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Corrigendum

Corrigendum to “Homogenisation of advective–diffusive transport in poroelastic media” [Mech. Res. Commun. 37 (2010) 133–136]

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The paper entitled “Homogenisation of advective–diffusive transport in poroelastic media”, published in Mechanics Research Communication, Vol. 37, pp. 133–136, contains unfortunately an error in the pore scale description of solute transport. Eq. (11), which expresses the no-mass flux condition over the solid/fluid boundary should include the convection term. This latter does not vanish as a result of solid deformation. This is corrected hereby. A slight modification of the macroscopic coupling term of mechanically induced solute transport is obtained.

1. Introduction

At the pore scale, *i.e.* within the periodic cell, the equations that govern solute transport by convection and diffusion in a deforming medium are the following:

$$\begin{cases} \frac{\partial c}{\partial t} - \nabla \cdot \bar{q} = 0 & \text{within } \Omega_f, \\ \bar{q} \cdot \bar{n} = 0 & \text{over } \Gamma, \\ \bar{v}_f = \frac{\partial \bar{u}_s}{\partial t} & \text{over } \Gamma, \end{cases} \quad (1)$$

where the flux \bar{q} is given by

$$\bar{q} = -D\bar{\nabla}c + c\bar{v}_f. \quad (4)$$

Therefore, Eq. (2) yields

$$(D\bar{\nabla}c - c\bar{v}_f) \cdot \bar{n} = 0 \quad \text{over } \Gamma. \quad (5)$$

Consequently, Eq. (11) in the original paper [1] should be replaced by Eq. (5).

2. Homogenisation of the transport equations

The dimensionless local scale description to be homogenised is therefore the following:

$$\begin{cases} \frac{\partial c}{\partial t} - \nabla \cdot (D\bar{\nabla}c - c\bar{v}_f) = 0 & \text{within } \Omega_f, \\ (D\bar{\nabla}c - c\bar{v}_f) \cdot \bar{n} = 0 & \text{over } \Gamma, \\ \bar{v}_f = \frac{\partial \bar{u}_s}{\partial t} & \text{over } \Gamma. \end{cases} \quad (6)$$

Let us now analyse the boundary-value problems that arise at the successive orders of ε , once the asymptotic expansions for c , \bar{v}_f and \bar{u}_s and the expression of the dimensionless gradient operator (Eq. (2) in the original paper) have been incorporated.

2.1. Determination of $c^{(0)}$

The boundary-value problem for $c^{(0)}$ is unmodified. It remains identical to that obtained for a rigid porous medium, which leads to:

$$c^{(0)} = c^{(0)}(\bar{x}, t). \quad (9)$$

2.2. Determination of $c^{(1)}$

Noticing that¹

$$\frac{\partial v_{f_i}^{(0)}}{\partial y_i} = 0, \quad (10)$$

we get the following boundary-value problem for $c^{(1)}$:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial y_i} \left[D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) \right] = 0 \quad \text{within } \Omega_f, \\ \left[D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) \right] n_i \\ - \left(c^{(0)} \frac{\partial u_{s_i}^{(0)}}{\partial t} \right) n_i = 0 \quad \text{over } \Gamma. \end{array} \right. \quad (11)$$

The solution $c^{(1)}$ depends on two forcing terms: $\bar{\nabla}_x c^{(0)}(\bar{x}, t)$ and² $c^{(0)}(\partial u_{s_i}^{(0)}/\partial t)(\bar{x}, t)$. By virtue of linearity, the solution for $c^{(1)}$ is a linear combination of both:

$$c^{(1)} = \chi_j(\bar{y}) \frac{\partial c^{(0)}}{\partial x_j} - \pi_j(\bar{y}) c^{(0)} \frac{\partial u_{s_j}^0}{\partial t} + \bar{c}^{(1)}(\bar{x}, t), \quad (13)$$

where $\bar{c}^{(1)}$ is an arbitrary function. The definition of $\bar{\chi}$ which is given by Eqs. (21) in the original paper remains valid, while the vector $\bar{\pi}$ is defined by the following local problem:

$$\left\{ \begin{array}{l} \nabla_y \cdot (D \nabla_y \bar{\pi}) = \bar{\mathbf{0}} \quad \text{in } \Omega_f, \\ (D \nabla_y \bar{\pi}) \cdot \bar{\mathbf{n}} = \bar{\mathbf{0}} \quad \text{on } \Gamma, \\ \langle \bar{\pi} \rangle^f = \bar{\mathbf{0}}, \\ \bar{\pi} : \Omega\text{-periodic.} \end{array} \right. \quad (14)$$

2.3. Derivation of the first-order macroscopic description

At the third order, the transport equations lead to the following system:

$$\left\{ \begin{array}{l} \frac{\partial c^{(0)}}{\partial t} - \nabla_y \cdot (D \nabla_y c^{(2)} + \nabla_x c^{(1)}) \\ + \nabla_y \cdot (c^{(0)} \bar{v}_f^{(1)} + c^{(1)} \bar{v}_f^{(0)}) \\ - \nabla_x \cdot (D \nabla_y c^{(1)} + \nabla_x c^{(0)}) \\ + \nabla_x \cdot (c^{(0)} \bar{v}_f^{(0)}) = 0 \quad \text{within } \Omega_f, \\ [D (\nabla_y c^{(2)} + \nabla_x c^{(1)})] \cdot \bar{\mathbf{n}} \\ - [c^{(0)} \bar{v}_f^{(1)} + c^{(1)} \bar{v}_f^{(0)}] \cdot \bar{\mathbf{n}} = 0 \quad \text{over } \Gamma. \end{array} \right. \quad (15)$$

The first-order macroscopic description is obtained by integrating the balance equation (Eq. (15)) over the periodic cell:

$$\phi \frac{\partial c^{(0)}}{\partial t} - \frac{1}{|\Omega|} \int_{\Omega_f} \frac{\partial}{\partial y_i} \left[D \left(\frac{\partial c^{(2)}}{\partial y_i} + \frac{\partial c^{(1)}}{\partial x_i} \right) - c^{(0)} v_{f_i}^{(1)} - c^{(1)} v_{f_i}^{(0)} \right] d\Omega - \frac{1}{|\Omega|} \int_{\Omega_f} \frac{\partial}{\partial x_i} \left[D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) - c^{(0)} v_{f_i}^0 \right] d\Omega = 0, \quad (17)$$

where

$$\phi = \frac{|\Omega_f|}{|\Omega|} \quad (18)$$

denotes the porosity. Using Gauss' theorem and boundary-condition (Eq. (16)), while keeping in mind the condition of periodicity, the second term of (Eq. (17)) vanishes and it thus reduces to:

$$\phi \frac{\partial c^{(0)}}{\partial t} - \frac{\partial}{\partial x_i} \left[\left\langle D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) \right\rangle^f \right] + \frac{\partial}{\partial x_i} [c^{(0)} \langle v_{f_i}^{(0)} \rangle^f] = 0. \quad (19)$$

Using the expression obtained for $c^{(1)}$ (Eq. (13)), we deduce that:

$$\left\langle D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) \right\rangle^f = D_{ij}^{\text{eff}} \frac{\partial c^{(0)}}{\partial x_j} - \left\langle D \frac{\partial \pi_j}{\partial y_i} \right\rangle^f c^{(0)} \frac{\partial u_{s_j}^0}{\partial t}, \quad (20)$$

where \mathbf{D}^{eff} represents the effective tensor of diffusion, and is defined by

$$\mathbf{D}^{\text{eff}} = \phi \mathbf{D}, \quad (21)$$

with

$$\mathbf{T} = \frac{1}{|\Omega_f|} \int_{\Omega_f} (\nabla_y \bar{\chi} + \mathbf{I}) d\Omega, \quad (22)$$

where \mathbf{T} denotes the tortuosity tensor.

Eq. (19) now reads:

$$\begin{aligned} \phi \frac{\partial c^{(0)}}{\partial t} - \frac{\partial}{\partial x_i} \left(D_{ij}^{\text{eff}} \frac{\partial c^{(0)}}{\partial x_j} \right) + \frac{\partial}{\partial x_i} (c^{(0)} \langle v_{f_i}^{(0)} \rangle^f) \\ = - \frac{\partial}{\partial x_i} \left[\left\langle D \frac{\partial \pi_j}{\partial y_i} \right\rangle^f c^{(0)} \frac{\partial u_{s_j}^0}{\partial t} \right]. \end{aligned} \quad (23)$$

Noticing that the macroscopic relative fluid velocity³ is:

$$\langle v_{f_i}^{(0)} \rangle^f - \phi \frac{\partial u_{s_i}^{(0)}}{\partial t},$$

Eq. (23) can be rewritten as follows:

$$\begin{aligned} \underbrace{\phi \frac{\partial c^{(0)}}{\partial t}}_{\text{accumulation term}} - \underbrace{\nabla_x \cdot (\mathbf{D}^{\text{eff}} \nabla_x c^{(0)})}_{\text{effective diffusion}} + \underbrace{\left(\langle \bar{v}_f^{(0)} \rangle^f - \phi \frac{\partial \bar{u}_s^{(0)}}{\partial t} \right) \cdot \nabla_x c^{(0)}}_{\text{advection}} \\ + \underbrace{\mathbf{A} \nabla_x \cdot \left(c^{(0)} \frac{\partial \bar{u}_s^{(0)}}{\partial t} \right)}_{\text{mechanically induced solute transport}} = 0, \end{aligned} \quad (24)$$

where

$$\mathbf{A} = \langle D \nabla_y \bar{\pi} + \mathbf{I} \rangle^f. \quad (25)$$

Tensor \mathbf{A} characterises a coupling between transport and deformation mechanisms, which is induced by the fluid convection.

3. Properties of the coupling tensor \mathbf{A}

By definition, the variational formulation of the boundary-value problem (Eqs. (11) and (12)) reads:

$$\int_{\Omega_f} \alpha \frac{\partial}{\partial y_i} \left[D \left(\frac{\partial c^{(1)}}{\partial y_i} + \frac{\partial c^{(0)}}{\partial x_i} \right) \right] d\Omega = 0, \quad (26)$$

¹ First-order expansion of the fluid continuity equation: $\bar{\nabla} \cdot \bar{v}_f = 0$.

² It has been shown earlier that $\bar{u}_{s_i}^{(0)} = \bar{u}_{s_i}^{(0)}(\bar{x}, t)$.

³ See the macroscopic poroelastic model in the original paper.

and can be transformed as follows:

$$\int_{\Omega_f} \frac{\partial \alpha}{\partial y_i} D \frac{\partial c^{(1)}}{\partial y_i} d\Omega = - \int_{\Omega_f} \frac{\partial \alpha}{\partial y_i} D \frac{\partial c^{(0)}}{\partial x_i} d\Omega + \int_{\Omega_f} \frac{\partial \alpha}{\partial y_i} c^{(0)} \frac{\partial u_{s_i}^{(0)}}{\partial t} d\Omega. \quad (27)$$

By taking

$$\begin{cases} c^{(1)} = \chi_j, & \frac{\partial c^{(0)}}{\partial x_i} = I_{ij}, \\ \alpha = \pi_j, & c^{(0)} \frac{\partial u_{s_i}^{(0)}}{\partial t} = 0, \end{cases} \quad (28)$$

into Eq. (27), we get:

$$\int_{\Omega_f} \frac{\partial \pi_j}{\partial y_i} D \frac{\partial \chi_j}{\partial y_i} d\Omega = - \int_{\Omega_f} D \frac{\partial \pi_j}{\partial y_i} d\Omega. \quad (30)$$

Now, by considering

$$\begin{cases} c^{(1)} = \pi_j, & c^{(0)} \frac{\partial u_{s_i}^{(0)}}{\partial t} = -I_{ij}, \\ \alpha = \chi_j, & \frac{\partial c^{(0)}}{\partial x_i} = 0, \end{cases} \quad (31)$$

into Eq. (27), we obtain:

$$\int_{\Omega_f} \frac{\partial \chi_j}{\partial y_i} D \frac{\partial \pi_j}{\partial y_i} d\Omega = - \int_{\Omega_f} \frac{\partial \chi_j}{\partial y_i} d\Omega. \quad (33)$$

From Eqs. (30) and (33), we deduce:

$$\int_{\Omega_f} D \frac{\partial \pi_j}{\partial y_i} d\Omega = \int_{\Omega_f} \frac{\partial \chi_j}{\partial y_i} d\Omega. \quad (34)$$

Consequently, we have:

$$\mathbf{A} = \langle D \nabla_y \bar{\pi} + \mathbf{I} \rangle^f = \langle \nabla_y \bar{\chi} + \mathbf{I} \rangle^f = \phi \mathbf{T}, \quad (35)$$

where \mathbf{T} is the tensor of tortuosity, defined by (Eq. (22)).

4. Conclusion

The macroscopic transport equation is written as:

$$\phi \frac{\partial c}{\partial t} + \phi (\bar{V}_f - \bar{V}_s) \cdot \nabla c + \phi \mathbf{T} \nabla \cdot (c \bar{V}_s) = \nabla \cdot (\mathbf{D}^{\text{eff}} \nabla c),$$

where \bar{V}_f and \bar{V}_s denote the fluid and solid intrinsic macroscopic velocities, respectively. Therefore, the only difference with the original paper is that the coupling term emerging from the scale transition is $\phi \mathbf{T} \nabla \cdot (c \bar{V}_s)$ instead of $\phi \bar{V}_s \cdot \mathbf{T} \nabla c$. All the other conclusions mentioned in the original paper remain valid.

Reference

- [1] P. Royer, F. Cherblanc, Homogenisation of advective–diffusive transport in poroelastic media, *Mech. Res. Commun.* 37 (2010) 133–136.