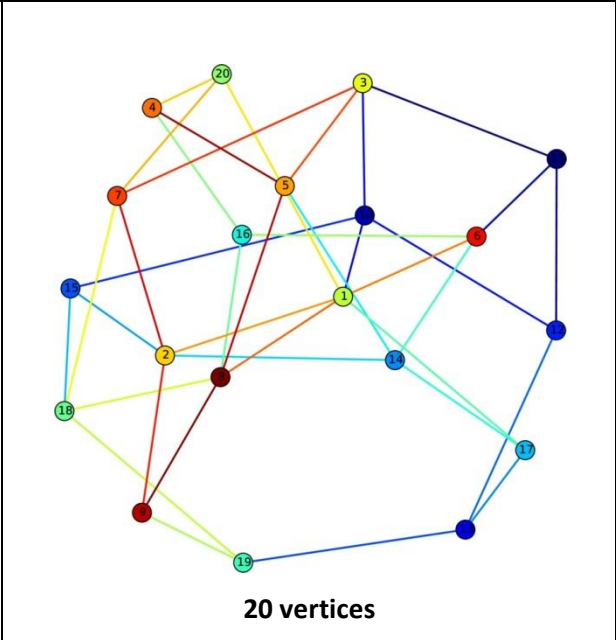
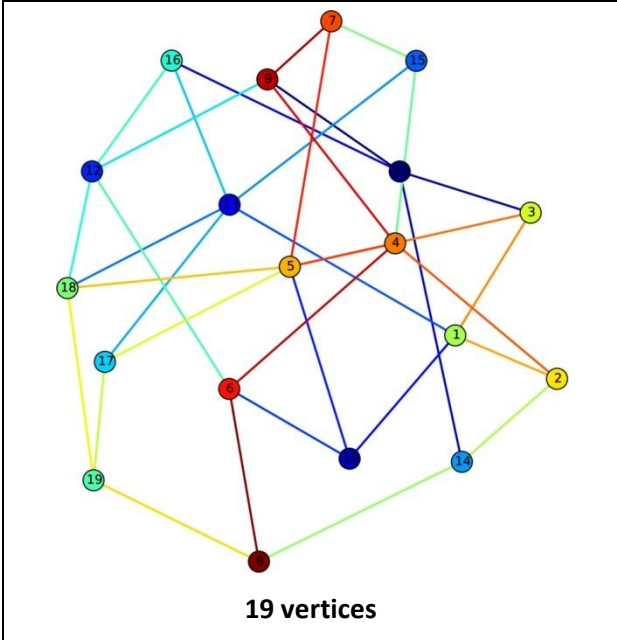
16 vertices



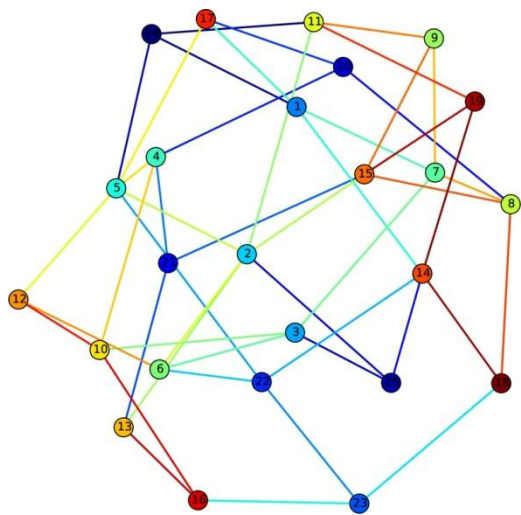
17 vertices



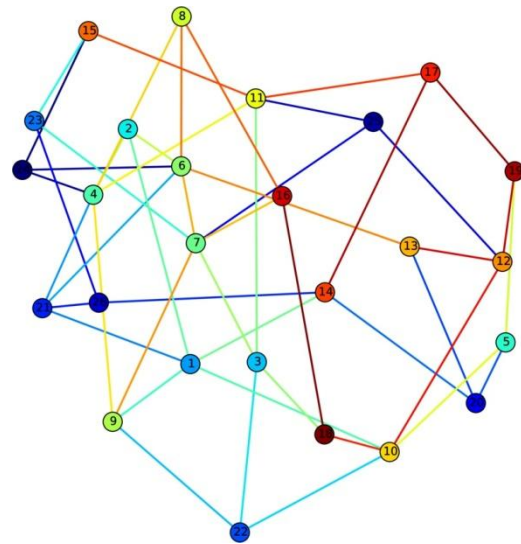
18 vertices



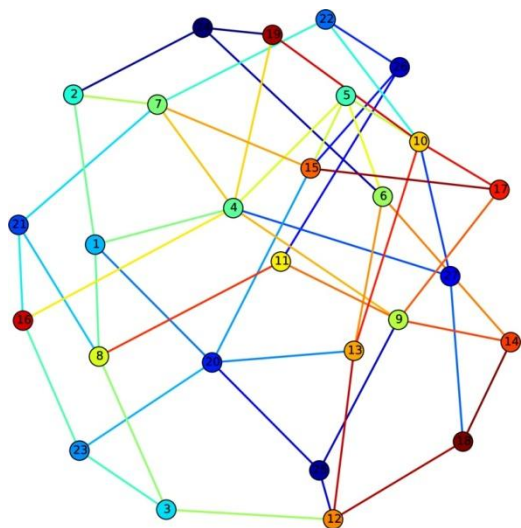




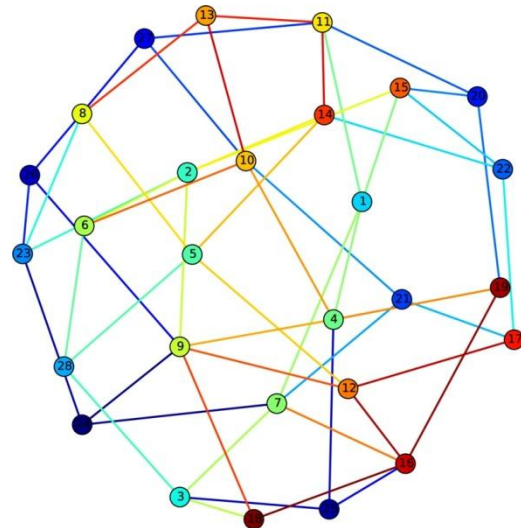
25 vertices



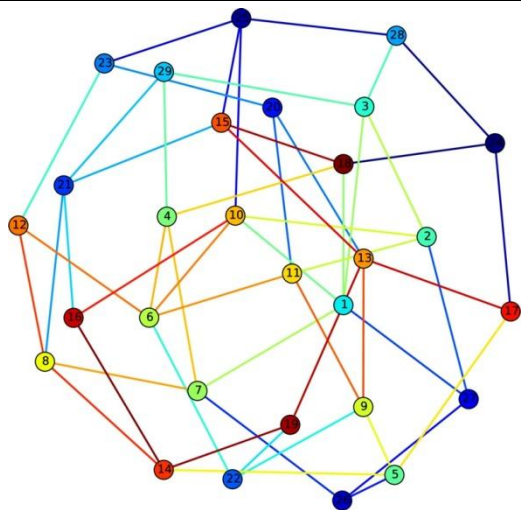
26 vertices



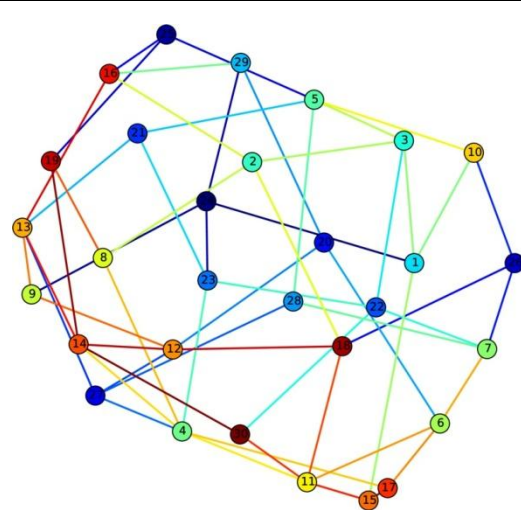
27 vertices



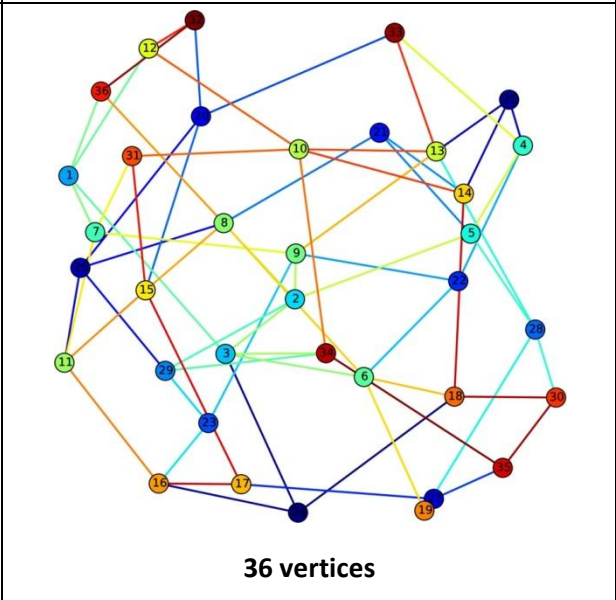
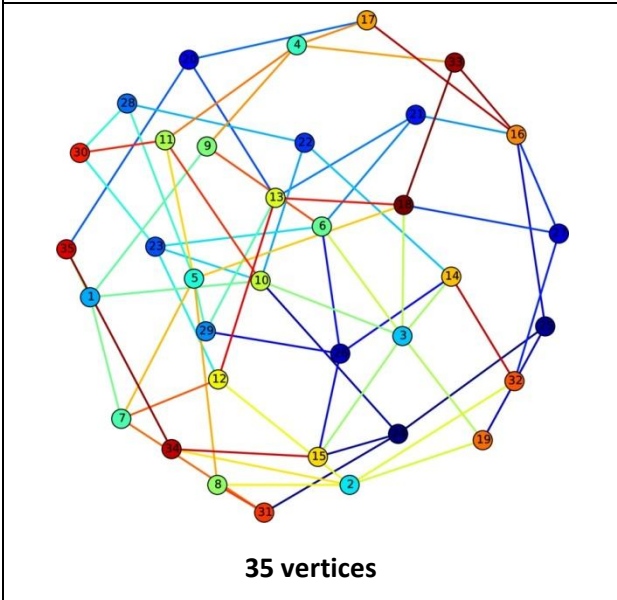
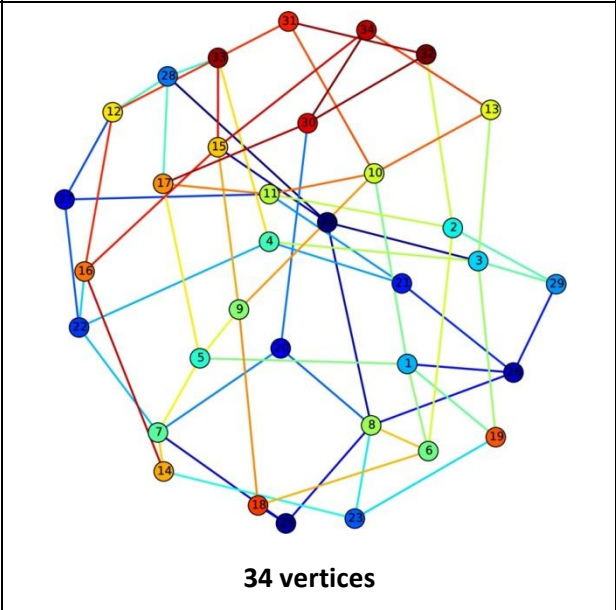
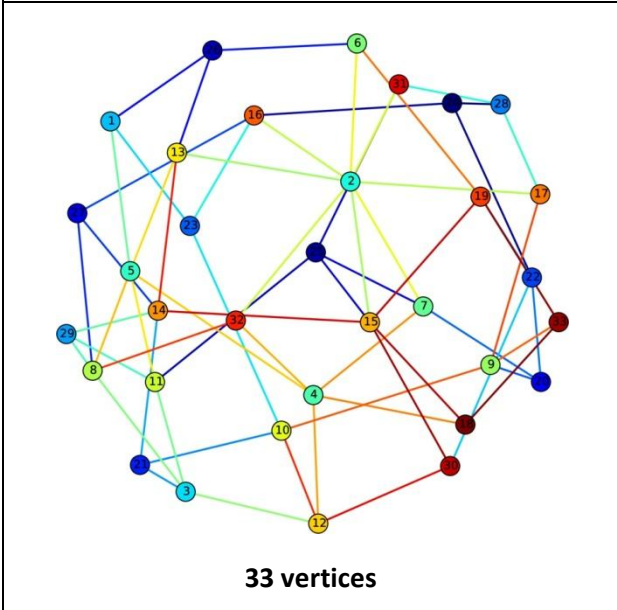
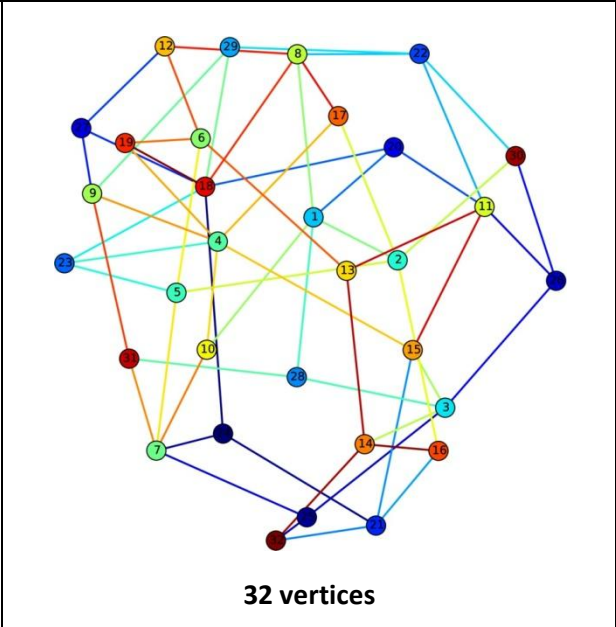
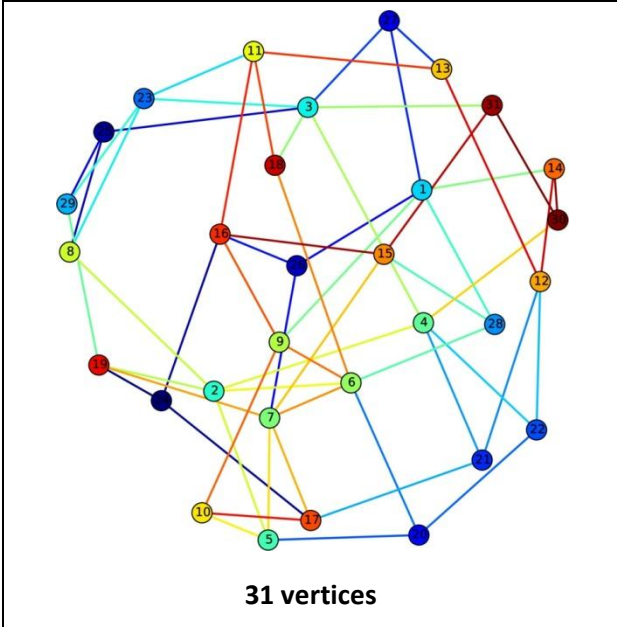
28 vertices



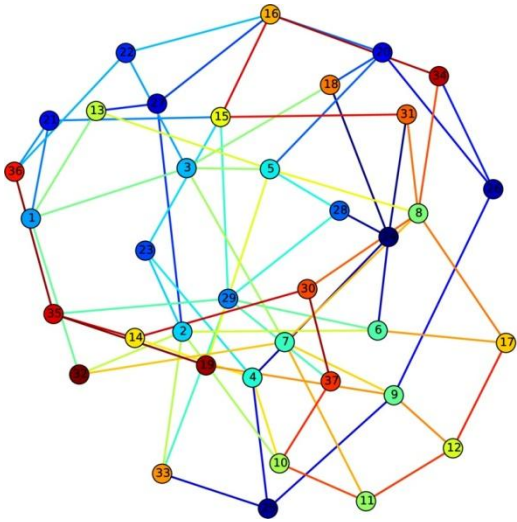
29 vertices



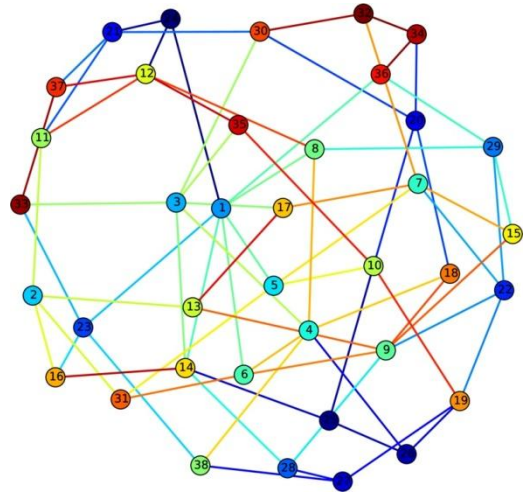
30 vertices



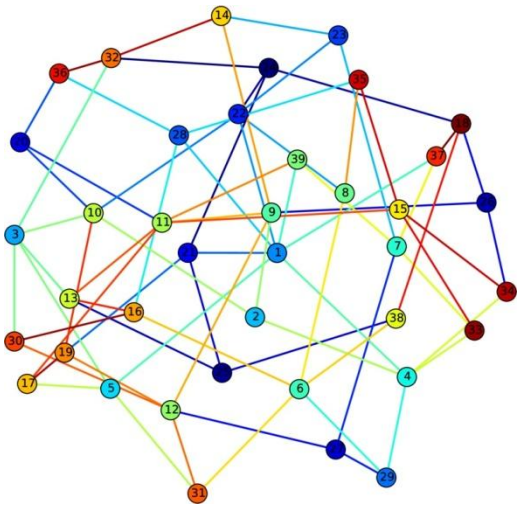




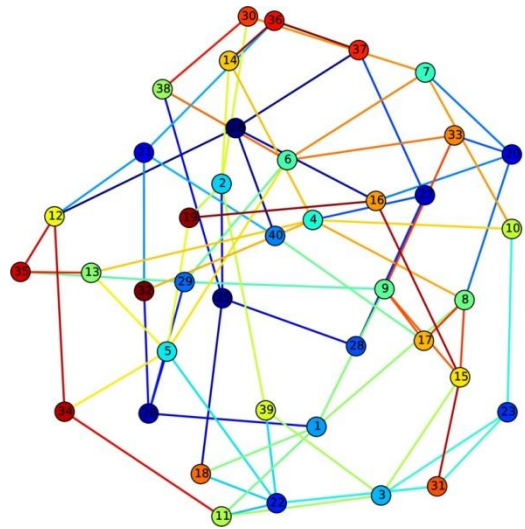
**37 vertices**



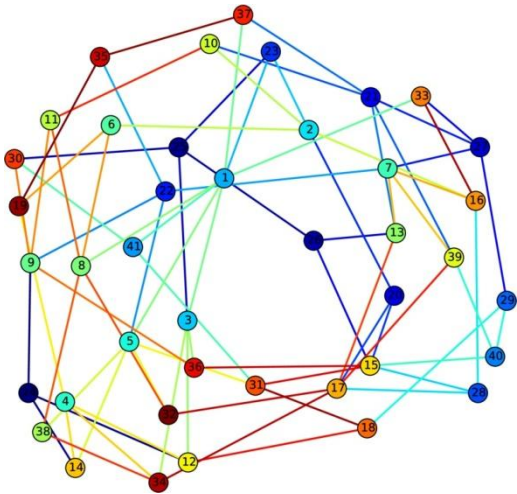
**38 vertices**



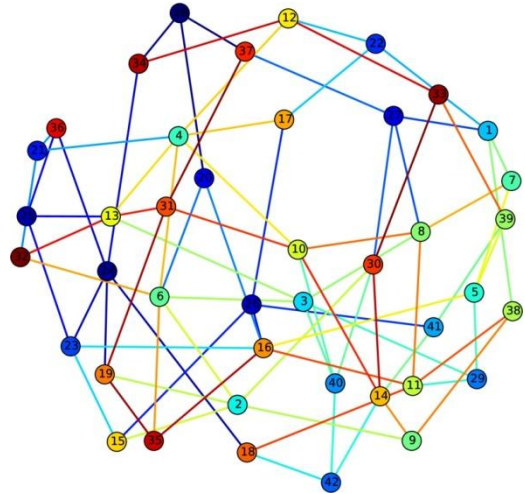
**39 vertices**



**40 vertices**

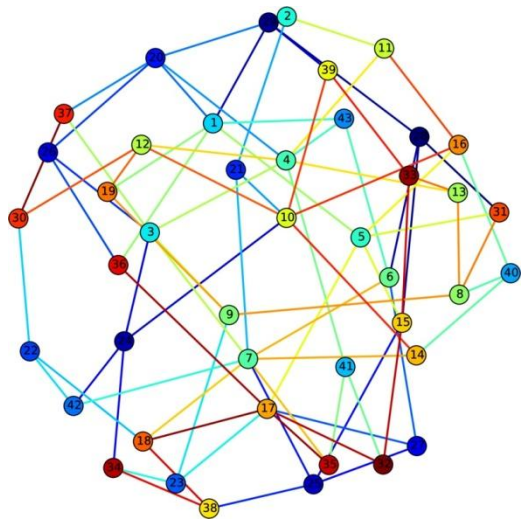


**41 vertices**

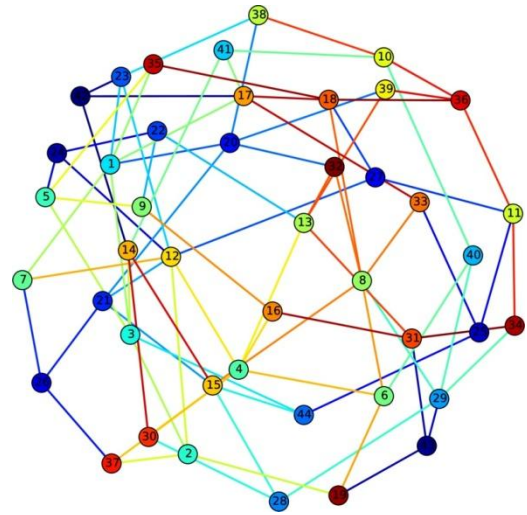


**42 vertices**

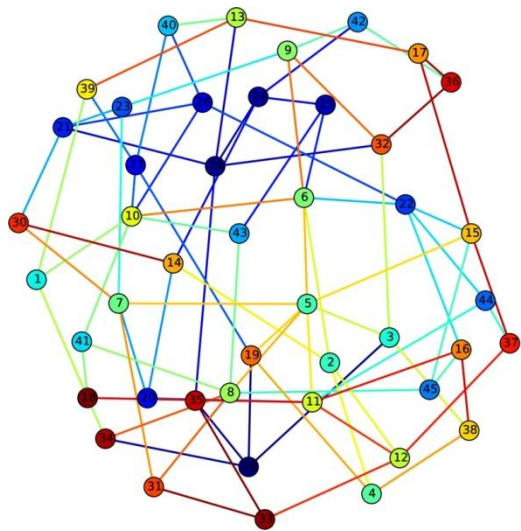




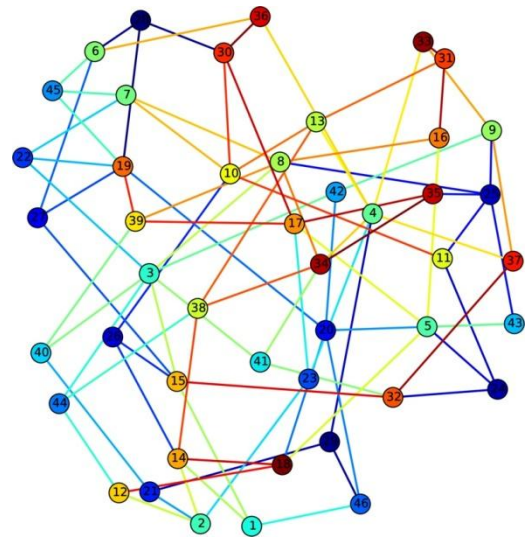
43 vertices



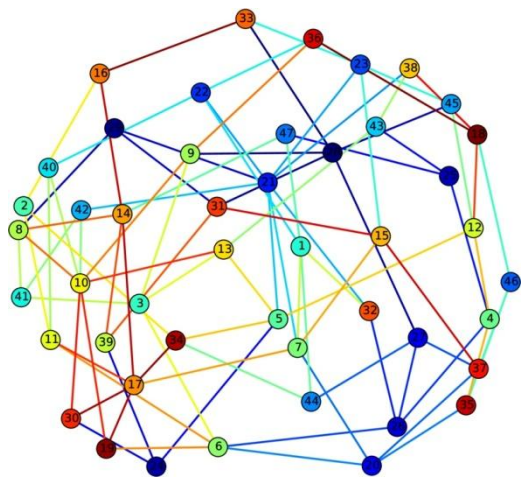
44 vertices



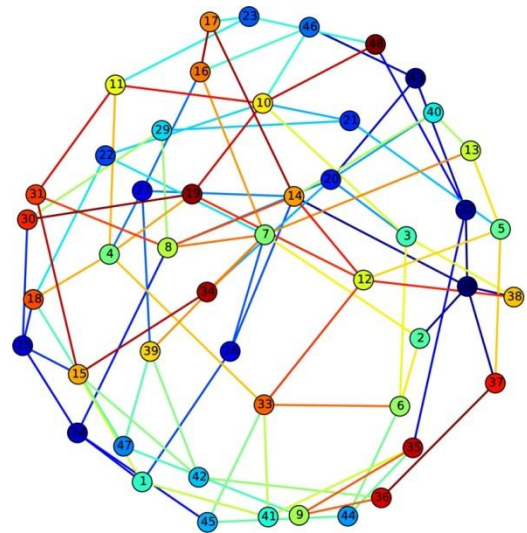
45 vertices



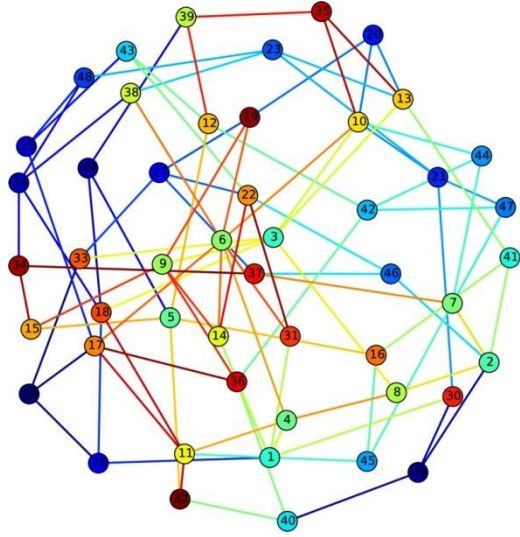
46 vertices



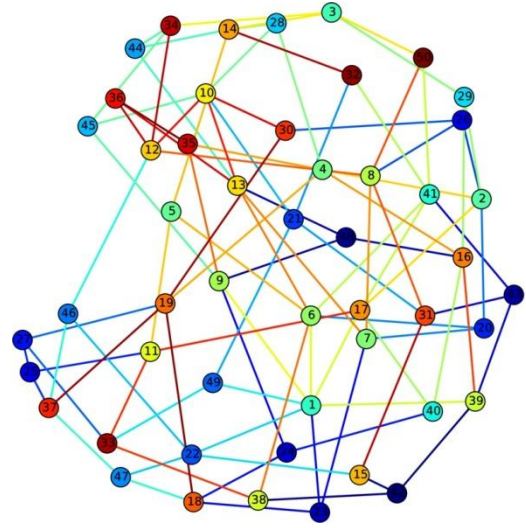
47 vertices



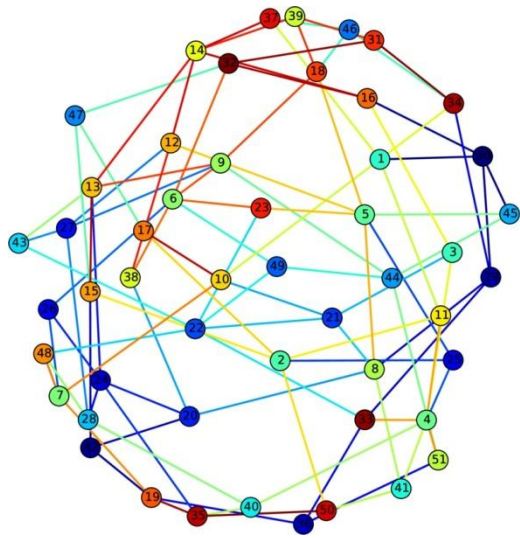
48 vertices



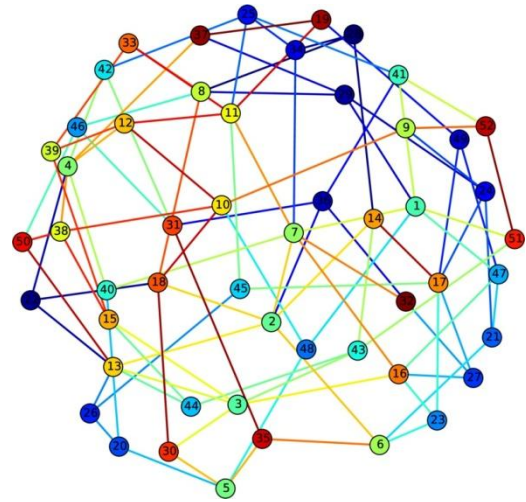
49 vertices



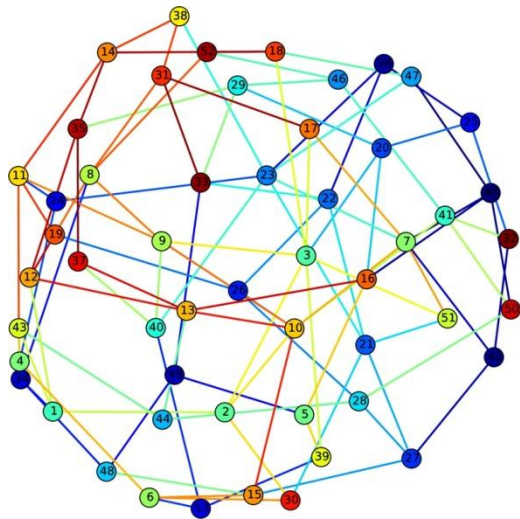
50 vertices



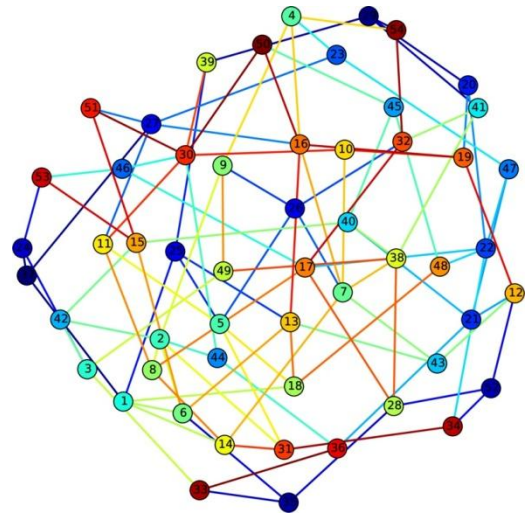
51 vertices



52 vertices

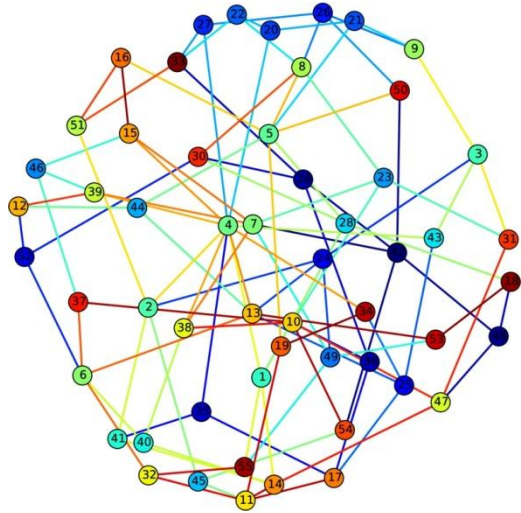


53 vertices

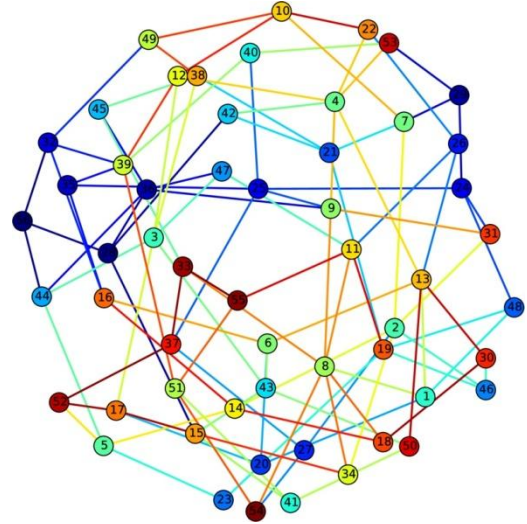


54 vertices

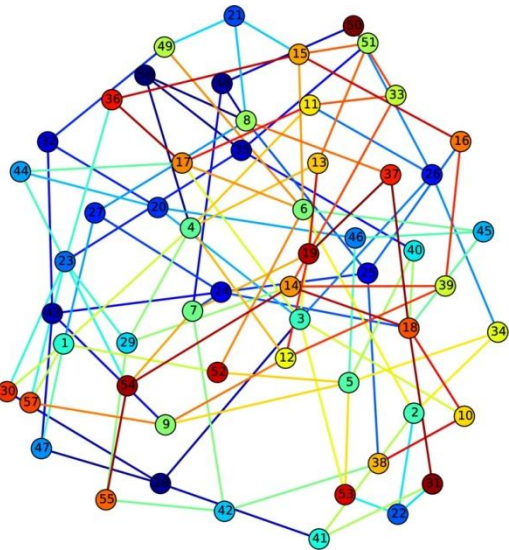




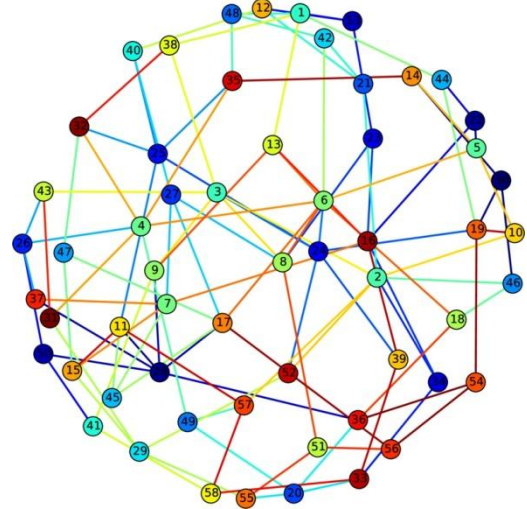
55 vertices



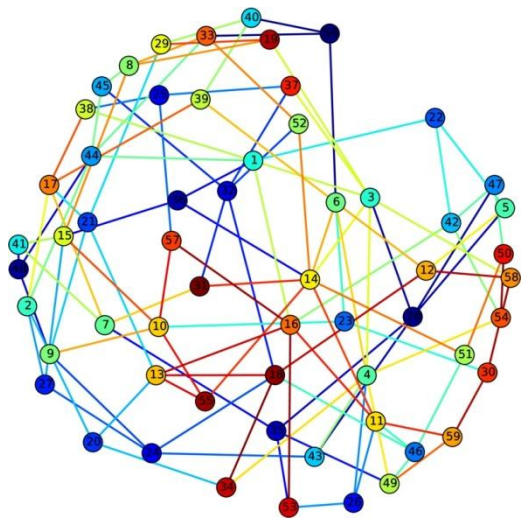
56 vertices



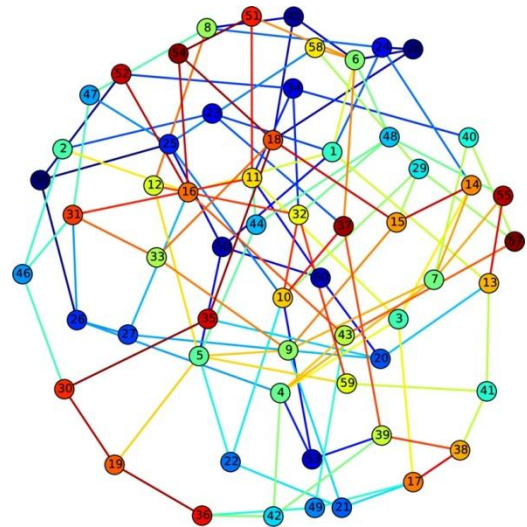
57 vertices



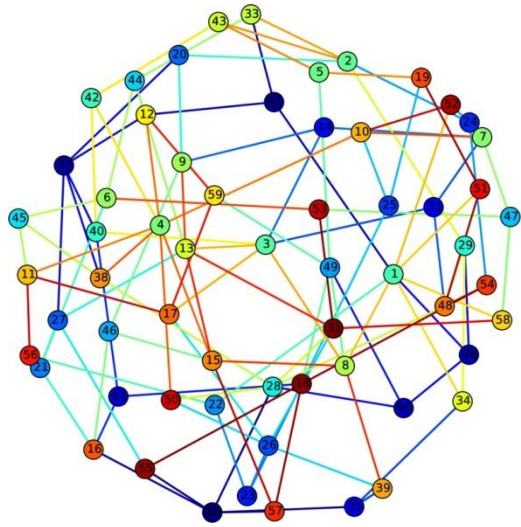
58 vertices



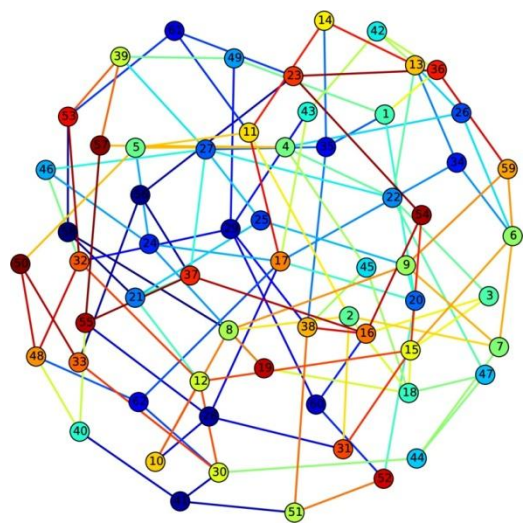
59 vertices



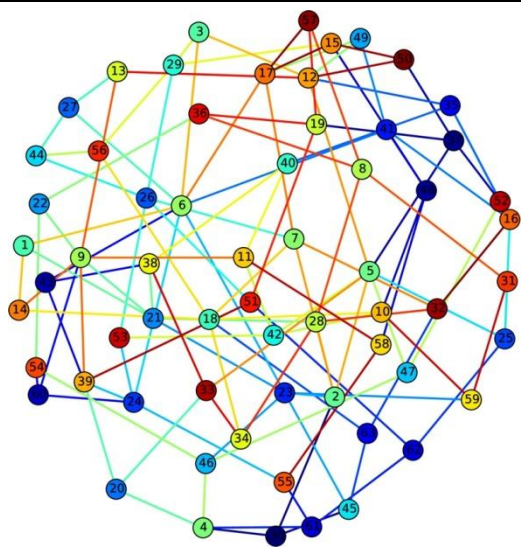
60 vertices



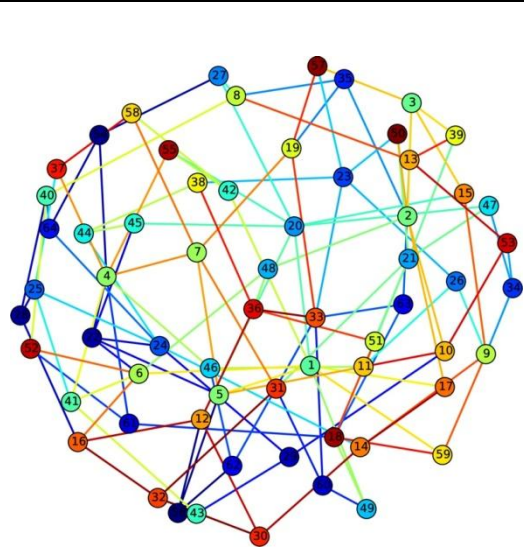
61 vertices



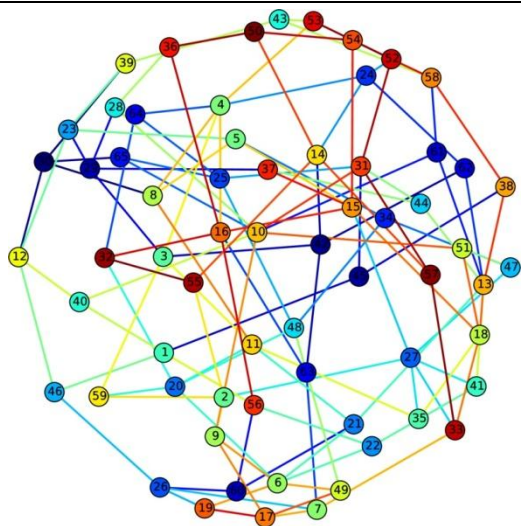
62 vertices



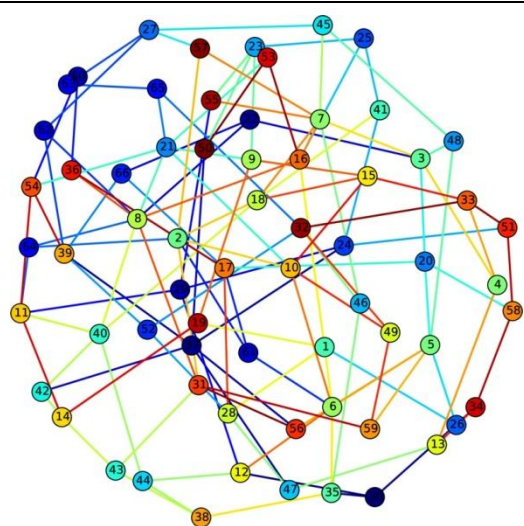
63 vertices



64 vertices

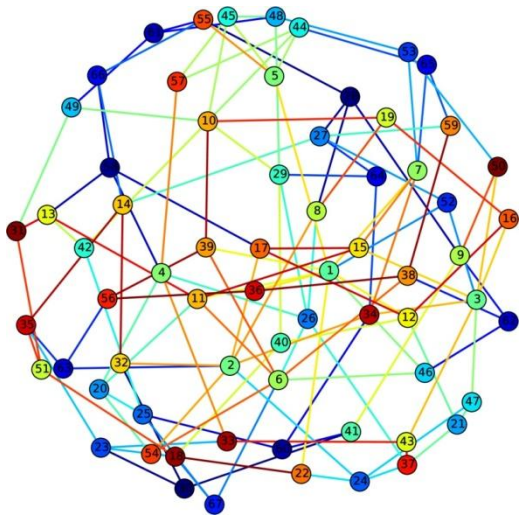


65 vertices

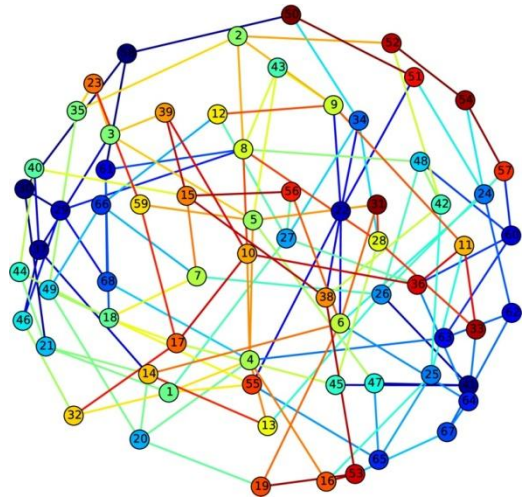


66 vertices

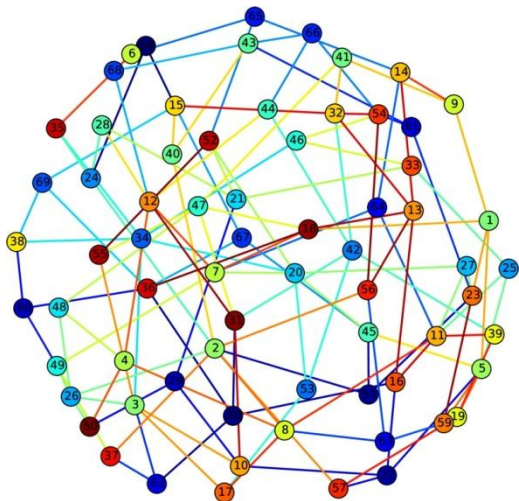




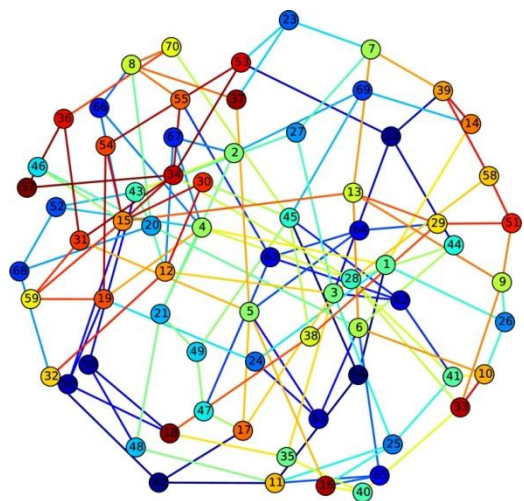
67 vertices



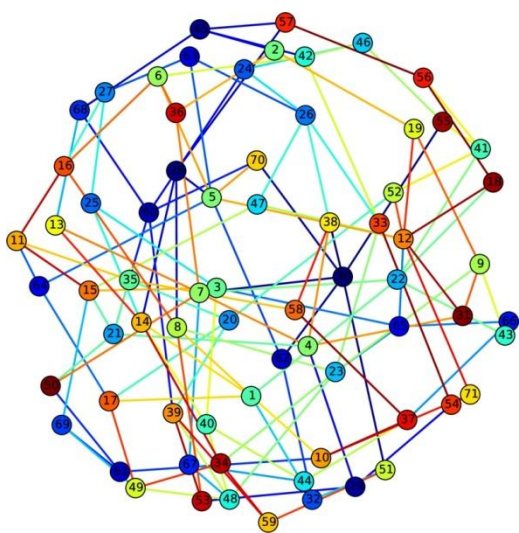
68 vertices



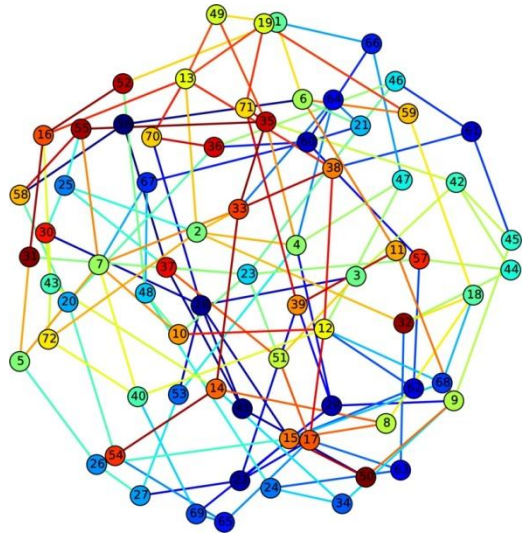
69 vertices



70 vertices

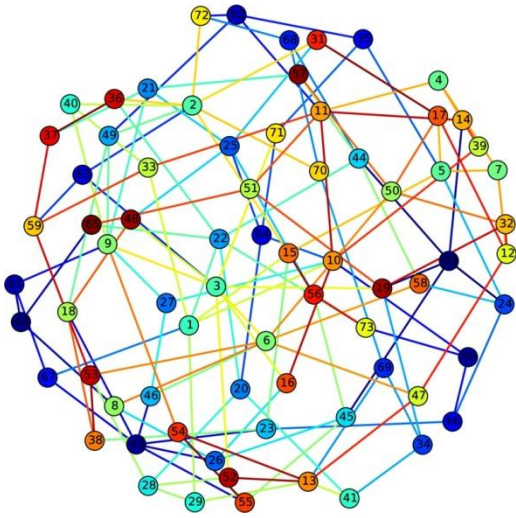


71 vertices

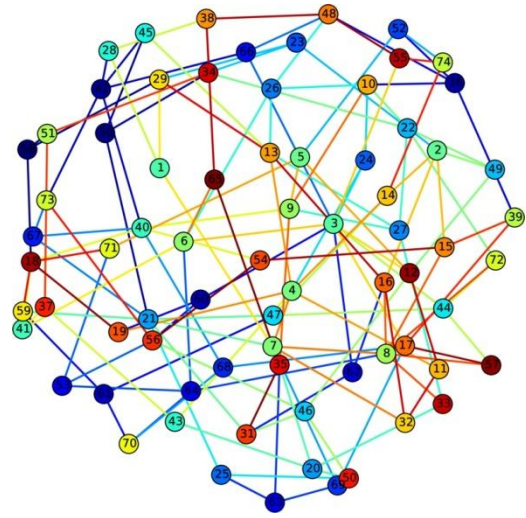


72 vertices

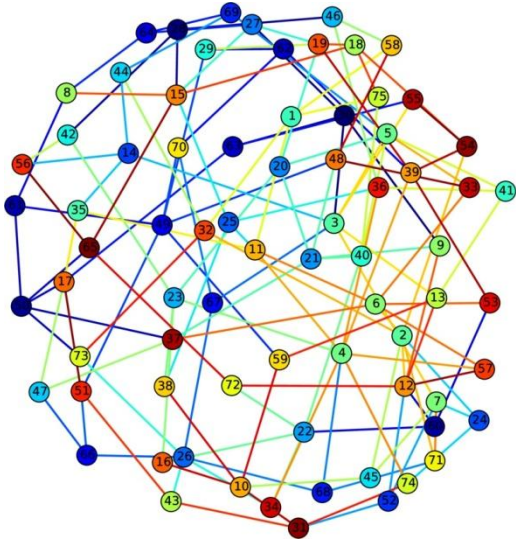




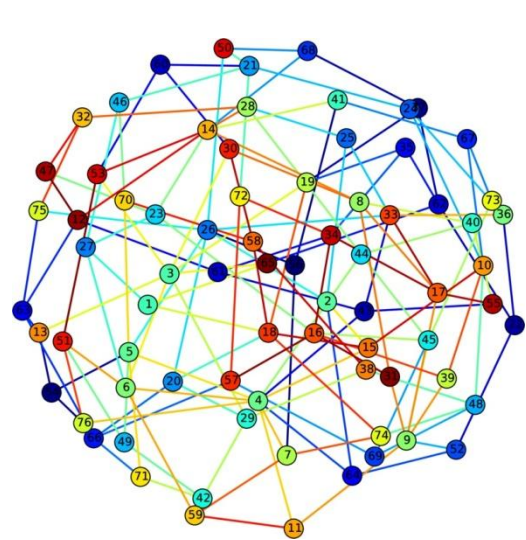
73 vertices



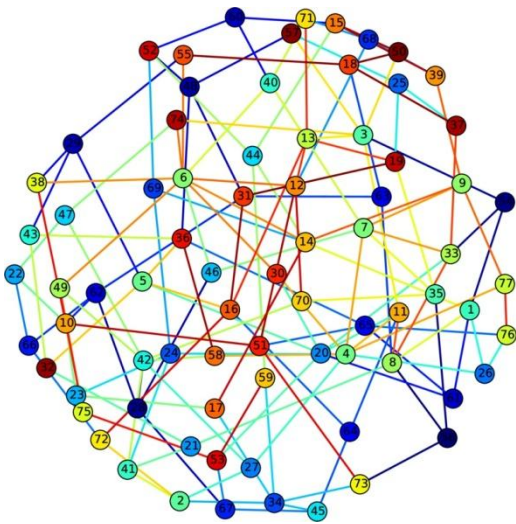
74 vertices



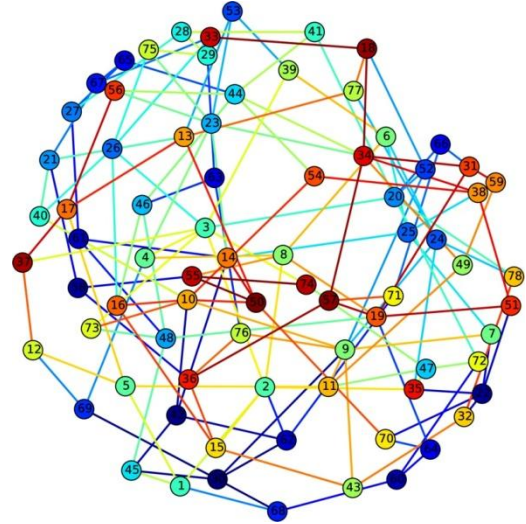
75 vertices



76 vertices

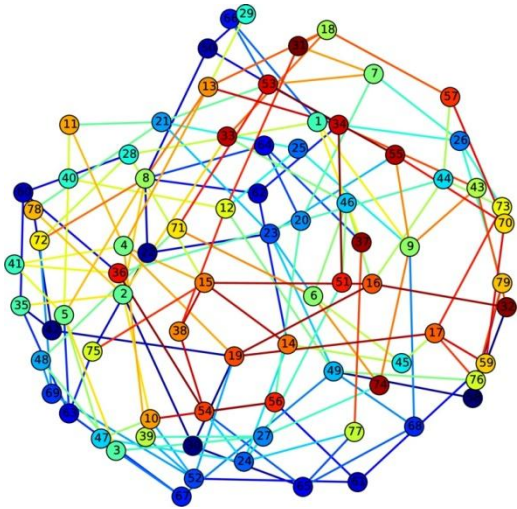


77 vertices

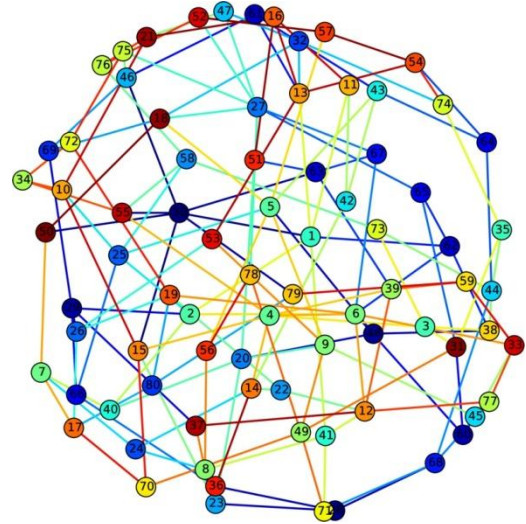


78 vertices

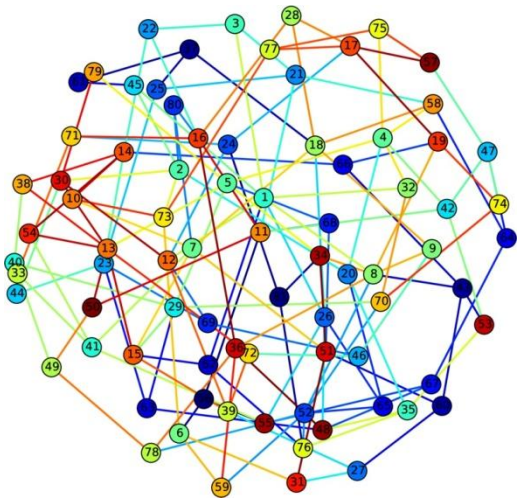




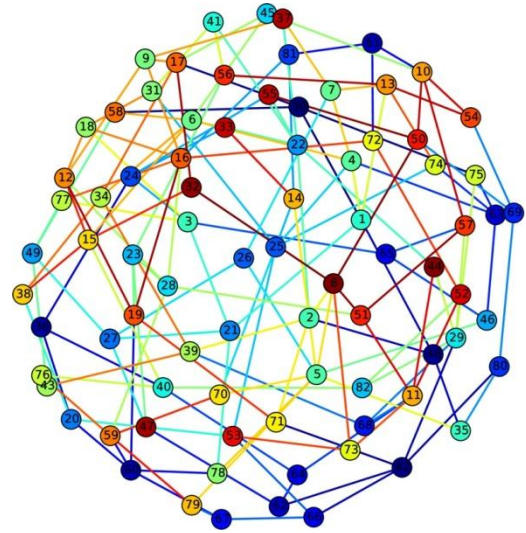
79 vertices



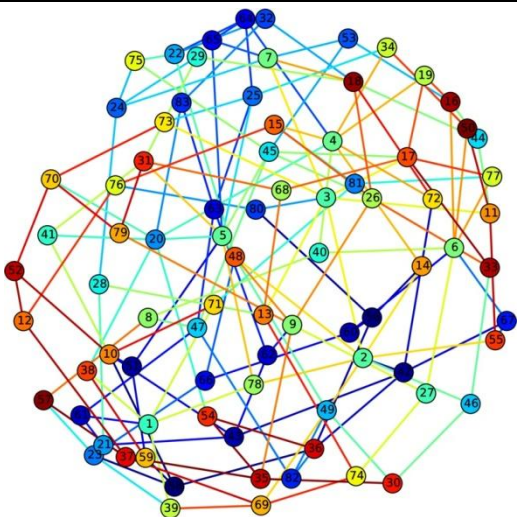
80 vertices



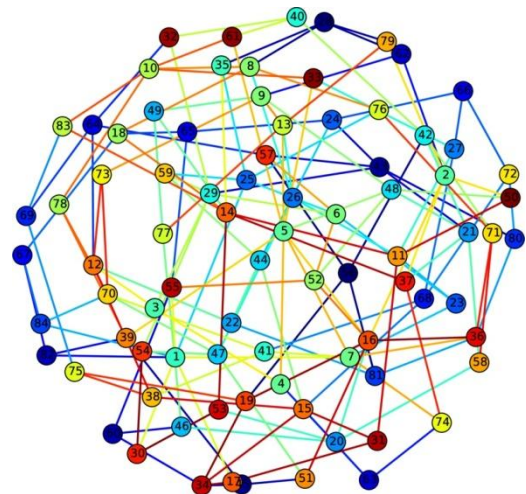
81 vertices



82 vertices

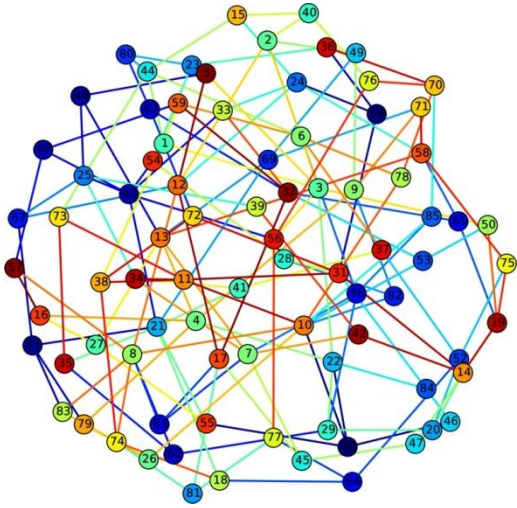


83 vertices

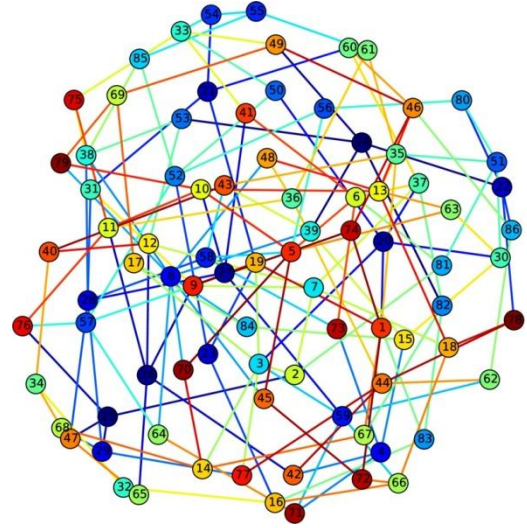


84 vertices

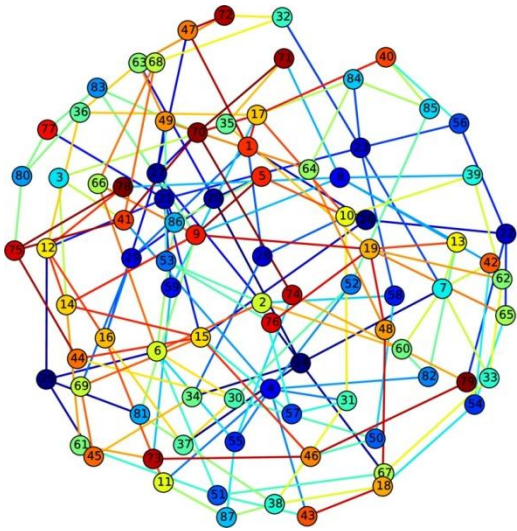




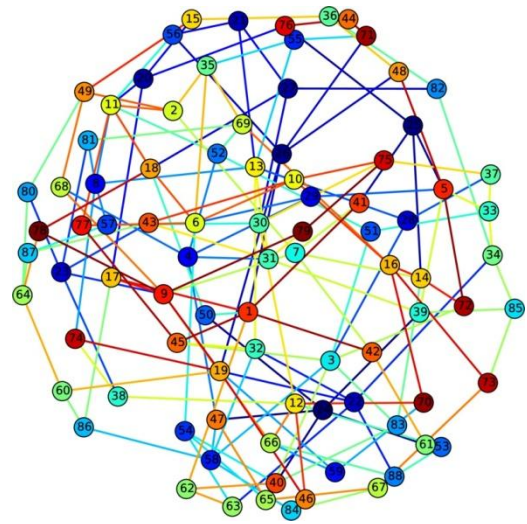
85 vertices



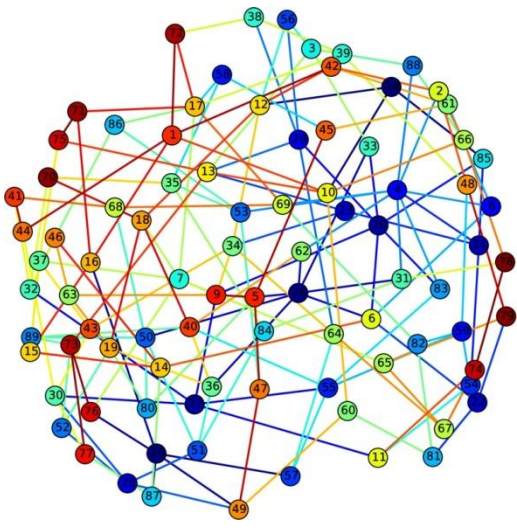
86 vertices



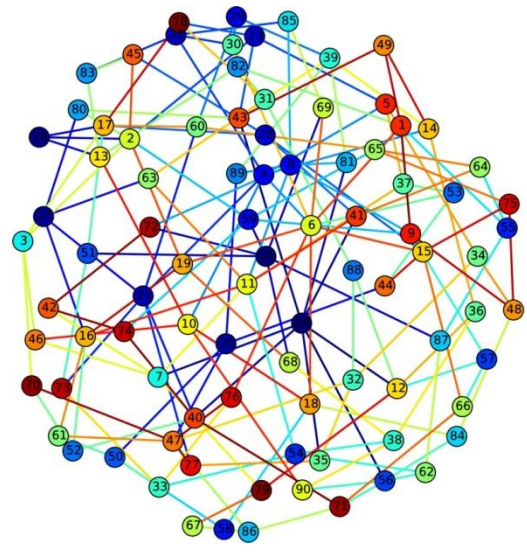
87 vertices



88 vertices

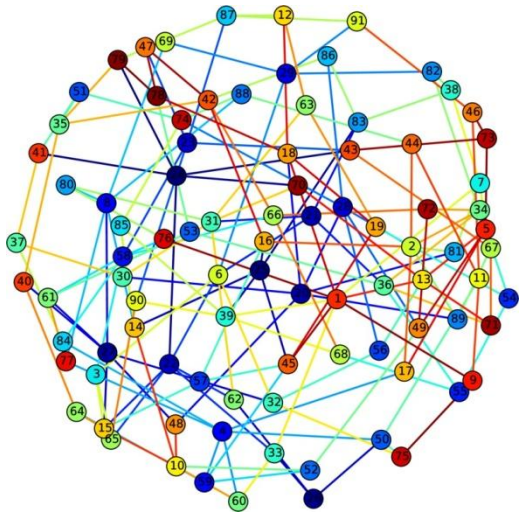


89 vertices

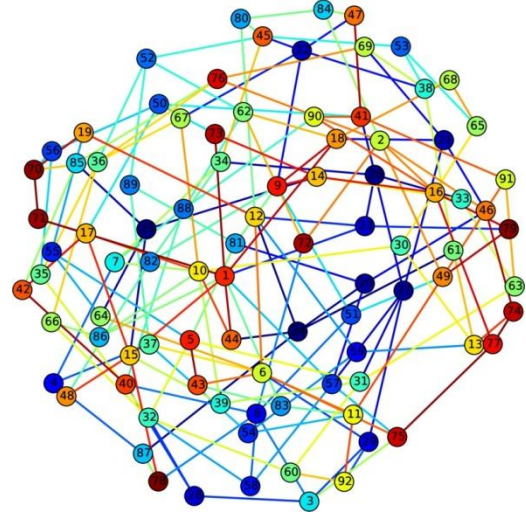


90 vertices

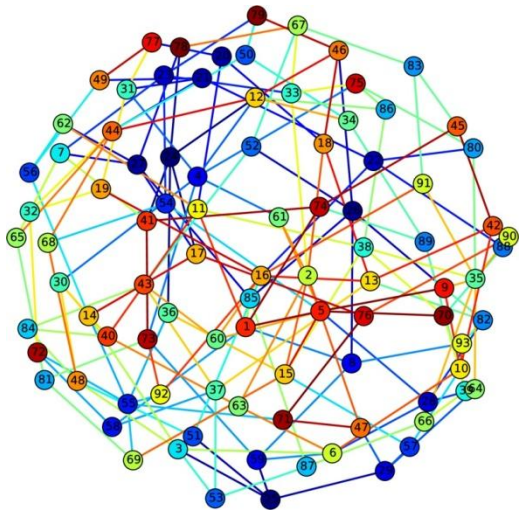




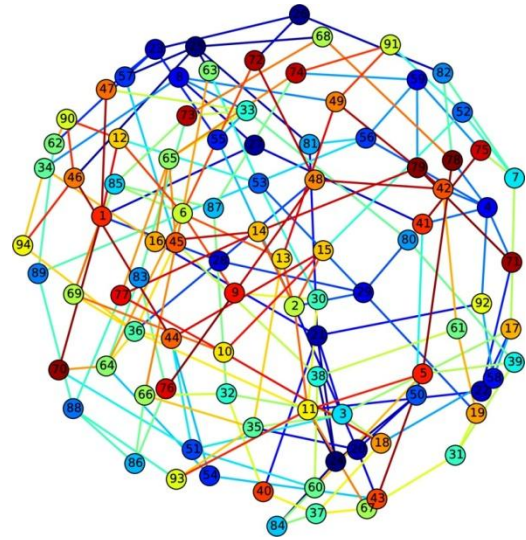
91 vertices



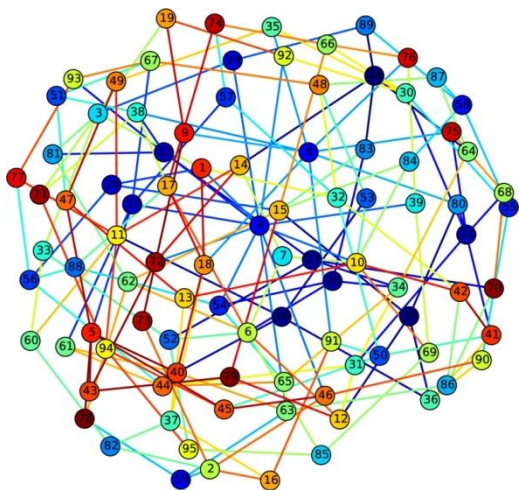
92 vertices



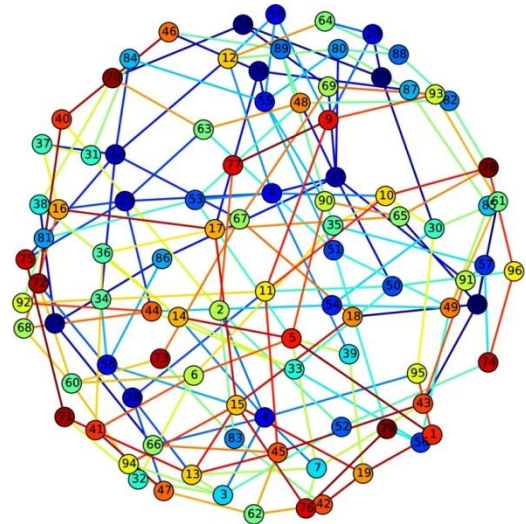
93 vertices



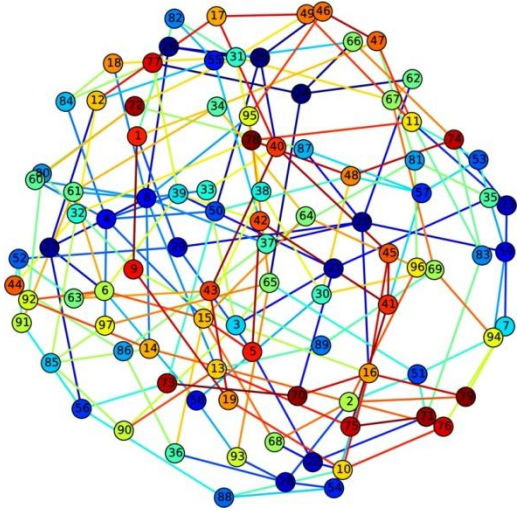
94 vertices



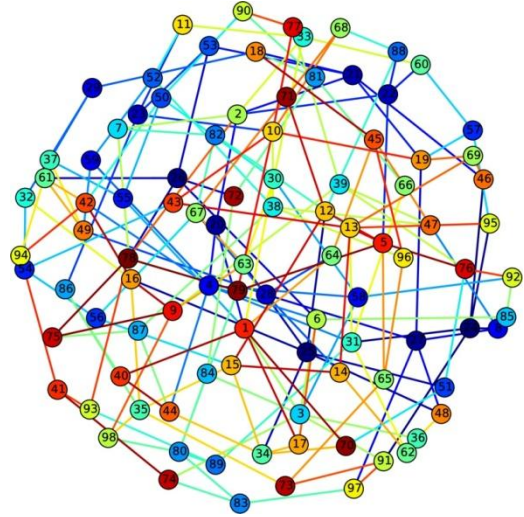
95 vertices



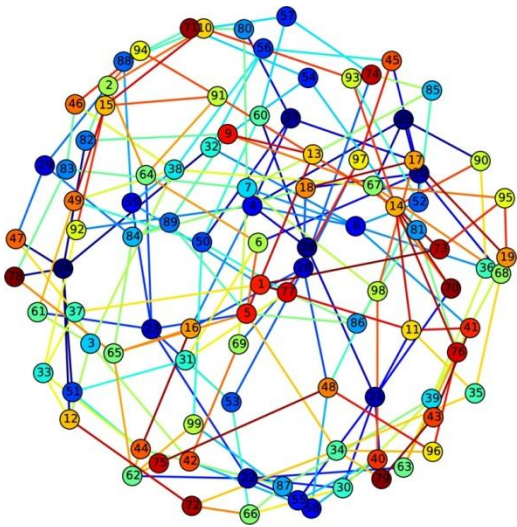
96 vertices



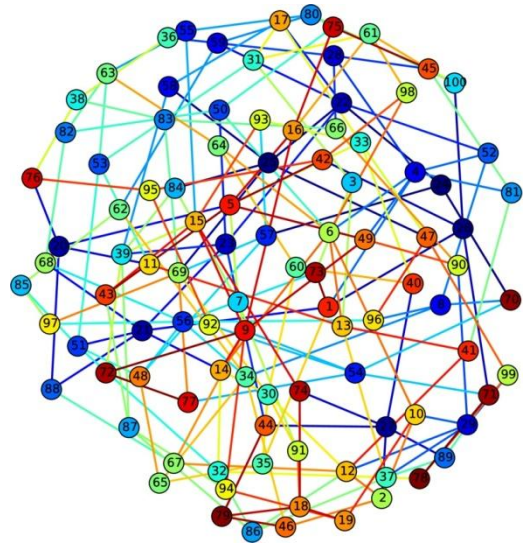
97 vertices



98 vertices



99 vertices



100 vertices