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Stochastic heterogeneous material modeling for granular material. Application to wave propagation in ballasted railway track

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Abstract — In this work we consider the description of a granular material as an heterogeneous linear continuum medium and the wave propagation in this continuum medium. The purpose of this modeling is to represent the equivalent stress field obtained in a discrete granular model through a continuum stochastic field of mechanical proprieties. One model was build in a finite element method to identify this continuum stochastic proprieties. We applied this approach in a ballasted railway track. The dynamic equations are solved using a Spectral Element approach using one probabilistic model.

Mots clés — Heterogeneous medium, Wave propagation, Ballasted railway track.

1 Introduction

The dynamical loads caused by the passage of high-speed trains accelerate track deterioration and damage neighbor buildings [1]. Two classes of numerical models are used to estimate the behavior of these dynamical systems: (1) discrete approaches, in which each grain of the ballast is represented by a rigid body and interacts with its neighbors through nonlinear contact forces (with non-smooth contact dynamics and software LMGC90 [2]); and (2) continuum approaches, in which the ballast is replaced by a homogenized continuum and the classical Finite Element Method (FEM, or similar) is used. Discrete approaches are today capable of solving a few meters-length of ballast, and the coupling with the underlying layers of soil remains an open problem. On the other hand, homogenized approaches are not capable of representing the heterogeneity of strains and stresses within the ballast.

In this work we investigate an alternative approach using a heterogeneous continuum model, that can be solved with a FE-like method while retaining some degree of heterogeneity. The present work is divided into two parts: (1) the statistical identification of the parameters of the continuum material; (2) wave propagation in a ballasted railway track. The first part identifies the parameters of our continuum model (average, correlation length, and variance of a random field of Young's parameter) on small cylindrical samples of discrete ballast (solved using LMGC90), with confinement pressure, gravity and a top pressure. The second part concentrates on the solution of the dynamical equations on a large model of a ballasted railway track with the Spectral Element Method (SEM) [3].

2 Statistical identification of the parameters

We compute the discrete mechanical behavior within 29 cubic samples of dimension: $48 \times 48 \times 35 \text{ cm}^3$, taken from cylinders with radius of 35 cm and 39 cm high. Each cylinder contains about 2700 particles. The particles are convex polyhedra with diameters between 2.5 cm and 5 cm. An isotropic confinement pressure around 60 kPa is applied on all samples, along with a vertical load of 63 kN on the upper face (corresponding to a mean pressure of 163 kPa). Gravity is considered within all samples, and the density of the particles is $\rho_0 = 2700 \text{ kg/m}^3$.

The continuum model has one cubic sample of $48 \times 48 \times 35 \text{ cm}^3$, taken from cylinders with radius of 35 cm and 39 cm high. A radial pressure of 60 kPa is applied, a pressure of 163 kPa was applied on the upper face. Gravity is considered, the density was assumed as $\rho = \phi \rho_0$, ϕ is the average volume ratio of the discrete model, $\rho \approx 1500 \text{ kg/m}^3$.

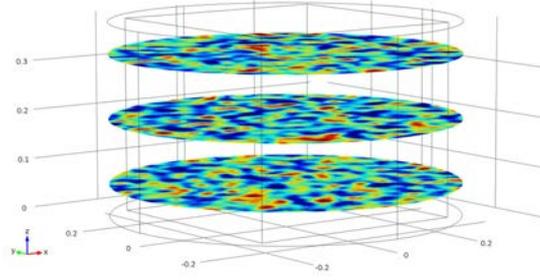


Figure 1: A sample random field in slices, with normalized variance $\sigma_E^2/\bar{E}^2 = 20$. The color map represent the Young's modulus.

In terms of first-order marginal law, we will model Young's modulus field with a log-normal distribution. This model requires the definition of a mean value \bar{E} , a variance σ^2 , and a correlation model [5]. The definitions required for the description are the average value, taken as 80 MPa (from literature [6]), variance (estimate below) and correlation length. A theoretical correlation model was used here to estimate the correlation length, the Percus-Yevick model [7]. The value of the Poisson ratio is deterministic and also taken from literature [6], $\nu = 0.23$. One sample of this continuum field are plotted in the Figure 1.

The identification of the variance in the first-order marginal has been done by the minimization of the distance between the probability density function of σ_{zz} . We compare the probability density function obtained from the discrete model (reference) and the continuous (target). To measure the distance two norms are used: (1) L^2 ; (2) the entropy norm (Kullback–Leibler divergence) [8].

A classical L^2 norm:

$$L_r = \int_{\mathfrak{R}^+} (p(x) - p_r(x))^2 dx \quad (1)$$

the entropy norm:

$$S_r = \int_{\mathfrak{R}^+} p(x) \log \left(\frac{p(x)}{p_r(x)} \right) dx \quad (2)$$

where $p_r(x)$ is the probability density function of the discrete model and $p(x)$ is the probability density function of the continuum model. Just one realization, for each variance, of the stochastic medium was made to perform this identification.

To be able to compare the results, we used the statistics definition for equivalent stress fields [4]. Given a network of contact forces f_i^c (at contact points c and with coordinates i) in a discrete medium, it is possible to define an equivalent stress field σ_{ij} as:

$$\sigma_{ij} = \frac{1}{V} \sum_{c=1}^{N_c} f_i^c l_j^c \quad (3)$$

where l_j^c is the vector linking the centers of the two particles in contact at c , the sum is on the N_c contact points in the averaging volume V . The Equation 3 shows that this measure depends on the size of averaging volume. The equation above was evaluated in a cube with edges in the range of 1 cm to 10 cm. We chose the $V = 10^3 \text{ cm}^3$ for compare the results.

The identification results are plotted in the Figure 2. The minimum distance is at $\sigma_E^2/\bar{E}^2 = 20$ for both types of measure. For this variance a comparison in log-log scale was present in the Figure 3, where a good approximation for the above-average values of stress was noted, the exponential behavior was well fitted on this part of the curve, however the match for below-average stress was not perfectly.

3 Dynamical Modeling

This study include two parts: (1) modeling the complex geometry of the railway ballasted track with homogeneous materials. This allow us understand the phenomena of geometric attenuation of the waves

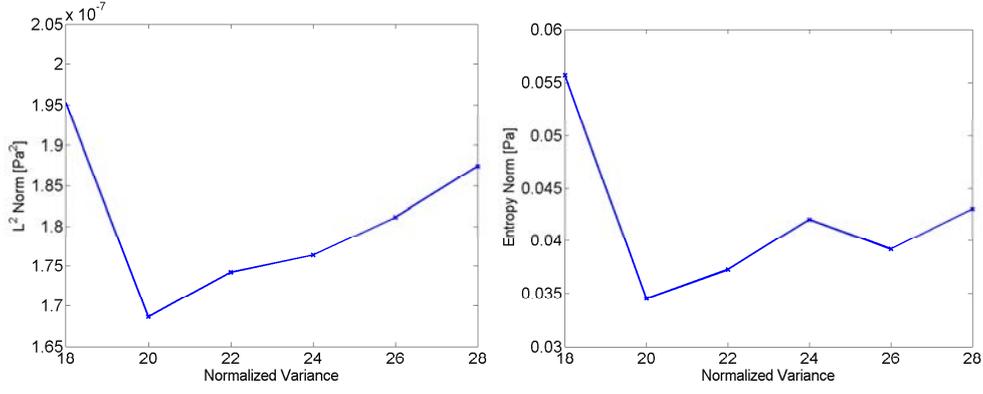


Figure 2: Relative distance between the probability distribution function σ_{zz} . The L^2 norm in the left figure and the S_r in the right. The normalized variance was used, σ_E^2/\bar{E}^2 , for $V = 10^3 \text{ cm}^3$.

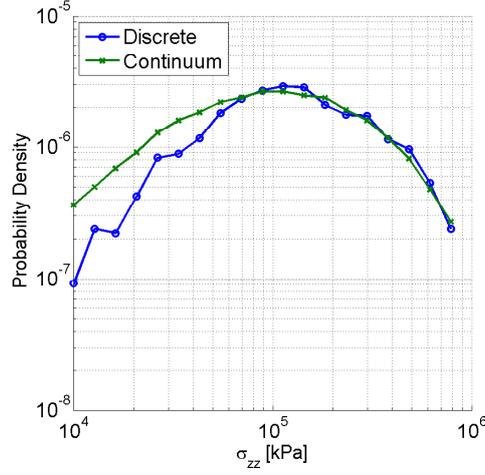


Figure 3: Comparison of the probability distribution function for σ_{zz} , with normalized variance $\sigma_E^2/\bar{E}^2 = 20$

in the neighborhood of the railway track; (2) modeling the same geometry with the correspondent heterogeneous materials. The heterogeneous medium introduces a wave scattering phenomena. Scattered waves are superimposed on the primary wave and cause fluctuations of the characteristics of the total field (fluctuations of amplitude, phase, etc.).

Some preliminaries results are obtained using the SEM. The random field was generated by a classical spectral method [5]. This results allow us compare directly the dispersion caused by the heterogeneity of the ballast and the soil with the homogeneous model. The wave propagation plotted in the Figure 4 is caused by a gaussian pulse. The geometry is a ballasted railway track without sleepers and only the ballast was modeled with a heterogeneous medium. The scattered wave can be clearly noted in this model.

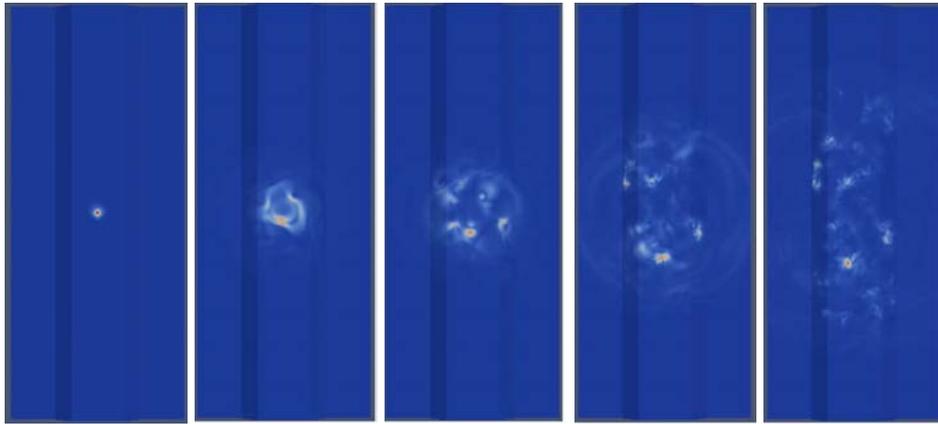


Figure 4: Wave propagation in a stochastic heterogeneous medium. To left to right: first time step, second, third, fifth, and seventh. The color scale represent the displacement magnitude.

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