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## Model order reduction applied to metal forming

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### Résumé —

Model reduction is nowadays considered as an elegant tool to transform any high-dimensional model into a low-dimensional approximation. The latter can easily decrease the computational time. The present paper is fully dedicated to metal forming applications, where the computing time is an important factor. A Proper Orthogonal Decomposition (POD) is implemented in the commercial software Forge. The proposed reduced order model involves a reduced basis for velocity field and an other one for the pressure field to account for incompressibility constraint. The newer version called Forge POD is validated with the classic version of Forge.

**Mots clés** — Model Order Reduction, Proper Orthogonal Decomposition (POD), Material Forming, Forge POD.

## 1 Introduction

Modeling techniques have brought an important contribution dealing with different engineering problems in the last decades. Despite major progress in numerical methods and computers capacities, an accurate solution requires a refined space discretization, then increasing the dimension of the problem. Model order reduction seeks to overcome this problem by approximating the high dimensional problem with a lower one. Nowadays, several techniques are available in the literature [1],[2]. Our chosen tool is the proper orthogonal decomposition (POD).

Our focus is not to develop a variant of the POD technique but instead to couple it with an industrial software dedicated to metal forming called Forge3. The whole procedure is detailed, then followed by a numerical application validating the used approach.

## 2 Problem description

Let us consider an incompressible strip denoted  $\Omega \subset \mathbb{R}^3$ . A pressure  $p_0$  is applied on the bottom of the strip whereas the edges are supposed to be clamped as illustrated in Figure 1.

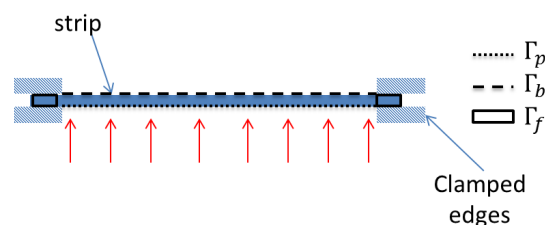


FIGURE 1 – A side view of the strip under pressure.

It is important to mention that no contact condition is taken into account in our problem. Instead, the edges of the strip are fixed via Dirichlet boundary conditions imposed on  $\Gamma_b$  (the zone between the dashed tools (see Figure 1)). The domain boundary  $\partial\Omega$  can be written as  $\Gamma_p \cup \Gamma_b \cup \Gamma_f$ .  $\Gamma_p$  and  $\Gamma_b$  represents respectively the surface on which the pressure and Dirichlet conditions are applied.  $\Gamma_f = \partial\Omega \setminus (\Gamma_p \cup \Gamma_b)$  is the free surface. The system of equations is expressed by :

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} = \rho \mathbf{g} & \text{in } \Omega & (1a) \\ \nabla \cdot \mathbf{v} = 0 & \text{in } \Omega & (1b) \\ \boldsymbol{\sigma} \cdot \mathbf{n} = 0 & \text{over } \partial\Omega \setminus (\Gamma_p \cup \partial\Gamma_b) & (1c) \\ \boldsymbol{\sigma} \cdot \mathbf{n} = p_0 & \text{over } \Gamma_p & (1d) \\ \mathbf{v} = 0 & \text{in } \Gamma_b & (1e) \end{cases}$$

in which  $\mathbf{v}$  is the velocity and  $p$  is the pressure.  $\rho$  and  $\mathbf{g}$  represent respectively the material density and the gravity.  $\mathbf{n}$  is the outer normal. The Cauchy stress tensor  $\boldsymbol{\sigma}$  obeys a Norton-Hoff behavior law :

$$\boldsymbol{\sigma} = 2K(\sqrt{3}\dot{\boldsymbol{\varepsilon}})^{m-1}\boldsymbol{\varepsilon}(\mathbf{v}) - p\mathbf{I} \quad (2)$$

where  $K$  is the consistency of the material and  $m$  is the strain rate sensitivity coefficient. The strain rate tensor is defined by  $\boldsymbol{\varepsilon}(\mathbf{v}) = (\nabla\mathbf{v} + {}^t\nabla\mathbf{v})/2$ , while  $\dot{\boldsymbol{\varepsilon}} = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}(\mathbf{v}) : \boldsymbol{\varepsilon}(\mathbf{v})}$ .

## 2.1 Matrix Formulation in Forge3

The computational domain  $\Omega$  is discretized using an unstructured mesh. The equations of the system above are discretized using a Mini-Element method (P1+P1)[3].

Solving the system is equivalent to finding  $(v_i, p_i)_{i=1}^{\mathcal{N}}$  such that  $v(x, t) = \sum_{i=1}^{\mathcal{N}} N_i(x)v_i(t)$  and  $p(x, t) = \sum_{i=1}^{\mathcal{N}} N_i(x)p_i(t)$  verifying the weak formulation. Since our problem is non linear, a Newton-Raphson algorithm is needed to ensure the accuracy of the solution. If  $\mathbf{R}$  is the residual of the finite element equations and  $\mathbf{J}$  its Jacobian, then the matrix formulation can be written as follows :

$$\text{Find the correction } (\delta q_i)_{i=1}^{\mathcal{N}} = \begin{bmatrix} \delta v_i \\ \delta p_i \end{bmatrix} \text{ verifying } \begin{bmatrix} \mathbf{J}^{vv} & \mathbf{J}^{vp} \\ \mathbf{J}^{pv} & \mathbf{J}^{pp} \end{bmatrix} \begin{bmatrix} \delta v_i \\ \delta p_i \end{bmatrix} = -\mathbf{R} \quad (3)$$

To improve the convergence rate of the Newton-Raphson algorithm, a line search algorithm is put in place. This technique consists on controlling the computed correction using a parameter  $\alpha$ . Then the solution is given as follows :

$$\begin{bmatrix} v \\ p \end{bmatrix} = \begin{bmatrix} v \\ p \end{bmatrix} + \alpha \begin{bmatrix} \delta v \\ \delta p \end{bmatrix} \quad (4)$$

Finally,  $\alpha$  is chosen to minimize the norm of the residual  $\mathbf{R}$  :

$$\underset{\alpha \in [0,1]}{\operatorname{argmin}} \|\mathbf{R}(q + \alpha\delta q)\| \quad (5)$$

## 2.2 Matrix Formulation in a POD

The objective of POD methods in general is to find a reduced equivalent system to solve while preserving as much as possible the accuracy of the solution. In other words, the aim is to build a reduced orthonormal basis vectors  $(N_r)_{r=1}^{mod}$  ( $mod < \mathcal{N}$ ) that minimizes the sum of the squared projection error over the chosen snapshots.

We applied the snapshot POD introduced by Sirovich in [4]. At every snapshot time  $t_i$ ,  $i = 1, \dots, n_f$ , the velocity and pressure fields are stocked respectively into two matrix  $\mathbf{Q}^v$  and  $\mathbf{Q}^p$ .

To determine the reduced basis, we use the singular value decomposition algorithm (SVD) :

$$\mathbf{Q}^j = U^j \boldsymbol{\Sigma}^j (V^j)^t; \quad j = \{v, p\} \quad (6)$$

where  $U^j$  contains the basis vector  $N_r$ .

Therefore,  $\begin{bmatrix} v \\ p \end{bmatrix} = A_r \begin{bmatrix} v_r \\ p_r \end{bmatrix}$  in which  $A_r$  is constructed as follows  $\begin{bmatrix} U^v & 0 \\ 0 & U^p \end{bmatrix}$  and  $q_r = (v_r, p_r)^t$  denotes the solution in the reduced basis. Then the reduced matrix formulation is expressed as follows :

$$\begin{bmatrix} (U^v)^t \mathbf{J}^{vv} U^v & (U^v)^t \mathbf{J}^{vp} U^p \\ (U^p)^t \mathbf{J}^{pv} U^v & (U^p)^t \mathbf{J}^{pp} U^p \end{bmatrix} \begin{bmatrix} \delta v_r \\ \delta p_r \end{bmatrix} = \begin{bmatrix} (U^v)^t \mathbf{R}_v \\ (U^p)^t \mathbf{R}_p \end{bmatrix} \quad (7)$$

The linear search is expressed and solved in the reduced basis as well :

$$\underset{\alpha \in [0,1]}{\operatorname{argmin}} \|\mathbf{R}(q_r + \alpha \delta q_r)\| \quad (8)$$

After determining the solution  $q_r$  in the reduced basis, the solution  $q$  in the initial basis is established via the relation mentioned above.

### 2.3 Application

In this section we use the problem detailed earlier as a numerical application for the described theory. We consider a cylindrical strip of 1 mm thick and a radius of 50 mm. The consistency is equal to 201,79 MPa.s and  $m = 0.15$ . An initial pressure of  $-0.028$  MPa is applied. It evolves linearly during the simulation. The outer edges are clamped. Only one fourth of the piece is considered due to the symmetry.

To begin with, snapshots are carried out for every time step. Figure 2 shows the norm of the velocity modes obtained by the SVD decomposition. The modes are plotted on a radial distance. The first mode ( in blue ) illustrates the shape of the deflected strip. The other two bring some corrections to the deformation.

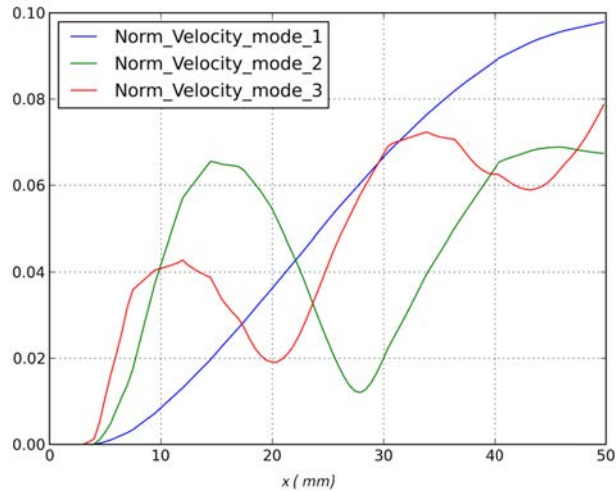


FIGURE 2 – The first three velocity modes plotted on a radial distance.

The same simulation is carried out in both Forge versions (Forge3 and Forge POD). Note that we used a total of 8 modes divided equally between the velocity and the pressure. In particular velocity components  $v_x$  and  $v_z$  are compared in Figure 3 for  $t = 2.68$  s. The superposition shows a good agreement. Forge POD gives the same results saving 40% of the computing time.

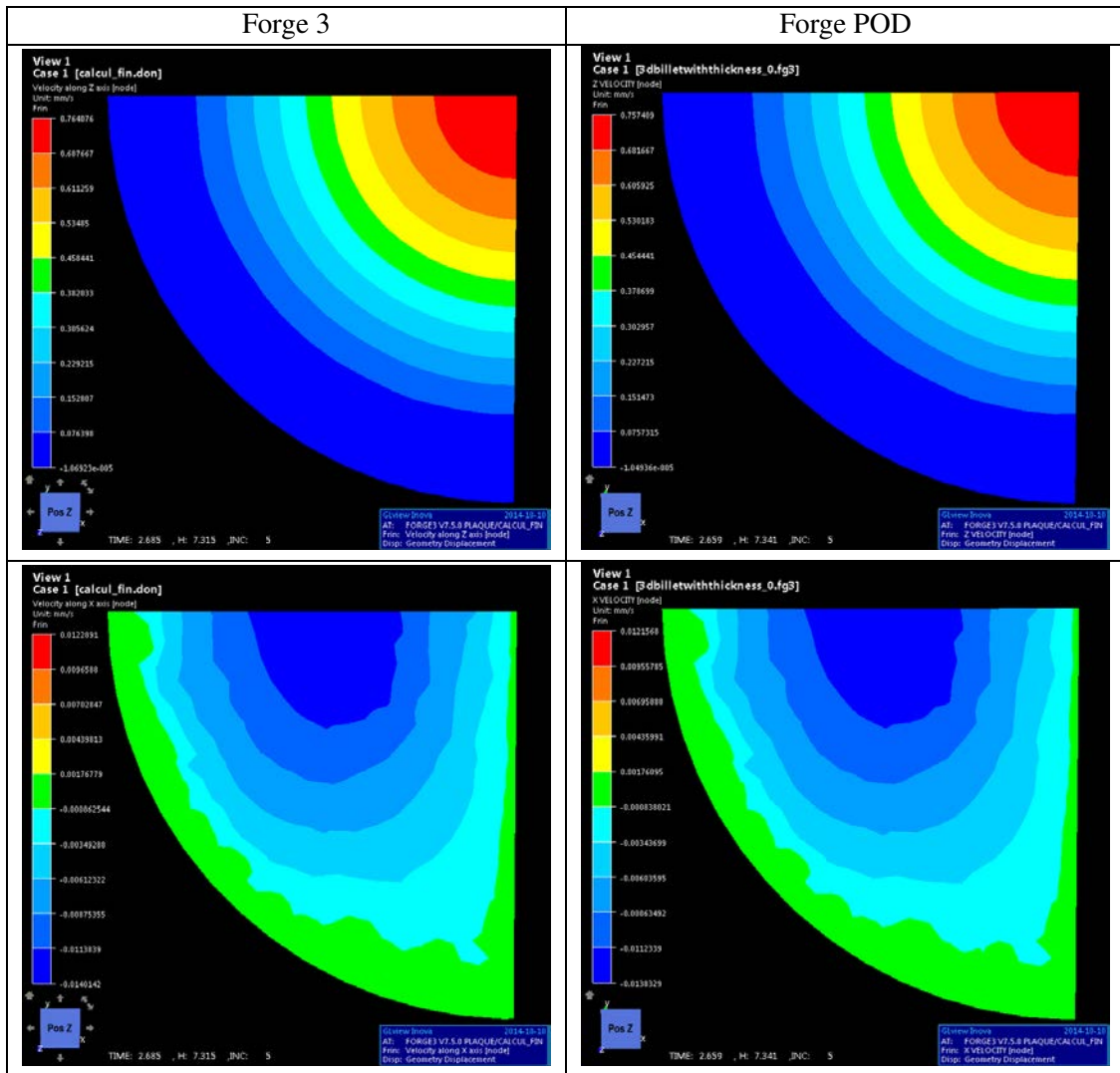


FIGURE 3 – A comparison between the velocities maps computed by Forge3 (on the left) and Forge POD (on the right) for  $t = 2.68s$ .

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