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An iterative method for selecting decision variables in analytical optimization problem

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Abstract. In this paper, we present an iterative method to assist the designer in the setting of decision variables in an optimization problem with analytical models. This method is based on the Design Structure Matrix (DSM) which allows a clear representation of interactions between variables. Such a process becomes particularly useful for complex optimal design for which choosing the right decision variables to efficiently solve the problem is not so trivial. This approach is applied to the geometrical model of a High Speed Permanent Magnet Synchronous Machine (HSPMSM).

Keywords: DSM (Design Structure Matrix), decision variables, optimization, model structuring

1. Introduction

One of the major issues in embedded systems is to optimize design with respect to objectives as weight and energy consumption also fulfilling a set of optimization constraints. Previous studies [1–3] show that strong couplings between the different parts of a given system require an integrated design optimization approach instead of the traditional local approaches where each part of the system is separately optimized. The integrated design optimization of a complex multidisciplinary system using finite models is highly expensive on time computation. In this context, using models with a lower computational cost as analytical models is hugely recommended. However, a great attention should be given to "how to write the equations and to choice the decision variables of the optimization problem" to prevent all kind of couplings (possibility of algebraic loops) in order to minimize the implicit blocks in the model and to make easier the code parallelization.

The Design Structure Matrix (DSM) is a simple compact and visual representation of a system or project in the form of a matrix [4]. It is also referred to as dependency structure matrix, problem solving matrix (PSM), incidence matrix, N^2 matrix, interaction matrix, dependency map or design precedence matrix. The DSM has been used to manage projects since the 1960s [5] and has been extended to several fields after the 1990s: in automotive [6], aerospace [7], electronics industries [8], etc. For a design problem, it helps in the structuring and clearly shows the interactions and the dependencies between the different system variables [4]. The main goal is to optimize the coding of the model equations in order to have more explicit formulation with reduced computation time, putting forward code parallelization capabilities.

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Parallel

	A	B
A		0
B	0	

Sequential

	A	B
A		1
B	0	

Coupled

	A	B
A		1
B	1	

Fig. 1. Different possibilities of dependence.

In addition, among the set of variables in an analytical model involved in a design problem, coupled by a set of equations, a plurality of decision variable set choice (i.e. inputs of the design model) are possible. The DSM approach can be used to guide and manage the choice of the right decision variables based on selection criteria such as the "occurrence frequency" or the importance of variables, etc.

In the first part of this paper, we focus on the structuring of the analytical model; the different steps necessary for the construction of the "DSM" are describ and illustrated on a simple mathematical model.

After that, we propose a method that helps for the selection of decision variables in an optimization problem based on the matrix representation of the model detailed in the first part.

Finally, the selection method for decision variables is applied to an actual analytical model constituted of a HSPMSM (High Speed Permanent Magnet Synchronous Machine) for an aeronautic application [2].

2. The DSM for structuring analytical models

DSM stands for "Design Structure Matrix" is a modeling tool used in design projects and derived from the binary matrices with the introduction of new capabilities and possibilities of interaction representation. In engineering, it can be used to represent interactions between the different variables of a system represented by an analytical model. The difference between a binary matrix and the "DSM" is the ability to quantify the interactions between variables: instead of "1" and "0" in the binary matrix, couplings can be represented by the equation number which binds two variables, or it may quantify the level of dependency between.

The objective when analyzing the "DSM" is to rearrange the elements (variables) of the system in rows and columns in order to identify a hierarchy of these elements to maximize the exchange of information and to reduce the overall processing time of the design process by parallelizing the decoupled tasks. This reorganization is performed by sequencing and partitioning algorithms looking to triangulate the "DSM".

For pedagogical reasons, the "DSM" process will be applied to the following simplified mathematical model:

$$x_4 = x_7 - 2 \cdot x_3 \tag{1}$$

$$x_5 = 2 \cdot x_2 \tag{2}$$

$$x_1 = 2 \cdot x_4 - x_6 \tag{3}$$

$$x_7 = x_3 \cdot x_6 \tag{4}$$

The construction of the "DSM" involves three steps:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1				3		3	
x_2					2		
x_3				1		4	4
x_4	3		1			3	1
x_5		2					
x_6	3		4	3			4
x_7			4	1		4	

Fig. 2. Non-oriented and non-sequenced “DSM”.

	x_2	x_3	x_6	x_1	x_4	x_5	x_7
x_2						2	
x_3					1		4
x_6				3			4
x_1							
x_4				3			
x_5							
x_7					1		

Fig. 3. Oriented and non-sequenced “DSM”.

Step 1: Non oriented and non sequenced “DSM”

Firstly, all the equations of the model are listed and numbered (here 1 to 4). Each parameter or variable of the design model is represented by a row and a column identically labeled (for example the variable “x_2” is represented by the second row and the second column) in Fig. 2. The aim of the first step is to represent all the interactions between the model variables. Figure 1 shows the possibilities of dependence between two variables: the “1” and “0” model the presence or the absence of dependence:

- Parallel configuration: means the absence of relation between variables,
- Sequential configuration: means that dependency is defined in only one direction, that is to say for a given causality assignment, there is only one variable that depends on the other.
- Coupled configuration: dependence is defined in both directions.

As shown in Fig. 2, the non-oriented and non-sequenced “DSM” is a square symmetric matrix, similar to the undirected binary matrix. However, in the “DSM”; the dependencies of a variable with respect to other variables are modeled by the number of the equation(s) which bind them (for example: dependence between “x_2” and “x_5” is modeled by “2” referred to the Eq. (2).

Step 2: Oriented and non-sequenced “DSM”

The number of variables in a design problem is always greater than the number of equations. To define the optimal design problem, a specific number of variables called “decision variables” should be selected. The number of decision variables is usually given by this formula:

$$N_{dv} = N_v - (N_{eq} + N_d) \tag{5}$$

$$\text{where} \begin{cases} N_{dv} : \text{Number of decision variables} \\ N_v : \text{Number of variables in the model} \\ N_{eq} : \text{Number of equations} \\ N_d : \text{Number of data} \end{cases}$$

In this step, decision variables are selected, and causalities of equations are assigned (oriented calculation); each unknown variable will be calculated only by one equation (each variable must be mentioned only one time in the left side of the equations).

For this example, $[x_2, x_3, x_6]$ are chosen as decision variables (how to choose these variables is treated in the second part of this paper). Then, in each column, only the information given by the equations

	L_0			L_1		L_2	L_3
	x_2	x_3	x_6	x_5	x_7	x_4	x_1
x_2	■			2			
x_3		■			4	1	
x_6			■		4		3
x_5				■			
x_7					■	1	
x_4						■	3
x_1							■

Fig. 4. Oriented and sequenced “DSM”.

		L_0		L_1			
	x_1	x_2	x_3	x_5	x_4	x_6	x_7
x_1	■					3	
x_2		■		2			
x_3			■		1		4
x_5				■			
x_4					■	3	4
x_6						■	4
x_7					1		■

Fig. 5. Oriented and sequenced “DSM” with “coupling problem”.

dedicated to calculate the concerned variables is represented (for example, in the 4$^{\mathrm{th}}$ column, only the dependencies of the variable “x_1” given by the Eq. (3) are represented).

The orientation of the “DSM” leads to remove all repeated information in the non-oriented and non-sequenced “DSM”. This allows a minimum number of calculations for determining all model variables and avoids the use of implicit relations by optimally exploiting the causal relations between variables.

Step 3: Oriented and sequenced “DSM”

In this step, we try to triangulate the oriented and non-sequenced “DSM” using partitioning and sequencing algorithms, with the aim of rationalizing model and favoring its parallelization (identification of the different levels L_i of calculation) to minimize the computational cost.

In the partitioning process, rows and columns of the oriented and non-sequenced “DSM” are interchanged in order to find the fastest sequence of calculation: variables of the i^{th} level are those which can be determined by the variables of previous levels (from the level “0” to the level “it1”). In our example, “x_5” and “x_7” can be calculated only from the decision variables, so they form the 1st level “L_1”. “x_4” forms the second level “L_2” and “x_1” forms the third level “L_3”.

The triangular form of the oriented and sequenced “DSM” means the absence of any coupling in the model. In the case where the triangulation of the matrix is impossible, the choice of decision variables must be rectified to eliminate all forms of couplings in the model.

The interest of the “DSM” approach consists in structuring, simplifying and reducing the representation of the analytical model in which all the important information is clearly shown: dependencies, couplings and causalities. The definition of several levels of calculation can be used to systematically parallelize the calculation process which can reduce significantly the computational time of the model. For example in the matrix of Fig. 4, the Eqs (2) and (4) involving both x_5 and x_7 variables belonging to the same level L_1 may be parallelized.

3. Iterative method for selecting decision variables

The good choice of the input variables of the design model in an optimization problem is highly important to avoid couplings and causality problems involving algebraic loops. A bad choice of decision

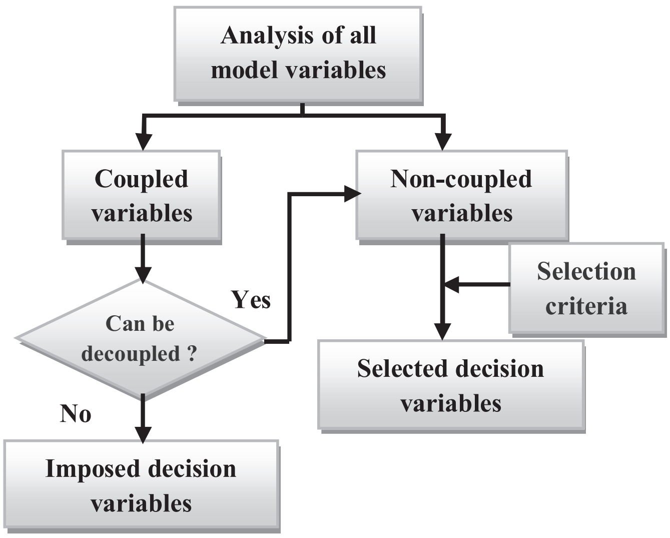

Fig. 6. Selection method algorithm.

variables that would generate implicit blocks in the model may lead to an over-specified design problem. Generally, to overcome coupling problems, two solutions are possible: the first one is to increase the size of the design variable vector; however, such a solution can lead to inconsistencies and redundancies in the model. The second solution is to impose additional simplifying assumptions which may decrease the accuracy of the model.

Suppose we choose [x_1, x_2, x_3] as decision variables for the simple mathematical example treated in the previous section. Figure 5 shows that such a choice of decision variables would lead to a coupling problem between "x_4, x_6 and x_7": to calculate "x_4" we should know the value of "x_7" and to define "x_7" we should have set "x_6" which requires the knowledge of "x_4".

This example shows the importance of a good choice of the decision variables in design problems. In this section, an iterative method based on the "DSM" is proposed to make systematic the selection of decision variables in analytical optimization problem, in order to avoid coupling problems and minimize implicit blocks in the model.

Figure 6 resumes the algorithm of the selection method. A preliminary step is recommended to reduce the processing time; it consists in dividing the model in several independent submodels. In the mathematical example, the Eq. (2) can be treated separately because it is not coupled with the rest of the model. The method requires two steps:

Step 1: Model variables are divided on coupled and non-coupled variables. Here, coupled variables means that they have exactly the same dependencies with the rest of the model variables. To treat these variables, we can try to rewrite the equations of the model in order to break the existing couplings, but a great attention must be given when doing these modifications because of causality problems. Practically, this solution is very hard to implement for complex models with a huge number of equations and variables. If the first solution is not possible, we should select imperatively the necessary number of decision variables from these coupled variables to overcome these coupling problems.

Step 2: Here, the decision variables are selected from the non-coupled variables of the model. In that case, the choice is based on a selection criteria which can be the "occurrence frequency" (i.e. the number of times a variable appears in the model) or the importance of variables ("importance" is here seen with its physical meaning: for example, the rated torque is an "important" degree of freedom for electric

Table 1
Occurrence frequency of model variables (Index_1)

	Submodel "1"		Submodel "2"				
Variable	x_2	x_5	x_1	x_3	x_4	x_6	x_7
Index_1	1	1	1	2	2	2	2

Table 2
Degree of model equations (Index_2)

Equation	(1)	(2)	(3)	(4)
Index_2	3	2	3	3

	x_1	x_3	x_4	x_6	x_7
x_1			3	3	
x_3					
x_4	3			3	1
x_6	3		3		4
x_7			1	4	

Variable	x_1	x_3	x_4	x_6	x_7
Index_1	1	2	2	2	2

Equation	(1)	(2)	(3)	(4)
Index_2	**2**	2	3	**2**

Fig. 7. DSM evolution after the 1$^{\mathrm{st}}$ iteration.

machine sizing). It can be also a choice of the expert to satisfy technical or environmental conditions. Remember that decision variables are systematically constrained in a certain range.

As mentioned before, this method is based on the non-oriented and non-sequenced "DSM" which contains all the interactions in the model.To guide the choice of the decision variables and to structure the method, two indexes are defined:

Index_1 in linked with the "occurrence frequency" of a variable: the number of equations that contain this variable. For example, "x_2" appears only in the Eq. (2), so $\mathrm{Index}_1(x_2) = 1$. In the other hand, "$x_3$" appears in two Eqs (1) and (4), so $\mathrm{index}_1(x_3) = 2$. On the "DSM", this index is equal to the number of equations in the column or the row of the concerned variable excluding repetitions. For a variable with a high occurrence frequency, we have interest to fix it as an input variable (decision variable "level 0") to minimize the number of indeterminations and to facilitate the calculation process. Table 1 gives the index_1 of the variables of the mathematical example.

Index_2 is linked with the equation size: it represents the number of variables in each equation of the model: it reflects the complexity for each equation. In the "DSM", it is given by the number of rows or columns which contain the number of the concerned equation. Table 2 gives the index_2 of the variables of the mathematical example.

The selection of the decision variables is based on the Index_1 which reflects the selection criteria adopted for this example. However, the second index is required to track the degree of equations each iteration. If the Index_2 is equal to "1" this means that we can directly deduce the value of a variable (i.e. a typical equation such as '$f(\mathrm{x}) = 0$)'. Therefore, we check if there is such a particular state after each iteration.

In the first iteration of the submodel "2", we select one of the variables "x_3", "x_4", "x_6" and "x_7" because each of these variables has the highest index_1. After having selected the first decision variable (for example "x_3"), we should update the "DSM" and tables; the index_2 of the equations which contain the variable "x_3" is decreased by "1" and the information of the column and the row which represent

	x_1	x_3	x_4	x_6	x_7
x_1				3	
x_3					
x_4					
x_6	3				4
x_7				4	

Variable	x_1	x_3	x_4	x_6	x_7
Index_1	1	2	2	2	2

Equation	(1)	(2)	(3)	(4)
Index_2	**1**	2	**2**	2

Fig. 8. DSM evolution after the 2^{nd} iteration.

the selected decision variable are deleted (only information between unknown variables should be represented). The DSM evolution after the first iteration is given in Fig. 7.

In the second iteration, we select "x_4" and we update the "DSM" and tables. The Index_2 of the equation (1) is thus reduced to "1", only involving x_7 (except the decision variables x_3, x_4); this means that the value of the variable "x_7" can be deduced, excluding it from the selection process. The DSM evolution after the second iteration is given in Fig. 8.

After 3 more iterations, we conclude that only two decision variables are required to calculate all the other variables in the submodel "2".

4. Example of the design model of a HSPMSM machine

4.1. Multi-physics analytical model

In this part, the DSM based selection method of decision variables is applied to a multi-physics analytical model of a High Speed Permanent Magnet Synchronous Machine (HSPMSM) dedicated to optimal design for an aeronautic application. This model is derived from a previous PMSM model devoted to ground applications [9,10]. The modelling has been extended in order to take all features related to the PMSM behavior at high speed operation. It includes:

- A geometry model that sets all characteristics related to the HSPMSM architecture: material types in each region (iron, magnet, sleeve, copper) and associated geometry parameters (i.e. radius length ratio, slot depth, slot width, number of pole pairs, number of slots per pole per phase, equivalent gap, magnet filling coefficient, etc.). This model also allows the computation of the HSPMSM weight from the mass density of each material and from the HSPMSM geometrical features;
- An electric model based on the HSPMSM electrical variables (resistance, leakage and main inductances, magnetic flux, voltage) calculated from the HSPMSM geometrical features;
- A magnetic model specifying HSPMSM electromagnetic behavior in each region (yoke, teethes, air gap, magnet) and magnet demagnetization characteristics;
- The computation of all HSPMSM power losses divided in Joule losses, iron losses [11], aerodynamic losses [12], magnet losses [13]. It allows establishing the energy efficiency of the whole process which is one of the key objectives of optimization;
- A thermal model giving the temperature in each HSPMSM part (copper, insulator, yoke, sleeve, magnet) from the corresponding power losses and from the external temperature imposed by the cooling plate. This model also provides the weight estimation of the HSPMSM cooling system;
- A HSPMSM control strategy allowing maximum torque per Ampere with field weakening mode. Together with the ''electric model", it allows verifying the satisfaction of the flight mission profile in the torque-speed plan.

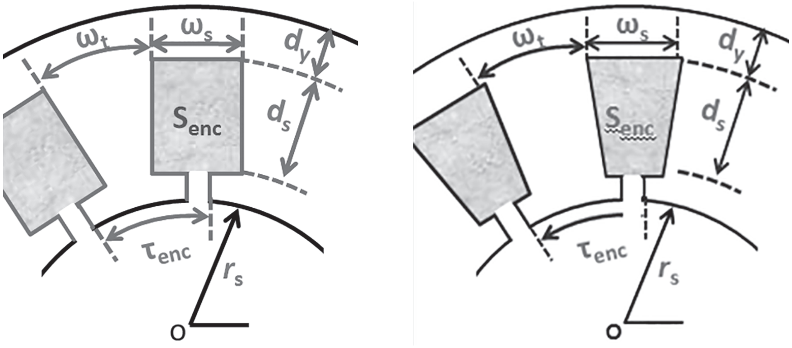

Fig. 9. Difference between rectangular slots (left) and trapezoidal slots (right).

4.2. Optimization problem

The optimal sizing of the HSPMSM has been initially formulated into an optimization problem with:

- Eleven design variables related to the HSPMSM geometric features (number of pole pairs, number of slot per pole per phase, radius length ratio, equivalent gap, magnet filling coefficient) electromagnetic variables (yoke induction, current density) and mechanical characteristics (base speed and base torque);
- Eleven constraints associated with geometrical variables (minimum and maximum sleeve thickness, minimum and maximum of copper windings per slots), technological limits (maximum rotational speed, magnet demagnetization) and temperature limits in the different motor parts (magnet, insulator, yoke);
- Two objectives to be minimized: the HSPMSM weight and the total losses estimated over the flight mission. Total HSPMSM losses are computed by weighting all losses on each mission point according to its occurrence during the flight.

Decision variable selection was initially based on expert knowledge and was inspired from a previous work of Slemon [14] where the studied machine was a PMSM with rectangular slots. However, for technical reasons, trapezoidal slots have been adopted for our application (see Fig. 9). Consequently some modifications are made in the HSPMSM model. These modifications have induced an implicit block in the design model which forced the expert to impose some simplifying assumptions to avoid this coupling problem.

Figure 10 shows the different steps followed in the calculation of the machine radius “r_s”. The different equations are given as follows:

$$S_{slot}^{REC} = \omega_s \cdot d_s \tag{6}$$

$$S_{slot}^{TRA} = [(\omega_s + \omega_t) + \tau_{slot}] \cdot \frac{d_s}{2} - (\omega_t \cdot d_s) \tag{7}$$

$$B = \frac{J_r}{r_m \cdot \mu_r \cdot \frac{K_c \cdot g}{l_m} + \frac{k_{Hop}}{K_p} \cdot \frac{1}{1-(g/r_s)}} \tag{8}$$

$$r_s = \sqrt{\frac{R_{lr} \cdot T_{bp}}{N_{slot} \cdot K_r \cdot K_{b1} \cdot S_{slot} \cdot B \cdot J_s}} \tag{9}$$

where: S_{enc}^{REC}, S_{enc}^{TRA} are section of rectangular and trapezoidal slots, ω_s, ω_t are slot and tooth widths,

Table 3
New and original decision variables vector comparison

Original decision variables (expert choice)		New decision variables with "DSM" analysis	
Variables	Symbol	Variables	Symbol
Rated torque	T_{bp}	Rated torque	T_{bp}
Rated speed	N_{bp}	Rated speed	N_{bp}
Number of slots/pole/phase	N_{epp}	Number of slots/pole/phase	N_{epp}
Number of pole pairs	P	Number of pole pairs	p
Equivalent Gap	$\boldsymbol{G}$	Equivalent Gap	$\boldsymbol{g}$
Radius length ratio	$\boldsymbol{R_{lr}}$	Length	$\boldsymbol{l_r}$
Magnet filling coefficient	$\boldsymbol{K_p}$	Magnet width	$\boldsymbol{l_m}$
Slot depth radius ratio	$\boldsymbol{R_{dr}}$	Slot depth	$\boldsymbol{d_s}$
Yoke induction	$\boldsymbol{B_y}$	Radius	$\boldsymbol{r_s}$
Current density	$\boldsymbol{J_s}$	Copper surface	$\boldsymbol{S_{cu}}$
Slot filling coefficient	$\boldsymbol{K_r}$	Slot width	$\boldsymbol{w_t}$

Table 4
Compared Pareto fronts between models based new and original decision variables

Original decision variables (expert choice)		Mixed decision variables	
Variables	Symbol	Variables	Symbol
Radius length ratio	R_{lr}	Length	l_r
Slot depth radius ratio	R_{dr}	Radius	r_s

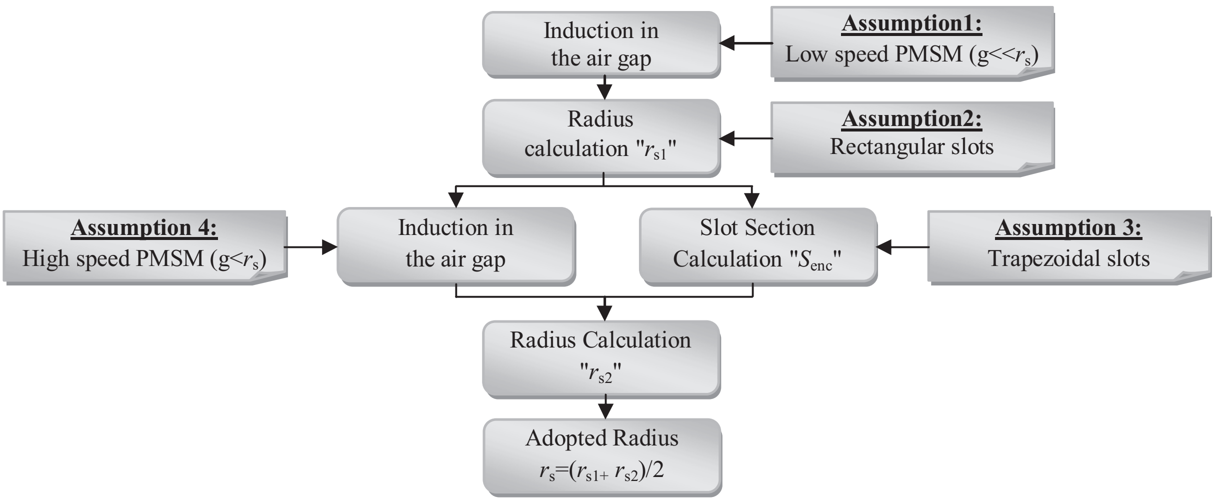

Fig. 10. Radius calculation steps involving implicit blocks.

τ_{slot} is the pole pitch,
N_{slot} is the number of slots,
B is the gap induction
$K_{b1}, K_c, k_{Hop}, r_m$ are coefficients

To avoid this problem, the selection method of decision variables is applied to this design model; Table 3 shows a comparison between the new (based on the DSM process) and the expert decision variables.

The first set of variables represents the common decision variables between the original and the new vectors and variables in bold characters represent the differences. Coupling problem on "r_s" calculation

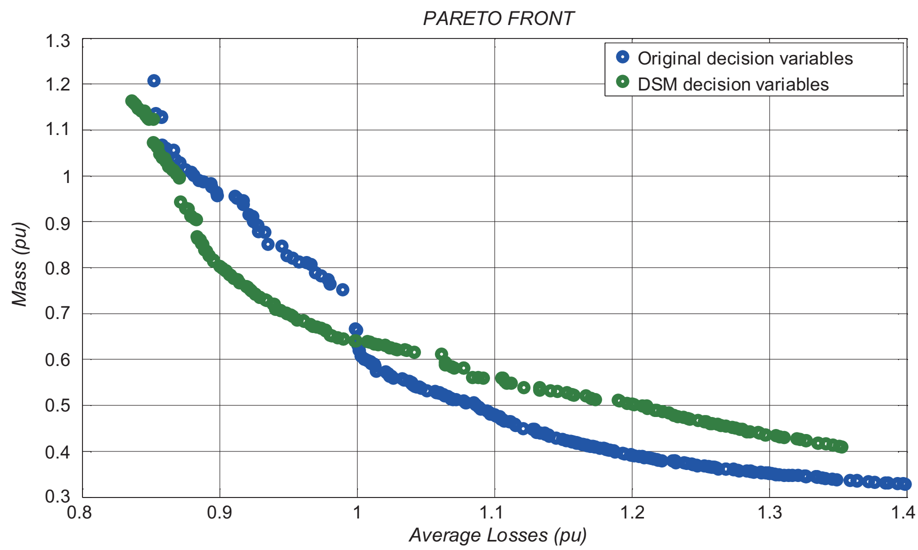

Fig. 11. Compared Pareto fronts between models based New and Original decision variables.

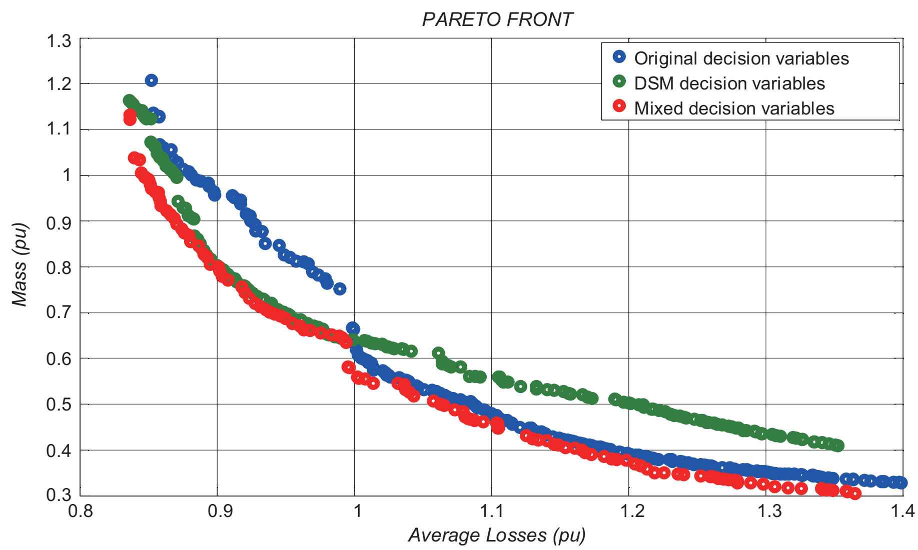

Fig. 12. Compared Pareto fronts of the different optimization problems.

as illustrated in Fig. 10 is overcome and the number of equations in the model is reduced by "40" equations; this reduction is related to the approximate calculation of the radius "r_s" in the initial model. However, the number of constraints in the new optimization problem is increased to 17 instead of 11 in the original problem to take account of the bounds of the changed decision variables (R_{lr}, K_p, R_{dr}, B_y, J_s, K_r).

To compare both decision variables vector, two optimization of the HSPMSM are performed with an NSGA-II evolutionary algorithm with a population size of 100 and with a number of generations of 500, by using a self-adaptive recombination [15]. In Fig. 11, it can be seen that the Pareto front of the original optimization problem outperforms one part (light machines) of the Pareto front of the new optimization problem. These results can be explained by the additional constraints in the new optimization problem which increase its complexity, and by the additional simplifying assumptions used in the original problem.

In Fig. 10, another compromise decision variables vector, so called "mixed decision variable" is proposed in order to reduce the complexity of the new optimization problem. The idea is to try reducing as much as possible the number of additional constraints and to keep the benefits of the "DSM" analysis (elimination of implicit blocks in the model). Table 4 shows the differences between the "mixed" and the "original" decision variables vector.

The new design problem with the "mixed decision variables" contains 13 constraints (instead of 17 in the previous DSM based decision variables) and retains the previous advantages of the model provided from the "DSM" analysis with the gain of "40" equations compared to the original model. The Pareto front with the "mixed design variables" clearly outperforms the two others (see Fig. 12).

5. Conclusion

We have presented in this paper a method based on the Direct Structure Matrix (DSM) which allows a clear representation of interactions between variables. Such a process has been proved to be particularly useful for complex optimal design for which choosing the right decision variables to efficiently solve the problem is not so trivial.

The results obtained in the typical case of a high speed electromechanical actuator shows that the selection of decision variables based on "occurrence frequencies" can solve coupling problems in the model. However, as illustrated in the case of "mixed decision variable" (Fig. 12) case, a selection based only on this criteria cannot guaranty systematically the optimal choice of the decision variables and certain expert choice must complete the selection.

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