t-résilient snapshot immédiat
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Dans un système de $n$ processus communicant par mémoire partagée, un snapshot immédiat est un état de la mémoire assurant que si l’écriture réalisée par $q$ est dans le snapshot de $p$ alors le snapshot de $p$ contient le snapshot obtenu par $q$. Un snapshot immédiat peut être réalisé dans un système où au plus $n-1$ processus peuvent tomber en panne. Dans un système où $t$ processus peuvent tomber en panne, un processus peut obtenir un snapshot contenant les valeurs écrites par jusqu’à $n-t$ processus. On définit ainsi le $t$-résilient snapshot immédiat et on établit des liens entre ce problème et les problèmes d’accord comme le consensus et le $k$-accord.

Mots-clés : Mémoire partagé, snapshot immédiat, tolérance aux pannes, consensus, $k$-accord.

1 Introduction

We consider a distributed computing model of $n$ asynchronous processes among which any subset of up to $t$ processes may crash communicating by shared memory.

The snapshot object was first proposed over a decade ago [1, 2] and has since been intensively studied by the distributed algorithms community. A snapshot object can be seen as a data structure initially empty, which can then contain at most $n$ pairs (one per process), each made up of a process index and a value. This object provides the processes with two operations denoted update and snap. The invocation update($v$) by a process $p$ adds the pair $⟨i, v⟩$ to the data structure and the operation snap returns all the pairs already written in the data structure.

The immediate snapshot (IS) object has been introduced in [3, 11], and later investigated in [5, 15]. It is a variant of the snapshot object. An immediate snapshot provides the processes with a single operation write_snapshot() that a process may invoke at most once. The invocation write_snapshot($v$) by a process $p_j$ adds the pair $⟨i, v⟩$ to the object and returns a set of pairs view$_i$ belonging to the object such that if $⟨j, w⟩$ is in view$_i$ then view$_j$ ⊆ view$_i$.

The noteworthy feature of the iterated immediate snapshot model is the following. It has been shown by Borowsky and Gafni in [5], that this model is equivalent to the usual read/write wait-free model ($(n-1)$-crash model) for task solvability with the wait-freedom progress condition (any non-faulty process obtains a result). Its advantage lies in the fact that its runs are more structured and easier to analyze than the runs in the basic read/write shared memory model [14]. It is also the basis of the combinatorial topology approach for distributed computing (e.g., [10]). Hence, IS objects constitute the algorithmic foundation of distributed iterated computing models.

When considering the $t$-crash $n$-process model where $t < n$, and assuming that each correct process writes a value, a process may wait for values written by $(n-t)$ processes without risking being blocked forever.

This naturally leads to the notion of a $t$-immediate snapshot object, which generalizes the basic $(n-1)$-immediate snapshot object. More precisely, when considering a $t$-immediate snapshot object in a $t$-crash $n$-process model, an invocation of write_snapshot() by a process returns a set including at least $(n-t)$ pairs

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(while it would return a set of \( x \) pairs with \( 1 \leq x \leq n \) if the object was an immediate snapshot object). Hence, a \( t \)-immediate snapshot object allows processes to obtain as much information as possible from the other processes while guaranteeing progress.

The obvious question is then the implementability of a \( t \)-immediate snapshot object in the \( t \)-crash \( n \)-process model.

Implementations of an \((n-1)\)-immediate snapshot object is described in [3]. For the other values of \( t \) (\( 0 < t < n-1 \)), this question is answered in this paper, which shows that it is impossible to implement a \( t \)-IS object in a \( t \)-crash \( n \)-process model when \( 0 < t < n-1 \). More precisely we prove that implementing a \( t \)-IS object is equivalent \(^\dagger\) to implementing consensus when \( t < n/2 \) and enables to implement \((2t-n+2)\)-Set agreement when \( n/2 \leq t < n-1 \).

At first glance, this impossibility result may seem surprising. First, an IS object is a snapshot object (a) whose operations update and snap are glued together in a single operation write\_snapshot(), and (b) satisfying an additional property linking the sets of pairs returned by concurrent invocations. Then, as already indicated, a \( t \)-IS object is an IS object such that the sets returned by write\_snapshot() contain at least \((n-t)\) pairs. This property on the sets returned by a snapshot object can be trivially implemented in a \( t \)-crash \( n \)-process model. Hence, while a \( t \)-snapshot object can be implemented in the \( t \)-crash \( n \)-process model, a \( t \)-IS object cannot when \( 0 < t < n-1 \). Second, because in general smallest is the number of possible faults easiest is the implementation. But in the \( t \)-IS, a small number of failure, induces a harder problem each output set must have the size at least \( n-t \).

2 Model

We consider a distributed computing model of \( n \geq 3 \) asynchronous sequential processes denoted \( p_1, \ldots, p_n \) among which any subset of up to \( t \) (\( 0 < t < n \)) processes may crash. The processes cooperate by reading and writing Single-Writer Multi-Reader atomic read/write registers [13].

One-shot immediate snapshot object. An immediate snapshot object (IS) is a set, initially empty, that will contain pairs made up of a process index and a value. Let us consider a process \( p_i \) that invokes write\_snapshot(v). This invocation adds the pair \((i,v)\) to the object, and returns to \( p_i \) a set, called view and denoted view_i, such that the sets returned to the processes collectively satisfy the following properties.

- Termination. The invocation of write\_snapshot() by a correct process terminates.
- Self-inclusion. \( \forall i : (i,v) \in \text{view}_i \).
- Validity. \( \forall i : ((j,v) \in \text{view}_j) \Rightarrow p_j \text{ invoked write\_snapshot}(v) \).
- Containment. \( \forall i,j : (\text{view}_i \subseteq \text{view}_j) \lor (\text{view}_j \subseteq \text{view}_i) \).
- Immediacy. \( \forall i,j : ((i,v) \in \text{view}_j) \Rightarrow (\text{view}_i \subseteq \text{view}_j) \).

\( k \)-Set agreement. \( k \)-Set agreement was introduced by S. Chaudhuri [6], it generalizes consensus which corresponds to the case \( k = 1 \). A \( k \)-Set agreement object is a one-shot object that provides the processes with a single operation \text{propose}_k(\cdot). This operation allows the invoking process \( p_i \) to propose a value it passes as an input parameter (called \text{proposed} value), and obtain a value (called \text{decided} value). The object is defined by the following set of properties.

- Termination. The invocation of \text{propose}_k(\cdot) by a correct process terminates.
- Validity. A decided value is a proposed value.
- Agreement. No more than \( k \) different values are decided.

It is shown in [8, 4, 11, 16] that the problem is impossible to solve if \( k \leq t \).

\( t \)-Immediate Snapshot. A \( t \)-immediate snapshot object (denoted by \( t \)-IS) is an immediate snapshot object with the following additional property.

- Output size. The set view obtained by a process is such that \( |\text{view}| \geq n-t \).

3 Results

\( t \)-Immediate Snapshot is Impossible if \( 0 < t < n-1 \). This section presents an algorithm Figure 1 to achieve (1) consensus from \( t \)-IS for \( 0 < t < n/2 \), and (2) \( k \)-Set agreement (in short \( k \)-SA) from \( t \)-IS for

\(^\dagger\) A is equivalent to B if A can be (computationally) reduced to B and reciprocally.
**t-resilient immediate snapshot**

```plaintext
1 PROPose($v$) begin 
2 view ← $I$.write_snapshot($v$); /* I shared immediate snapshot*/ 
3 VIEW[$i$] ← view; /* VIEW is a shared array*/ 
4 wait(|{ $j$ such that VIEW[$j$] ≠ ⊥}| = $t + 1$); 
5 let view be the smallest of the previous ($t + 1$) views; 
6 return(smallest proposed value in view) 
end PROPose
```

**Figure 1:** Solving consensus from t-IS if $0 < t < n/2$ (code for $p_i$)

```plaintext
1 write_snapshot($v_i$) begin 
2 $S$.update(($i$, $v_i$)); /* $S$ shared snapshot*/ 
3 view ← $\emptyset$; dec ← $\emptyset$; $k$ ← $-1$; launch the tasks $T1$ and $T2$. 
4 Task $T1$:
5 repeat $k$ ← $k + 1$
6 do aux ← $S$.snapshot() 
7 until ($dec$ ∩ $aux$ ∩ $|aux|$ ≥ $n - t$) 
8 dec ← CONS[$k$],propose$_i$(aux) 
9 if ($\langle i, v_i \rangle \in dec$) ∧ (view = $\emptyset$) then view ← dec end if 
10 until $|aux| = n$ 
end task $T1$
12 Task $T2$ : wait(view ≠ $\emptyset$); return(view) end task $T2$. 
14 end write_snapshot
```

**Figure 2:** Implementing t-IS with consensus (code for $p_i$)

$k = 2t - n + 2$ (e.g., $(n - 2)$-SA agreement from $(n - 2)$-IS if $t = n - 2$). As these problems are impossible to solve [4, 11, 16], we show that it is impossible to implement a t-IS object when $0 < t < n - 1$.

Intuitively this algorithm works because there is a set of at least $\ell \geq n - t$ processes, that obtained the same view min_view (or crashed before returning from write_snapshot()), and this view is the smallest view obtained by a process and its size is $|min_view| = \ell$. If $0 < t < n/2$, as $\ell \geq n - t$ and $(n - t) + (t + 1) > n$, it follows from the waiting predicate, that, any process that executes line 5, obtains a copy of min_view, and consequently we have view = min_view at line 5. It follows that no two processes can decide different values. If $n/2 \leq t < n - 1$, we have $n - t \leq t$. The $m = (n - t) - 1$ biggest views will never be selected by the processes, and consequently these processes obtain at most $t - m = t - ((n - t) - 1) = 2t - n + 1$ different smallest views. Hence, these processes may decide at most $2t - n + 1$ different values.

**Theorem 1** A t-IS object cannot be implemented if $0 < t < n - 1$.

**From Consensus to t-IS if $0 < t \leq n - 1$**. While a snapshot object is atomic (operations on a snapshot object can be linearized [12]), an IS object and a $k$-immediate snapshot objects are not atomic (its operations cannot always be linearized).

So we cannot apply the universality result of Herlihy [9], and we have to write a specific algorithm given Figure 2.

**t-Immediate Snapshot and k-Set agreement.** With the algorithms Figures 1 and 2, we get:

**Theorem 2** Consensus and t-IS are equivalent if $0 < t < n/2$.

We have shown that from t-IS when $n/2 \leq t < n - 2$ we can implement $(2t - n + 2)$-Set agreement. Can we do consensus as in the case $0 < t < n/2$? By a simulation argument, we show that consensus is not solvable with t-immediate snapshot when $n/2 \leq t < n$ proving that the computational power of t-immediate snapshot when $0 < t < n/2$ is strictly stronger than the computational power of t-immediate snapshot when $n/2 \leq t < n$. 

```
Theorem 3 If $0 < t < n/2$ then $t$-IS can implement $(2t - n + 2)$-Set agreement and cannot implement consensus. Consensus implements $t$-IS.

<table>
<thead>
<tr>
<th>$1 \leq t &lt; n/2$</th>
<th>$n/2 \leq t &lt; n-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$-IS and consensus are equivalent</td>
<td>$t$-IS implements $(2t - n + 2)$-Set agreement</td>
</tr>
<tr>
<td>$t$-IS does not implement consensus</td>
<td>consensus implements $t$-IS</td>
</tr>
</tbody>
</table>

| TABLE 1: Summary of results presented in the paper |

4 Conclusion

The paper has shown that, while it is possible to build an $(n-1)$-IS object in the asynchronous read/write $(n-1)$-crash model, it is impossible to build a $t$-IS object in an asynchronous read/write $t$-crash model when $0 < t < n - 1$. It follows that the notion of an Iterative immediate snapshot distributed model seems inoperable for these values of $t$. The results of the paper are summarized in Table 1.

Interestingly, this study shows that there are two contrasting impossibility results in asynchronous read/write $t$-crash $n$-process systems. Consensus is impossible as soon as $t > 0$, while $t$-immediate snapshot is impossible as soon as $t < n - 1$.

As a final remark, some computability problems remain open. As an example, is it possible to implement a $t$-IS object from $(2t - n + 2)$-Set agreement?

Références


