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A Model-Based Controller for Interactive Delayed Force Feedback Virtual Environments

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Abstract. This paper addresses the stability of time-delayed force-reflecting displays used in virtual reality interactive systems. A novel predictive-like approach is proposed. The developed solution is stable and robust. Neither time delay estimation nor time delay behavior's knowledge are required. The controller applies to constant or time-varying delays without any adaptation. In this research, efforts are devoted towards making results easy to implement in commercial haptic libraries and interface build-in controllers. Moreover, although this study focuses on virtual environments haptics, it can easily spreads to force feedback teleoperators.

Keywords: virtual environment haptics, time-delayed force feedback, stability.

1. Introduction

Virtual reality techniques refer typically to human-in-the-loop or human centered advanced simulation or prototyping systems. The original feature of the concept lies in the multi-modality of the man-machine interaction involving all the human sensory modalities. Among these capabilities, haptic feedback is an important issue allowing the human operator to experience manipulation and touching of virtual objects with realistic sensations of stiffness, roughness, temperature, shape, weight, contact forces, etc. In real worlds, these parameters are collected then interpreted by the human haptic sense through direct touch (contact). Virtual environments (VE) are visually rendered to the human operator through computer screens, head mounted displays, workbenches, etc. To display 3D virtual sound simple headphones can be used. In the contrary to vision and auditory modalities, haptics requires active displays. Indeed, the used interfaces must be able to constraint human desired motions or to apply forces on the involved part (e.g. the human hand). Haptic displays are typically robotic-like devices which: (i) track hand motion (or applied forces) to be sent to the VE engine, and (ii) render subsequent VE reaction forces. Reaction forces are calculated thanks to computer haptics algorithms (collision detection, dynamic contact computation, etc.). There are many applications that use haptic feedback technology. Among the well known ones: interactive surgi-

cal simulators, interactive driving simulators, interactive games, VE based teleoperation and a great demand in industry virtual prototyping. The last one would extend to concurrent engineering and would make available haptic interaction among a group of users sharing the same VE over a network. It is well known that the haptic loop requires a high bandwidth of 1 kHz, ideally, to guarantee the stability and the transparency of the haptic interaction.

Data transmission time delay may compromise both stability and transparency of the haptic loop. Indeed the VE and the haptic interface may be distant from each other. This is the case of some applications such as concurrent engineering or shared VE. Moreover, heavy computational haptics may cause time-delay in the simulation loop. There is few work addressing the stability of VE delayed haptics since, in most applications, the user is not distant from the interactive simulation. This papers proposes a simple and efficient solution to deal with this problem. As it will be discussed, the proposed solution holds for constant and time-varying delay without any change or adaptation of the controller.

2. Time-Delay Force Reflecting Controller

VE haptic display controllers are mainly an adaptation of force reflecting teleoperators controllers. Nowadays, time delay is still known to be one of the most hard problems in force reflecting teleoperation. Many solutions, based on control theory, were proposed to deal with this problem. Some of the most attractive ones are based on passivity. Anderson and Spong derived a stable controller from scattering network theory (Anderson, 1992). Their proposed controller guarantees stability under any communication time delay. Niemeyer and Slotine also proposed a passivity-based controller thanks to a “wave variable” formulation of the problem (Niemeyer, 1991; Niemeyer, 1997) also derived from network theory. A controller using geometric scattering and based on a Hamiltonian modelling of the teleoperator was proposed in (Stramigoli, 2000). In (Yokokojhi, 1999; Yokokojhi, 2000) and (Niemeyer, 1998), extensions and adaptations were made to deal with time-varying delay. Besides passivity based controllers, there are other methods, obviously less conservative, that have been investigated to deal with delayed force feedback. Indeed some controllers have been derived from the well known Lyapunov stability theory, see (Eusebi, 1998; Oboe, 1998), whereas in (Leung, 1995), a μ -synthesis approach where time-delay is modelled as a disturbance was proposed. In (Kosuge, 1996) a simple buffering technique to deal with time-varying delay was also developed.

In the frame of VE haptic feedback, Hannaford *et al.* proposed a controller relaying on a “passivity observer” (Hannaford, 2002). The observer estimates the energy excess meanwhile, the controller cancels the estimated amount of energy so that force feedback is always passive. For delayed-process control, Smith prediction method is known since 1959 (Smith, 1959). However, it was not implemented in the frame of delayed force reflecting teleoperation. The reason that prohibits using Smith prediction lies in the practical impossibility to predict mainly (i) the remote environment behavior and, (ii) the operator desired trajectories, since they are given on-line.

2.1. MAIN RESULT

Before addressing the VE haptics context, let us recall some generic results that will be used in our proposed solution. Figure 1 illustrates any interconnected pair of passive systems defined respectively by their time-domain or frequency-domain linear mapping \mathbf{g}_1 and \mathbf{g}_2 . The external input signals of the interconnected systems are denoted respectively by e_1 and $e_2 \in \mathcal{L}_2^1$, the output signals by y_1 and $y_2 \in \mathcal{L}_2$, where as $u_1 = e_1 - y_2$ and $u_2 = e_2 + y_1$ are respectively the control signals. Eventually, the systems output signals y may be delayed by respectively τ_1 and τ_2 .

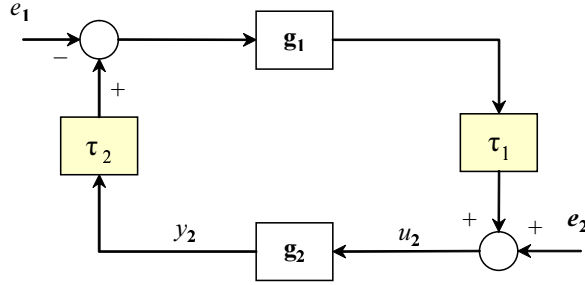


Figure 1. Interconnected systems with time delay.

THEOREM 1 (Shaft, 1996). *Considering $\tau_1 = \tau_2 = \mathbf{1}$ (identity mapping which means no time-delay), if $\mathbf{g}_1, \mathbf{g}_2$ are passive then the resulting system with inputs $(e_1, e_2) \in \mathcal{L}_2$ and outputs $(y_1, y_2) \in \mathcal{L}_2$ is also passive.*

Proof. see (Shaft, 1996; Vidyasagar, 1993) for the demonstration.

¹ $\mathcal{L}_2[0, \infty) = \mathcal{L}_2$ consists of all functions $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ which are measurable and satisfy $\int_0^\infty |f(t)|^2 dt < \infty$

This passivity property is not preserved when there exist a transmission delay in the closed loop system, i.e. $\tau_1(t) \neq 0$ or $\tau_2(t) \neq 0$ ■

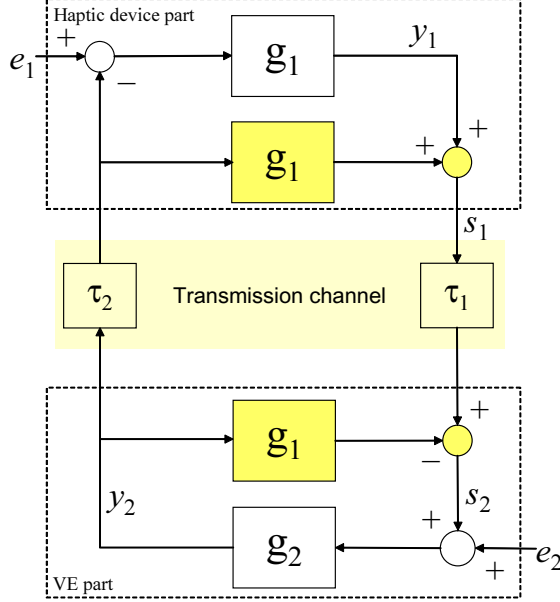


Figure 2. Model-based controller.

THEOREM 2. *The interconnected delayed system shown in figure 1 can be stabilized, using a process-model based control of either g_1 or g_2 as illustrated in figure 2.*

Proof. Let $f * g$ denotes the convolution product of the functions $f(t)$ and $g(t)$:

$$f * g = \int_{-\infty}^{\infty} f(t - \nu)g(\nu)d\nu$$

we recall that the convolution product is commutative.

Let $h(\tau_i, t)$ denotes the impulse response of the transmission channel (considered to be a delay-variable filter). The knowledge of this impulse response completely characterizes the transmission channel. In the case of none stationary channels (the case of variable delays), one can define relations similar to stationary ones: for example the convolution product. In this case however, the impulse response h depends on two parameters: $(\tau_i$ and $t)$, which denotes the system output at instant τ_i for an input at instant t . The convolution product can be generalized with the relation, known as Bello function (Bello, 1963):

$$z(t) = x(t) * h(\tau, t) = \int_{-\infty}^{\infty} x(t - \tau)h(\tau, t)d\tau \quad (1)$$

where $z(t)$ and $x(t)$ are respectively output and input signals of the transmission channel.

On the figure 2, at the s_1 output level we have:

$$s_1 = (e_1 - y_2 * h(\tau_2, t)) * g_1 + (y_2 * h(\tau_2, t)) * g_2 \quad (2)$$

Thus, it is easy to show that s_1 reduces to $s_1 = e_1 * g_1$ and the output of the channel τ_1 is simply $(e_1 * g_1) * h(\tau_1, t)$. Finally, the input of the last block, $s_2(t)$, is given by:

$$s_2 = (e_1 * g_1) * h(\tau_1, t) - y_2 * g_1 \quad (3)$$

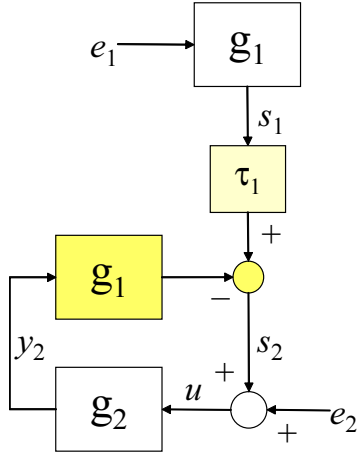


Figure 3. System reduction equivalence with the model-based controller.

At this level, the equivalent system obtained with the proposed controller is illustrated in figure 3. There is two cases to be considered.

Constant τ_1 : in this case, the impulse response $h(\tau_1, t)$ becomes a linear delay operator for which we can apply the known property $(f*g)_\tau = f_\tau*g = f*g_\tau$ and $s_2(t)$ simplifies to $s_2 = (e_1*g_1)_\tau - y_2*g_1$. By using this property, s_2 reduces to:

$$s_2 = e_{1\tau_1} * g_1 - y_2 * g_1 \quad (4)$$

In this case, the whole system reduces to the one illustrated by figure 4. Since the mapping \mathbf{g}_1 and \mathbf{g}_2 are passive the loop part is passive and the global system is stable.

Variable τ_1 : in this case, since $(e_1 * g_1) * h(\tau_1, t) \neq (e_1 * h(\tau_1, t)) * g_1$, the system (figure 3) can not be reduced. However, the system is

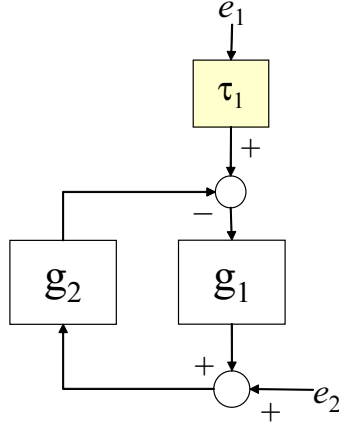


Figure 4. System reduction equivalence, constant τ_1 case.

stable if the loop $\mathbf{g}_1\text{-}\mathbf{g}_2$ is stable. Consequently, internal system stability is investigated and the transfer matrix:

$$\frac{1}{1 + G_1(s)G_2(s)} \begin{bmatrix} 1 & -G_1(s)G_2(s) \\ 1 & 1 \end{bmatrix} \quad (5)$$

must be stable. This is the case since the passivity of \mathbf{g}_1 and \mathbf{g}_2 implies the stability of $\frac{1}{1+G_1G_2}$

In a similar way, a \mathbf{g}_2 -based model prediction leads to the same demonstration ■

2.2. HAPTIC INTERACTION CASE

When applying this theorem to VE haptic interfaces, \mathbf{g}_1 is the model of the haptic device including the built-in controllers and \mathbf{g}_2 expresses the interactive VE model. The originality of the proposed solution is in the somehow prediction of the haptic device part within the VE (Arioui, 2002; Arioui, 2002). Hence, the developed equations lead to a scheme where only the haptic device model appears. The term “somehow prediction” is used to signify that in fact the proposed solution is not really a prediction in the sense of Smith since only the haptic device model is required. This means that no prediction or knowledge on operator behavior or trajectory is needed. Without loss of generality and to better understand the concept, a simple LTI model of a VE haptic interface is taken. Figure 5 shows the implementation of the proposed controller (gray part of the block diagram representation) within the haptic architecture. $M(s)$ is the haptic device transfer function, s is the Laplace transform variable, $E(s)$ is a continuous representation of the

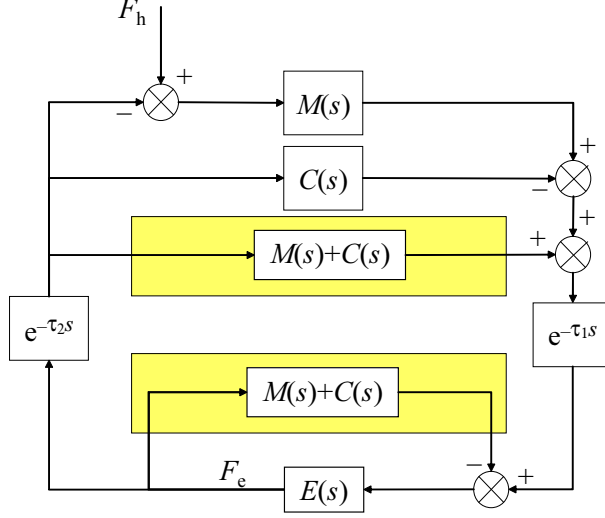


Figure 5. A practical implementation of the controller in the frame of VE haptics.

VE transfer function, F_e is the VE computed force, F_h is the operator applied force on the device, $C(s)$ is the commonly used virtual coupling (built-in controller) (Adams, 1999; Colgate, 1995); it guarantees unconditional stability of the haptic interaction in the absence of time-delay, and τ_1, τ_2 are respectively upwards and downwards time delays (on the figure 5, time delays are taken constant).

Estimation of the haptic device model do not require complicated techniques. Well known estimation methods (namely those developed in robotics) can be applied. The structure of the proposed controller (as opposed to a direct Smith prediction scheme) leads to interesting extensions that:

1. avoids the estimation of time-delay;
2. makes a straightforward extension to time-varying delay;
3. unburdens the VE from buffering to compute the controller.

3. Simulation results

This section presents simulation results of the developed controller. The haptic display is an actuated arm with a terminal point felt-mass $m = 0.2$ kg and a motion felt-friction of $b = 3$ Ns/m. The contact is performed between a rigid virtual probe and virtual walls of stiffness K_e . In this first simulation, time delays are taken constant but different,

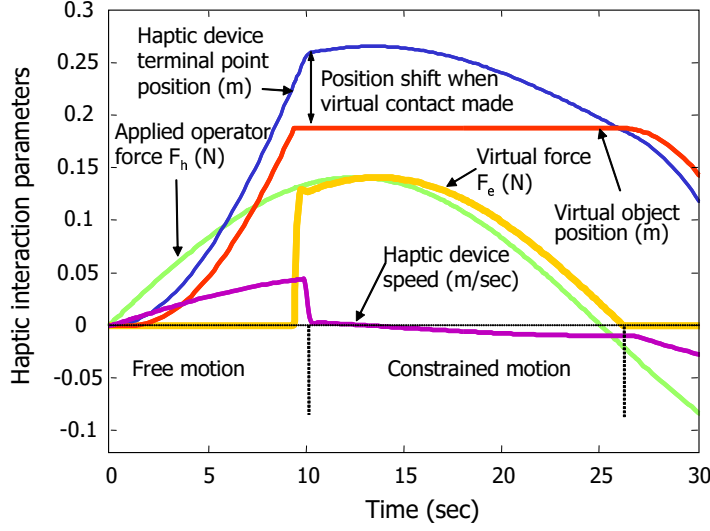


Figure 6. Simulation of delayed force reflecting virtual contact with a stiff wall.

indeed $\tau_1 = 1$ sec and $\tau_2 = 0.5$ sec. Figure 6 shows the tracking and force feedback behavior when the contact is made in the VE. Collision detection and force computation are performed quickly. The operator is supposed to apply a sinusoidal profile force F_h which drops the haptic device terminal point position to increase until a contact is made between the virtual probe (VP) and the virtual wall. This is done when the VP position reaches 20 cm. One can notice that when the contact is made, the master velocity v_m drops to zero and the feeded force F_e intensity increases correlatively to F_h . The position discrepancy (shift) appearing when the contact is made, is unavoidable whatever the controller is (unless a very prediction is made in the operator side). This is due to the undergo physical time-delay. Nevertheless, the VP position x_e is stably maintained by the operator during the contact. We conducted many other simulations with multiple hard and viscous contacts that show that the behavior of the VE haptic interaction is stable whatever the delay is. Obviously, one must not suspect that performances are acceptable for a real use in the presence of important time delays.

The figure 7 shows the result of the haptic interaction using similar parameters and time-varying delay. The variation of the delay $\tau_1(t)$ is plotted on the same figure 7. In this case, the simulation shows globally a stable behavior of the system. But, as it can be noticed, the dynamic of $\tau_1(t)$ affects the response behavior of force reflection. Indeed, figure 7 shows clearly that when a transition occurs between different $\tau_1(t)$ behaviors, i.e. between varying and constant delays, the force feedback

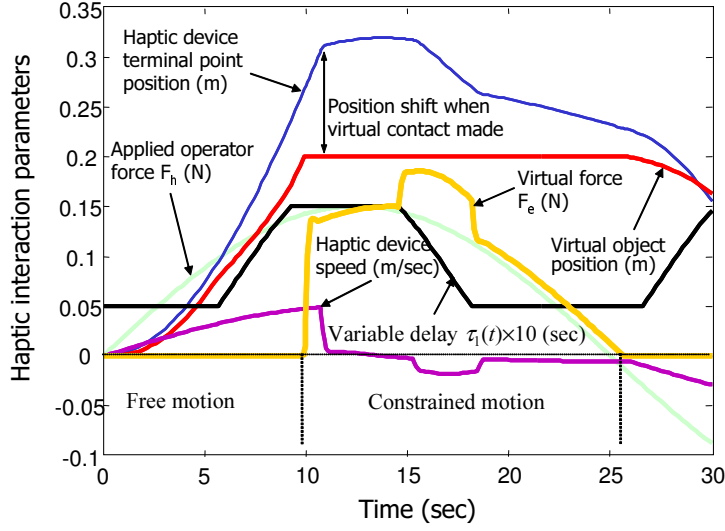


Figure 7. Simulation of haptic interaction under time-varying delay.

response switches respectively between two overall system behaviors. The transitions seem to be abrupt but do not affect the overall stability of the system.

To better understand these changes in the response behavior, we conducted a simple simulation where three responses were superimposed on the same figure 8:

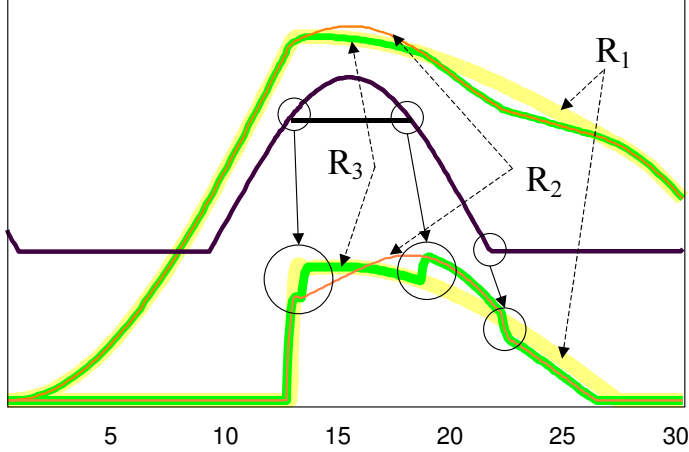


Figure 8. Correlation between force feedback and $\tau_1(t)$ behaviors.

1. the R_1 labelled signals (position/force) correspond to the case where both delay-operators are constant;

2. the R_2 labelled signals correspond to the case where $\tau_1(t)$ is defined as follow:

$$\begin{aligned} \tau_1(t) &= a \text{ [sec]} & t < 10 \text{ [sec]} \text{ and } t > 22 \text{ [sec]} \\ \tau_1(t) &= a + |\sin(t - t_i)| \text{ [sec]} & 10 \text{ [sec]} \leq t \leq 22 \text{ [sec]} \end{aligned}$$

where a is the value of the constant delay and $t_i \simeq 10$. The variation of the delay follows a sinusoidal profile;

3. the R_3 labelled signals correspond to the case where $\tau_1(t)$ is defined as previously in R_2 except that the delay becomes constant again for $13 \text{ [sec]} \leq t \leq 18 \text{ [sec]}$.

The time-delay variations (darker curve, figure 8) are causal. We can notice that the behavior of the computed force (controller) and the position of the haptic device terminal point are correlated to the time-delay behavior. The behavior transitions of the feeded force, big circles, correspond to constant-to-variable and variable-to-constant τ_1 transitions. Each time, the force signal switches from a shape to another according to the type (and not the value) of the delay (constant or variable).

3.1. ROBUSTNESS ANALYSIS

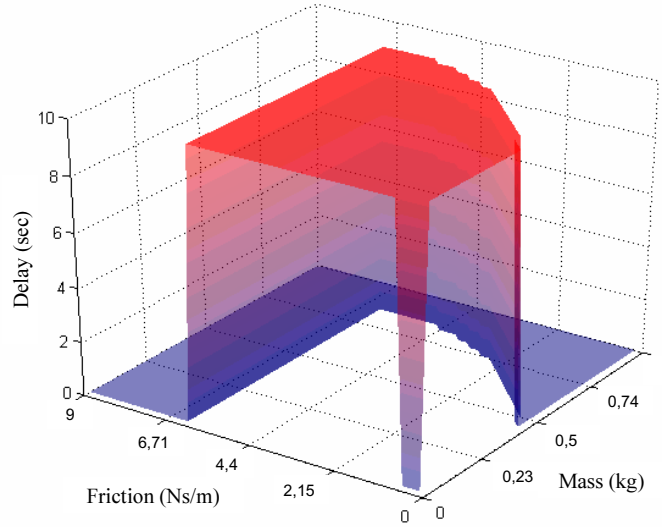


Figure 9. Stability margin expressing robustness of the control scheme.

To have a global idea on the controller robustness, the analysis was performed in the case of constant time delay. The time delays

were approximated using Padé's four order ($n = 4$) transfer function approximation², that is:

$$\exp(-s\tau) = \lim_{n \rightarrow \infty} \left(\frac{1 - \frac{\tau}{2n}s}{1 + \frac{\tau}{2n}s} \right)^n \frac{1}{1 + \frac{s}{n}}$$

We recall that when the model-based controller's parameters does not much the haptic device one, time delay terms remain in the characteristic equation of the closed loop system. And, this may lead to instability. The expression of the remaining transfer function delay-items are replaced by their Padé's approximations. A root locus of the closed loop characteristic equation of the system was performed by varying the controller's parameters.

A 3D plot of these parameter's variation and its influence on the stability margin, thus expressing robustness of the control scheme, is represented in the figure 9.

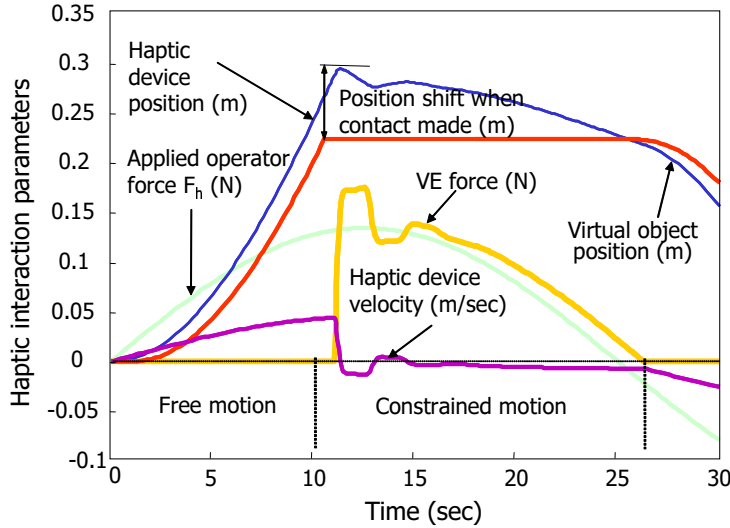


Figure 10. Simulation behavior with an error estimation of the master model parameters (mass and friction).

The figure 10 shows that within a determined margins, force reflection is still stable although some light oscillations appear in the master position x_m and in the reflected force F_e . In this simulation, figure 10, $\hat{m} = 0.3$ kg instead of the actual $m = 0.2$ kg and $\hat{b} = 5$ Nm/sec instead of the actual $b = 3$ Nm/sec are the estimated parameters, we can notice that the behavior of the interaction is still stable.

² Actually we performed a 40th order approximation of the delay, but the obtained margins are nearly the same as for a 4th order approximation.

3.2. COMPARISON WITH SCATTERING OR WAVE-BASED APPROACHES

Comparing to wave-based approaches, considered to be a standard solution for delayed force reflecting systems, this proposed solution is more transparent to the user (even for important delays. However, we recall that bilateral control would drastically decrease haptic interaction performance for delays up to 1 sec, (Hannaford, 1993; Hirzinger, 1993). The model-based approach would keep good performance since the controller is conceived with no corrupting dampers. This is the case in wave-base controllers when force and flux parameters are transformed into wave variables. Nevertheless, the price to be paid, when adopting a model-based controller, is in the importance of the position discrepancy (or shift) between the haptic device position and the virtual probe's one when the contact occurs. Indeed, position shift is more important in the model-based control case comparing to wave-based methods. In fact, in wave-based method, the artificial damping increases with speed (in free motion). This prevents important position discrepancies, but adding additional felt forces that are not related to actual remote contact forces. Thus a wave-based approach is more conservative but less transparent. Moreover, our proposed method applies to both constant and varying time delay without any change in the controller. This is not the case of wave-based passive techniques where an entire reformulation of the controller is necessary to deal with the varying delay cases, see (Yokokojhi, 2000; Niemeyer, 1998).

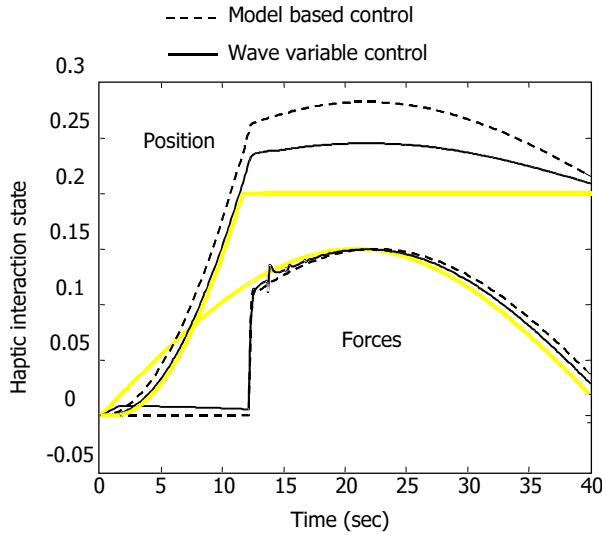


Figure 11. Wave-base and model-based controllers comparative simulation.

In few words, the main advantages of the proposed method comparing to wave-based ones are: a better fidelity and transparency of the reflected force, a simple computation of the controllers see figure 11, a straightforward extension to varying time delay.

The drawbacks are: a less conservative method, stability may be lost if the haptic device parameters are not well estimated, a more important position discrepancy between the haptic device and the virtual probe.

4. Experimental Results

The proposed controller was experimented thanks to an actual haptic feedback setup. We used an SGI workstation with OpenGL-based VE

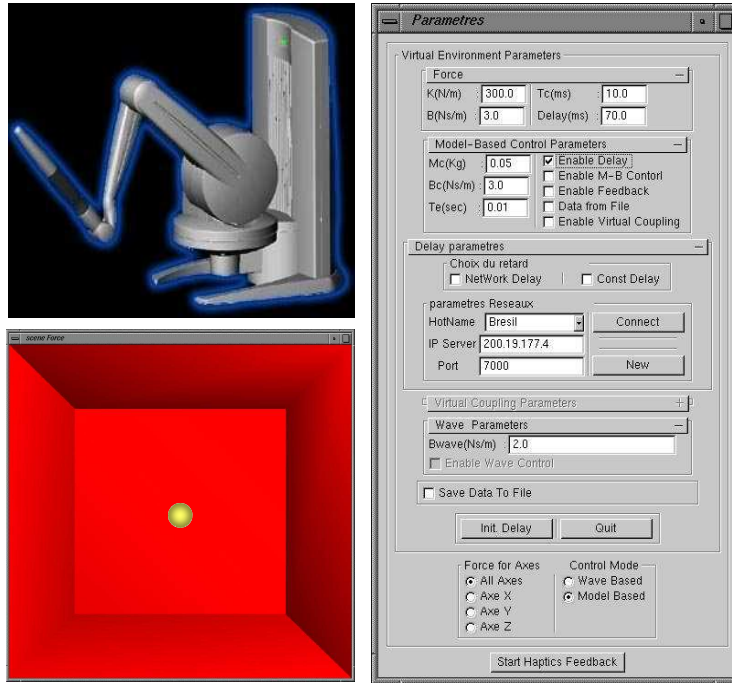


Figure 12. Experimental set-up: the PHANTOM haptic device (left-up), the VE (left-down), the control panel (right).

consisting of a simple virtual cube with rigid walls. Inside this cube, the operator moves a zero mass virtual sphere and performs contacts with virtual walls. Virtual sphere trajectory and computed force feedback are delivered via the PHANTOM haptic device. The PHANTOM Desktop is able to track all operator desired position and orientation but displays forces only on the three translational axes. A control panel was designed

to set-up virtual environment, controller and transmission channel parameters. In fact the haptic device - VE link allows achieving high bandwidth communication. To simulate constant time-delay, additional buffering software was implemented. The operator can setup any virtual constant delay through the control panel. In order to realize variable time delay, we used Internet-based relay servers at different locations (Berlin, Rio-de-Janeiro and Athens). These servers relay, through Internet, data sent by the haptic device to the VE and *vice versa*. Internet is known to be a very none time-deterministic protocol. All the defined experimental setup components are illustrated on the figure 12.

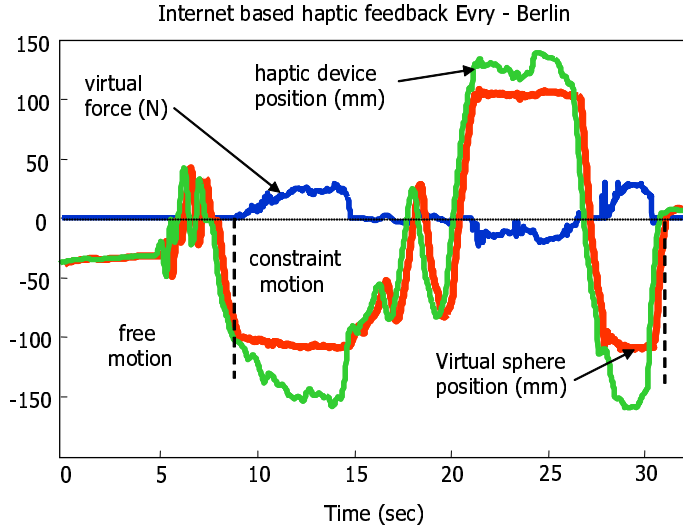


Figure 13. Internet-based stable haptic interaction (Berlin's server).

Many experiments were performed for constant and variable time delay. When the proposed controller is off, small transmission delays (e.g. $\tau_1 = \tau_2 = 100$ ms) destabilize the haptic interaction. When the proposed controller is made active, all experiments were conforming theoretical expectations. Indeed, the haptic interactions were stable whatever constant or variable delays are. Figure13 shows the results of a stable haptic interaction through the Internet. In this experimental case, the used server was in Berlin, Germany. These experiments show the validity of the proposed scheme. Robustness of the controller was also experienced for constant and varying time-delay. Obviously allowed estimation errors and stability margins are less than that found in theory especially for varying time-delay case.

5. Conclusion

A novel model-based controller to stabilize delayed virtual reality haptic feedback is presented. The proposed method lies in an astute adaptation and implementation of the well known Smith prediction scheme. In this study only the haptic device model is required. Moreover, the estimation of both (upwards and downwards) delays is not needed. The simulation results confirmed a stable virtual environment haptic interaction in the presence of both constant and time-varying delays. A robustness analysis of the proposed controller was also conducted. The error margins which guarantee the stability of the haptic interaction are found to be large enough to state robustness of the approach. Comparing to wave-based methods, this proposed solution is more transparent to the user, since there is no additional corrupting damping. The price to be paid is in the position discrepancy between the haptic device and the virtual probe when the contact is made. Many performed experiments showed the validity of the proposed theory. Future work is focused in improving performances by exploring virtual reality prediction based on computer haptics algorithms and a prediction in the wave passive space.

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