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Abstract

The image segmentation methods based on Markovian assumption consist in optimizing a Gibbs energy function which depends on the observation field and the segmented field. This energy function can be represented as a sum of potentials defined on cliques which are subsets of the grid of sites. The Potts model is the most commonly used to represent the segmented field. However, this model expressed just a potential on the classes for nearest neighbour pixels. In this paper, we propose the integration of global informations, like the size of a region, in the local potentials of the Gibbs energy. To extract these informations, we use a representation model well known in geometric modeling: the topological map. Results on synthetic and natural images are provided showing improvements in the obtained segmented fields.

1 Introduction

The main objective of image segmentation methods is to find areas of homogeneous pixels. In such context, there is two main researches axes: the boundary and the region based methods [1]. Fusions of these two approaches have also been proposed. Nevertheless, whatever approach you are using, a partitioned image is the result of the segmentation operation. This partition is composed of subsets called "classes" for a set of homogeneous pixels, and "cluster" for a set of homogeneous and related pixels.

The interest of such treatment can be found in many fields like image compression [2], biomedical image analysis [3],...

For two decades, Monte-Carlo Markov Chain (MCMC) methods have received increased interest. This is due to a rigorous mathematical background and the growing power of computers. From the original work of Geman & Geman on image restoration [4], these methods have been applied successfully to the problem of image segmentation using different implementations, $i.e.$ using stochastic or deterministic algorithms [5] [6].

In such approaches, the image is considered as a hierarchical field composed by an observation field and a segmented or label field. Different parametric models have been proposed to represent observation field's textures. [7] [8]. But in most cases, the commonly used Potts model [9] is proposed for represent the label field. This model only takes into account the label values of the nearest neighbours in the Gibbs energy function. Geman & Geman [4] tried to add more informations to the Potts model by using the boundaries estimated during the optimization process. Some boundary configurations were penalized. In [10], this idea has been extensively used. A drawback can be seen to this approach in the sense that $a\ priori$ weighted values were given to each boundary configurations.

In this paper, we propose to add informations to the energy function from the label field's geometry and topology. These informations, like the form or the size of a cluster, could be considered as "global" as opposed to "local" information provided by the Potts model. Of course, the Potts model will be still used cause of its regularization effect but potentials functions taking account "global" information will be added in the Gibbs energy function.

To get these additional informations, we propose the use of a combinatorial model: the topological maps. This structure encode all topological and geometrical informations contains in the image in an efficient way.

Our segmentation algorithm consists in two steps and it is unsupervised considering the number of classes and the model parameters. A first step allows to estimate the number of classes and the model parameters of a gaussian mixture. A Stochastic Expectation Maximization (SEM) algorithm is used [11] as in [12]. During a second step, a simulated annealing is done in order to obtain the final label field. This is the step in which we will introduce topological or geometrical informations obtained from the topological map.

In the second part, we start with a brief summary on the principles of markovian segmentation. Then, after having explained in the third part the operation of topological maps, we will propose in fourth part an integration model to include this map in a markovian segmentation algorithm. In

the last, we propose some results on synthetic and natural images.

2 Markovian segmentation algorithm

2.1 Definitions

Let $X = \{X_s, s \in S\}$ a family of random variables on a regular grid $S = \{s_1, s_2, \ldots, s_N\}$ which is a finite subset of $\mathcal{Z}$ and an image $x = \{x_s, s \in S\}$, realization of X. Each X_s has the same state space which can be $\Omega_x = \{0, 1, \ldots, 255\}$, the set of gray pixel values, $\Omega_x = \mathcal{R}$, ... We suppose now that there is an another random field $Z = \{Z_s, s \in S\}$ called the label field with state space $\Omega_z = \{c_1, c_2, \ldots, c_K\}$ for which a realization will be $z = \{z_s, s \in S\}$. In our case, a label represents the class to which the site belongs.

From this definition, we suppose that X and Z are defined in a hierarchical way : for each given z_s, we have a conditional model for X_s which can be, for example an AR model or a Gaussian distribution [5] [8].

The likelihood method defines the *a posteriori* law $P(Z = z/X = x)$ as the probability to get one specific realization of the labels field knowing the observations field:

$$P(Z = z/X = x) = \frac{P(X = x, Z = z)}{P(X = x)} \tag{1}$$

Using the Bayes formula, we can also write equation 1 in function of the *a priori* law:

$$\begin{aligned} P(Z = z/X = x) &= \frac{P(X=x/Z=z)P(Z=z)}{P(X=x)} \\ &\propto P(X = x/Z = z)P(Z = z) \end{aligned} \tag{2}$$

since we consider the observations field's probability $P(X = x)$ as a constant value.

From now, we will simply write:

$$P(z/x) \propto P(x/z)P(z) \tag{3}$$

Maximizing this *a posteriori* law (*MAP*) is equivalent to getting the most probable labels field's realization, knowing the observations field.

2.2 Markov field and Gibbs distribution

We call $\mathcal{V} = \{v_s\}$ a site neighborhood system for S. A neighborhood system has the two following properties: $s \notin v_s$ and $s \in v_r \Leftrightarrow r \in v_s$. A subset $C \subseteq S$ is a clique if every pair of distinct sites in C are neighbors and $\mathcal{C}$ is the set of cliques. In our study, we will only use the neighboring system based on the four nearest neighbors (4-connecivity) and the eight nearest neighbors (8-connectivity). A random field $Y = \{Y_s, s \in S\}$ is a markovian field associated to the system v_s if and only if:

$$\begin{aligned} &P(Y = y) = P(y) > 0, \forall y \in \Omega_Y \\ &P(Y_s/Y_r, r \in S - \{s\}) = P(Y_s/Y_r, r \in v_s) \end{aligned} \tag{4}$$

From Hammersley-Clifford's theorem, the distribution function of a Markov random field follows a Gibbs' law:

$$P(y) = \frac{\exp\left(-U\left(y\right)\right)}{Z_y} \tag{5}$$

in which U is an "energy function". U can be written as:

$$U\left(y\right) = \sum_{C \in \mathcal{C}} \Phi_C\left(y\right) \tag{6}$$

where $\Phi_C : \Omega_y^{|C|} \rightarrow \mathcal{R}$ is the potential corresponding at C with $\Phi_C(y) = \Phi_C(y_C)$. $|A|$ is the cardinality of the A set and in the special case of a clique, it is called its order. y_C denotes the set $\{y/y \in C\}$.
Therefore, if the markovian assumption is done about the hierarchical field of the part 2.1, (X, Z), the equation 3 can be written in a general way as follows:

$$P\left(z/x\right) = \frac{\exp(-U_1(x, z) - U_2(z))}{Z(x)} \tag{7}$$

in which U_1 is an "energy function", depending on the conditional observation fields, and U_2 another energy function depending on the label field. $Z(x)$ is a normalization constant depending on x. Let define now $U(z) = U_1(x, z) + U_2(z)$ as x is the given observation field.
The MAP estimated of z (see part 2.1) can be written now:

$$\hat{z} = \arg\max_{z \in \Omega_z} \left[P\left(z/x\right)\right] = \arg\min_{z \in \Omega_z} \left[U_1\left(x, z\right) + U_2\left(z\right)\right] \tag{8}$$

We have to optimize $U : \Omega_z^{|S|} \rightarrow \mathcal{R}$. Simulated annealing (SA) based on Gibbs sampler has been shown to be adapted to such an optimization problem. In the next part, we present the stochastic model we have chosen and the form of the corresponding potentials.

2.3 Stochastic modeling

The conditional field X is now supposed to be composed of K textures. So, we have K probability laws describing the gray pixel value variations in x for the different clusters belonging to the K textures. Different choices can be done for this probability laws [8]. In this paper, we are making the assumption that our observation field is a mixture of K i. i. d. (independently and identically distributed) gaussian process. This assumption allows us to use Maximum Likelihood based algorithm like SEM [11] for estimating K and the different model parameters, $\theta_k = \{\mu_k, \sigma_k\}$, $k = 1 \ldots K$, where μ_k and σ_k are respectively the average

and the standard deviation of the texture k.
Now, we suppose that K and $\theta = \{\theta_k, k = 1 \dots K\}$ are known. Then, we will have:

$$P(x/z) = \prod_{s \in S} f(x_s/z_s) \tag{9}$$

with $f(x_s/z_s)$ is a gaussian law $\mathcal{N}(\mu_{z_s}, \sigma_{z_s})$. Therefore, we can write $U_1(x,z)$ as:

$$U_1(x,z) = \sum_{s \in S} \left(\frac{(x_s - \mu_{z_s})^2}{\sigma_{z_s}^2} + \log(\sigma_{z_s}) \right) \tag{10}$$

This energy can be seen as a sum of local potential based on one order cliques:

$$\Phi_s^1(x_s, z_s) = \frac{(x_s - \mu_{z_s})}{2\sigma_{z_s}^2} + \log(\sigma_{z_s}) \tag{11}$$

It remains to propose a model for the label field. In many papers, the Potts model is used [8]. This assumption corresponds to a general hypothesis on the label field: the different classes can be permuted without any influence on the probability law. We propose to use a more general formalism mixing the Potts model, as it is used in [5], and the general auto-model from Besag [13] in order to include some additional informations to Potts model:

$$\begin{array}{c} U_2(z) = \sum_{s \in S} \Phi_s(z_s) + \beta \sum_{\langle i,j \rangle_1} (1 - \delta(z_i, z_j)) + \\ \frac{\beta}{\sqrt{2}} \sum_{\langle i,j \rangle_2} (1 - \delta(z_i, z_j)) \end{array} \tag{12}$$

where for $p = 1, 2$:

$$\begin{array}{l} (i,j) \in S, i \neq j : \langle i,j \rangle_p \Leftrightarrow \\ \exists C \in \mathcal{C} / \{i,j\} \in C, \|i - j\|_2 = \sqrt{p} \end{array} \tag{13}$$

So, it is possible now to express the local probability law at site $s \in S$:

$$p_s(z_s/x_s, z^s) = p_s(z_s/x_s, v_s) = \frac{\exp(-U_s(x_s, z_s))}{Z_s(x_s, v_s)} \tag{14}$$

with $z^s = \{z_s / s \in S - \{s\}\}$ and

$$\begin{array}{l} U_s = \Phi_s^1(x_s, z_s) + \Phi_s(z_s) + \Phi_s^P(z_s) \\ \Phi_s^P(z_s) = \beta \sum_{r \in v_s : \langle s,r \rangle_1} (1 - \delta(z_s, z_r)) + \\ \qquad \frac{\beta}{\sqrt{2}} \sum_{r \in v_s : \langle s,r \rangle_2} (1 - \delta(z_s, z_r)) \\ Z_s(x_s, v_s) = \sum_{z_s \in \Omega_z} \exp(-U_s(x_s, z_s)) \end{array} \tag{15}$$

Φ_s must be chosen to respect the context of applicability of MCMC methods like SA [4]. Moreover, SA implies the introduction of a temperature T in the Gibbs distribution. The local probability can then be written as follows:

$$p_s(z_s/x_s, z^s) = p_s(z_s/x_s, v_s) = \frac{\exp(-U_s(x_s, z_s/T))}{Z_s(x_s, v_s, T)} \tag{16}$$

Φ_s will allow us to integer geometrical and/or topological informations, $i.e.$ "global" informations, on the segmented field at each iteration. Let's present now the topological map.

3 Topological map

Topological maps allow to represent the nD regions segmented images. They encode at the same time topology and geometry of images. Topological maps are combinatorial maps with particular properties. So we are beginning by recalling the notion of combinatorial map before present the topological map structure.

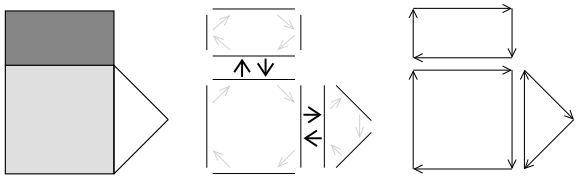
3.1 Combinatorial maps

Combinatorial maps were introduced in the sixties by [14], at first as a planar graph representation model. They were extended by [15] in dimension n to represent orientable or not-orientable quasi-manifold. They encode space subdivisions and all the incidency relations. They are made of abstract elements, called *darts*, on which are defined application, called β_i. We are giving here the combinatorial map definition in n dimensions, that we can find for example in [16]:

Definition 1 (combinatorial maps) *Let $n \geq 0$. A n combinatorial map, (or n-map) is an $(n+1)$-tuple $M = (B, \beta_1, \dots, \beta_n)$ where :*

1. *B is a finite set of darts;*
2. *β_1 is a* permutation *on B;*
3. *$\forall\, 2 \leq i \leq n$, β_i is an* involution *on B;*
4. *$\forall\, 1 \leq i \leq n-2$, $\forall\, i+2 \leq j \leq n$, $\beta_i \circ \beta_j$ is an* involution.

In this definition, there is an application β_i for each space dimension which puts in relation two i dimension oriented cells. When two darts are linked with β_i, they are said β_i-*sewed*. Each space cell is implicitly represented by a set of darts. We can see figure 1.*a* an example of an image and figure 1.*b* the corresponding combinatorial map. Each dart is represented by a segment, the β_1 relation by light grey arrows and the β_2 relation by dark grey arrows. β_1 put in relation a dart and the next dart of the same face. For example, the light grey face of the image is represented by four β_1-sewed darts in the map. The adjacency between this face and the dark grey face is represented by two darts β_2-sewed together. We are using the simplified representation (figure 1.*c*) that does not represent explicitly the applications, because it is more understandable.



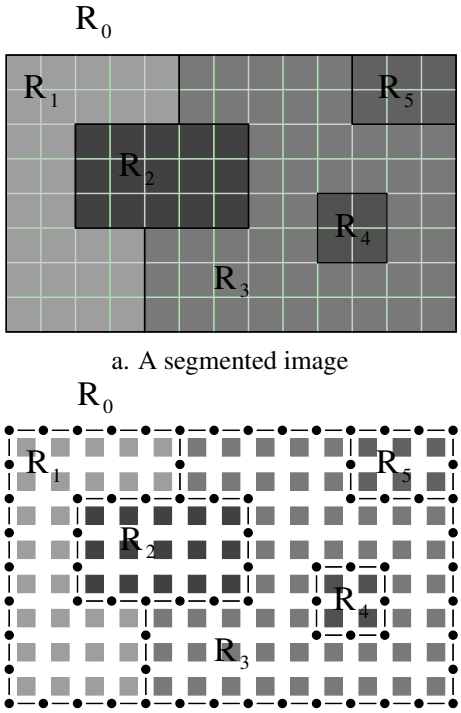
a. A 2D object. b. Corresponding map. c. Simplified representation.

Figure 1: An object and the corresponding combinatorial map represented by two different ways.

3.2 Topological map

A topological map's construction needs the cluster-based partition of the image, as opposed to its classes-bases partition we get from the markovian segmentation algorithm. So its construction will be preceded by a clusters growth algorithm, introduced to split all clusters belonging to the same class.

The topological map's construction consists in taking the pixels' border elements, to coherently define the image's topology: the space is subdivided in elements and borders' elements, with which are provided incidence and adjacency relationships. Figure 2 represents a clusters-based segmented image and its interpixel borders. [17, 18] proposes to redefine the topological map in the 2-dimensional space, introduced by Luc Brun in [19], a simpler way, so it makes it easily expendable in n-dimensional spaces. This definition uses simplification levels we show below:



a. A segmented image

b. Its clusters' interpixel borders

Figure 2: Interpixel Vision

1. Level 0: Full map
   It is the starting point of the process. This map doesn't code the image's borders but simply all its interpixel elements. It is composed by squared faces, each of them represent a pixel of the image (figure 3).

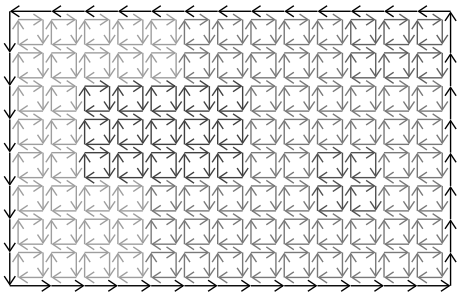


Figure 3: Level 0: Full map

2. Level 1: Linel map
   Level 1 map is obtained from the level 0 map and encodes the image's interpixel borders. This is done by removing every edge of that level 0 that doesn't belong to a border line of the image (figure 4).

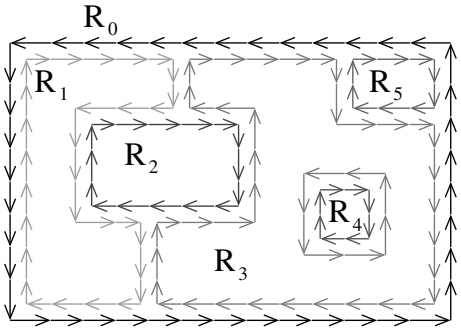


Figure 4: Level 1: Linel map

3. Level 2: Borders map
   It simplifies the linel map, by using edges to represent parts of a straight-line border instead of interpixels belonging to a border line of the image (figure 5).

4. Level 3: Topological map
   From this last simplification level results the topological map, which now associates a single edge to each border (figure 6).

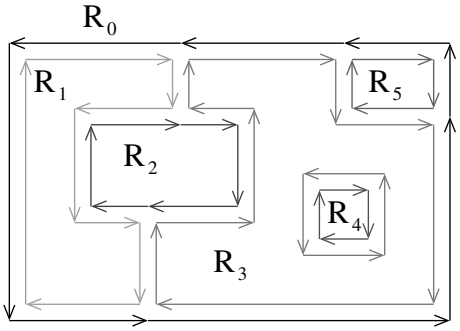


Figure 5: Level 2: Borders map

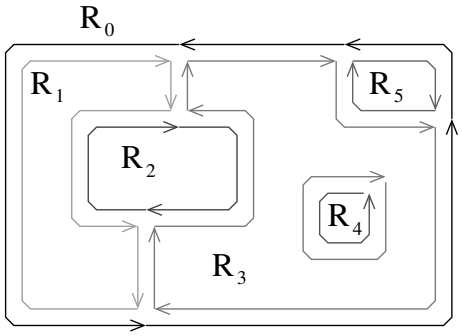


Figure 6: Level 3: Topological map

This last level is the topological map which encodes all topological and geometrical information of the represented image. Moreover, it is minimal, stable for rigid transformations and unique. So a topological map is characteristic of an image. These properties ensure that this model is an efficient way to encode information contains in an image. So we are going to use this model to extract some global informations, for instance size or shape of clusters, number of adjacent clusters, clusters totally included into another... The following part proposes an exploitation of some of these informations.

4 Integration of geometrical and topological informations

4.1 Unsupervised segmentation

The main purpose of this work is to make an algorithm able to treat any image. So we are looking for wanted or unwanted geometrical or topological features that could appear in the estimated label field.

One of these unwanted features is the presence of isolated pixels or small clusters. So, the first potential Φ_s (see equations 14 and 15) we propose is a "size" potential: It imposes a minimal size, s_{min}, under which the probability of pixel s to belong to a cluster will be disadvantaged. s_{min} threshold's can be chosen proportional to the size of the image to segment.

Disadvantaging a site's membership to a cluster also equals minimizing its *a posteriori* law, or maximizing the *U* energy of the Gibbs probability. The chosen "size" potential function (figure 7) is written:

$$\Phi_s^{size} = \gamma \left(1 + \mathbf{1}_{(|A_s(z_s)>s_{\min}|)} \times \left(\exp\left(-\kappa\left(|A_s\left(z_s\right) - s_{\min}|\right)\right) - 1\right)\right) \quad (17)$$

with $\mathbf{1}_{(|A_s(z_s)|>s_{\min})} = \begin{cases} 1 \text{ if } |A_s(z_s)| > s_{\min} \\ 0 \text{ otherwise} \end{cases}$ and $A_s(z_s)$ is the connected set of sites r such as $z_r = z_s$, $i.e.$ the cluster at which the site s belongs. γ and κ are two constants which represents respectively the weight of the potential in the Gibbs energy function and the exponential decrease of Φ_s^{size}.

Figure 7: Φ_s^{size} Size potential evolution in function of cluster size

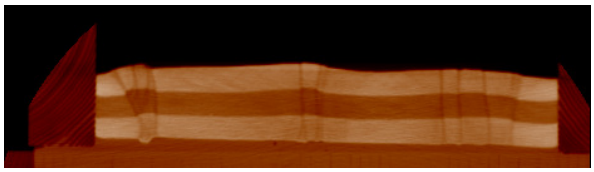
To introduce this part, we have evoqued our intention to make an unsupervised algorithm. We are then now facing the difficulty to find wanted or unwanted features on the label field other than small clusters: we want the topological map to give us informations about the shape or the adjacency of clusters and use these informations to segment the image. But, advantaging or disadvantaging a cluster in function of its shape or adjacency is a choice that can only be made by the user, for a given image. So, this is the purpose of the next part.

4.2 Semi-supervised segmentation

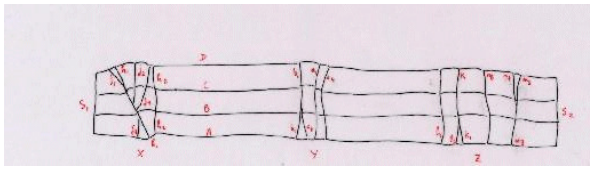
Let's take two examples: an image representing geological cuts (figure 8) and the "House" image (figure 9).

In the first case, the purpose of segmentation is to keep the identified seismic faults (see 8.b). Our algorithm, at its present state, is unable to get a satisfying segmentation of that image. But if we decide to use the fact that fails represent relatively long and narrow, mainly vertical, clusters, and ask it to advantage this type of clusters, the result could become convincing.

In the case of the "House" image, the window to our left is a rather small cluster that could disappear on during the segmentation, although we would like to keep it. This cluster has an interesting propriety: it is adjacent to only one cluster (the wall). Here is another information we can easily get from the topological map. To reduce the risk to see this



a. Geological cut



b. Seismic faults

Figure 8: Geological cut

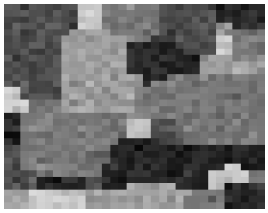
Figure 9: "House" Image

cluster disappear, we have to provide the information to advantage clusters that are adjacent to only one cluster.
In this paper, we limit our presentation to the unsupervised case. So, the following part gives results of the segmentation algorithm with and without the use of the size potential (see equation 17).

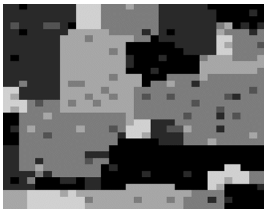
5 Segmentation on synthetic and natural images

5.1 Synthetic image

Figure 10.a shows a synthetic (32×32) image which is a realization of different gaussian i.i.d. process. Figure 10.b represents the last iteration of the SEM algorithm. The number of classes and model parameters have been correctly estimated. In MCMC methods, the final result of the Markov Chain process depends not on the initial law on X. So, we used the last iteration of the SEM algorithm as initialization of the SA. We start from an initial temperature $T_0 = 1$ and use the classical decrease scheme: $T_k = 0.99^k T_0$. We fix $\beta = 1$.
Figure 10.c shows the result of the SA after 200 iterations without taking account the potential on clusters. We can



a. Original image.



b. First step segmentation : SEM's last iteration.

c. SA Without size potential.

d. SA With size potential.

Figure 10: Synthetic image

notice on this first segmented image a few incoherences: badly-defined borders, a few "isolated" pixels, that don't look meaningless when showing at the original image, but that we don't want to show off on the final partition.
To increase β is not a solution because it will produce a decrease in the accuracy of boundary estimation. However, additional informations about the image's clusters, such as their size, or adjacency, could be used to perform these kinds of enhancements. These informations, we call "global", as opposed to our previously used "local" informations, can be extracted from the topological map.
Figure 10.d shows the estimated label field when using SA with the size potential with $s_{min} = 2$, $\kappa = 1.5$ and $\gamma = 10$. We can notice on figure 10.d the disappearing of isolated pixels and still exact boundaries. So, the size potential allows an enhancement of the segmentation process. Let's see now on a natural image.

5.2 Natural image

We used the "House" (128×128) image (see figure 11.a). Figure 11.b show the last iteration of the SEM algorithm we

used as an initialization for SA algorithm. On figure 11.c and 11.d, we can notice that the noise on the wall without the use of the size potential disappears when using it.

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6 Conclusion

The algorithm we have proposed shows up as a markovian relaxation algorithm, to which are added global informations about the image, extracted from the topological map, as energy potentials, thanks to the Gibbs distribution. It should be able to deal with any image, as long as the user provides it an information, as simple as it can be, about the cluster he wants to advantage or disadvantage's characteristics. A possible way to handle this would be a user interface, on which choices such as "advantage long and narrow clusters", "advantage vertical clusters", "advantage clusters adjacent to only one cluster", etc. In the case of "difficult" images, we could also remove the learning phase (*SEM*) and directly provide the relaxation algorithm the image's stochastic model's parameters. Then it would be supervised segmentation. Moreover, there are many different possibilities to add some more topological or geometrical informations to the simulated annealing. We currently work at the definition of some criterion based on some of these informations. We think that these new criterion can strongly improve the result of our final segmentation.

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a. Original image.

b. SA Without size potential.

c. SA With size potential.