
Transfert in the multimodel Arlequin framework

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RÉSUMÉ. L'un des piliers principaux du cadre multi-modèle Arlequin est la transmission inter-modèles. Ce point est discuté et analysé dans cette contribution et son lien avec la partition des énergies est souligné.

ABSTRACT. One of the main pillar of the multimodel Arlequin framework is the transmission between models. This point is discussed and analyzed in this contribution and its link with the energy partition is underlined.

MOTS-CLÉS : multi-modèle, multi-échelle, couplage de modèles, Méthode Arlequin.

KEYWORDS: Multimodel, multiscale, bridging models, Arlequin method.

1. Introduction

Based on a super-imposition technique, the Arlequin method (Ben Dhia, 1998, Ben Dhia, 1999) stands for a convenient and flexible tool for the practical modeling of multimodel and multiscale problems. Sticking to a pre-defined goal, it consists basically in adapting locally by a priori knowledge or, more accurately, by a posteriori error estimates, a global model, aiming to represent the behaviour of a material system occupying a domain Ω . The Arlequin adaptation consists in superposing and gluing the appropriate local model to the locally inappropriate global one. The resulting Arlequin solution is obtained as a partition of the solutions of the super-imposed models.

Relying on basic pillars that will be recalled in the sequel, the Arlequin family of methods actually constructs a (*multi*)-modeling framework that extends the classical (*mono*)-modeling one. It has a large range of applications in many fields of interest. In the computational mechanical field, in particular, it leads to formulations of mechanical problems that allow for local discrete multimodel and multiscale analyses (e.g. (Ben Dhia *et al.*, 2002, Ben Dhia *et al.*, 2005)).

This contribution aims at giving basically an overview of the Arlequin framework with further insights concerning its essential ingredients, particularly the transmission of information between the different models. Some theoretical analyses of the Arlequin problems with defined transmissions and some other possible applications are also given.

2. The Arlequin framework

2.1. Preliminaries, definitions and notations

Let us assume that the domain Ω is partitioned into two overlapping domains Ω_1 and Ω_2 . That is :

$$\Omega = \Omega_1 \cup \Omega_2 \quad [1]$$

The overlap denoted by S , also called the superposition zone, is defined by : (see Figure 1)

$$S = \Omega_1 \cap \Omega_2 \quad [2]$$

with $meas(S) > 0$.

The superposition zone S is also partitioned either into two non overlapping volume domains (the *gluing* configuration) or into the boundary ∂S of S and its interior, (a

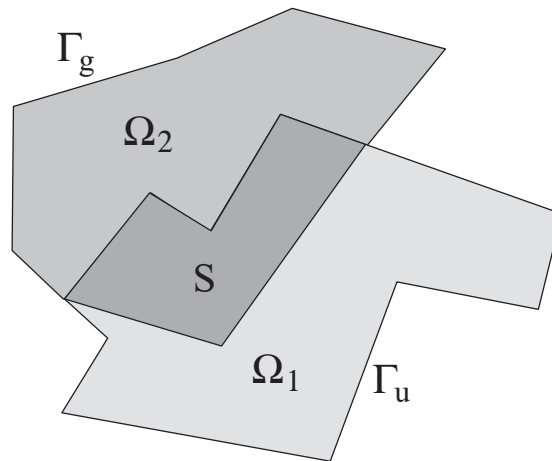


Figure 1. *Overlapping domains*

more classical *welding* configuration). In both cases, we use the same notations to refer to the two parts. The first part is denoted by S_c . That is the models coupling or transmission zone whose boundary has to satisfy : (see Figure 2 in the right)

$$\partial S \subset \partial S_c \quad [3]$$

The second, denoted by S_f , is the free zone. That is generally the part of interest in enrichment applications, namely the part in which the underlying model is changed. In this second zone, the models are super-imposed but no *communication* is activated between them.

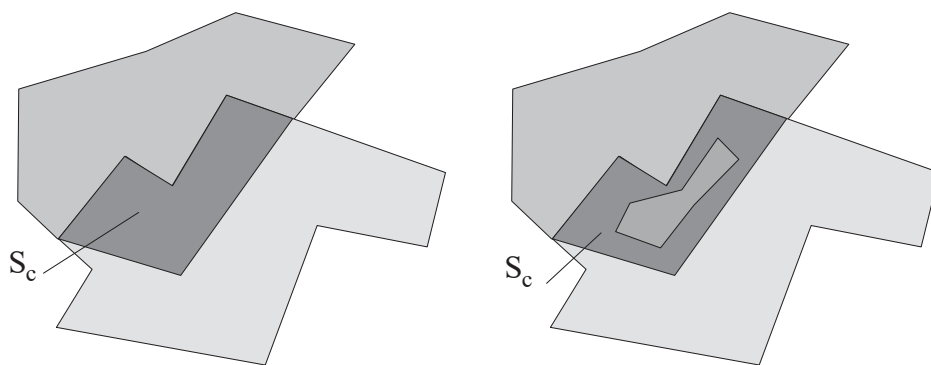


Figure 2. *Transmission zones S_c in the hole overlap in the left and in a sub-part of the overlap in the right*

2.2. Transmission in the Arlequin framework

The first pillar of the Arlequin method consists in allowing for the co-existence in S of two models having (roughly) all their own attributes (geometry, material properties, behaviour laws, mechanical hypotheses, etc.). Notice that more than two models can be super-imposed with no particular difficulty (Ben Dhia, 1999, Ben Dhia *et al.*, 2002).

As a rather natural consequence of the co-existence of models in S , weight parameter functions are activated, as a second pillar of the approach, to let the energies in S or more generally the virtual works of actions in S (Ben Dhia, 1998) be shared between the co-existent models. For instance, when static mechanical problems are considered, internal and external couples of weight parameter functions are introduced. They are denoted by (α_1, α_2) and (β_1, β_2) , respectively. The weight parameter functions α_i and β_i are defined in Ω_i , $i = 1, 2$ and verify a kind of partition of unity : (see Figure 3 for a representation of the couple (α_1, α_2))

$$\alpha_i \geq 0 \quad \text{in} \quad \Omega_i \quad [4]$$

$$\alpha_i = 1 \quad \text{in} \quad \Omega_i \setminus S \quad [5]$$

$$\alpha_1 + \alpha_2 = 1 \quad \text{in} \quad S \quad [6]$$

$$\beta_i \geq 0 \quad \text{in} \quad \Omega_i \quad [7]$$

$$\beta_i = 1 \quad \text{in} \quad \Omega_i \setminus S \quad [8]$$

$$\beta_1 + \beta_2 = 1 \quad \text{in} \quad S \quad [9]$$

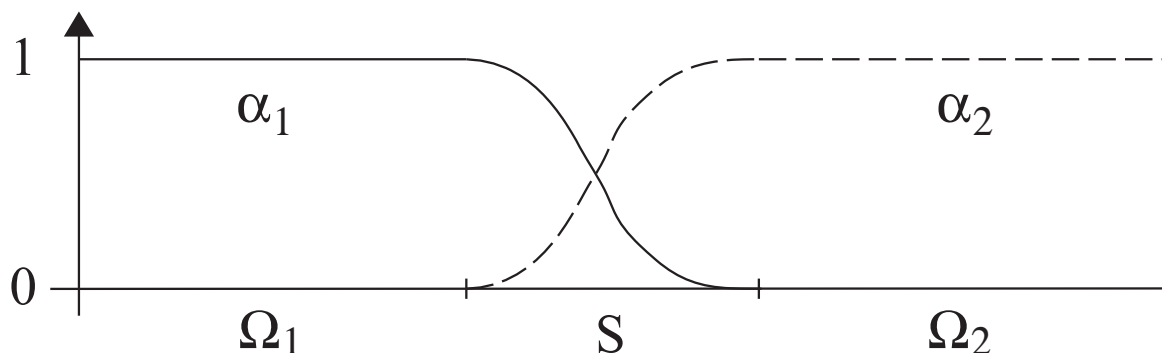


Figure 3. Weight parameter functions α_1 and α_2

Other couples of weight parameter functions have to be introduced when one takes into account other virtual mechanical actions such as virtual works of inertial forces

in dynamic regimes or virtual works of dumping forces, etc.. Notice that if ever necessary, the weight parameter functions may be vector fields or even second order tensors.

The choice of the *weight* parameter functions plays a central part in the Arlequin framework. As it could be guessed from the label "*weight*" parameters, it is expected that the more a model is stressed by choosing the associated *weight* parameter function near from one, the more the Arlequin solution is near from the solution of this stressed model. This expected result, depending also on the transfert operators, third pillar of the approach, has been proved for the superposition of a quite representative local altered model to a global sound one (Ben Dhia, 2007, submitted).

The transmission between the super-imposed models is the third and probably more sensitive pillar of the Arlequin framework. Many transmission models were designed in (Ben Dhia, 1998) and (Ben Dhia, 1999). When activated with the energies repartitions, these transmission models are required to satisfy the following criteria :

- (stability requirement) when the local model is similar to the global one, the Arlequin problem has to have the same stability than the *monomodel* problem,
- (consistence requirement) the *monomodel* solution of the considered problem can be recovered (at least as closely as desired) in the Arlequin framework,
- (extension requirement) the Arlequin framework extends indeed the classical mono-modeling framework in the sense that it leads, by local alterations, to efficient and flexible local treatments of multimodel and multiscale problems that can hardly be treated (if ever possible) in mono-modeling frameworks.

As a matter of fact, an effective transmission is achieved within two steps. The first consists in listing the transmitted mechanical quantities between models in the transmission zone. The second consists in describing the way this transmission is realized. For a model static elasticity problem in the continuum mechanics framework, it has been suggested to equate (weakly) the primal fields and/or the associated deformation fields of the two models. More generally, it has been suggested to equate a transformation of these fields by means of operators called *compatibility operators* and denoted by Π_S^Ω in (Ben Dhia, 1998, Ben Dhia, 1999). Such compatibility or *accomodation* operators may prove to be important in situations where a continuum model has to "*dialogue*" with another continuum but coarser or finer one in the transmission zone. These operators are even more essential for the the coupling of a discrete model (say granular or atomistic model) and a continuum one.

To clarify, let us denote by $(\mathbf{E}_i), i = 1, 2$, the states or parts of the states of $(model)_i, i = 1, 2$, we wish to transmit in S_c . The restriction to S_c of these (unknown, in general) states, referring in each model to a representation of the same concept of mechanical quantity (say the displacement, velocity, forces or stresses, etc.), are in spaces we denote by $\mathbf{IS}_i, i = 1, 2$. These spaces may be incompatible. Indeed, the two spaces may rely on incompatible meshes, or one space may be discrete (think to the space of placement of discrete particles in a given granular or atomistic medium)

and the other continuum (think to the space of continuous motions, defined as fields for the continuum representation of the same medium). In such cases, what is suggested in the Arlequin framework is to associate to $IS_i, i = 1, 2$, a couple of other spaces that are compatible or merely compatible with each other (see (Ben Dhia *et al.*, 2003) for a notion of $(1 - \epsilon)$ -compatible spaces). We denote these compatible or merely compatible spaces by $CS_i, i = 1, 2$. We assume, without significant restrictions, that the dual space of (say) CS_1 is included in the dual space of CS_2 . We denote the former by M and call it the *Mediator* space. Two compatibility operators, denoted by $\Pi_{1 \rightarrow 2}$ and $\Pi_{2 \rightarrow 1}$, ranging from IS_i , into $CS_i, i = 1, 2$, are then defined. Observe that the introduction of these two operators is an immediate extension of the introduction of the single compatibility operator Π_S^Ω in our first references where, for the sake of clarity, a model (the local one) was assumed to be "finer" than the other (global) one.

Once the previous spaces and operators are fixed, the transmission is achieved by activating a density of actions field, denoted by Φ . The latter is designed to control (weakly) the deviation in S_c between $\Pi_{1 \rightarrow 2}E_1$ and $\Pi_{2 \rightarrow 1}E_2$. It belongs to the mediator space of actions M . The mediator field is derived from action models that can be either of elastic type, rigid type (Lagrange multiplier), rigid-elastic type (Ben Dhia, 1998, Ben Dhia, 1999) or of more "fuzzy" types, aiming to couple as "peacefully" and as accurately as possible the super-imposed models.

All these aspects will be exemplified and the effectiveness of our choices studied theoretically during the conference.

3. Bibliographie

- Ben Dhia H., « Problèmes mécaniques multi-échelles : la méthode Arlequin », *C.R.A.S. Paris Série Iib 326* : 899-904., 1998.
- Ben Dhia H., « Numerical modelling of multiscale problems : the Arlequin method », in , Wunderlich (ed.), *Proceedings of the First Conference in Computational Mechanics*, Munchen, Germany, 1999.
- Ben Dhia H., Balmès E., « Mesure de compatibilité et application aux problèmes de sous-structuration », *Actes du Sixième Colloque en Calcul des Structures*, vol. 3, Giens, France, 2003.
- Ben Dhia H., Rateau G., « Application of the Arlequin method to some structures with defects », *R.E.E.F. 11, n° 2,3,4* : 291-304, 2002.
- Ben Dhia H., Rateau G., « The Arlequin method as a flexible engineering design tool », *Int. J. Numer. Meth. Engng*, vol. 62, p. 1442-1462, 2005.