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Analytical Homogenization for Honeycomb Sandwich Plates with Skin and Height Effects

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Abstract — Numerical modeling of honeycomb sandwich plates is too tedious and time consuming. The analytical homogenization enables to obtain an equivalent homogeneous solid and its elastic stiffness. In this paper, the skin effect is considered for the in-plane shear and torsion problems, in which the two skins are relatively rigid. Homogenization models using trigonometric function series and energy method is proposed to study the influence of the honeycomb height on these moduli. The comparison between our H-model and Abaqus 3D modeling has shown very good agreement.
Key words —Analytical homogenization, Honeycomb sandwich plate, Skin and height effects.

1. Introduction

The numerical modeling of honeycomb sandwich plates is too tedious and time consuming. The analytic homogenization enables to obtain an equivalent homogeneous solid leading to efficient simulations. The book of Gibson and Ashby [1] is the first systematic literature in this field. Further refinements have been done by Masters and Evans [2]. However, these models are based on pure cellular structures without considering the strengthening effect of the skin faces. Since the skins' constraints significantly alter the core's local deformation, the core properties are sensitive to the core thickness, called thickness effect by Becker [3]. An interesting two-step approach was proposed by Xu et al. [4] to homogenize a honeycomb cell with the skin effect. Chen *et al.* [5] have proposed some trigonometric and hyperbolic function series based on the equilibrium solution. However, this solution is limited by boundary condition assumptions. Hoang *et al.* [6] have proposed an analytical formulation to calculate the stretching and bending stiffnesses of a honeycomb core in a sandwich plate considering the skin and core height effects. In this study, we deal with the in-plane shear and torsion problems. Since the core's deformation is constrained by two skins, the membrane stretching is dominant. The energetic method is used to determine the in-plane shear and torsion moduli of the equivalent solid. A homogenization formulation using trigonometric function series is proposed by taking into account the stress redistribution between honeycomb walls. The minimization of the strain energy allows obtaining the series parameters, the internal strain energy and then the elastic moduli. The comparison between our H-model and Abaqus 3D modeling has shown very good agreement.

2. Methodology of the analytic homogenization models

The homogenization consists in replacing the honeycomb structure by an equivalent homogenized solid. An energetic homogenization method is used to determine the shear and torsion moduli of the equivalent solid. Since the skins are very rigid, the honeycomb walls are constrained by the skin deformation, and undergo stretching, shearing, bending and torsion deformations. A REV of honeycomb cell is shown in Fig. 1. Due to the triple symmetry, 1/8 REV (Fig. 2) is considered. The internal strain energy of the three walls can be defined in their local references as follows:

$$\pi_{\text{int}} = \frac{1}{2}\iint\left[\frac{Et}{1-\nu^2}\left(\varepsilon_x^2+\varepsilon_y^2+2\nu\varepsilon_x\varepsilon_y+\frac{1-\nu}{2}\gamma_{xy}^2\right)+D\left(w_{,xx}^2+w_{,yy}^2+2\nu w_{,xx}w_{,yy}+2(1-\nu)w_{,xy}^2\right)\right]dxdy \qquad (1)$$

Once the displacement and strain fields in the three walls are defined, the internal strain energy can be calculated, then the shear and torsion moduli G_{XY}^{S*} and G_{XY}^{T*} can be obtained.

3. Homogenization formulation for in-plane shear moduli

The basic displacement field of the honeycomb REV is given by the imposed skin shear deformation (Fig. 1). Due to the continuity condition with the adjacent cells, the displacements are horizontal, and do not lead to the stretching deformation in the vertical walls. However, the different horizontal displacements at the two ends of the inclined wall induce to a normal strain in this wall:

$$\varepsilon_s = \frac{(U_{03} - U_{02})\cos\theta}{l} = \left[\gamma_{XY}\frac{h}{2} - \gamma_{XY}\left(\frac{h}{2} + l\sin\theta\right)\right]\frac{\cos\theta}{l} = -\gamma_{XY}\sin\theta\cos\theta \tag{2}$$

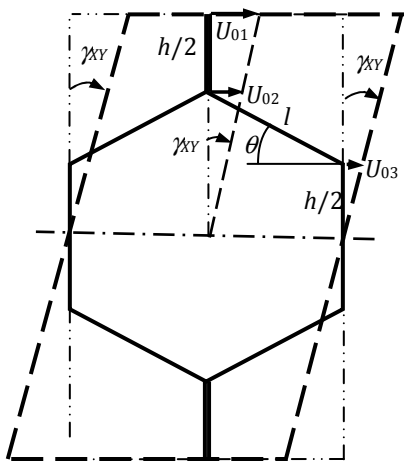

Figure 1 – Displacements imposed by the skins

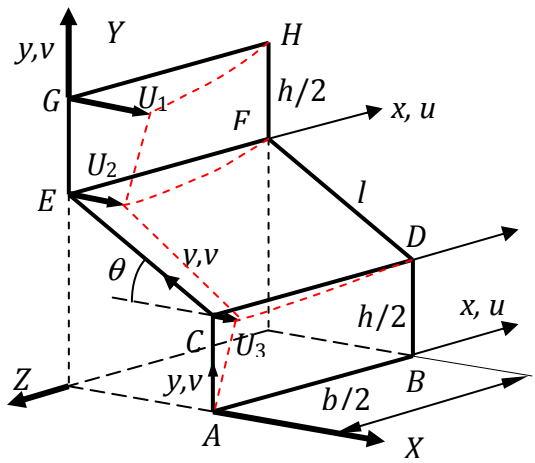

Figure 2 – Additional displacements on the walls

The basic displacement field cannot satisfy the equilibrium condition between the 3 walls. The stress redistribution between them gives an additional displacement field (Fig. 2). This largely reduces the shear and torsion stiffnesses. According to the boundary conditions, the following additional displacement fields are proposed in the local references (denoted xy in Fig. 2) of the upper, inclined and lower walls, respectively:

$$u = a_0\frac{2}{b}x \text{ (for 3 walls)} \;;\; v = 0 \;;\; w = \sum_{i=1}^{N}\left[a_i\frac{2y}{h} + b_i\left(1 - \frac{2y}{h}\right)\right]\cos\frac{m\pi x}{b} \;;\; m = 2i - 1 \tag{3}$$

$$v = -\cos\theta\sum_{i=1}^{N}\left[b_i\frac{y}{l} + c_i\left(1 - \frac{y}{l}\right)\right]\cos\frac{m\pi x}{b} \;;\; w = \sin\theta\sum_{i=1}^{N}\left[b_i\frac{y}{l} + c_i\left(1 - \frac{y}{l}\right)\right]\cos\frac{m\pi x}{b} \tag{4}$$

$$v = 0 \;;\; w = \sum_{i=1}^{N} c_i\frac{2y}{h}\cos\frac{m\pi x}{b} \tag{5}$$

where a_0, a_i, b_i and c_i are the parameters to be determined, *u* and *v* give the membrane strains, *w* gives the bending curves.

The total internal strain energy in 1/8 REV can be defined by using Eqs. (1-5). The work of the external forces is nil during the stress redistribution. The minimization of the strain energy leads to a linear system of equations to calculate the unknown parameters a_0, a_i, b_i and c_i.

Then the strain energy π_{int} in function of the core height *b* can be calculated. This energy should be equal to the strain energy of the equivalent homogenized solid:

$$\frac{1}{2}G_{XY}^{S*}\gamma_{XY}^{2}\frac{b}{2}l\cos\theta\left(h+l\sin\theta\right)=\pi_{\text{int}} \quad \Rightarrow \quad G_{XY}^{S*} \tag{6}$$

The above formulation may give the upper and lower bounds of G_{XY}^{S*} by taking $b \to 0$ and ∞.

4. Homogenization formulation for torsion stiffness

The above procedure is also adopted for the torsion problem. We consider the basic deformations and then additional ones. In Fig. 3-4, the rotation β_X imposed by the skins gives a normal strain on the intersection of the inclined wall and skin, and this strain varies linearly along *x*:

$$\gamma_{XY}=\frac{\beta_X b/2}{h+l\sin\theta}=\beta_{X'Y}\frac{b}{2}; \quad \varepsilon_s=-\alpha\gamma_{XY}=-\alpha\beta_{X'Y}\frac{b}{2} \tag{7}$$

$$\varepsilon_s(x)=-\alpha\beta_{X'Y}\frac{b}{2}\frac{2}{b}x=-\alpha\kappa x \qquad (\kappa=\beta_{X'Y}\text{ half of torsion curvature}) \tag{8}$$

According to the observation of numerical simulation results, the additional displacements on *CD*, *EF*, *GH* are only along the horizontal direction and have a sinusoidal form. We propose the following additional displacement fields for the upper, inclined and lower walls, respectively:

$$u=\frac{a_0}{b}x^2 \text{ (for 3 walls)} \;;\; v=0 \;;\; w=\sum_{i=1}^{N}\left[a_i\frac{2y}{h}+b_i\left(1-\frac{2y}{h}\right)\right]\sin\frac{m\pi x}{b} \;;\; m=2i \tag{9}$$

$$v=-\cos\theta\sum_{i=1}^{N}\left[b_i\frac{y}{l}+c_i\left(1-\frac{y}{l}\right)\right]\sin\frac{m\pi x}{b} \;;\; w=\sin\theta\sum_{i=1}^{N}\left[b_i\frac{y}{l}+c_i\left(1-\frac{y}{l}\right)\right]\sin\frac{m\pi x}{b} \tag{10}$$

$$v=0 \;;\; w=\sum_{i=1}^{N}c_i\frac{2y}{h}\sin\frac{m\pi x}{b} \tag{11}$$

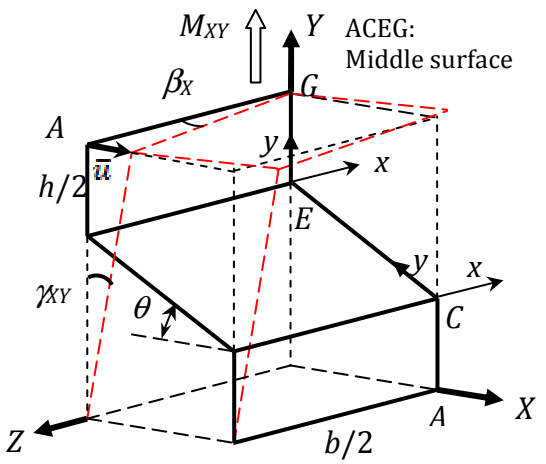

Figure 3 - Torsion moment M_{XY} in 1/8 REV

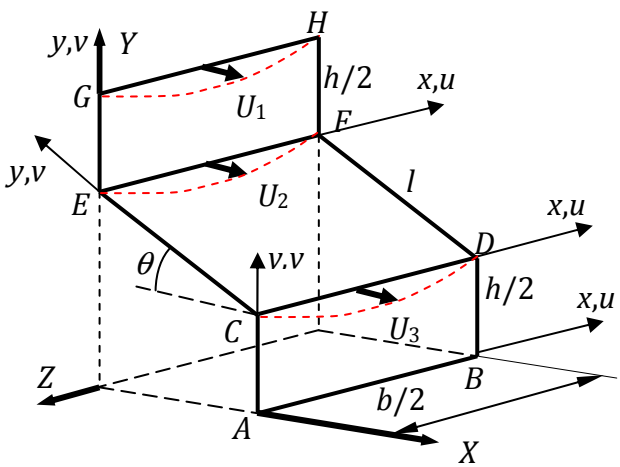

Figure 4 - Additional displacements due to torsion

Thus the total internal strain energy in the 1/8 honeycomb REV can be defined. The unknown parameters a_0, a_i, b_i and c_i can be obtained by minimization of the internal strain energy. For the torsion problem, the strain energy of the homogenized solid is defined to obtain the modulus G_{XY}^{T*}:

$$\pi_{\text{int}}^{*}=\frac{1}{2}G_{XY}^{T*}\frac{1}{2}\frac{b^3}{12}l\cos\theta\left(h+l\sin\theta\right)\kappa^2=\pi_{\text{int}} \quad \Rightarrow \quad G_{XY}^{T*} \tag{12}$$

The upper and lower bounds of the strain energy are found by taking $b \to 0$ and $b \to \infty$. It is noted that the upper and lower bounds of the moduli are identical for the shear and torsion problems.

5. Validation of the H-models for shear moduli and torsion moduli

Our H-models are validated by FE simulations using the thin shell element 'S4R' of Abaqus. The stretching problem in [4, 6] is taken for shearing and torsion problems: E=72.2 GPa, t=0.05 mm, ν=0.34, h=l=4 mm, θ=30°, b=0.5-10 mm. The shear and torsion moduli versus the honeycomb height are shown in Fig. 5. It is observed that the analytical curves are very close to the numerical curves of Abaqus. The shear and torsion moduli have the same upper and lower bounds; but the torsion modulus decrease more slowly with respect to shear one due to its different stress redistribution.

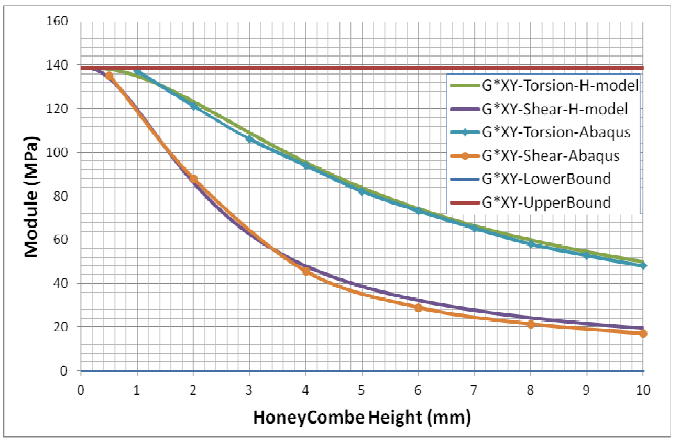

Figure 5 - Shear and torsion moduli versus the honeycomb height

6. Conclusions

Analytical homogenization models for honeycomb cores in sandwich plates are developed based on the strain energy method. The proposed formulation, using trigonometric function series, takes into account the skin and core height effects to determine the in-plane shear and torsion moduli. It leads to a great improvement compared to classical homogenization models. The obtained shear and torsion moduli are in very good agreement with those got by shell FE simulations (Abaqus).

The present H-models are quite easy to use and enable to largely reduce the CAD and mesh preprocessing, the memory storage and the computation time.

7. References

[1] L.J. Gibson, M.F. Ashby. Cellular Solids: Structure and Properties. Pergamon, Oxford, 1988.

[2] I.G. Masters, K.E. Evans. Models for the elastic deformation of honeycombs. Composite Structures, Vol. 35, p403–422, 1996.

[3] W. Becker. The in-plane stiffness of a honeycomb core including the thickness effect. Arch. Appl. Mech. Vol. 68, Vol. 68, p334–341, 1998.

[4] F.X. Xu, P. Qiao. Homogenized elastic properties of honeycomb sandwich with skin effect. Int. J. of Solids and Structures, Vol. 39, p2153–2188, 2002.

[5] A. Chen, J.F. Davalos. A solution including skin effect for stiffness and stress field of sandwich honeycomb core. Int. J. of Solids and Structures, Vol. 42, p2711–2739, 2005.

[6] M. P. Hoang, Y.M. Li, B. Abbes, F. Abbes and Y.Q. Guo. Analytical homogenization of honeycomb with skin and height effects in sandwich plates. ECCM16, Seville, Spain, 2014.