

# Weak Trefftz Discontinuous Galerkin method: an approach to resolve low and mid frequency mixed vibration problems

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**Résumé** — In this paper, an innovative method called Weak Trefftz Discontinuous Galerkin Method is proposed to solve the vibration problems composed by low and mid frequency band. Several methods have been developed to solve low frequency problems, the most common used of them is the Finite Element Method. Facing to mid frequency problems, Variational Theory of Complex Rays (VTCR) is developed, which is revisited as a discontinuous Galerkin method. By weakening the Trefftz constraint, this Weak Trefftz Discontinuous Galerkin Method enables hybrid FEM and VTCR approaches applied together to easily resolve engineering problems.

**Mots clés** — Weak Trefftz, Discontinuous Galerkin methods, FEM, VTCR, low and mid frequency

## 1 Introductions

The standard Galerkin Finite Element method FEM [1] is a well developed computer-aided engineering tool commonly used for the analysis of time-harmonic dynamic problems. Its uses of continuous, piecewise polynomial shape functions will require very large numerical model when frequency of vibration increases. So this approach is restricted to low frequency vibration problems. Facing to mid frequency vibration problems, an approach called Variational Theory of Complex Rays (VTCR) [2, 3], which is based on Trefftz methods [4], is introduced. Since Trefftz methods use the exact solution of the governing differential equations as shape functions, the restriction of FEM is overcome by this means. All the transmission conditions and boundaries conditions are satisfied by integration terms in variational formulation. By weakening the satisfaction condition of governing equation, an innovative technique named Weak Trefftz Discontinuous Galerkin method [5] is established, which enables the use of FEM and the use of VTCR together. The combination use of FEM and VTCR can be realised by this method to resolve more complicated engineering problems. In these cases, vibrations are the superposition of terms of different frequency. FEM is used to simulate the low frequency part or slowly varied part and VTCR is used for mid frequency part. A considerable reduction in model size and computational effort is realised by this method. The new shape function contains in mean time the polynomial and exponential parts.

### 1.1 Reference Problem

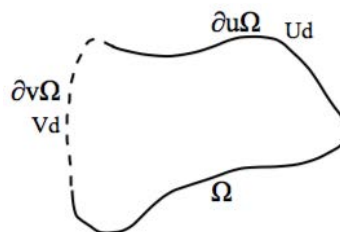


FIGURE 1 – Definition of computational domain  $\Omega$

Helmholtz function with exact solution is :

$$U = \cos(k_1 \cdot x) + \cos(k_2 \cdot x)$$

$$\begin{aligned} \Delta U + k^2 U &= F \text{ over } \Omega \\ U &= U_d \text{ over } \partial_1 \Omega \\ \underline{\underline{\text{grad} U}} \cdot \underline{n} &= \underline{V}_d \text{ over } \partial_2 \Omega \end{aligned} \tag{1}$$

## 1.2 Result

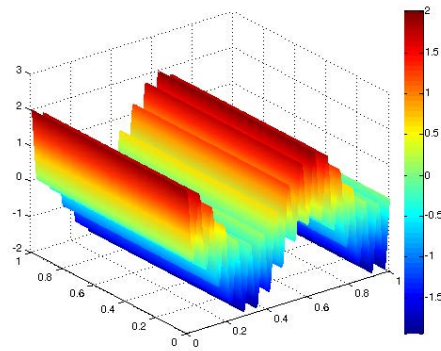


FIGURE 2 – Simulation result by Weak Trefftz Discontinuous Galerkin method

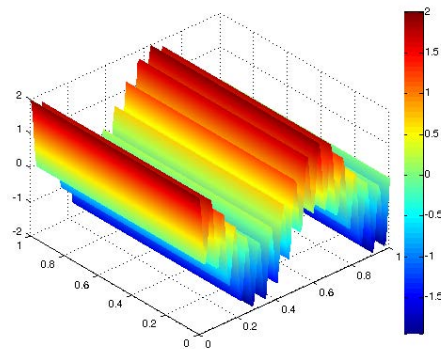


FIGURE 3 – Exact Solution

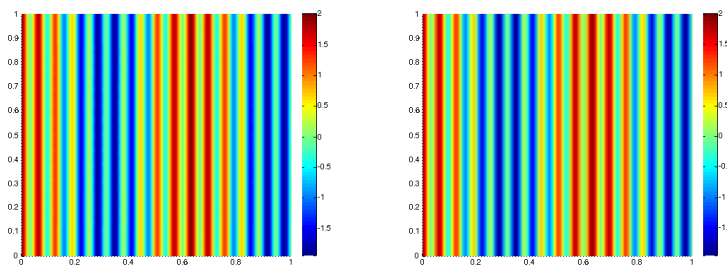


FIGURE 4 – Comparison : left : simulation result. right : exact solution

## Références

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