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# A filter-based computational homogenization method for handling non-separated scales problems

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**Résumé** — In this work, a computational homogenization framework based on filters is proposed to handle homogenization with arbitrary scale separation. Classical homogenization is extended by using low-pass filters instead of averaging operators, leading to a naturally nonlocal elastic framework, where the kernel function is fully constructed by means of computations on a unit cell describing the microstructure. An associated Finite Element Framework is proposed to deal with heterogeneous structures containing inclusions with characteristic length of the same order than loading fluctuation wavelength.

**Mots clés** — Homogenization, Non-separated scales, Filter-based homogenization.

## 1 Introduction

Classical homogenization assumes separation of scales. This assumption is only valid when the characteristic fluctuation wavelength of an applied strain field over a given microstructure is supposed to be large compared with the scale of the microstructure. Otherwise, the scales are considered as *non-separated* and the overall effective behavior at a given point depends on the strain states at other points in the neighborhood, defining a nonlocal effective behavior. Several families of methodologies have been proposed to handle lack of scale separation [1, 2, 3, 4]. The objective of this work is to provide a new computational homogenization method able to handle arbitrary scale separation.

## 2 Mesoscopic description of mechanical fields by numerical filters

In [5, 6], a framework for homogenization able to handle arbitrary scale separation has been proposed by replacing averaging operators by numerical filters (Gaussian, or least-square-based).

First, we define two scales, one scale associated to fine scale strain and stress fields  $\boldsymbol{\varepsilon}(\mathbf{x})$  and  $\boldsymbol{\sigma}(\mathbf{x})$ , and another (upper) scale called "Mesoscale", associated to strain and stress fields denoted by  $\hat{\boldsymbol{\varepsilon}}(\mathbf{x})$  and  $\hat{\boldsymbol{\sigma}}(\mathbf{x})$ , respectively, and called "Mesoscopic" strain and stress fields. These fields are related to each other through the following relations :

$$\hat{\boldsymbol{\varepsilon}}(\mathbf{x}) = \mathcal{F}(\boldsymbol{\varepsilon}(\mathbf{x})), \quad \hat{\boldsymbol{\sigma}}(\mathbf{x}) = \mathcal{F}(\boldsymbol{\sigma}(\mathbf{x})), \quad (1)$$

where  $\mathcal{F}(\cdot)$  is a linear operator, acting as a low-pass filter on the fine scale fluctuations. Associating  $\mathcal{F}$  with a characteristic length  $h$  related to the fluctuations observation at the mesoscale, the following properties are assumed for  $\mathcal{F}$  :

$$\lim_{h \rightarrow 0} \mathcal{F}^h(\boldsymbol{\varepsilon}(\mathbf{x})) = \boldsymbol{\varepsilon}(\mathbf{x}), \quad \lim_{h \rightarrow \infty} \mathcal{F}^h(\boldsymbol{\varepsilon}(\mathbf{x})) = \langle \boldsymbol{\varepsilon}(\mathbf{x}) \rangle. \quad (2)$$

In [6], we introduced a least-square-based filter in the form :

$$\mathcal{F}(\boldsymbol{\varepsilon}_{ij}(\mathbf{x})) = \sum_{p=1}^P M^p(\mathbf{x}) \hat{\boldsymbol{\varepsilon}}_{ij}^p, \quad (3)$$

where  $M^p(\mathbf{x})$  are piece-wise polynomial basis functions (e.g. finite element shape functions) and  $\hat{\boldsymbol{\varepsilon}}_{ij}^p$  are coefficients, interpreted as the nodal values of the mesoscopic strain field at some nodes of a coarse grid

associated with the mesoscopic mesh (see [6] for more details), for each component  $ij$ . These values are assumed associated with a compatible mesoscopic strain field.

Given the fine scale strain field  $\varepsilon(\mathbf{x})$  on a discrete fine mesh composed of  $N$  nodes  $\mathbf{x}^m$ ,  $m = 1, \dots, N$ , the unknown coefficients  $\hat{\varepsilon}_{ij}^p$  are found by minimizing the following functional :

$$U = \sum_{m=1}^N \left( \sum_{p=1}^P M^p(\mathbf{x}^m) \hat{\varepsilon}_{ij}^p - \varepsilon_{ij}(\mathbf{x}^m) \right)^2. \quad (4)$$

Following [5], we then introduce the following split of microscopic strain into a filtered (mesoscopic) part and a remaining fluctuation  $\tilde{\varepsilon}(\mathbf{x})$  :

$$\varepsilon(\mathbf{x}) = \hat{\varepsilon}(\mathbf{x}) + \tilde{\varepsilon}(\mathbf{x}). \quad (5)$$

Let us consider a unit cell characterizing the microstructure defined in a domain  $\Omega \in \mathbb{R}^D$ ,  $D$  being the dimension of the space, with boundary  $\partial\Omega$ . We define the following localization problem on the unit cell for non-separated scales : assuming known an applied (non-constant) mesoscopic strain field  $\hat{\varepsilon}(\mathbf{x})$ , find  $\varepsilon(\mathbf{x})$  satisfying :

$$\nabla \cdot (\boldsymbol{\sigma}(\mathbf{x})) = 0 \quad \text{in } \Omega \quad (6)$$

and

$$\mathcal{F}(\boldsymbol{\varepsilon}(\mathbf{x})) = \hat{\varepsilon}(\mathbf{x}) \quad \text{in } \Omega \quad (7)$$

with

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbb{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) \quad (8)$$

where  $\mathbb{C}(\mathbf{x})$  is a fourth-order elasticity tensor. Condition (7) is an extension of the classical averaging condition of homogenization in the case of separated scales. It can be interpreted as follows : the filtered part of the compatible strain field satisfying (6)-(8) must match  $\hat{\varepsilon}(\mathbf{x}) \forall \mathbf{x} \in \Omega$ . An iterative scheme has been proposed in [6] to enforce this condition.

### 3 Mesoscopic homogenized model

In this framework, we have shown in [5] that the constitutive law at the meso scale is given by :

$$\hat{\boldsymbol{\sigma}}(\mathbf{x}) = \int_{\Omega} \hat{\mathbb{C}}(\mathbf{x}, \mathbf{y}) : \hat{\boldsymbol{\varepsilon}}(\mathbf{y}) d\mathbf{y}, \quad \hat{\mathbb{C}}(\mathbf{x}, \mathbf{y}) = \mathcal{F} \{ \mathbb{C}(\mathbf{x}) : \hat{\mathbb{A}}(\mathbf{x}, \mathbf{y}) \}, \quad (9)$$

where  $\hat{\mathbb{C}}(\mathbf{x}, \mathbf{y})$  is a nonlocal elastic tensor defined in  $\Omega$ . Note that the constitutive law (9) is general for linear filter operators satisfying conditions (2), and then remains valid when  $\mathcal{F}$  is chosen e.g. as a Gaussian filter. Another remark is that in contrast to classical nonlocal elasticity [1], the present nonlocal elasticity operator *does not have translational invariance*, i.e.  $\hat{\mathbb{C}}(\mathbf{x}, \mathbf{y}) \neq \hat{\mathbb{C}}(\mathbf{x} - \mathbf{y})$ . This constitutes a key feature of the the model to continuously define mesoscopic constitutive laws for arbitrary scale separation.

A discrete framework can be formulated as follows. Using (3), we define  $\hat{\varepsilon}(\mathbf{x})$  in the form

$$\hat{\varepsilon}_{ij}(\mathbf{x}) \simeq \sum_p M^p(\mathbf{x}) \hat{\varepsilon}_{ij}^p \quad (10)$$

where the nodal values  $\hat{\varepsilon}_{ij}^p$  are defined on a coarse grid discretizing the structure at the mesoscopic scale and  $M^p$  are interpolations functions. Using the filter operator  $\mathcal{F}$  defined by (3), we obtain the following relationships :

$$\hat{\boldsymbol{\sigma}}(\mathbf{x}) = \sum_p \hat{\mathbb{C}}^p(\mathbf{x}) : \hat{\boldsymbol{\varepsilon}}^p, \quad \hat{\mathbb{C}}^p(\mathbf{x}) = \mathcal{F} \{ \mathbb{C}(\mathbf{x}) : \hat{\mathbb{A}}^p(\mathbf{x}) \}, \quad (11)$$

and

$$\hat{\mathbb{A}}^p(\mathbf{x}) = M^p(\mathbf{x}) \mathbb{I} + \boldsymbol{\Psi}^p(\mathbf{x}) - \mathcal{F}(\boldsymbol{\Psi}^p(\mathbf{x})), \quad (12)$$

where  $\Psi^p(\mathbf{x})$  are transformation tensors associated to each node  $p$  of the coarse mesh on the unit cell, defined by :

$$\Psi_{ijkl}^p(\mathbf{x}) = e_{ij}^{p,(kl)}(\mathbf{x}), \quad (13)$$

where  $e^{p,(kl)}$  is the strain solution field obtained by solving on the unit cell an elementary problem (see details in [6]) and where  $\chi^{p,(kl)}(\mathbf{x})$  is a non-uniform eigenstrain prescribed over the support of  $M^p(\mathbf{x})$  and defined as

$$\chi^{p,(kl)}(\mathbf{x}) = \frac{M^p(\mathbf{x})}{2} (\mathbf{v}_k \otimes \mathbf{v}_l + \mathbf{v}_l \otimes \mathbf{v}_k) \quad (14)$$

where  $\mathbf{v}_i$  are the base vectors of a Cartesian coordinate system. Then the local strain field can be re-localized by the relationship :

$$\varepsilon(\mathbf{x}) = \sum_p \hat{\mathbb{A}}^p(\mathbf{x}) \hat{\varepsilon}^p. \quad (15)$$

## 4 Numerical examples

The methodology is illustrated in the following example. A heterogeneous structure, as depicted in Figure 1 (b), is subjected to 4 points bending. The unit cell described in Fig. 1 (a) is used to compute the nonlocal constitutive operators described in section 3. Results in Figures 2 and 3 show that the proposed method allows reproducing both the simplified model (mesoscopic model) computed on a coarse grid, but also provides a good approximation for local fields, which can be obtained by post-treatment through localization rule (15). We also show the limits of classical homogenization methods and extended versions, in such situations when scales cannot be separated.

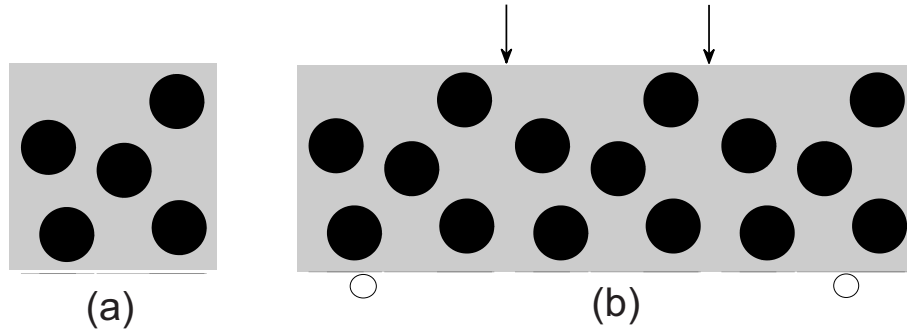


FIGURE 1 – (a) Unit cell ; (b) heterogeneous structure subjected to 4 points bending.

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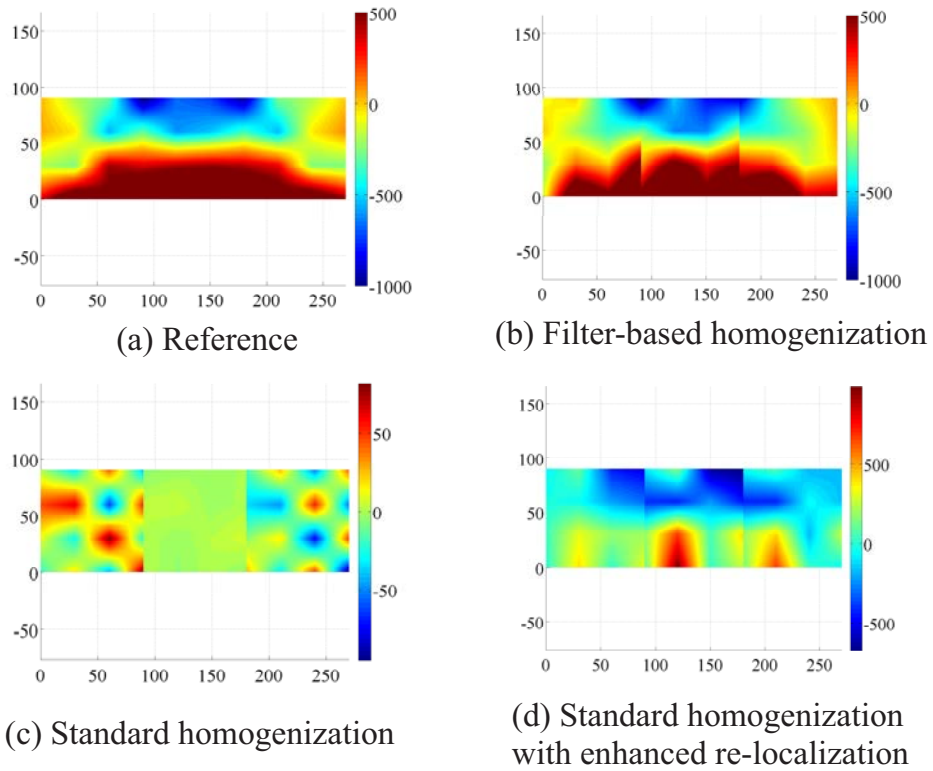


FIGURE 2 – Stress fields obtained at the mesoscopic scale (on the coarse grid) : comparisons between several models.

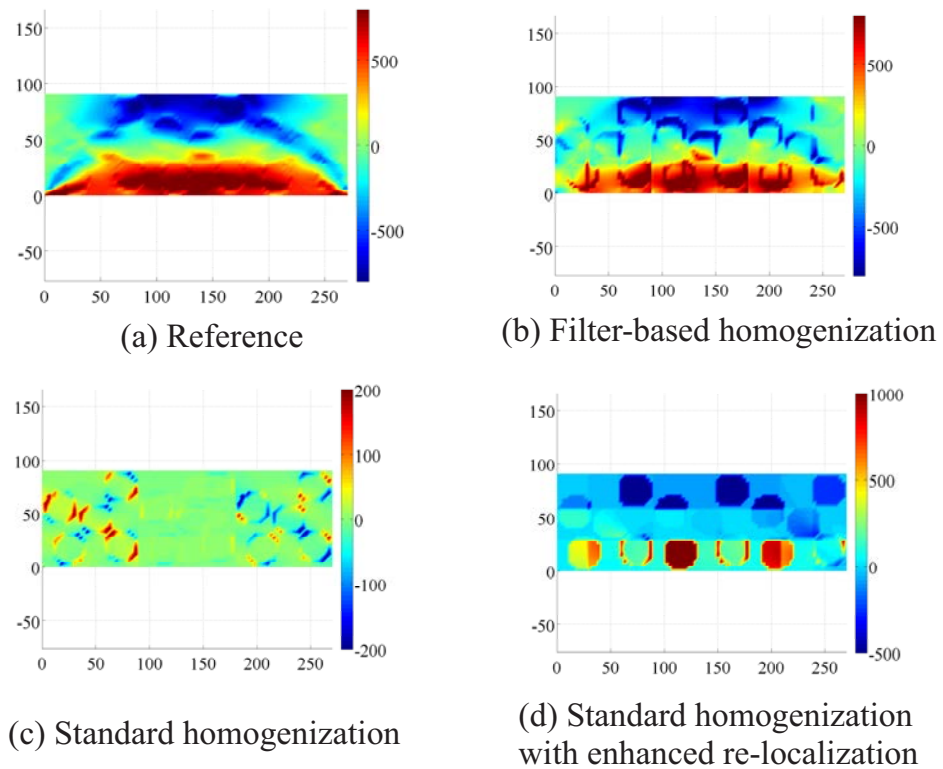


FIGURE 3 – Stress fields obtained at the microscopic scale (on the finest grid) : comparisons between several models.