



**HAL**  
open science

# Optimal input design for parameter estimation of nonlinear systems: case study of an unstable delta wing

Jun Qian, Madiha Nadri, Pascal Dufour

## ► To cite this version:

Jun Qian, Madiha Nadri, Pascal Dufour. Optimal input design for parameter estimation of nonlinear systems: case study of an unstable delta wing. *International Journal of Control*, 2017, 90 (4), pp.873-887. 10.1080/00207179.2016.1225990 . hal-01518400

**HAL Id: hal-01518400**

**<https://hal.science/hal-01518400>**

Submitted on 4 May 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 SUBMISSION, SPECIAL ISSUE: IDENTIFICATION AND CONTROL OF  
2 NONLINEAR ELECTRO-MECHANICAL SYSTEMS

3 Optimal input design for parameter estimation of nonlinear systems:  
4 case study of an unstable delta wing

5 J. Qian<sup>a,b</sup>, M. Nadri<sup>a</sup> \* and P. Dufour<sup>a</sup>

6 <sup>a</sup>*Université de Lyon, Lyon F-69003, Université Lyon 1, CNRS UMR 5007, Laboratory of Process Control*  
7 *and Chemical Engineering (LAGEP), Villeurbanne 69100, France;*

8 <sup>b</sup>*Acsystème, 4 rue René Dumont, 35700 Rennes, France*

9 (*Received 00 Month 20XX; accepted 00 Month 20XX*)

10 A closed loop optimal experiment design for on-line parameter identification approach is developed  
11 for nonlinear dynamic systems. The goal of the observer and the nonlinear model predictive control  
12 theories is here to perform online computation of the optimal time-varying input and to estimate the  
13 unknown model parameters online. The main contribution consists in combining Lyapunov stability  
14 theory with an existing closed loop identification approach, in order to maximize the information  
15 content in the experiment and meanwhile to asymptotically stabilize the closed loop system. To  
16 illustrate the proposed approach, the case of an open loop unstable aerodynamic mechanical system is  
17 discussed. The simulation results show that the proposed algorithm allows to estimate all unknown pa-  
18 rameters, which was not possible according to previous work, while keeping the closed loop system stable.  
19

20 **Keywords:** closed loop identification; optimal experiment design; model predictive control; stability;  
21 nonlinear observer; persistent excitation, delta wing; mechanical system.

22 **1. Introduction**

23 Accurate modeling is required for simulation, optimization or control of dynamic processes. There-  
24 fore, the identification of unknown model parameters can not be avoided. Badly designed exper-  
25 iments for model parameter identification can be costly (e.g. materials fed at the process inlet,  
26 energy consumption during such experiments, output materials with undesired properties) and  
27 increase the time needed for pure model identification. Optimal experiment design (OED) is a  
28 classical technique for parameter identification purposes (Goodwin et al., 1977; Ljung, 1999). How-  
29 ever, in a large part of the existing literature on OED for parameter identification, optimal input  
30 design is separated from parameter estimation. In that case, the experimental data is gathered  
31 from previous experiences for further offline model parameter estimation (Barz et al., 2012; Ljung,  
32 1999; Walter and Pronzato, 1994). Moreover, for many decades OED for parameter estimation has  
33 only been applied to linear or approximated linearized models (Franceschini and Macchietto, 2008;  
34 Keviczky, 1975; Ng et al., 1977), whereas in areas such as biological and chemical processes, models  
35 are highly nonlinear, even sometimes unstable.

36 Recently, coupled closed loop OED and online parameter estimation approaches have been de-  
37 veloped by several authors for nonlinear multivariable stable systems. The basic objective in this  
38 kind of approach is to maximize the information content of the experiments to improve the ac-

---

\*Corresponding author. Email: nadri@lagep.univ-lyon1.fr

39 curacy of parameter estimation, which is usually described by a sensitivity criterion based on the  
 40 Fisher information matrix (FIM)(Goodwin et al., 1977; Ljung, 1999; Walter and Pronzato, 1994).  
 41 In (Jayasankar et al. , 2010), the authors developed an OED for on-line parameter estimation in  
 42 the multivariable case but without any concern with the closed loop stability. In (Zhu and Huang,  
 43 2011), the authors used steady state analysis to add linear equality constraints to an extended  
 44 Kalman filter based approach to reduce the influence of poor initial parameter guesses.

45 These techniques for closed-loop identification purposes were addressed for open loop stable  
 46 nonlinear systems. However, for industrial applications it is often necessary to account for process  
 47 constraints. Model Predictive Control (MPC) strategy is usually considered for solving this kind  
 48 of optimal control under constraints, which classically aims to drive the process state to a known  
 49 target value (i.e. set-point or trajectory tracking). Recently, the field of economic MPC (EMPC)  
 50 has emerged (see: (Rawlings et al., 2012), the recent special issue on this topic in the Journal of  
 51 Process Control 24(8) and the references in (Ellis et al., 2014)). In contrary to classical MPC,  
 52 EMPC aims to drive (most of the time) the process state in a particular time-varying state (that  
 53 is unknown, in contrary to the MPC set-point) and which optimizes the defined economic cost  
 54 function. In MPC and EMPC, some works are also dealing on stabilization issues (see for example  
 55 (Huang et al., 2011; Zavala and Biegler, 2009)).

56 In this paper a closed loop on-line optimal identification approach for a class of dynamic systems  
 57 is proposed, which optimizes an economic cost function and in the meantime maintains the closed  
 58 loop asymptotically stable. Combining observer theory and nonlinear predictive control theory,  
 59 the basis of the closed-loop OED for online identification approach proposed here was initially  
 60 introduced in (Flila et al., 2008). In this initial work, the nonlinear model was linearized and the  
 61 authors considered only the mono-variable case (a single input, a single measured state and a single  
 62 unknown constant parameter) for stable nonlinear systems. In contrast to this previous work, the  
 63 approach proposed here is developed for a general multi-variable case of open loop unstable or stable  
 64 nonlinear systems, with input constraints and where the state may not be entirely measured. The  
 65 novelty in the present paper is in the extension of the previous approach of (Qian et al., 2013) to  
 66 guarantee local stability of the closed loop of nonlinear dynamic system by integrating a Lyapunov  
 67 stability criterion (Calvet et al., 1989; Castillo et al., 2012) into the optimal control problem.

68 The paper is organized as follows. An outline of basic components and requirements needed  
 69 for the proposed approach is presented in Section 2. Then in Section 3, the closed loop optimal  
 70 identification approach for nonlinear dynamic model is developed which also stabilizes the system  
 71 in a neighborhood of a steady state. This approach is illustrated in Section 4 through an example  
 72 of a nonlinear open loop unstable dynamic system: a mechanical rolling delta wing. The obtained  
 73 simulation results and analysis are given in Section 5.

## 74 2. Problem statement

### 75 2.1 *Class of systems considered*

76 The proposed approach is dedicated to processes that feature some dynamic behavior. Meanwhile,  
 77 at least one output  $y_p$  must be available as an on-line measure and the manipulation of at least  
 78 one exogenous input  $u$  must be possible on-line by a controller. Some constraints may be specified  
 79 on the magnitude and velocity of the manipulated input<sup>1</sup>. Hence, this covers a very large number  
 80 of potential applications.

81 Models considered here are nonlinear (or linear) in terms of state representation and/or in terms of  
 82 model parameters. This dynamic multivariable model is described by ordinary differential equations

---

<sup>1</sup>Other constraints may be specified on the measured outputs or estimation of the process states (dealing with safety, set-point tracking within error bounds, production, ...). In order to handle the stability with the method detailed here, such constraints are not included.

83 as follows:

$$\begin{cases} \dot{x}(t) &= f(x(t), \theta, u(t)) \\ y(t) &= h(x(t), \theta, u(t)), \end{cases} \quad (1)$$

84 where  $x \in \mathcal{R}^n$  is the state vector,  $y \in \mathcal{R}^p$  is the output vector,  $u \in \mathcal{U} \subset \mathcal{R}^m$  denotes the vector  
85 of manipulated inputs,  $\theta \in \mathcal{R}^q$  is the unknown constant model parameter vector,  $f$  and  $h$  are  
86 nonlinear functions of suitable dimensions.

87 **Assumption 1:** *In system (1),  $f$  and  $h$  are  $C^\infty$  with respect to their arguments.*

## 88 2.2 Problem formulation

89 The use of model (1) for control or/and simulation requires an identification step to get the value of  
90 the unknown model parameter vector  $\theta$ . Here, as motivated before, we are interested in a coupled  
91 closed loop OED and online parameter estimation approach. This consists in on-line designing the  
92 optimal experiment by an optimal design of the input vector  $u$  (which is constrained by physical  
93 limitations), while also on-line estimating the unknown parameter vector  $\theta$  of the model based on  
94 the on-line process measures  $y_p$ . In order to do this, the following two main tools are needed.

### 95 2.2.1 Online optimal input design

96 MPC strategy is usually considered for solving such optimal control problems with input con-  
97 straints. The idea of an MPC strategy is to on-line solve a constrained optimization problem at  
98 each current sampled time  $t_k$ <sup>2</sup> in order to minimize (or maximize) a criterion  $J$ :

$$J(\tilde{u}(l|k)) = \sum_{l=k+1}^{k+N_p} \phi(y_p(k), y(l), u(l)), \quad (2)$$

99 where the prediction horizon is  $N_p$ .  $y(l)$  is the prediction of the process output vector over this  
100 prediction horizon, obtained from the model (1) where the initial state  $x(k)$  is the last measured or  
101 estimated process state. At the current time  $k$ , to optimize  $J$ , this controller aims to determine an  
102 optimal sequence of inputs over the prediction horizon ( $\tilde{u}(l|k) = \{u(k), \dots, u(l), \dots, u(k + N_p)\}$ )  
103 under input constraints:

$$\begin{cases} u_{min} \leq u(l) \leq u_{max} \\ \Delta u_{min} \leq \frac{u(l) - u(l-1)}{T_s} \leq \Delta u_{max} \end{cases} \quad (3)$$

104 The first control objective is to design a suitable cost function  $\phi$  in the criterion (2): here, it aims  
105 to maximize the information content of the experiments to improve the accuracy of the parameter  
106 estimation and ensure closed loop stability. To design the cost function  $\phi$ , we are here inspired  
107 by the recent concept of EMPC which aims to optimize the economic performance of the process  
108 directly in real time, instead of solving a set-point or reference trajectory tracking. With EMPC,  
109 in recent works (Angeli et al., 2012), (Zanon et al., 2014) and (Alangar et al., 2015), the initial  
110 economic cost function is penalized by a quadratic term. This allows us to guarantee the closed  
111 loop stability in a neighborhood of a steady state. Meanwhile, in these works, it is either assumed

---

<sup>2</sup>To simplify the notation in the following discrete formulations,  $s(k) = s(t_k)$  (resp.  $s(l) = s(t_l)$ ) represents the value of the signal  $s$  at the current (resp. future) discrete time  $t = k \times T_s$  (resp.  $t = l \times T_s$ ), where  $T_s$  is the constant sampling time and  $k$  (resp.  $l$ ) is a time index (i.e. an integer). For the input, a zero order hold is used between two consecutive sampling time instants. The various models are still formulated in a continuous framework and are solved numerically. Hence, sampled values may be taken at any discrete time. It is assumed that process data may also be sampled at the same rate.

112 that all the model parameters are known, or that the state disturbance vector is bounded.  
 113 Based on this, the second control objective is to propose a method to solve online this constrained  
 114 optimization problem (2-3) to find the optimal input  $u(k)$  to apply at each time on the real process.  
 115 The prediction of the output  $y(l)$  in the cost function (2) depends also on the value of the parameter  
 116 vector  $\theta$  in the model (1). Here, since such parameters are unknown, they must be estimated online.  
 117 In the field of parameter identification, one of the very common techniques in the engineering  
 118 literature is based on the Kalman filter. In the following we recall main lines of this technique.

119 *2.2.2 Observer design for online identification*

120 This technique consists first in extending model (1) with its unknown constant parameters  $\theta$  to be  
 121 estimated (see for instance (Cox, 1964); (Nelson and Stear, 1976); (Ljung, 1979)):

$$\begin{cases} \dot{x}(t) &= f(x(t), \theta, u(t)) \\ \dot{\theta} &= 0 \\ y(t) &= h(x(t), \theta, u(t)). \end{cases} \quad (4)$$

122 In the following, the augmented state vector is noted  $x_a = [x \ \theta]^T \in \mathcal{R}^{n+q}$  and the vector  
 123 function is  $f_a = [f \ 0]^T$ .

124 In the nonlinear case, different observer design techniques have been proposed such as: high gain  
 125 observer (Gauthier et al., 1992), extended Kalman filter (EKF) (Besançon, 2007b) or adaptive-gain  
 126 observer (Boizot et al., 2010; Nadri et al., 2013). The choice of the observer structure depends on  
 127 the model structure and its observability property.

128 The nonlinear system (4) considered here may *a priori* admit inputs for which observability is  
 129 not guaranteed (i.e. is not uniformly observable). Consequently, to design an observer for nonlinear  
 130 systems requires the observability property which generally depends on the applied inputs (i.e.  
 131 the sensitivity of the measurements with respect to the inputs). This addresses the well known  
 132 important issue of persistency of excitation, which is still an open issue in the nonlinear case.

133 For state affine nonlinear systems, appropriate inputs have been characterized. In this case,  
 134 observability corresponds to the notion of regularly persistent inputs which is clearly formulated  
 135 using the Gramian of observability (Bornard et al. (1988)). We can find also some earlier results  
 136 on input conditions to guarantee a possible observer design, depending on the system structure in  
 137 (Besançon (2007c); Dufour et al. (2012)). However, the design of such inputs is still difficult and  
 138 and in practice done heuristically.

139 Under observability conditions, a global observer can be designed for the augmented system (4):

140 **Definition 1:** *A global observer based on the augmented model (4) can be given by a dynamical*  
 141 *system as:*

$$\begin{cases} \dot{\hat{x}}_a(t) &= f_a(\hat{x}_a(t), u(t)) + g_a(t, h(\hat{x}_a(t), u(t)) - y_p(t)) \\ \text{with: } &g_a(t, 0) = 0, \end{cases} \quad (5)$$

142 where  $g_a$  is a function (of the output estimation error) to be designed and  $y_p$  is the process output  
 143 vector (on-line measures) such that

- 144 i) if  $\hat{x}_a(0) = x_a(0)$ , then  $\hat{x}_a(t) = x_a(t)$ ,  $\forall t \geq 0$ ;  
 145 ii) if  $\forall x_a(0)$ ,  $\forall \hat{x}_a(0)$ , then  $\lim_{t \rightarrow +\infty} \|\hat{x}_a(t) - x_a(t)\| = 0$ .

146 The estimation problem consists in determining a gain structure  $g_a$  and tuning the observer  
 147 parameters such that the estimation error  $e(t) = x_a(t) - \hat{x}_a(t)$  converges asymptotically to zero.  
 148 Note that this convergence is also related to appropriate input excitation.

### 149 2.2.3 Problem considered

150 From (5), the observer design problem may be turned into an optimization problem in the sense  
 151 that we have to find an optimal input excitation  $u$  (corresponding to the observability of the  
 152 system) which guarantees the convergence of the estimation error.

153 Based on model (1) with unknown constant parameters  $\theta$ , a new optimal approach is proposed to  
 154 guarantee the observability of system (4) through an optimal persistent input excitation calculated  
 155 on-line. As a first step, an initial cost function is designed based on the sensitivity of the measure-  
 156 ments with respect to the inputs to guarantee the persistence of the input. Then, the problem of  
 157 closed-loop stability is considered. To guarantee this property, a second cost function is introduced  
 158 into the optimization problem.

159 **Remark 1:** - Note that the proposed approach can be also relevant in the case of uniformly ob-  
 160 servable systems in the sense that the designed optimal input should improve the robustness of the  
 161 observer with respect to measurement noise and model uncertainties.

162 - For a simple illustration of the approach, we assume in what follows that all the states are mea-  
 163 sured. However, in general the method does not require all states to be measured.

## 164 3. A closed loop optimal identification approach

### 165 3.1 Structure for coupled control and online parameter identification

166 In this paper, the developed general framework combines a closed loop OED with an observer for a  
 167 nonlinear dynamic system, which can be open loop unstable. Based on the chosen model structure,  
 168 an observer is designed for the augmented system.

169 Then, a sensitivity model is developed to capture the dynamics of the sensitivities of the state  
 170 vector to the unknown model parameters. Finally, the outputs of the four components (a process,  
 171 a model, an observer and a sensitivity model) are fed back into the optimal control problem which  
 172 is solved by the EMPC strategy taking into account the stability analysis.

173 Fig. 1 shows how the components needed in the proposed closed loop identification algorithm  
 174 are combined. In the following, the remaining components for the proposed closed loop optimal  
 175 identification approach are presented, leading to the final optimal control problem.

### 176 3.2 Sensitivity model

177 The sensitivity model is explicitly deduced from model (1). Sensitivity analysis tells us how the  
 178 unknown parameter vector  $\theta$  affects the model state  $x$  and the model output  $y$ .

179 Using the definition of the sensitivity function  $(\cdot)_\theta = \frac{\partial(\cdot)}{\partial\theta}$  of a variable  $(\cdot)$  with respect to the  
 180 parameters  $\theta$ , and the dynamical model (1), one gets the sensitivity model:

$$\begin{cases} \dot{x}_\theta(t) &= \frac{\partial f}{\partial x}(x(t), \theta, u(t)) x_\theta + \frac{\partial f}{\partial \theta}(x(t), \theta, u(t)) \\ y_\theta(t) &= \frac{\partial h}{\partial x}(x(t), \theta, u(t)) x_\theta + \frac{\partial h}{\partial \theta}(x(t), \theta, u(t)), \end{cases} \quad (6)$$

181 where  $x_\theta \in R^{n \times q}$  and  $y_\theta \in R^{p \times q}$  are the matrices of sensitivities of the states and the outputs,  
 182 respectively, with respect to the parameters.

### 183 3.3 Prediction model

184 Based on model (1), the prediction model (7) aims to predict, at the current time  $k$  and over the  
 185 prediction horizon  $N_p$ , the future model output  $y$ . It is based on the current estimation  $\hat{x}(k)$  of the

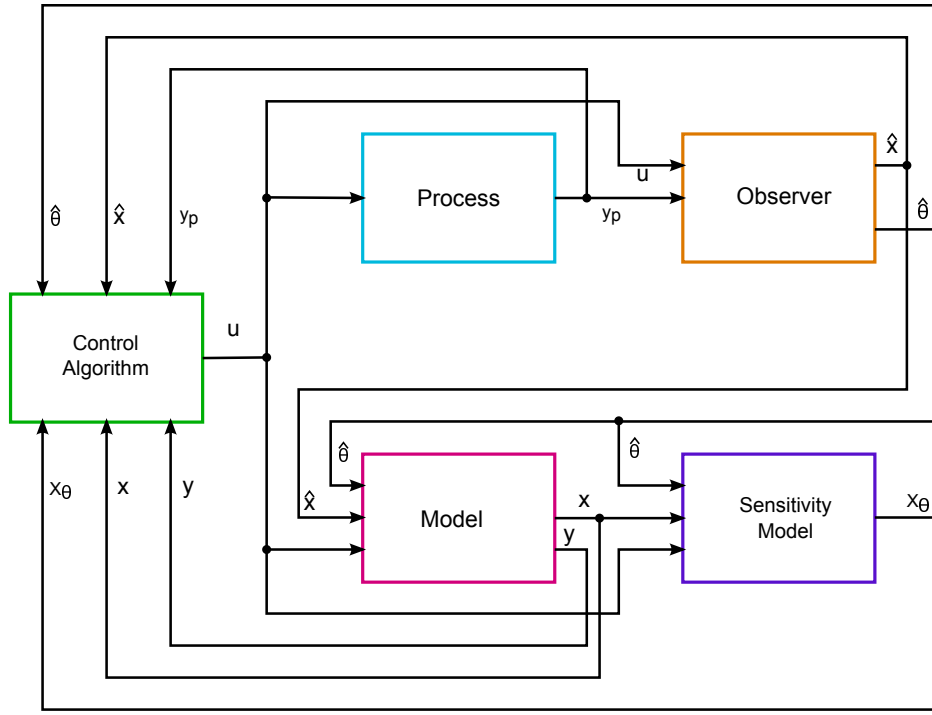


Figure 1. Closed loop control structure for on-line identification.

186 process state and the current estimation  $\hat{\theta}(k)$  of the unknown constant parameters given by the  
 187 observer (5):

$$\begin{cases} \dot{x}(t) = f(x(t), \hat{\theta}(k), u(t)) \quad \forall t \in ]t_k, t_{k+N_p}] \\ y(t) = h(x(t), \hat{\theta}(k), u(t)) \quad \forall t \in ]t_k, t_{k+N_p}] \\ x(t_k) = \hat{x}(k). \end{cases} \quad (7)$$

### 188 3.4 Prediction sensitivity model

189 Based on the sensitivity model (6), the prediction sensitivity model (8) aims to predict, at the  
 190 current time  $k$  and over the prediction horizon  $N_p$ , the future sensitivity  $y_\theta$  of the output of  
 191 the prediction model (7) with respect to the current estimation  $\hat{\theta}(k)$  of the unknown constant  
 192 parameters:

$$\begin{cases} \dot{x}_\theta(t) = \frac{\partial f}{\partial x} (x(t), \hat{\theta}(k), u(t)) x_\theta(t) + \frac{\partial f}{\partial \theta} (x(t), \hat{\theta}(k), u(t)), \quad \forall t \in ]t_k, t_{k+N_p}] \\ y_\theta(t) = \frac{\partial h}{\partial x} (x(t), \hat{\theta}(k), u(t)) x_\theta(t) + \frac{\partial h}{\partial \theta} (x(t), \hat{\theta}(k), u(t)), \quad \forall t \in ]t_k, t_{k+N_p}] \\ x_\theta(t_k) = \begin{cases} x_\theta(k|k-1), & \text{for } t_k > 0 \\ 0, & \text{for } t_k = 0, \end{cases} \end{cases} \quad (8)$$

193 where  $x_\theta(k|k-1)$  is the one step ahead solution of (8) at the time  $k-1$ .

194  
 195 Physical values involved in these sensitivities have usually different scales and units. So, in order  
 196 to re-scale the effects of the different parameters on the different outputs of the model, each

197 sensitivity is normalized with the relative-sensitivity function:

$$\begin{cases} \bar{x}_{i\theta_j} &= \frac{\hat{\theta}_j(k)}{x_i} x_{i\theta_j}; & i = 1, \dots, n; j = 1, \dots, q \\ \bar{y}_{i\theta_j} &= \frac{\hat{\theta}_j(k)}{y_i} y_{i\theta_j}; & i = 1, \dots, p; j = 1, \dots, q. \end{cases} \quad (9)$$

198 **3.5 Sensitivity criterion**

199 In the proposed approach, one of the objectives is to maximize the future information content of  
 200 the experiment which must be described by a sensitivity criterion. First, one defines a sensitivity  
 201 matrix  $\bar{y}_\theta(l)$ , which at the current instant  $k$  gives the prediction at a future time  $l > k$  of the  
 202 normalized output sensitivity matrix  $\bar{y}_\theta$  as

$$\bar{y}_\theta(l) = \begin{bmatrix} \bar{y}_{1\theta_1}(l) & \bar{y}_{1\theta_2}(l) & \dots & \bar{y}_{1\theta_q}(l) \\ \bar{y}_{2\theta_1}(l) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \bar{y}_{p\theta_1}(l) & \dots & \dots & \bar{y}_{p\theta_q}(l) \end{bmatrix}. \quad (10)$$

203 Using (10), we can now define the FIM as

$$M(l) = \|\bar{y}_\theta(l)\|^2. \quad (11)$$

204 This matrix contains the information of the experiment, at the current time  $k$ , for a future time  
 205  $l > k$ . Then, the classical  $E$ -optimality criterion

$$J_\theta(l) = \left| \frac{\lambda_{\min}(M(l))}{\lambda_{\max}(M(l))} \right|, \quad l \in ]k, k + N_p], \quad (12)$$

206 is defined to be used in a maximization problem within a EMPC framework to get an OED.

207 The criterion given by (12) is specific to the maximization of the information contained in the  
 208 FIM. Consequently, it does not take into account the eventual problem of system instability and  
 209 the resulting optimal control does not ensure closed-loop system stability. In the following, this  
 210 maximization problem is modified to take also this stability objective into account.

211 **3.6 Lyapunov local stability criterion**

212 The study of closed loop stability for nonlinear systems often relies on a Lyapunov function.  
 213 Concerning EMPC, in (Alangar et al., 2015), the decrease of the derivative of the Lyapunov function  
 214 is constrained by the predictive control resolution. But this relies on the perfect knowledge of the  
 215 model parameters. In the case of open loop unstable nonlinear dynamic systems with uncertain  
 216 parameters, it is always difficult to formulate an optimal control problem which can guarantee  
 217 closed loop stability. Following the nonlinear robust control design methodology (Başar et al.  
 218 (1995)), solving this problem relies on finding a positive definite and proper smooth Lyapunov  
 219 function satisfying the nonlinear Hamilton Jacobi Bellman equation, which can be difficult (or  
 220 impossible) to solve. However, if we focus on the linear approximation of the nonlinear system,  
 221 then this problem can be solved locally.



222 Now, let us consider the following controlled uncertain nonlinear system:

$$\dot{x} = F(x, u) + d(t), \quad x(0) = x_0, \quad (13)$$

223 where  $x$  in  $\mathcal{R}^n$  is the state vector,  $u$  in  $\mathcal{R}^m$  is the control input, and  $F : \mathcal{R}^n \rightarrow \mathcal{R}^n$  is a  $C^1$  function  
 224 such that  $F(0, 0) = 0$  and  $d$  in  $C_0(\mathcal{R}; \mathcal{R}^n)$  is an unknown external disturbance. The function  $F$   
 225 being smooth, we can introduce the two matrices  $(A(t), B(t))$  in  $\mathcal{R}^{n \times n} \times \mathcal{R}^{n \times m}$  with  $A(t) = \frac{\partial F}{\partial x} |_{(x,u)}$   
 226 and  $B(t) = \frac{\partial F}{\partial u} |_{(x,u)}$  to describe the first order approximation of system (13) at  $(x(t), u(t))$ . All  
 227 along this paper, it is assumed that the system (13) satisfies the following assumption:

228 **Assumption 2 (First order controllability):** *The pair of matrices  $(A(t), B(t))$  is controllable*  
 229  $\forall t$ .

230 Based on Assumption 2, we focus on the design of an optimal local controller for the linear time  
 231 varying system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + g(x, t), \quad (14)$$

232 where  $A(\cdot)$  and  $B(\cdot)$  are time varying state matrices given by (13).  $g(x, t)$  is a locally Lipschitz  
 233 continuous function nearby  $x = 0$  such that  $g(0, t) = 0, \forall t > 0$ , and which can be considered as a  
 234 “disturbance” of the system:

- 235 • if  $g(x, t) = 0$ , the system (14) is non-autonomous linear.
- 236 • if  $\lim_{\|x\| \rightarrow 0, x \neq 0} \frac{g(x, t)}{\|x\|} = 0$  uniformly with respect to  $t$ , the system (14) is non-autonomous quasi-  
 237 linear system (Calvet et al., 1989; Willems, 1970).

238 In the context of MPC technique, at time  $k$ , an update of the model parameters  $\hat{\theta}(k)$  is done and  
 239 the time varying matrices  $A(t)$  and  $B(t)$  of the linearized system (14) are updated  $(A(k), B(k))$   
 240 and considered constant over the prediction horizon  $N_p$ . Consequently, at time  $k$ , the Hamilton  
 241 Jacobi Bellman equality is an algebraic equation which is easy to solve :

$$P(k)A(k) + A(k)^T P(k) - P(k)B(k)R^{-1}B(k)^T P(k) + Q = 0, \quad (15)$$

242 where the unique solution  $P(k)$  is a positive definite matrix in  $\mathcal{R}^{n \times n}$ , with both  $Q$  and  $R$  are  
 243 positive definite matrices.

244 Now, let us assume that  $(A(k), B(k))$  is stabilizable and  $(A(k), Q^{\frac{1}{2}})$  is detectable, then  
 245  $V(x) = x^T P(k)x$  is a Lyapunov function for system (14).  
 246

247 Based on this, the optimal input  $u$  minimizing the cost :  
 248

$$J(u) = x^T(t)P(k)x(t), \quad (16)$$

249 stabilizes the solution of system (14) to the origin.  
 250

251 Then, based on the prediction of the model states  $x(l)$ , a stability criterion is established as a  
 252 quadratic Lyapunov function as follows:

$$J_L(l) = x^T(l)P(k)x(l), \quad l \in ]k, k + N_p]. \quad (17)$$

---

<sup>3</sup>As usual, the same development can be done for any pair  $(x, u) \neq (0, 0)$  that represents a steady state in (13).

253 By construction, this criterion  $J_L$  is therefore a positive definite and decreasing function. Con-  
 254 sequently, the input sequence  $\tilde{u}(l|k)$  minimizing  $J_L$  allows to stabilize locally asymptotically the  
 255 system (13) in closed loop.

256 In the following, the stability criterion  $J_L$  (17) based on  $P(k)$  has to be combined with the  
 257 sensitivity criterion  $J_\theta$  (12) to get the final criterion to maximize such that the EMPC can also  
 258 handle local stability. Otherwise, since that linearization is done around the augmented state vector,  
 259 robust stabilization particularly depends on the quality of convergence of the estimation error.  
 260 However, based on the update of the linearization at every time  $k$  and thanks to the optimization  
 261 algorithm which allows to increase the robustness of the observer to the measurements noise an  
 262 uncertainties, the local stability in closed loop is handled.

### 263 3.7 Final optimal control problem formulation

264 In (Qian et al., 2013) a first approach was designed based on an EMPC, where the cost function  
 265 aimed only to maximize the FIM of the experiment, while some fictitious output constraints were  
 266 designed by trial and error to stabilize the process in closed loop. The main drawback is that  
 267 design and tuning of output constraints is not easy. Compared to this result, in the present work,  
 268 instead of using fictitious output constraints, a dual cost function is formulated in an EMPC which  
 269 combines two parts in a constrained maximization problem: one part is, as previously (Qian et  
 270 al., 2013), a sensitivity criterion (12) to improve parameter estimation, and the second part (the  
 271 new part) is based on a Lyapunov function (17) deduced from Lemma 1 to locally asymptotically  
 272 stabilize the process in closed loop. Indeed, on one hand, if only the maximization of the FIM is  
 273 considered, the closed loop system may become unstable. On the other hand, if only the closed  
 274 loop stability of the system is considered, the information content in the experience may not be  
 275 rich enough to guarantee accuracy of the parameter estimation step. Similarly to the discussion  
 276 on the theorem 3 in (Angeli et al., 2012), the solution adopted here is to find the optimal control  
 277 which leads to a balance between parameter estimation and the stability of the system in closed  
 278 loop. Hence, one formulates the final optimal identification problem as follows:

$$\left\{ \begin{array}{l}
 \tilde{u}^*(l|k) = \arg \max_{\tilde{u}(l|k)} J \\
 \tilde{u}(l|k) = \{u(k) \cdots u(l) \cdots u(k + N_p)\}, l \in [k \ k + N_p] \\
 J = \frac{1}{N_p} \sum_{l=k+1}^{k+N_p} \left( \beta \frac{J_\theta(l)}{w_\theta} - (1 - \beta) \frac{J_L(l)}{w_L} \right) \\
 \text{under the input constraints, } (\forall k > 0) : \\
 \left\{ \begin{array}{l}
 u_{min} \leq u(k) \leq u_{max} \\
 \Delta u_{min} \leq \frac{u(k) - u(k-1)}{T_s} \leq \Delta u_{max}
 \end{array} \right. \\
 \text{based on the observer (5),} \\
 \text{the prediction model (7), the sensitivity model (8),} \\
 \text{the sensitivity criterion (12) and the stability criterion (17).}
 \end{array} \right. \quad (18)$$

279 **Remark 2:** Note that this formulation does not aim to solve a set-point regulation problem but  
 280 the OED. Therefore, only the input is constrained. Indeed, adding tight state constraints can be  
 281 penalizing for such a design of persistent excitation and handling such constraints requires more  
 282 knowledge on the system. Meanwhile, to ensure closed loop stability, the Lyapunov term (17) re-  
 283 places (in the sense of closed loop stability) the previous fictitious constraints on the output (Qian  
 284 et al., 2013).

285 Here,  $J$  is the final optimization criterion to maximize,  $\beta$  is the real weighting tuning parameter  
 286 in  $[0, 1]$  used to mix the two competitive parts involved in the cost function: the sensitivity criterion

287  $J_\theta$  and the Lyapunov stability criterion  $J_L$ <sup>4</sup>. Since these two criteria may have different scales, two  
 288 weights  $w_\theta$  and  $w_L$  are defined, to normalize  $J_\theta$  and  $J_L$  respectively in  $J$ . Hence, the pure sensitivity  
 289 maximization is one of the two extremes (i.e.  $\beta = 1$ ) where the outputs contain the maximum of  
 290 information in all the unknown model parameters but closed loop stability is not ensured. On  
 291 the other extreme (i.e.  $\beta = 0$ ), the pure stabilization problem is shown to have a well defined  
 292 Lyapunov function. In the meantime, the information content in the outputs might be not rich  
 293 enough, which may render the accurate estimation of all parameters of the model impossible. This  
 294 will be illustrated with the tuning of  $\beta$  in the case study in Section 5. Hence an optimal control  
 295  $\tilde{u}^*(l|k)$  is obtained that in a unique experience maximizes the content of information fed into the  
 296 observer used for the online parameter estimation, and meanwhile locally stabilize the system and  
 297 solve the disturbance attenuation in closed loop.

298 Moreover, in contrary to the case studied in (Angeli et al., 2012), the weighting matrix  $P$  involved  
 299 in the stability criteria  $J_L$  is not constant but is adapted online according to the estimation of the  
 300 model parameters.

301 Note also that, we do not need here to specify a weighting matrix to penalize the deviation from  
 302 the control input  $u$  away from 0 (the steady input).

303 The robustness of the proposed approach to measurements noise and model uncertainties relies  
 304 on the choice of the sensitivity cost function (12). However, to preserve this performance, the choice  
 305 of the parameter  $\beta$  is important and should be done based on prior knowledge on the real system.

### 306 3.8 Implementation of the algorithm

307 Based on the nonlinear constrained optimal problem (18) to be solved online, the implementation  
 308 of the EMPC strategy can be summarized as follows, at the current instant  $k$ :

- 309 • Step 1: update the input/output measures and apply at the process input the first component  
 310 of  $\tilde{u}^*(l|k-1)$  (with a zero order hold).
- 311 • Step 2: integrate the observer (5) to get an estimate of the state  $\hat{x}(k)$  and the unknown  
 312 constant parameters  $\hat{\theta}(k)$ , based on the current input and output measures.
- 313 • Step 3: solve online the nonlinear constrained optimization problem (18) to get the optimal  
 314 control sequence  $\tilde{u}^*(l|k)$ . This requires integrating the prediction model (7) and the prediction  
 315 sensitivity model (8) over the prediction horizon for all control sequence guesses  $\tilde{u}(l|k)$ . It is  
 316 based on the current input and output measures, the unknown parameter estimations  $\hat{\theta}(k)$   
 317 (considered as constant over the prediction horizon) and the state estimations  $\hat{x}(k)$
- 318 • Step 4: The first element  $u^*(k|k)$  of the optimal control sequence  $\tilde{u}^*(l|k)$  is applied at the  
 319 next sampling time  $k+1$  (via a zero order blocker).

320 At the next discrete time the whole procedure is repeated. Finally, this optimal control problem  
 321 is solved to estimate the unknown constant parameters, excite the system and stabilize the closed  
 322 loop.

323 This algorithm has been implemented in Matlab and under the name *ODOE4OPE* software<sup>5</sup>. The  
 324 models are solved with the Matlab ODE solvers. The local solution of the constrained optimization  
 325 problem is obtained by the Matlab *fmincon* routine.

326 It has to be noticed that this method may lead to a computational burden, especially for large  
 327 size systems with relatively short sampling times. Indeed, the search for the optimal  $\tilde{u}^*(l|k)$  at each  
 328 time  $k$  may be time consuming since the prediction model, the prediction sensitivity model, the  
 329 minimum and maximum eigenvalues of the predicted FIM must be computed repeatedly for each  
 330 guess of  $\tilde{u}(l|k)$ .

<sup>4</sup>As discussed previously, since it is a Lyapunov function and since the cost function  $J$  has to be maximized, one adds a minus sign before  $J_L$ .

<sup>5</sup>More information on [odoe4ope.univ-lyon1.fr](http://odoe4ope.univ-lyon1.fr)

331 **4. A rolling delta wing: the step by step procedure**

332 In this section, the developed closed loop on-line parameter identification approach is illustrated  
 333 step by step for an open loop unstable dynamic system: a mechanical rolling delta wing. This  
 334 application has been studied in (Jain et al., 2005) and in (Qian et al., 2013), whose results will be  
 335 compared to the simulation results obtained with the approach presented here.

336 **4.1 Step 1: Model, open loop analysis and recent works**

337 The nonlinear model features a single dimensionless input  $u(t)$ , a two dimensionless component  
 338 state vector  $x(t)$  and the five unknown dimensionless constant parameter vector  $\theta$  (Jain et al.,  
 339 2005):

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \alpha_1\theta_1x_1(t) + (\alpha_1\theta_2 - \alpha_2)x_2(t) \dots \\ &\dots + \alpha_1\theta_3x_1^3(t) + \alpha_1\theta_4x_1^2x_2(t) + \alpha_1\theta_5x_1x_2^2(t) + \alpha_3u(t), \end{cases} \quad (19)$$

340 where  $t$  is the dimensionless time and  $\alpha$  is the known constant parameter vector. The dimensionless  
 341 numerical values of constant known parameters and target values of the unknown parameters  
 obtained from (Nayfeh et al., 1989) are listed in the table 1. To underline the open loop instability

Table 1. Constant parameters: unknown values (top) and known values (bottom) (Nayfeh et al., 1989).

| index    | 1        | 2       | 3       | 4       | 5      |
|----------|----------|---------|---------|---------|--------|
| $\theta$ | -0.05686 | 0.03254 | 0.07334 | -0.3597 | 1.4681 |

| index    | 1     | 2     | 3 |
|----------|-------|-------|---|
| $\alpha$ | 0.354 | 0.001 | 1 |

342 of the system, the nonlinear system (19) is first linearized around the steady state  $(u^0, x^0)$ , with  
 343  $u^0 \in [u_{min} \ u_{max}] = [-0.01 \ 0.01]$  (which is in the following the domain of interest for this system).

344 Figure 2 shows each pair of steady states  $[x_1^0 \ x_2^0]$  and the real part of each eigenvalue of the  
 345 linearized system for the corresponding input  $u^0$ . The real parts of two eigenvalues are always  
 346 positive, which presents the instability of the delta wing model in open loop, and therefore the  
 347 need to design a stabilizing controller.  
 348

349 In previous work (Jain et al., 2005), a closed loop identification of the five unknown model  
 350 parameters is discussed. A feedback linearizing control is used such that the closed-loop behavior  
 351 matches with a specified second order linear reference model response with damped sinusoidal input  
 352 reference. Both designs of this model and its input reference are not really discussed in a general  
 353 framework. With their approach, the convergence of the two parameter estimations ( $\theta_1$  and  $\theta_2$ ) to  
 354 their targets is reported, while the convergence of the three remaining parameter estimations ( $\theta_3$ ,  
 355  $\theta_4$  and  $\theta_5$ ) to their target is not possible.

356 In 2013 (Qian et al., 2013), the sensitivity criterion E-optimality is used as the cost function of  
 357 the optimal control problem. To stabilize the system in a prescribed region, two fictitious output  
 358 constraints were imposed and designed by trial and error method. The convergence of the five

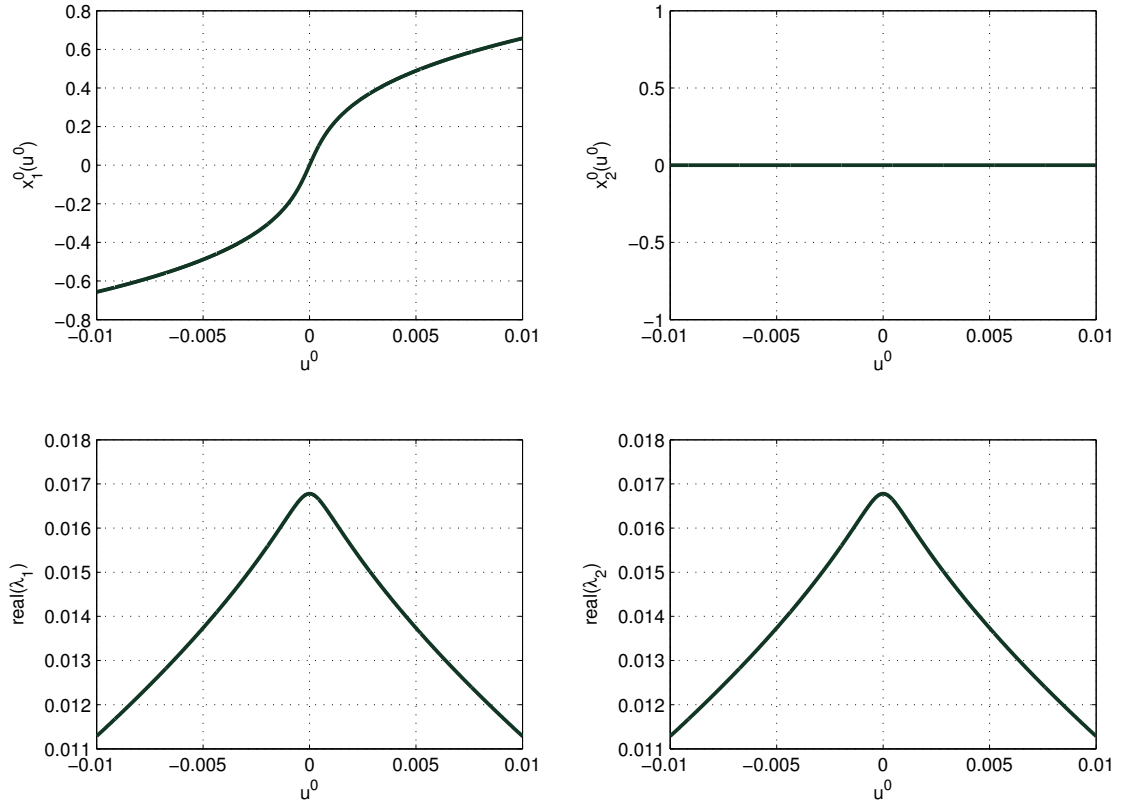


Figure 2. Linearized system of (19), for  $u^0$  in  $[-0.01, 0.01]$ : Steady state  $x_1^0$  (top left) and  $x_2^0$  (top right), real part of each eigenvalue (bottom left and right).

359 unknown constant parameter estimations to their target is obtained with good accuracy and the  
 360 system is stable in closed loop.

361 **4.2 Step 2: Observer design**

362 Similarly to the work of (Jain et al., 2005), both the states here are on-line measured. Hence, the  
 363 purpose of the observer to be designed is to estimate on-line the vector of unknown parameters  
 364  $\theta_i (i = 1, \dots, 5)$ . To do so, according to the augmented model (4), system (19) is extended by the  
 365 vector  $\theta = 0$ . Consequently, the obtained augmented model is a state affine system up to nonlinear  
 366 output injection in the following form

$$\begin{cases} \dot{x}_a(t) &= A_a(y(t))x_a(t) + B_a(u(t)) \\ y(t) &= C_a x_a(t), \end{cases} \quad (20)$$

$$367 \text{ where } x_a(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}; B_a(u(t)) = \begin{bmatrix} 0 \\ -\alpha_3 u(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; C_a^T = \begin{bmatrix} I_{2 \times 2} \\ 0_{5 \times 2} \end{bmatrix};$$

$$368 \text{ with } A_a(y(t)) = \begin{bmatrix} 0_{2 \times 1} & A(y(t)) \\ 0_{5 \times 1} & 0_{5 \times 6} \end{bmatrix}; A^T(y(t)) = \begin{bmatrix} 1 & -\alpha_2 \\ 0 & \alpha_1 y_1(t) \\ 0 & \alpha_1 y_2(t) \\ 0 & \alpha_1 y_1^3(t) \\ 0 & \alpha_1 y_1^2(t) y_2(t) \\ 0 & \alpha_1 y_1(t) y_2^2(t) \end{bmatrix},$$

369  $I_{2 \times 2}$  is the  $2 \times 2$  identity matrix,  $0_{a \times b}$  is the  $a \times b$  matrix of zeros.  
 370 Then, a high gain observer based on the augmented state  $x_a(t)$  can be designed as follows (see  
 371 (Hammouri and Morales, 1990) and (Besançon, 2007a) for more details). For any  $x_a(0)$ , the system  
 372 (20) admits an exponential observer of the form  
 373

$$\begin{cases} \dot{\hat{x}}_a(t) = A_a(y(t))\hat{x}_a(t) + B_a(u(t)) - \Gamma S_\mu(t)^{-1} C_a^T (C_a \hat{x}_a(t) - y_p(t)) \\ \dot{S}_\mu(t) = -\mu S_\mu(t) - A_a(y(t))^T S_\mu(t) - S_\mu(t) A_a(y(t)) + C_a^T \Gamma C_a, \end{cases} \quad (21)$$

374 where  $S_\mu$  is a  $7 \times 7$  symmetric positive definite matrix, the positive constant  $\mu > 0$  and  $\Gamma > 1$  are  
 375 the observer tuning parameters.

### 376 4.3 Step 3: Sensitivity model

377 Based on the model (19), the sensitivity model (6) simply becomes:

$$\begin{cases} \dot{x}_{1\theta_1} = x_{2\theta_1} \\ \dot{x}_{1\theta_2} = x_{2\theta_2} \\ \dot{x}_{1\theta_3} = x_{2\theta_3} \\ \dot{x}_{1\theta_4} = x_{2\theta_4} \\ \dot{x}_{1\theta_5} = x_{2\theta_5} \\ \dot{x}_{2\theta_1} = \alpha_1(x_1 + \theta_1 x_{1\theta_1}) + (\alpha_1 \theta_2 - \alpha_2) x_{2\theta_1} + 3\alpha_1 \theta_3 x_1^2 x_{1\theta_1} \dots \\ \dots + \alpha_1 \theta_4 (2x_1 x_{1\theta_1} x_2 + x_1^2 x_{2\theta_1}) + \alpha_1 \theta_5 (x_{1\theta_1} x_2^2 + 2x_1 x_2 x_{2\theta_1}) \\ \dot{x}_{2\theta_2} = \alpha_1 \theta_1 x_{1\theta_2} + (\alpha_1(x_2 + (\theta_2 x_{2\theta_2}) - \alpha_2 x_{2\theta_2})) + 3\alpha_1 \theta_3 x_1^2 x_{1\theta_2} \dots \\ \dots + \alpha_1 \theta_4 (2x_1 x_{1\theta_2} x_2 + x_1^2 x_{2\theta_2}) + \alpha_1 \theta_5 (x_{1\theta_2} x_2^2 + 2x_1 x_2 x_{2\theta_2}) \\ \dot{x}_{2\theta_3} = \alpha_1 \theta_1 x_{1\theta_3} + (\alpha_1 \theta_2 - \alpha_2) x_{2\theta_3} + \alpha_1 (x_1^3 + 3\theta_3 x_1^2 x_{1\theta_3}) \dots \\ \dots + \alpha_1 \theta_4 (2x_1 x_{1\theta_3} x_2 + x_1^2 x_{2\theta_3}) + \alpha_1 \theta_5 (x_{1\theta_3} x_2^2 + 2x_1 x_2 x_{2\theta_3}) \\ \dot{x}_{2\theta_4} = \alpha_1 \theta_1 x_{1\theta_4} + (\alpha_1 \theta_2 - \alpha_2) x_{2\theta_4} + 3\alpha_1 \theta_3 x_1^2 x_{1\theta_4} \dots \\ \dots + \alpha_1 (x_1^2 x_2 + \theta_4 (2x_{1\theta_4} x_1 x_2 + x_1^2 x_{2\theta_4})) + \alpha_1 \theta_5 (x_{1\theta_4} x_2^2 + 2x_1 x_2 x_{2\theta_4}) \\ \dot{x}_{2\theta_5} = \alpha_1 \theta_1 x_{1\theta_5} + (\alpha_1 \theta_2 - \alpha_2) x_{2\theta_5} + 3\alpha_1 \theta_3 x_1^2 x_{1\theta_5} + \dots \\ \dots \alpha_1 \theta_4 (2x_{1\theta_5} x_1 x_2 + x_1^2 x_{2\theta_5}) + \alpha_1 (x_1 x_2^2 + \theta_5 (x_{1\theta_5} x_2^2 + 2x_1 x_2 x_{2\theta_5})). \end{cases} \quad (22)$$

### 378 4.4 Step 4: Control design

379 As described in section 3, the outputs of the process  $y_p$ , the state  $x$  of the model (19), the state  
 380  $\hat{x}_a$  of the observer (20) and the state  $x_\theta$  of the sensitivity model (22) are fed back into the control  
 381 law. In (18), the cost function is built from two parts: the sensitivity criterion and the Lyapunov  
 382 stability criterion. The outputs of the sensitivity model are normalized using the relative-sensitivity

383 function (9) in order to establish the sensitivity criterion (12). At the same time, a Lyapunov  
 384 stability analysis is also needed to stabilize the system in closed loop. For determining the Lyapunov  
 385 function (17), let us first consider system (19) as a quasi-linear system (14) in which

$$\left\{ \begin{array}{l} \text{At the current time } k, \text{ over the prediction horizon } N_p: \\ A(k) = \begin{bmatrix} 0 & 1 \\ \alpha_1 \hat{\theta}_1(k) & \alpha_1 \hat{\theta}_2(k) - \alpha_2 \end{bmatrix} \\ B(k) = \begin{bmatrix} 0 \\ \alpha_3 \end{bmatrix} \\ g(x, k) = \begin{bmatrix} 0 \\ \alpha_1 \hat{\theta}_3(k) \hat{x}_1^3(k) + \alpha_1 \hat{\theta}_4(k) \hat{x}_1^2(k) \hat{x}_2(k) \dots \\ \dots + \alpha_1 \hat{\theta}_5(k) \hat{x}_1(k) \hat{x}_2^2(k) \end{bmatrix}, \end{array} \right. \quad (23)$$

386 where the function vector  $g(x, k)$  is assumed to be a "small disturbance" of the system and respects  
 387 the condition of non-autonomous quasi-linear systems (Calvet et al., 1989). Then based on the  
 388 constant matrices  $A(k)$  and  $B(k)$  over the prediction horizon, one computes the positive semi-  
 389 definite matrix  $P(k)$  by solving the algebraic Riccati equation (15). Finally, using the computed  
 390  $P(k)$  and the prediction of model states  $x(l)$ , one gets the Lyapunov function (17). Effects of tuning  
 391 the weighting value  $\beta$  are discussed in the following section.

## 392 5. A rolling delta wing: numerical simulation results

### 393 5.1 Numerical conditions

394 All simulation runs are performed under the following conditions where all values are dimensionless<sup>6</sup>

$$\left\{ \begin{array}{l} \text{input constraints: } -0.01 \leq u(k) \leq 0.01 \\ \text{prediction horizon: } N_p = 5 \\ \text{sampling time: } T_s = 1 \\ \text{observer tuning parameters: } \mu = 0.03, S_\mu = I_{7 \times 7} \text{ and } \Gamma = 2 \\ \text{parameters in Riccati equation: } Q = 0.01 I_{2 \times 2} \text{ and } R = I \\ \text{weights in the cost function: } w_\theta = 10^{-14} \text{ and } w_L = 1. \end{array} \right. \quad (24)$$

395 Initial estimation errors for  $\theta_i$  are listed in table 2: in order to see the robustness of the approach,  
 396 large initial errors in the estimation of  $\theta_i$  are introduced, including even sign errors.

Table 2.  $\theta$ : initial estimation errors.

| $\theta$          | 1  | 2    | 3   | 4  | 5    |
|-------------------|----|------|-----|----|------|
| Initial error (%) | 80 | -200 | 200 | 80 | -200 |

396 The simulation runs are performed under the *ODOE4OPE software*<sup>7</sup> based on Matlab.  
 397

### 398 5.2 Influence of the weighting value $\beta$

399 As discussed before, the weighting value  $\beta$ , ranging between 0 and 1, balances the dual cost function  
 400 to obtain the convergence of the five unknown parameter estimation and also stabilizes the system in  
 401 closed loop. In order to study the effects of the weighting value  $\beta$ , a series of simulations is presented

<sup>6</sup>The time, the input and the states are also dimensionless.

<sup>7</sup>To use this software, please visit <http://odoe4ope.univ-lyon1.fr>

402 with different values of  $\beta$  to compare the convergence of the unknown parameter estimations to  
 403 their targets and the closed loop stability. Moreover, to see the impact of noise measurement, two  
 404 cases are studied to show the robustness of the proposed closed loop on-line optimal identification  
 405 approach defined in this paper:

- 406 • Case 1: without noise.
- 407 • Case 2: with a 5% Gaussian noise on the output measurements.

408 Table 3 presents the mean value (resp. the norm of the mean value) of the last 60 values of the  
 409 estimation of the five parameters in the two cases. As it can be seen here, in the case without noise,  
 410 for several weighting values  $\beta$  (i.e. 0.3, 0.5, 0.6 and 0.7), and in spite of large initial error, the final  
 411 estimation error of  $\theta_i$  tends to zero (the maximal final estimation error is less than 9% (often less  
 412 than 1%) and the minimum is less than 1%), which means the convergence of the five parameter  
 413 estimations to their targets is achieved. With  $\beta$  tuned between 0.2 and 0.8 in the case without noise,  
 414 it is robust for both the parameter estimations and the closed loop stability. In the case with noise,  
 415 table 3 shows that for  $\beta = 0.6$ , the most accurate parameter estimations are obtained: all five final  
 416 estimation errors are less than 3%. The closer the weighting value  $\beta$  approaches its limits 0 or 1, the  
 417 larger the parameter estimation errors are: Indeed, for  $\beta = 0$  (i.e., the cost function is only dealing  
 418 with the stability) in the two cases, both input  $u$  and model state  $x$  tend as expected towards zero,  
 419 but there is not enough information for parameter estimation: the estimation convergence for the  
 420 two first components of the parameter vector is obtained, but the estimation for the other three  
 421 other parameters is not possible (see table 3). For  $\beta \geq 0.8$  in the case without noise or for  $\beta \geq 0.7$   
 422 in the case with noise (i.e. maximizing sensitivity criterion is more important than stabilizing the  
 423 system), the system response is completely divergent. In other words the system is unstable in  
 424 closed loop, so parameter estimation is impossible.

Table 3. Influence of the tuning of  $\beta$ : Final mean parameter estimation error (%) for each parameters and for the whole parameter vector, and states stability.

| $\beta$                  | 1            | 2          | $\theta_i$<br>3 | 4         | 5          | Vector<br>$\theta$ | $x_1$         | $x_2$         |
|--------------------------|--------------|------------|-----------------|-----------|------------|--------------------|---------------|---------------|
| 0 without noise          | -0.003       | 8          | 179             | 79        | -200       | 125                | stable        | stable        |
| 0 with noise             | 0.5          | -13        | 178             | 79        | -200       | 125                | stable        | stable        |
| 0.3 without noise        | 0.01         | 2          | 0.2             | -1        | -1         | 1                  | stable        | stable        |
| 0.3 with noise           | 0.01         | 2          | 16              | 4         | -62        | 27                 | stable        | stable        |
| <b>0.5 without noise</b> | <b>-0.06</b> | <b>0.4</b> | <b>-0.5</b>     | <b>1</b>  | <b>2</b>   | <b>1</b>           | <b>stable</b> | <b>stable</b> |
| 0.5 with noise           | 0.4          | 7          | 4               | 14        | -4         | 7                  | stable        | stable        |
| 0.6 without noise        | -0.2         | 3          | 0.2             | 8         | -5         | 5                  | stable        | stable        |
| <b>0.6 with noise</b>    | <b>-0.05</b> | <b>-1</b>  | <b>-0.06</b>    | <b>-2</b> | <b>0.7</b> | <b>1</b>           | <b>stable</b> | <b>stable</b> |
| 0.7 without noise        | 0.1          | -2         | 0.04            | -0.4      | 2          | 1                  | stable        | stable        |
| 0.7 with noise           | -0.3         | 16         | 2               | 2         | -6         | 8                  | unstable      | unstable      |
| 0.8 without noise        | 1            | 8          | 3               | 0.2       | -19        | 9                  | unstable      | unstable      |
| 0.8 with noise           | 2            | -19        | 8               | 2         | -63        | 30                 | unstable      | unstable      |
| 1 without noise          | 359          | -250       | 792             | 827       | 22         | 548                | unstable      | unstable      |
| 1 with noise             | 48           | 120        | 50              | -16       | -140       | 88                 | unstable      | unstable      |

425 The optimal tuning for  $\beta$  is almost similar in the two cases: 0.5 for the case without noise, and 0.6  
 426 for the case with noise. This tuning is therefore robust with respect to the noise.

### 427 5.3 Simulation results for $\beta = 0.6$ , with output noise

428 Applying the proposed developed approach on the delta wing system, the results for the case of  
 429  $\beta = 0.6$  with output noise are presented here in more details. Figure 3 (a zoom is shown in Figure  
 430 4) presents the closed loop optimal control time behavior within input constraints which stabilizes



431 the rolling delta wing behavior as shown in Figure 5. The first component  $y_{p_1}$  of the output  $y_p$   
 432 is stabilized in the region between  $-0.6$  and  $0.8$  (which is larger than the output constraint set  
 433 defined by trial and error in (Qian et al., 2013)), and the second component  $y_{p_2}$  of the output  
 434  $y_p$  is maintained between  $-0.1$  and  $0.1$ . In Figures 6 and 7, the five parameter targets and time  
 435 evolution of the estimations are normalized (hence, 1 is the target): all unknown parameters are  
 436 estimated. Meanwhile, the parameter  $\theta_1$  reaches its target in  $t = 200$ , the parameter  $\theta_2$  reaches its  
 437 target before  $t = 600$  (Figure 6), and the other three parameters converge to their targets around  
 438  $t = 500$  (Figure 7). Therefore, the optimal control stabilizes the system in closed loop and helps to  
 439 estimate all five unknown parameters with high accuracy.

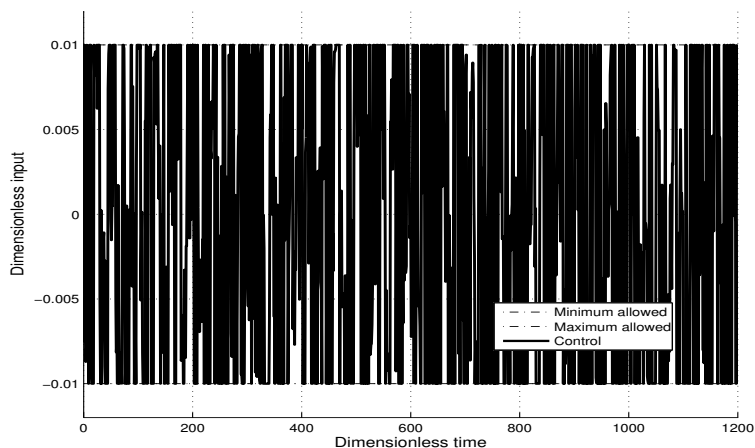


Figure 3. Closed loop optimal input for  $\beta = 0.6$  with output noise.

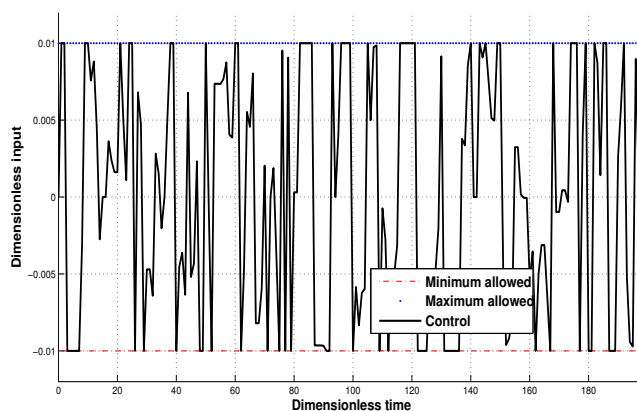


Figure 4. Closed loop optimal input for  $\beta = 0.6$  with output noise (zoom on  $0 \leq t \leq 200$ ).

440 **6. Conclusion**

441 In this paper, a closed loop controller for on-line parameter identification was designed for a general  
 442 class of nonlinear dynamic systems. An optimization problem was formulated based on a sensitivity  
 443 criterion and a Lyapunov function. Indeed, based on the observer theory and the MPC strategy,  
 444 a cost function was maximized online to get a trade-off between local stability and the observer  
 445 robustness by tuning a unique controller parameter. Consequently, stability was guaranteed in

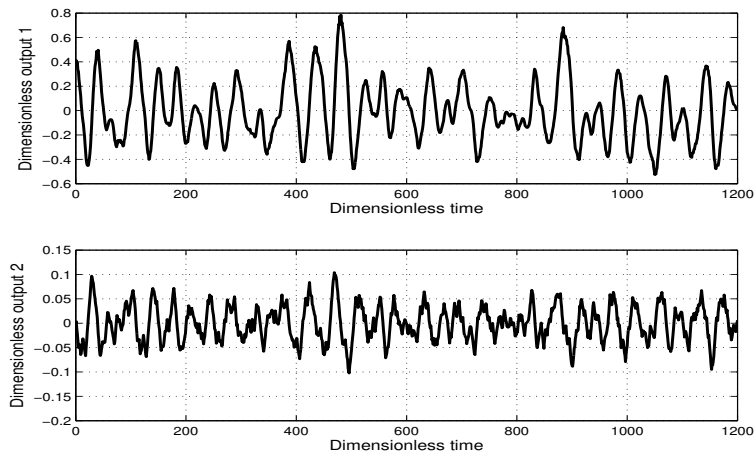


Figure 5. Closed loop outputs for  $\beta = 0.6$  (top:  $y_{p1}$ , bottom:  $y_{p2}$ ) with output noise.

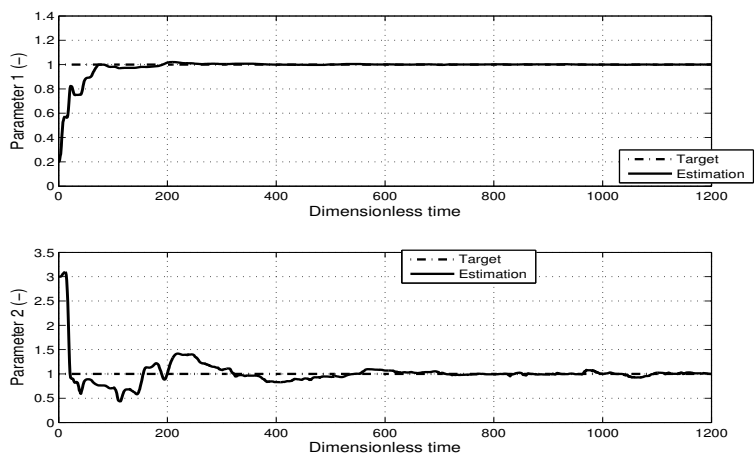


Figure 6. Closed loop estimation for  $\beta = 0.6$  (from the top to the bottom:  $\theta_1$  and  $\theta_2$ ) with output noise.

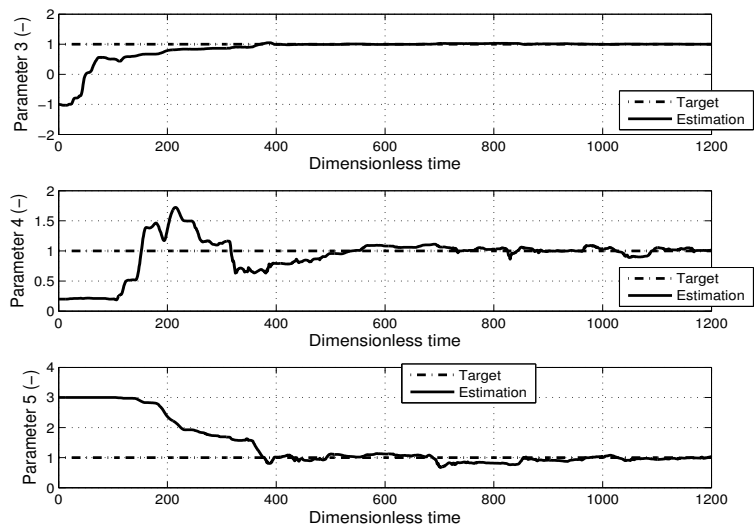


Figure 7. Closed loop estimation for  $\beta = 0.6$  (from the top to the bottom:  $\theta_3$ ,  $\theta_4$  and  $\theta_5$ ) with output noise.

446 closed loop by Lyapunov analysis (while previous results were based on a trial and error method)  
 447 and the estimation error converged asymptotically to zero. This generic approach was illustrated  
 448 step by step with simulations on an aircraft example (an open loop unstable mechanical delta wing  
 449 system). In contrary to a previous work, the convergence of all five parameter estimations to their  
 450 target and the stability of closed loop were shown with noisy output measurements.  
 451 As possible future works, it could be discussed how to add output constraints to keep also the  
 452 system in a desired production area. It should then be considered how this competes with the  
 453 stability criteria. Also, a formal proof of robustness needs to be addressed, to minimize the impact  
 454 of the transient estimation error on the closed loop control objectives. Interval observers, as a tool  
 455 to give the estimation bounds, may be considered.

## 456 Funding

457 This work was a part of the PhD thesis CIFRE 2011/0876 of J. Qian between the french company  
 458 Acsystème and the french ministry of higher education and research, which is acknowledged for  
 459 the funding support.

## 460 References

- 461 Alanqar, A., Ellis, M., & Christofides, P.D. (2015). Economic model predictive control of nonlinear process  
 462 systems using empirical models. *AIChE Journal*, 61(3), 816–830.
- 463 Angeli, D., Amrit, R., & Rawlings, J.B. (2012). On average performance and stability of economic model  
 464 predictive control. *IEEE Transactions On Automatic Control*, 57(7), 1615–1626.
- 465 Barz, T., Cárdenas, D.C.L., Arellano-Garcia, H., & Wozny, G. (2012). Experimental evaluation of an ap-  
 466 proach to online redesign of experiments for parameter determination. *AIChE Journal*, 59(6), 1981–1995.
- 467 Başar, T. & Bernhard, P. (1995).  $H_\infty$ -optimal control and related minimax design problems: a dynamic  
 468 game approach. Birkhauser.
- 469 Besançon, G. (2007a). Identification of parametric models from experimental data. *Lecture Notes in Control  
 470 and Information Sciences*, 363, Berlin Heidelberg:Springer-Verlag.
- 471 Besançon, G. (2007b). Nonlinear observers and application. *Lecture Notes in Control and Information Sci-  
 472 ences*, 363, Berlin Heidelberg:Springer-Verlag.
- 473 Besançon, G. & Ticlea, A. (2007). An immersion-based observer design for rank-observable nonlinear sys-  
 474 tems. *IEEE Transactions on Automatic Control*, 52(1), 83–88.
- 475 Blanc, D., Dufour, P., Toure, Y., & Laurent, P. (2003). On nonlinear distributed parameter model predictive  
 476 control strategy: on-line calculation time reduction and application to an experimental drying process.  
 477 *Computers and Chemical Engineering*, 27, 1533–1542.
- 478 Boizot, N., Busvelle, E., & Gauthier, J.-P. (2010). An adaptive high-gain observer for nonlinear systems.  
 479 *Automatica*, 46(9), 1483–1488.
- 480 Bornard, G., Couenne, N. & Celle, F. (1988). Regularly persistent observers for bilinear system. *Lecture  
 481 Notes in Control and Information Sciences*, Springer, 122, 130–140.
- 482 Calvet, J.-P., & Arkun, Y. (1989). Stabilization of feedback linearized nonlinear processes under bounded  
 483 perturbations. *American Control Conference*, Pittsburgh, PA, (pp. 747–752).
- 484 Castillo, F., Witrant, E., Prieur, C., & Dugard, L. (2012). Dynamic boundary stabilization of linear and  
 485 quasi-linear hyperbolic systems. *IEEE Conference on Decision and Control CDC*, Maui, HI, (pp. 2952–  
 486 2957).
- 487 Cox, H.(1964). On the estimation of state variables and parameters for noisy dynamic systems. *IEEE Trans-  
 488 actions On Automatic Control*, (9), 5–12.
- 489 Dufour, P., Flila, S., & Hammouri, H. (2012). Observer design for MIMO non-uniformly observable systems.  
 490 *IEEE Transactions On Automatic Control*, 57(2), 511–516.
- 491 Ellis, M., Durand, H., & Christofides, P.D. (2014). A tutorial review of economic model predictive control  
 492 methods. *Journal of Process Control*, 24(8), 1156–1178.

- 493 Flila, S., Dufour, P., & Hammouri, H. (2008). Optimal input design for on-line identification: a coupled  
 494 observer-MPC approach. *IFAC World Congress*, Seoul, South Korea, (pp. 11457-11462).
- 495 Forssell, U., & Ljung, L. (1998). Identification of unstable systems using output error and box-jenkins model  
 496 structures, in: Proceedings of the 29th IEEE CSS Chinese Control Conference, (pp. 3932-3937).
- 497 Franceschini, G., & Macchietto, S., (2008). Optimal experiment design for linear systems with input-output  
 498 constraints: State of the art. *Chemical Engineering Science*, 63, 4846-4872.
- 499 Gauthier, J.P., Hammouri, H., & Othman, S. (1992). A simple observer for nonlinear systems applications  
 500 to bioreactors. *IEEE Transactions on Automatic Control*, 37(6), 875-880.
- 501 Goodwin, G.C., & Payne, R.L. (1977). Dynamic system identification: experiment design and data analysis.  
 502 *Mathematics in Science and Engineering*, Vol. 136, Academic Press.
- 503 Hammouri, H. & Morales, J.D.L (1990). Observer synthesis for state-affine systems. *IEEE Conference on*  
 504 *Decision and Control*, Honolulu, HI, vol. 2, (pp. 784-785).
- 505 Huang, R., Harinath, E., & Biegler, L.T., (2011). Lyapunov stability of economically oriented NMPC for  
 506 cyclic processes. *Journal of Process Control*, 21(4), 501-509.
- 507 Jain, H., Kaul, V., & Ananthkrishnan, N. (2005). Parameter estimation of unstable, limit cycling systems us-  
 508 ing adaptive feedback linearization: example of delta wing roll dynamics. *Journal of Sound and Vibration*,  
 509 287, 939-960.
- 510 Jayasankar, B., Huang, B., & Ben-Zv, A. (2010). Receding horizon experiment design with application in  
 511 SOFC parameter estimation. *Dynamics and Control of Process Systems*, Leuven, Belgium, (pp. 527-532).
- 512 Keviczky, L. (1975). Design of experiments for the identification of linear dynamic systems. *Technomet-*  
 513 *rics*, 17(3), 303-308.
- 514 Ljung, L. (1979). Asymptotic behavior of the extended kalman filter as a parameter estimator for linear  
 515 systems, *IEEE Transactions on Automatic Control*, 24, 36-50.
- 516 Ljung, L. (1999). *System identification: Theory for the user*, (2nd ed.), Prentice Hall.
- 517 Nadri, M., Hammouri, H., & Grajales, R. (2013). Observer design for uniformly observable systems with  
 518 sampled measurements. *IEEE Transactions on Automatic Control*, 58(3), 757-762.
- 519 Nayfeh, A.H., Elzeba, J.M., & Mook, D.T. (1989). Analytical study of the subsonic wing-rock phenomenon  
 520 for slender delta wings. *Journal of Aircraft*, 26(9), 805-809.
- 521 Nelson, L., & Stear, E. (1976). The simultaneous on-line estimation of parameters and states in linear  
 522 systems, *IEEE Transactions on Automatic Control*, 21, 94-98.
- 523 Ng, T.S., Goodwin, G.C., & Söderström, T. (1997). Optimal experiment design for linear systems with  
 524 input-output constraints. *Automatica*, 13(6), 571-577.
- 525 Qian, J., Dufour, P., & Nadri, M. (2013). Observer and model predictive control for on-line parameter  
 526 identification in nonlinear systems. *IFAC Dynamics and Control of Process Systems*, Mumbai, India, (pp.  
 527 571-576).
- 528 Qian, J., Nadri, M., Moroşan, P.D.M., & Dufour, P. (2014). Closed loop optimal experiment design for on-line  
 529 parameter estimation. *IFAC IEEE European Control Conference*, Strasbourg, France, (pp. 1813-1818).
- 530 Rawlings, J.B., Angeli, D., & Bates, C.N. (2012). Fundamentals of economic Model Predictive Control.  
 531 *IEEE Conference on Decision and Control*, (pp. 3851-3861).
- 532 Walter, E., & Pronzato, L. (1994). *Identification of parametric models from experimental data*, Springer-  
 533 Verlag.
- 534 Willems, J.L. (1970). A general stability criterion for non-linear time-varying feedback systems. *International*  
 535 *Journal of Control*, 11(4), 625-631.
- 536 Zanon, M., Gros, S., & Diehl, M. (2014). Indefinite linear MPC and approximated economic MPC for  
 537 nonlinear systems. *Journal of Process Control*, 24(8), 1273-1281.
- 538 Zavala, V.M., & Biegler, L.T., (2009). The advanced-step NMPC controller: Optimality, stability and ro-  
 539 bustness. *Automatica*, 45(1), 86-93.
- 540 Zhu, Y., & Huang, B. (2011). Constrained receding-horizon experiment design and parameter estimation in  
 541 the presence of poor initial conditions. *AIChE Journal*, 57(10), 2808-2820.