



HAL
open science

Changes in the dynamical properties of the North Atlantic atmospheric circulation in the past 150 years

David Rodrigues, M Carmen Alvarez-Castro, Gabriele Messori, Pascal Yiou, Yoann Robin, Davide Faranda

► **To cite this version:**

David Rodrigues, M Carmen Alvarez-Castro, Gabriele Messori, Pascal Yiou, Yoann Robin, et al.. Changes in the dynamical properties of the North Atlantic atmospheric circulation in the past 150 years. 2017. hal-01504478v1

HAL Id: hal-01504478

<https://hal.science/hal-01504478v1>

Preprint submitted on 10 Apr 2017 (v1), last revised 30 Oct 2017 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

1 **Changes in the dynamical properties of the North Atlantic atmospheric**
2 **circulation in the past 150 years**

3 David Rodrigues¹, M. Carmen Alvarez-Castro¹, Gabriele Messori²,

4 Pascal Yiou¹, Yoann Robin¹, Davide Faranda^{1*}

5 ¹*Laboratoire de Sciences du Climat et de l'Environnement, UMR 8212 CEA-CNRS-UVSQ,IPSL,*

6 *Universite Paris-Saclay, 91191 Gif-sur-Yvette, France.*

7 ²*Department of Meteorology and Bolin Centre for Climate Research, Stockholm University,*

8 *Stockholm, Sweden.*

9 **Corresponding author address: Laboratoire de Sciences du Climat et de l'Environnement, UMR*
10 *8212 CEA-CNRS-UVSQ,IPSL, Universite Paris-Saclay, 91191 Gif-sur-Yvette, France.*

11 *E-mail: davide.faranda@lsce.ipsl.fr*

ABSTRACT

13 It is of fundamental importance to evaluate the ability of climate models to
14 capture the large-scale atmospheric circulation patterns. In the context of a
15 rapidly changing climate, it is equally crucial to quantify the robustness of the
16 modeled changes in the large-scale atmospheric dynamics. Here we approach
17 this problem from an innovative point of view based on dynamical systems
18 theory. We characterize the atmospheric circulation over the North Atlantic
19 in the CMIP5 historical simulations (1851 to 2000) in terms of two instan-
20 taneous metrics: local dimension of the attractor and stability of trajectories.
21 We then use these metrics to compare the models to the 20CR reanalysis over
22 the same historical period. The comparison suggests that: i) all the models
23 capture the mean attractor properties and models with finer grids perform bet-
24 ter; ii) extremes of the dynamical systems metrics match the same large-scale
25 patterns in most of the models; iii) changes in the attractor properties observed
26 for the 20CR reanalysis - studied by dividing the 1851-2000 period into 3 sub-
27 periods of 50 years each - are not reproduced by the models; iv) some models
28 present significant changes in the dynamical systems metrics over time but
29 there is no agreement on the direction and on the intensity of the shifts.

30 1. Introduction

31 One of the main sources of uncertainty in determining the impact of climate change on ex-
32 treme events is the forced response of atmospheric dynamics (Shepherd 2014; Field 2012). While
33 changes in observables such as surface temperature are easily diagnosed, shifts in the mid-latitude
34 atmospheric patterns have proved very difficult to quantify. Some advances have been made by
35 focussing on specific features such as atmospheric blocking (Kay et al. 2015; Cassou and Cattiaux
36 2016; Faranda et al. 2016b), which in turn influence the occurrence of European cold spells and
37 heat waves, but the broader appreciation of circulation changes is still unsatisfactory. Here we
38 address this knowledge gap by using a dynamical systems framework. We illustrate the power of
39 such an approach by considering the well-known Lorenz (1963) system, a conceptual model of
40 atmospheric convection consisting of three differential equations:

$$\dot{x} = \sigma(y - x) \quad \dot{y} = rx - y - xz \quad \dot{z} = xy - bz \quad (1)$$

41 where x , y , z represent respectively the convection strength, the temperature difference between
42 the surface and the top of the troposphere and the asymmetry of the convection cells. The
43 parameters σ , r are the Prandtl and the Rayleigh numbers, while b is a ratio of critical parameters.
44 A trajectory of the Lorenz (1963) attractor is shown in blue in figure 1. The figure consists of
45 2000 points obtained by iterating the Lorenz equations with $\Delta t \simeq 0.035$, $\sigma = 28$, $r = 10$, $b = 8/3$
46 with a Runge Kutta scheme of order 4.

47
48 To study the effects of an external forcing, we increase σ by 2% with respect to the classical
49 value. Figure 1 shows a trajectory at $\sigma = 28.5$ in magenta. The magenta trajectory favours
50 higher values of the variable z , but the changes relative to the original trajectory depend on the
51 point being considered: some points are not displaced, while some others are mapped elsewhere.

52 Assuming no knowledge of the system other than the trajectories' paths, how could we determine
53 whether they both belong to the Lorenz attractors with two different forcings? To answer this
54 question we would need: i) to measure the dynamical properties of an ensemble of trajectories
55 representing the two configurations; ii) to estimate the distance between the obtained trajectories
56 and determine if the shift has changed the properties of the points in a detectable way.

57
58 The atmospheric equivalent of a point on the Lorenz attractor is the ensemble of instantaneous
59 fields describing the atmosphere at a time t . To study the atmospheric circulation over the North
60 Atlantic we will focus on a single field: the sea-level pressure (SLP) over this region. The SLP
61 field reflects the major modes of variability affecting the North Atlantic (Hurrell 1995; Moore
62 et al. 2013) and can further be used to diagnose a wealth of other atmospheric features, ranging
63 from teleconnection patterns to storm track activity to atmospheric blocking e.g. (Rogers 1997;
64 Comas-Bru and McDermott 2014). The trajectories of our dynamical systems are the succession
65 of daily SLP fields from 27 CMIP5 models and the 20CR reanalysis over the period 1851 to
66 2000. In order to measure changes in the systems, one must be able to specify at each point (each
67 day) the local (daily) dynamical properties and track their evolution. Recent contributions to
68 dynamical systems analysis have proven that local properties of the trajectories are characterized
69 by two quantities: the local dimension and stability of the field considered (Lucarini et al. 2016;
70 Faranda et al. 2017). They correspond respectively to the rarity and the typical persistence of the
71 configuration. Faranda et al. (2017) have also shown that these two metrics can be connected to
72 the predictability of a given atmospheric state and that their extremes match climate extremes.

73
74 In this work we will first assess whether the models and reanalysis present similar attractor prop-
75 erties over the full time period considered. To do this, we compute daily values for the dimension

76 and stability of the SLP fields and study their average and extreme properties. Next, we study the
77 attractors for three sub-periods of 50 years each and quantify their changes across the periods. We
78 then compare the changes seen in the models to those observed in the reanalysis. Finally, we use
79 statistical mechanics arguments to attribute the changes observed in the reanalysis to greenhouse
80 forcing.

81 **2. Data & Methods**

82 We use daily model output from the historical simulations of 27 CMIP5 models (see Table 1).
83 The data is publicly available from the CMIP5 archive (Taylor et al. 2012). We then compare
84 these to the 20th Century Reanalysis (20CRv2c) ensemble mean dataset (Compo et al. 2011). The
85 analysis focuses on the region $22.5^{\circ}N - 70^{\circ}N$ and $80^{\circ}W - 50^{\circ}E$.

86
87 In order to compute the dynamical systems metrics, we combine the statistical tools of extreme
88 value theory with the results obtained by Freitas et al. (2010) for Poincaré recurrences. The pa-
89 rameters mentioned in the introduction (local dimension d and stability θ) are computed for the
90 points ζ on the attractor obtained as sequence of states of the system. The dynamical indicators
91 are linked to the probability \mathcal{P} that a trajectory $x(t)$ of the distances of a trajectory emerging from
92 a sphere of center ζ and diameter 2ϵ , i.e. the recurrence rate of the configuration ζ . We briefly
93 outline the physical meaning of these quantities and the way they are computed below.

94 *(i) Local Dimensions:* The Freitas et al. (2010) theorem and its modification in Lucarini et al.
95 (2012) states that the probability \mathcal{P} for chaotic attractors is a generalized Pareto distribution
96 (Pickands III 1975). We first compute the distance δ between the SLP field ζ and all other obser-
97 vations along the trajectory. We then weight the time series of the distance:

$$g(x(t)) = -\log(\delta(x(t), \zeta)).$$

98 The reason for taking the logarithm is explained by Collet and Eckmann (2009): in the dy-
 99 namical system set-up the logarithm increases the discrimination of small values of $\delta(x, y)$ which
 100 correspond to large values of $g(x(t))$. The generalized Pareto distribution then reduces to:

$$\mathcal{P}(g(x(t)) > q, \zeta) \simeq \exp(-[x - \mu(\zeta)]/\beta)$$

101 namely an exponential law whose parameters μ and σ depend on the point ζ chosen on the
 102 attractor. Remarkably, $\beta(\zeta) = 1/d(\zeta)$, where $d(\zeta)$ is the local dimension around the point ζ .
 103 This result has recently been applied to SLP fields in Faranda et al. (2017).

104
 105 (ii) *Local Stability:* θ , the inverse of the residence time within a neighborhood of the config-
 106 uration, is exactly the extremal index introduced in extreme value theory to measure clustering
 107 (Freitas et al. 2012; Faranda et al. 2016a). As in the extreme value theory, θ varies between 0 and
 108 1. The value $\theta = 0$ corresponds to a stable fixed point of the dynamics where the observation ζ is
 109 repeated infinite times (as for a pendulum left in its equilibrium position). This is of course never
 110 observed in the atmospheric dynamics. A value of $\theta = 1$ indicates a point immediately leaving
 111 the neighborhood of ζ . Since θ is the inverse of a persistence time, it depends on the Δt used. If
 112 Δt is too large, the time dependence structure is hidden and θ will be close to 1. If Δt is too
 113 small, θ is close to zero. In Faranda et al. (2017) it has been observed that θ for SLP fields over
 114 the North Atlantic is between 0.3 and 0.5, when $\Delta t = 1$ day. In this work we use the same Δt .

115

116 Figure 2 illustrates of the meaning of the indicators: the local dimension d is the number of
117 degrees of freedom needed to describe the dynamics of the system linearized around the state ζ
118 and it is therefore proportional to the number of possible states resulting from ζ . The inverse of
119 the persistence time θ is linked to the probability that the trajectory follows a path where each
120 field resembles those of the previous and subsequent days.

121

122 Before beginning the analysis, it is necessary to outline how the method of recurrence deals with
123 changes in the attractor. There are few theoretical results on non-stationary statistics of dynamical
124 systems, as well as on non-stationary extreme value theory. Luckily, the recurrence technique
125 also allows to bypass most of the technical difficulties linked to non-stationarity because the
126 dynamical properties are measured with respect to each single state of the attractor. If the change
127 affects the neighbourhood of a state, it will change its dynamical properties d and θ . If most of
128 the states are affected by the changes in the dynamics, then the average dimension of the attractor
129 and the average persistence will change accordingly.

130

131 In order to test this idea, we again consider the two Lorenz (1963) systems discussed in the
132 Introduction and perform two sets of 30 realizations (trajectories) at $\sigma = 28$ and $\sigma = 28.5$, each
133 of them consisting of 50000 points. These values correspond respectively to the size of the
134 CMIP5 ensemble (about 30) and to the number of days in the period 1850-2000 (about 50000).
135 If the method of recurrence is capable to distinguish between the $\sigma = 28$ and $\sigma = 28.5$ Lorenz
136 attractors, then the d and θ distributions of the two sets of realizations should be significantly
137 different.

138

139 First, we have to define a metric to compare (d, θ) distributions. The simplest idea is to compute
140 the median of the d, θ distribution for each realization and verify that the clouds of median
141 centroids for $\sigma = 28$ and for $\sigma = 28.5$ are separated. This is shown in the top panel of Figure
142 3. As a further test, we can compute the empirical probability density functions $\Lambda(\sigma_{28})$ and
143 $\Lambda(\sigma_{28.5})$ of the pairwise distances between the d, θ median-based centroids. These distributions
144 should be significantly different from the $\Lambda(\sigma_{28})/\Lambda(\sigma_{28.5})$ distribution obtained by mixing
145 together the two sets of realizations. The visual inspection of Λ distributions (Figure 3-bottom
146 panel) suggests that distances computed when mixing the $\sigma = 28$ and $\sigma = 28.5$ realizations are
147 generally higher than those computed by realizations of the same attractor. This claim is sta-
148 tistically supported by the Kolmogorov-Smirnov test (Von Storch 1999) results reported in Table 2.

149
150 Table 2 also contains the results of the Kolmogorov-Smirnov test obtained when considering the
151 Wasserstein distances \mathscr{W}_2 (Villani 2008) between the full d, θ distributions. As described by Robin
152 et al. (2017), the Wasserstein distance is the proper tool to measure distance in multivariate set-ups
153 so that our median-based indicator should be tested against \mathscr{W}_2 . The test results suggest that the
154 median-based centroids are good proxies of the Wasserstein distances. We will see in the next
155 section that this result also hold for the CMIP5 ensemble. Since in the climate system we do not
156 have distinct trajectories (one before and one after climate change) we will divide the dataset in
157 three different periods imagining three separate trajectories obtained under different (greenhouse)
158 forcings.

159 3. Aggregate analysis of model and reanalysis attractors

160 We begin the analysis of the daily SLP fields from 1851 to 2000 by presenting the scatterplot
161 of d versus θ for the 20CR reanalysis (Figure 4). The average of d is proportional to the

162 number of degrees of freedom needed to represent the systems' dynamics while the average of
163 θ is the inverse of the mean persistence time. Maxima (minima) of d correspond to the most
164 complex (simple) trajectories of the system. Maxima (minima) of θ correspond to the most
165 unstable (stable) trajectories (Messori et al. 2017). Such extremes are associated to specific
166 weather patterns that closely resemble the canonical North-Atlantic weather regimes. The
167 top panels in Figure 4 show the composite SLP anomalies for days beyond the 0.98 and 0.02
168 quantiles of the d and θ distributions. Maxima of θ reproduce an Atlantic Ridge pattern,
169 while minima of θ correspond to a negative North Atlantic Oscillation (NAO) phase. Similarly,
170 maxima of d correspond to a Blocking pattern and minima to a positive NAO (Faranda et al. 2017).

171
172 We next compare the (d, θ) bivariate histograms obtained for the 20CR with those computed
173 for the CMIP5 models (Figure 5-right). Two different behaviors emerge: some of the models
174 (e.g. CMCC-CMS reported in Figure 5-centre) yield a single mode distribution resembling
175 that obtained for the 20CR ; other models show bimodal distributions (e.g. the IPCC-CM5A)
176 histograms reported in Figure 5-bottom. We find the different behaviors to be related to the
177 seasonal cycle: in Figure 5-left, we plot the (d, θ) diagrams for the same models by coloring
178 each point according to the month of the year. In the 20CR and the CMCC-CMS model, the
179 different seasons are spread across the cloud, although maxima of θ mostly occur in winter.
180 The IPSL-CM5A displays a much stronger seasonal discrimination, with two distinct (d, θ)
181 clouds for the winter and for the summer seasons corresponding to the different modes of the
182 bivariate histograms. This implies that both the bulk statistics and the extremes are modified by
183 the seasonal cycle.

184

185 Given the variety of the possible behaviors, we will analyze separately the mean and the
186 extreme behavior of the dynamical properties. We report the aggregate analysis in Table 3 and
187 in Figure 7. The dots correspond to the centres of the ellipses and represent the median values
188 for each model; the semiaxes correspond to the standard deviation of the mean. Models are
189 numbered as in Tables 1, 3 and are ordered by increasing horizontal resolution. In Table 3 we
190 also provide the distances $\delta(d)$ and $\delta(\theta)$ from the median of the 20CR, the relative distances
191 $R(d) = \delta(d)/\max \delta(d)$ and $R(\theta) = \delta(\theta)/\max \delta(\theta)$ with respect to the farthest model and a
192 global score $R_{tot} = (R(d) + R(\theta))/2$. To check the validity of this global score, we compare
193 R_{tot} with the Wasserstein distance \mathscr{W}_2 between 20CR and the CMIP5 ensemble, computed as
194 described in Robin et al. (2017). The results are displayed in Figure 6. The two indicators are
195 so similar (Pearson coefficient: $r_{pear} = 0.90$ and Spearman coefficient $r_{spear} = 0.85$ (Von Storch
196 1999)) that we will use the simpler R_{tot} when discussing our results. R_{tot} further indicates the
197 direction of the changes (larger or smaller d, θ) while \mathscr{W}_2 only provides this information if the
198 transport plan is computed (Villani 2008). the latter would be particularly complex to compute
199 for the dataset analysed here.

200

201 By using the R_{tot} metrics, we find that all the models are within one standard deviation of the
202 20CR ensemble mean. At the same time, most models display median values in d and θ which
203 are statistically different from those found in 20CR. The results of a Wilcoxon ranksum test
204 (Von Storch 1999) are reported in Table S1. We remark that both Figure 7 and Table S1 indicate
205 that models with a higher horizontal resolution have median values generally closer to those of
206 the 20CR.

207

208 Different models have different spreads in d and θ , making it worth to investigate the extremes
209 of these quantities and their relation with the weather regimes found in 20CR. Figures 8 and 9
210 display the composite SLP anomalies for the d and θ extremes - computed as in Figure 4 - for
211 the three closest and the three farthest models (in terms of R_{tot}) relative to the 20CR. The models
212 display similar composites independently their R_{tot} score. A quantitative analysis is reported in
213 Table S2 using the Root Mean Square Error (RMSE) between 20CR and CMIP5 SLP composite
214 anomalies. In general, we find the NAO- and NAO+ patterns to have higher RMSE whereas
215 Atlantic Ridge and Blocking pattern are better represented.

216

217 The aggregate analysis shows that many CMIP5 models provide a dynamical picture coherent
218 with that of the 20CR reanalysis. At the same time, most models reproduce statistically different
219 medians of the two dynamical systems metrics relative to what found in the reanalysis.

220 **4. Changes in the attractor properties**

221 We next investigate whether the SLP's dynamical indicators have changed as an effect of
222 past greenhouse forcing by separating the results into three periods: 1851-1900, 1901-1950 and
223 1951-2000. To provide a visually immediate depiction of the changes, we compute the joint
224 histograms of (d, θ) . The left-hand side and middle columns in Figure 10 display these for
225 CCSM4 (the model with the highest horizontal resolution among those analysed) and 20CR.
226 The median values are highlighted by magenta lines. The right-hand side panels show the
227 differences between 20CR and CCSM4. Results for all the other models are shown in Figures
228 S1-S26. In the reanalysis, both the median of the local dimension d and the median of θ
229 increase from the first to the second and the third period. These increases, although small
230 relative to the metrics' spread, are significant at the 95% level under a Wilcoxon ranksum test

231 (Table S1). For the models, the sign and significance of the changes depend on the period
232 considered and are often not significant (Table S1). As a general feature, models tend to repro-
233 duce the dynamical properties better for the period 1951-2000 (see Figure 10 and Figures S1-S26).

234

235 We also perform a simple analysis of extremes in d and θ for the three different periods. We
236 first compute the quantiles 0.02 and 0.98 for the d and θ distributions of the whole dataset. For the
237 20 CR these quantiles correspond to the black lines of Figure 4. Then we compute the number of
238 days falling beyond the 0.02 and 0.98 quantiles in each of the three different periods. The results
239 are shown in Figure 11. From top to bottom, we report the four extremes corresponding to Atlantic
240 Ridge (maxima of θ), NAO- (minima of θ), Blocking (maxima of d) and NAO+ (minima of d).
241 Again, the changes observed for the 20CR are generally larger than those observed for the models.
242 None of the models displays changes coherent with those observed in the reanalysis. The results
243 are stable with respect to reasonable changes in the quantiles.

244 5. Discussion and Conclusions

245 We have computed the instantaneous dynamical properties of the SLP fields for the 20CR and
246 the CMIP5 historical runs, over the period 1851-2000. The goal of our analysis was to assess
247 whether different models with different physics and resolutions quantitatively represent the *same*
248 dynamical system and therefore possess attractors with similar characteristics. The metrics we
249 used are the local dimension d and the inverse of the persistence time θ . As described in Faranda
250 et al. (2017), these two quantities give a complete characterization of the attractor of the system.

251

252 When the whole analysis period is considered, we find that the models successfully capture
253 some of the dynamical systems features identified in the reanalysis. For example, the range and

254 variability of the dynamical metrics are consistent across the datasets. At the same time, some
255 models exaggerate the effects of the seasonal cycle on the dynamical indicators, and the statistical
256 agreement in the median values of the metrics is generally poor. Models with higher horizontal
257 resolutions tend to perform better. The SLP fields corresponding to extremes in d and θ are
258 mostly similar across the models and reanalysis. The main differences are found for the minima
259 of θ and d , which in 20CR correspond to the NAO- and NAO+ patterns.

260
261 To detect the changes in the attractor properties with time, we have then divided the results into
262 three periods: 1851-1900, 1901-1950 and 1951-2000. We have analysed the joint histograms
263 of the d, θ variables and compared them to those obtained for the 20CR. The reanalysis shows
264 significant increases in d and θ throughout the time period analysed. These changes also reflect
265 in the number of days having extreme d and/or θ values. Days with Blocking and Atlantic Ridge
266 patterns increase in frequency with time, while days with NAO patterns are decreasing. This is
267 coherent with the results obtained by Alvarez-Castro et al. (2017). We note that the decrease in
268 the frequency of NAO patterns does not imply a more negative NAO index. Indeed, the frequency
269 of the NAO- pattern decreases more than that of the NAO+ pattern suggesting that – if anything
270 – the positive NAO phase becomes more dominant towards the end of the analysis period, in
271 agreement with SLP measurements (Hurrell et al. 2001). None of the models show comparable
272 changes: changes in d are mostly not significant and the shifts in θ are significantly smaller
273 than in the reanalysis. This is reflected in the fact that the median values of the two variables
274 in the models do not show a clear upward trend throughout the three sub-periods considered.
275 As a caveat we note that our analysis does not attempt to separate the forced variability from
276 natural low-frequency oscillations and that, especially during the first two periods analysed,
277 it is unclear whether the greenhouse forcing can be clearly discerned above the background

278 "climate noise" (Paeth et al. 1999; Lyu et al. 2015). We must therefore take into account the possi-
279 bility that the model's internal variability dominates over the trends for the time period considered.

280

281 As a further caveat we note that several studies (Krueger et al. 2013; Ferguson and Villarini
282 2012, 2014) have questioned the consistency of the 20CR dataset. So, how much can we trust
283 the results obtained when investigating the three separate periods? Most of the observations used
284 to constrain 20CR in the first part of the dataset are located in Europe or eastern North America
285 (Cram et al. 2015); the North Atlantic sector can therefore be expected to perform better than
286 elsewhere. The dataset has a sufficiently high horizontal resolution to obtain a good estimate of
287 the local dimension distribution (Faranda et al. 2017). However, the fact that the 20CR data is
288 increasingly constrained to follow the SLP observations as the time approaches the present day
289 causes a decrease of the ensemble spread with time, since the system is more closely pinned to
290 a specific manifold (the observations), without the possibility of exploring the full phase space.
291 This may explain the changes in the 20CR local dimension with time.

292

293 If the results obtained for 20CR do not depend on the quality of the dataset but have a real phys-
294 ical meaning, the increase in dimension with time could be explained by using the results obtained
295 in Faranda et al. (2013) for simple dynamical systems. In these conceptual models, the dimension
296 increases with the *temperature* of the system defined – following the Einstein model for Brownian
297 motion – as the variance of a stochastic noise term added to the deterministic dynamics. Schubert
298 and Lucarini (2015) have demonstrated that this modeling approach is relevant for climate and, in
299 general, for any multiscale system. Faranda et al. (2013) have further shown that $\theta \rightarrow 1$ when the
300 temperature is increased, coherently with what observed in the 20CR.

301 *Acknowledgments.* P.Yiou and D. Faranda were supported by ERC grant No. 338965, M.C.
302 Alvarez-Castro was supported by Swedish Research Council grant No. C0629701 and G. Messori
303 was supported by a grant from the Department of Meteorology of Stockholm University.

304 **References**

- 305 Alvarez-Castro, M., D. Faranda, and P. Yiou, 2017: Changes in the atmospheric dynamics leading
306 to west-european heatwaves since 1900. *Environmental Reserch Letters*, **in review**.
- 307 Cassou, C., and J. Cattiaux, 2016: Disruption of the european climate seasonal clock in a warming
308 world. *Nature Climate Change*.
- 309 Collet, P., and J.-P. Eckmann, 2009: *Iterated maps on the interval as dynamical systems*. Springer
310 Science & Business Media.
- 311 Comas-Bru, L., and F. McDermott, 2014: Impacts of the ea and sca patterns on the european
312 twentieth century nao–winter climate relationship. *Quarterly Journal of the Royal Meteorolog-*
313 *ical Society*, **140 (679)**, 354–363.
- 314 Compo, G. P., and Coauthors, 2011: The twentieth century reanalysis project. *Quarterly Journal*
315 *of the Royal Meteorological Society*, **137 (654)**, 1–28.
- 316 Cram, T. A., and Coauthors, 2015: The international surface pressure databank version 2. *Geo-*
317 *science Data Journal*, **2 (1)**, 31–46, doi:10.1002/gdj3.25, URL [http://dx.doi.org/10.1002/gdj3.](http://dx.doi.org/10.1002/gdj3.25)
318 **25**.
- 319 Faranda, D., M. C. Alvarez-Castro, and P. Yiou, 2016a: Return times of hot and cold days via re-
320 currences and extreme value theory. *Climate Dynamics*, 1–13, doi:10.1007/s00382-016-3042-6.

- 321 Faranda, D., J. M. Freitas, V. Lucarini, G. Turchetti, and S. Vaienti, 2013: Extreme value statistics
322 for dynamical systems with noise. *Nonlinearity*, **26 (9)**, 2597.
- 323 Faranda, D., G. Masato, N. Moloney, Y. Sato, F. Daviaud, B. Dubrulle, and P. Yiou, 2016b: The
324 switching between zonal and blocked mid-latitude atmospheric circulation: a dynamical system
325 perspective. *Climate Dynamics*, **47 (5-6)**, 1587–1599.
- 326 Faranda, D., G. Messori, and P. Yiou, 2017: Dynamical proxies of north atlantic predictability and
327 extremes. *Scientific Reports*, **7**, 41 278.
- 328 Ferguson, C. R., and G. Villarini, 2012: Detecting inhomogeneities in the twentieth century reanal-
329 ysis over the central united states. *Journal of Geophysical Research: Atmospheres*, **117 (D5)**,
330 n/a–n/a, doi:10.1029/2011JD016988, URL <http://dx.doi.org/10.1029/2011JD016988>, d05123.
- 331 Ferguson, C. R., and G. Villarini, 2014: An evaluation of the statistical homogeneity of
332 the twentieth century reanalysis. *Climate Dynamics*, **42 (11)**, 2841–2866, doi:10.1007/
333 s00382-013-1996-1, URL <http://dx.doi.org/10.1007/s00382-013-1996-1>.
- 334 Field, C. B., 2012: *Managing the risks of extreme events and disasters to advance climate change*
335 *adaptation: special report of the intergovernmental panel on climate change*. Cambridge Uni-
336 versity Press.
- 337 Freitas, A. C. M., J. M. Freitas, and M. Todd, 2010: Hitting time statistics and extreme value
338 theory. *Probability Theory and Related Fields*, **147 (3-4)**, 675–710.
- 339 Freitas, A. C. M., J. M. Freitas, and M. Todd, 2012: The extremal index, hitting time statistics and
340 periodicity. *Advances in Mathematics*, **231 (5)**, 2626–2665.
- 341 Hurrell, J. W., 1995: Decadal trends in the north atlantic oscillation: Regional tem-
342 peratures and precipitation. *Science*, **269 (5224)**, 676–679, doi:10.1126/science.269.5224.

343 676, URL <http://science.sciencemag.org/content/269/5224/676>, [http://science.sciencemag.org/](http://science.sciencemag.org/content/269/5224/676.full.pdf)
344 [content/269/5224/676.full.pdf](http://science.sciencemag.org/content/269/5224/676.full.pdf).

345 Hurrell, J. W., Y. Kushnir, and M. Visbeck, 2001: The north atlantic oscillation. *Science*,
346 **291 (5504)**, 603–605.

347 Kay, J., and Coauthors, 2015: The community earth system model (cesm) large ensemble project:
348 A community resource for studying climate change in the presence of internal climate variabil-
349 ity. *Bulletin of the American Meteorological Society*, **96 (8)**, 1333–1349.

350 Krueger, O., F. Schenk, F. Feser, and R. Weisse, 2013: Inconsistencies between long-term trends
351 in storminess derived from the 20cr reanalysis and observations. *Journal of Climate*, **26 (3)**,
352 868–874, doi:10.1175/JCLI-D-12-00309.1.

353 Lorenz, E. N., 1963: Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, **20 (2)**,
354 130–141.

355 Lucarini, V., D. Faranda, G. Turchetti, and S. Vaienti, 2012: Extreme value theory for singular
356 measures. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, **22 (2)**, 023 135.

357 Lucarini, V., and Coauthors, 2016: *Extremes and recurrence in dynamical systems*. John Wiley &
358 Sons.

359 Lyu, K., X. Zhang, J. A. Church, and J. Hu, 2015: Quantifying internally generated and ex-
360 ternally forced climate signals at regional scales in cmip5 models. *Geophysical Research*
361 *Letters*, **42 (21)**, 9394–9403, doi:10.1002/2015GL065508, URL [http://dx.doi.org/10.1002/](http://dx.doi.org/10.1002/2015GL065508)
362 [2015GL065508](http://dx.doi.org/10.1002/2015GL065508), 2015GL065508.

363 Messori, G., R. Caballero, and D. Faranda, 2017: A dynamical systems approach to studying
364 mid-latitude weather extremes. *Submitted: Geophysical Research Letters*.

- 365 Moore, G., I. A. Renfrew, and R. S. Pickart, 2013: Multidecadal mobility of the north atlantic
366 oscillation. *Journal of Climate*, **26** (8), 2453–2466.
- 367 Paeth, H., A. Hense, R. Glowienka-Hense, S. Voss, and U. Cubasch, 1999: The north at-
368 lantic oscillation as an indicator for greenhouse-gas induced regional climate change. *Cli-
369 mate Dynamics*, **15** (12), 953–960, doi:10.1007/s003820050324, URL [http://dx.doi.org/10.
370 1007/s003820050324](http://dx.doi.org/10.1007/s003820050324).
- 371 Pickands III, J., 1975: Statistical inference using extreme order statistics. *the Annals of Statistics*,
372 119–131.
- 373 Robin, Y., P. Yiou, and P. Naveau, 2017: Detecting changes in forced climate attractors with
374 wasserstein distance. *Nonlinear Processes in Geophysics Discussions*, **2017**, 1–19, doi:10.5194/
375 npg-2017-5, URL <http://www.nonlin-processes-geophys-discuss.net/npg-2017-5/>.
- 376 Rogers, J. C., 1997: North atlantic storm track variability and its association to the north atlantic
377 oscillation and climate variability of northern europe. *Journal of Climate*, **10** (7), 1635–1647,
378 doi:10.1175/1520-0442(1997)010<1635:NASTVA>2.0.CO;2.
- 379 Schubert, S., and V. Lucarini, 2015: Covariant lyapunov vectors of a quasi-geostrophic baroclinic
380 model: analysis of instabilities and feedbacks. *Quarterly Journal of the Royal Meteorological
381 Society*, **141** (693), 3040–3055.
- 382 Shepherd, T. G., 2014: Atmospheric circulation as a source of uncertainty in climate change
383 projections. *Nature Geoscience*, **7** (10), 703–708.
- 384 Taylor, K. E., R. J. Stouffer, and G. A. Meehl, 2012: An overview of CMIP5 and the experiment
385 design. *Bulletin of the American Meteorological Society*, **93** (4), 485–498.
- 386 Villani, C., 2008: *Optimal transport: old and new*, Vol. 338. Springer Science & Business Media.

³⁸⁷ Von Storch, H., 1999: Misuses of statistical analysis in climate research. *Analysis of Climate*
³⁸⁸ *Variability*, Springer, 11–26.

389 **LIST OF TABLES**

390 **Table 1.** List of CMIP5 Models Analysed and 20CR reanalysis from 1851 to 2001.
 391 Models are ordered by increasing in resolution. 20

392 **Table 2.** Two sample Kolmogorov Smirnov (KS) tests results and p-values for the dis-
 393 tribution of distances between realizations of the Lorenz system for two dif-
 394 ferent metrics: (\mathcal{W}_2) Wasserstein distance between full d, θ distributions, (m)
 395 distances between the medians of the d, θ distributions. The null hypothesis is
 396 that data in $\Lambda(1)$ and $\Lambda(2)$ are from the same continuous distribution, with a
 397 significance level of 95%. The null hypothesis is always rejected. 21

398 **Table 3.** List of median values for dimension d and inverse of persistence time θ ,
 399 distances $\delta(d)$ and $\delta(\theta)$ from the median of the 20CR , relative distances
 400 $R(d) = \delta(d)/\max \delta(d)$ and $R(\theta) = \delta(\theta)/\max \delta(\theta)$ with respect to the far-
 401 thest model and global score $R_{tot} = (R(d) + R(\theta))/2$ 22

402 TABLE 1. List of CMIP5 Models Analysed and 20CR reanalysis from 1851 to 2001. Models are ordered by
 403 increasing in resolution.

No. ¹	Model	Institution/ID	Country	Resolution ²
1	20CRv2c	NOAA-CIRES	USA	2 x 2
2	CMCC-CESM	Centro Euro-Mediterraneo sui Cambiamenti Climatici	Italy	3,75x3,75
3	CanESM2	Canadian Centre for Climate Modelling and Analysis, CCCMa	Canada	2,81x 2,79
4	MIROC-ESM-CHEM	MIROC ⁴	Japan	2,81x 2,79
5	MIROC-ESM	MIROC ⁴	Japan	2,81x 2,79
6	BCC-CSM1-1	Beijing Climate Center	China	2,81x 2,79
7	IPSL-CM5B-LR	Institute Pierre Simon Laplace, IPSL	France	3,75x1,89
8	NorESM1-M	Norwegian Climate Center	Norway	2,5x1,89
9	FGOALS-2	Institute of Atmospheric Physics, Chinese Academy of Sciences	China	2,81x2,81
10	MPI-ESM-P	Max Planck Institute for Meteorology, MPI	Germany	1,87x1,87
11	MPI-ESM-LR	Max Planck Institute for Meteorology, MPI	Germany	1,87x1,87
12	CSIRO-MK3-6-0	CSIRO-BOM ⁵	Australia	1,87x1,87
13	CMCC-CMS	Centro Euro-Mediterraneo sui Cambiamenti Climatici	Italy	1,87x1,87
14	MPI-ESM-MR	Max Planck Institute for Meteorology, MPI	Germany	1,87x1,87
15	IPSL-CM5A-MR	Institute Pierre Simon Laplace, IPSL	France	2,5x1,26
16	INM-CM4	Institute for Numerical Mathematics, INM	Russia	2x1,5
17	ACCESS 1-0	CSIRO-BOM ⁵	Australia	1,87x1,25
18	MIROC5	MIROC ⁴	Japan	1,40x1,40
19	CNRM-CM5	CNRM-CERFACS ³	France	1,40x1,40
20	MRI-ESM1	Meteorological Research Institute, MRI	Japan	1,125x1,125
21	BCC-CSM1-M	Beijing Climate Center	China	1,125x1,125
22	MRI-CGCM3	Meteorological Research Institute, MRI	Japan	1,125x1,125
23	EC-EARTH	Danish Meteorological Institute, DMI	Denmark	1,125x1,125
24	CESM1-FASTCHEM	Community Earth System Model Contributors, NCAR	USA	1,25x0,94
25	CESM1-CAM5	Community Earth System Model Contributors, NCAR	USA	1,25x0,94
26	CESM1-BGC	Community Earth System Model Contributors, NCAR	USA	1,25x0,94
27	CCSM4	National Center for Atmospheric Research, NCAR	USA	1,25x0,94

¹ Order by horizontal resolution (Decreasing)

² Longitude x Latitude (°)

³ Centre National de Recherches Meteorologiques - Centre Europeen de Recherche et de Formation Avancee en Calcul Scientifique

⁴ Atmosphere and Ocean Research Institute (University of Tokyo), National Institute for Environmental Studies, and Japan Agency for Marine-Earth Science and Technology

⁵ Commonwealth Scientific and Industrial Research Organisation(CSIRO), Bureau of Meteorology(BOM)

404 TABLE 2. Two sample Kolmogorov Smirnov (KS) tests results and p-values for the distribution of distances
 405 between realizations of the Lorenz system for two different metrics: (\mathcal{W}_2) Wasserstein distance between full d, θ
 406 distributions, (m) distances between the medians of the d, θ distributions. The null hypothesis is that data in
 407 $\Lambda(1)$ and $\Lambda(2)$ are from the same continuous distribution, with a significance level of 95%. The null hypothesis
 408 is always rejected.

	$\Lambda(\sigma_{28})/\Lambda(\sigma_{28/28.5})$	$\Lambda(\sigma_{28.5})/\Lambda(\sigma_{28/28.5})$
KS(\mathcal{W}_2)	0.16	0.22
p-value(\mathcal{W}_2)	$5 \cdot 10^{-10}$	$2 \cdot 10^{-20}$
KS(m)	0.29	0.4
p-value(m)	$3 \cdot 10^{-33}$	$1 \cdot 10^{-63}$

409 TABLE 3. List of median values for dimension d and inverse of persistence time θ , distances $\delta(d)$ and $\delta(\theta)$
410 from the median of the 20CR, relative distances $R(d) = \delta(d)/\max \delta(d)$ and $R(\theta) = \delta(\theta)/\max \delta(\theta)$ with
411 respect to the farthest model and global score $R_{tot} = (R(d) + R(\theta))/2$.

N.	Model	median(d)	$\delta(d)$	$R(d)$	median(θ)	$\delta(\theta)$	$R(\theta)$	R_{tot}
1	20CRv2c	11,56	–	–	0,5	–	–	–
2	CMCC-CESM	12,22	0,67	0,54	0,51	0,01	0,13	0,33
3	CanESM2	11,99	0,43	0,35	0,51	0	0,02	0,19
4	MIROC-ESM-CHEM	12,54	0,98	0,8	0,47	0,04	0,75	0,77
5	MIROC-ESM	12,48	0,92	0,75	0,47	0,04	0,76	0,76
6	BCC-CSM1	12,12	0,57	0,46	0,51	0,01	0,12	0,29
7	IPSL-CM5B	12,73	1,17	0,95	0,46	0,05	0,93	0,94
8	NorESM1-M	12,12	0,56	0,46	0,48	0,02	0,44	0,45
9	FGOALS-S2	11,63	0,07	0,06	0,45	0,05	1,00	0,53
10	MPI-ESM-P	12,17	0,61	0,5	0,51	0	0,06	0,28
11	MPI-ESM-LR	12,13	0,58	0,47	0,51	0,01	0,14	0,3
12	CSIRO-MK3-6-0	12,66	1,11	0,9	0,5	0	0,02	0,46
13	CMCC-CMS	11,95	0,39	0,32	0,52	0,01	0,22	0,27
14	MPI-ESM-MR	12,09	0,53	0,43	0,51	0	0,09	0,26
15	IPSL-CM5A	11,86	0,31	0,25	0,48	0,03	0,51	0,38
16	INM-CM4	12,79	1,23	1	0,47	0,04	0,70	0,85
17	ACCESS-1-0	11,74	0,19	0,15	0,49	0,02	0,31	0,23
18	MIROC5	12,58	1,02	0,83	0,49	0,02	0,33	0,58
19	CNRM-CM5	12,36	0,8	0,65	0,47	0,03	0,59	0,62
20	MRI-ESM1	11,72	0,16	0,13	0,51	0,01	0,12	0,13
21	BCC-CSM1-M	11,45	0,11	0,09	0,55	0,05	0,91	0,5
22	MRI-CGCM3	11,74	0,18	0,15	0,51	0	0,07	0,11
23	EC-EARTH	11,87	0,32	0,26	0,5	0	0,01	0,13
24	CESM1-FASTCHEM	11,56	0	0	0,51	0,01	0,18	0,09
25	CESM1-CAM5	11,88	0,32	0,26	0,51	0,01	0,13	0,2
26	CESM1-BGC	11,53	0,02	0,02	0,51	0,01	0,11	0,06
27	CCSM4	11,57	0,02	0,01	0,51	0,01	0,12	0,07

412 **LIST OF FIGURES**

413 **Fig. 1.** Two realizations of the Lorenz attractor. Blue: classic attractor $\Delta t \simeq 0.035$, $\sigma = 28$, $r = 10$,
414 $b = 8/3$; Violet: $\sigma = 28.5$ 25

415 **Fig. 2.** Schematic representation of the dynamical indicators: the local dimension d is proportional
416 to the number of possible configurations originating that of the day analyzed and resulting
417 from it. θ , is the inverse of the persistence time of a certain configuration. 26

418 **Fig. 3.** Top: Medians of d and θ for 30 realizations of the Lorenz attractor with $\sigma = 28$ (blue) and
419 $\sigma = 28.5$ (red); the crosses display the median of the ensemble of realizations. Bottom:
420 Empirical probability density functions (pdf) of the pairwise distances among d and θ
421 medians in $\sigma = 28$ realizations ($\Lambda(\sigma_{28})$), $\sigma = 28.5$ realizations ($\Lambda(\sigma_{28})$) and mixed realizations
422 ($\Lambda(\sigma_{28}/\sigma_{28.5})$). 27

423 **Fig. 4.** The scatter plot displays the daily values of the instantaneous dimension d and the persis-
424 tence θ of the field. The straight black lines mark the 0.02 and 0.98 quantiles of d and θ .
425 The composite anomalies in SLP for the four regions delimited by the black lines are plotted
426 as side panels and can be associated with known weather regimes: Atlantic Ridge (maxima
427 of θ), NAO- (minima of θ), Blocking (maxima of d), NAO+ (minima of d). The black lines
428 indicates regions where at least the 2/3 of the composite members display the same sign. . . . 28

429 **Fig. 5.** (d, θ) bivariate histograms (left) and scatter-plots (right) for the 20 CR (top), the CMCC-
430 CMS (center) and the IPSL-CM5A models(bottom). The color-scales on the left indicate
431 the frequency of observations in number of days. The colorscales on the right indicate the
432 month of the observation and show the dependence of the (d, θ) diagrams on the seasonal
433 cycle. 29

434 **Fig. 6.** Comparison between R_{tot} values (blue) and Wasserstein distances \mathcal{W}_2 (red) computed with
435 the procedure described in Robin et al. (2017) between the (d, θ) 20CR and CMIP5 distri-
436 butions. 30

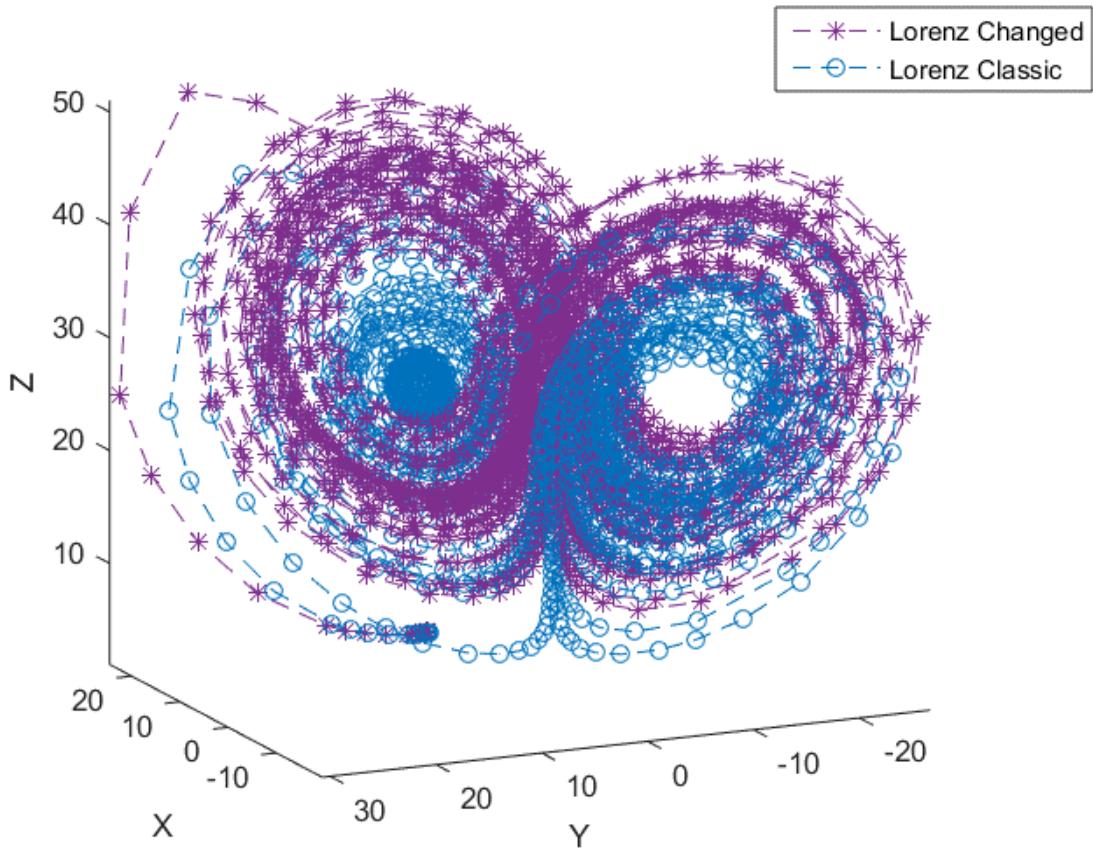
437 **Fig. 7.** Comparison between 20 CR median values of (d, θ) (denoted by 1) and all the other CMIP5
438 models (progressive numbers 2-27, see table 1 for the details). The semiaxes of each ellipse
439 represent one standard deviation of d and θ . Models 07 and 09 are highlighted by arrows
440 and a grey background to increase their visibility. 31

441 **Fig. 8.** Comparison between 20CR and the best 3 models (according to the metric R_{tot} , a measure
442 of distance from the median values of d, θ) of the composite anomalies in SLP for the four
443 regions of the (d, θ) diagram, highlighted in Fig. ???. The black lines indicate regions where
444 at least the 2/3 of the composite members display the same sign. 32

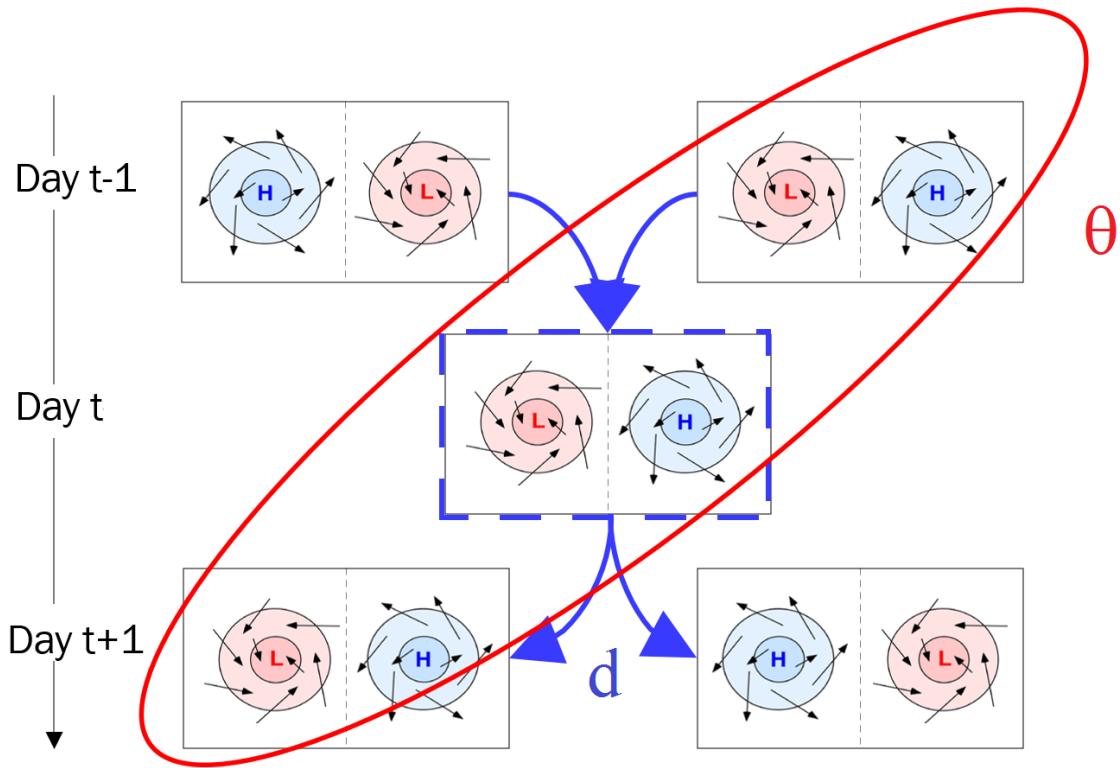
445 **Fig. 9.** Comparison between 20CR and the worst 3 models (according to the metric R_{tot} , a measure
446 of distance from the median values of d, θ) of the composite anomalies in SLP for the four
447 regions of the (d, θ) diagram, highlighted in Fig. ???. The black lines indicate regions where
448 at least the 2/3 of the composite members display the same sign. 33

449 **Fig. 10.** Bivariate histograms of (d, θ) for 20CR (left), the model CCSM4 (centre) and their differ-
450 ence Δ (right) for three periods: 1851-1900 (top), 1901-1950 (centre), 1951-2000 (bottom).
451 The violet lines indicate the median values. The colorbars represent the frequency of joint
452 d, θ observations in number of days. 34

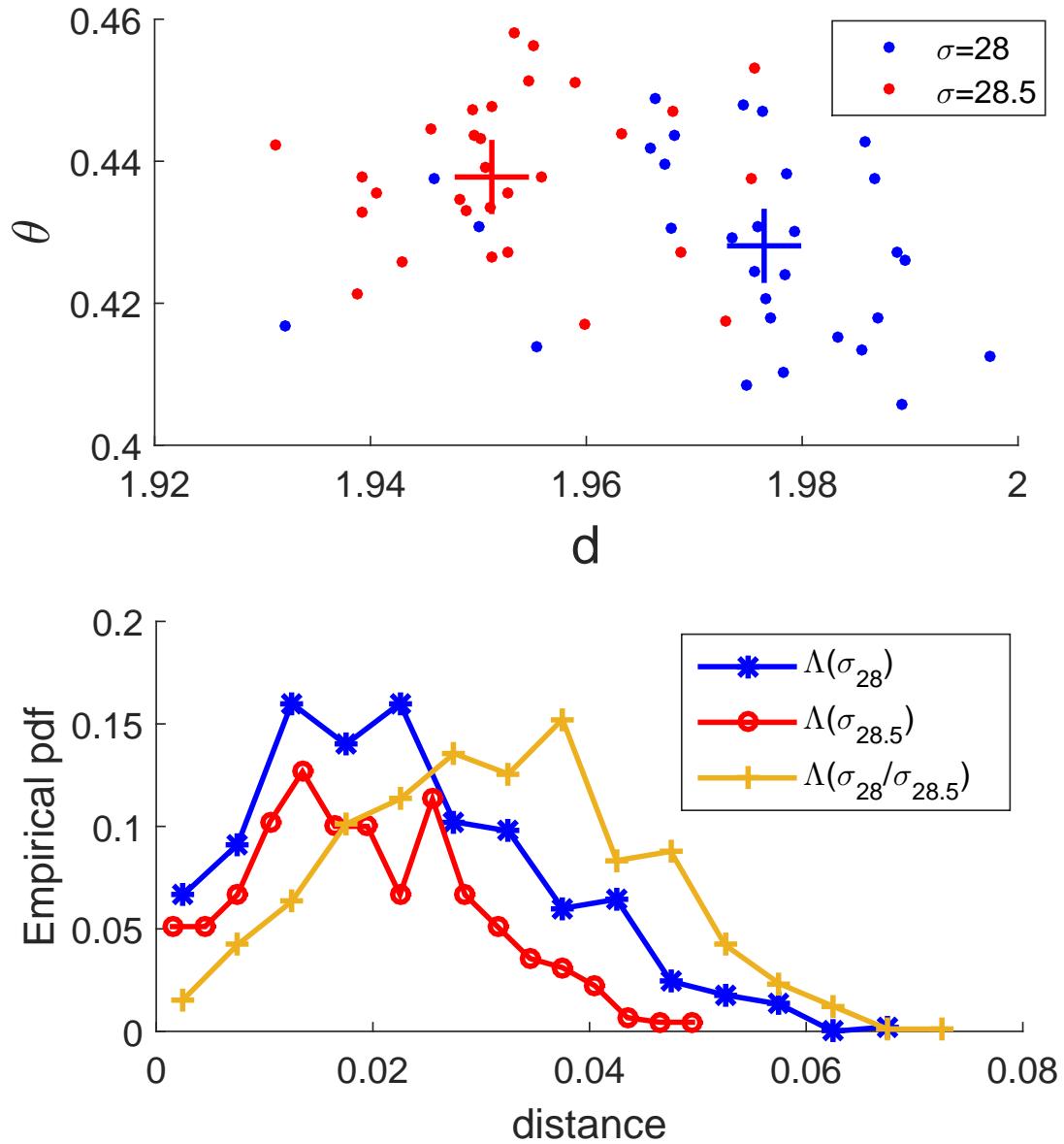
453 **Fig. 11.** Change in the number of extreme days, computed using the 0.98 and 0.02 quantiles of the
454 full time series for each model, for three different sub-periods. From top to bottom different
455 regimes: Atlantic Ridge (maxima of θ), NAO- (minima of θ), Blocking (maxima of d),
456 NAO+ (minima of d) 35



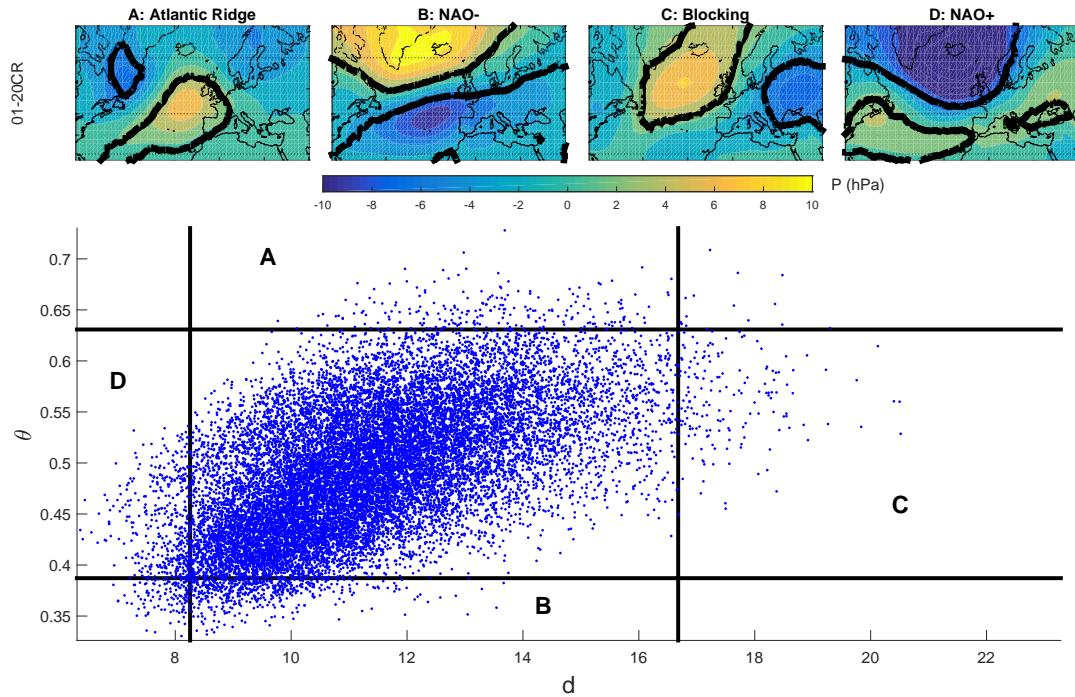
457 FIG. 1. Two realizations of the Lorenz attractor. Blue: classic attractor $\Delta t \simeq 0.035$, $\sigma = 28$, $r = 10$, $b = 8/3$;
 458 Violet: $\sigma = 28.5$



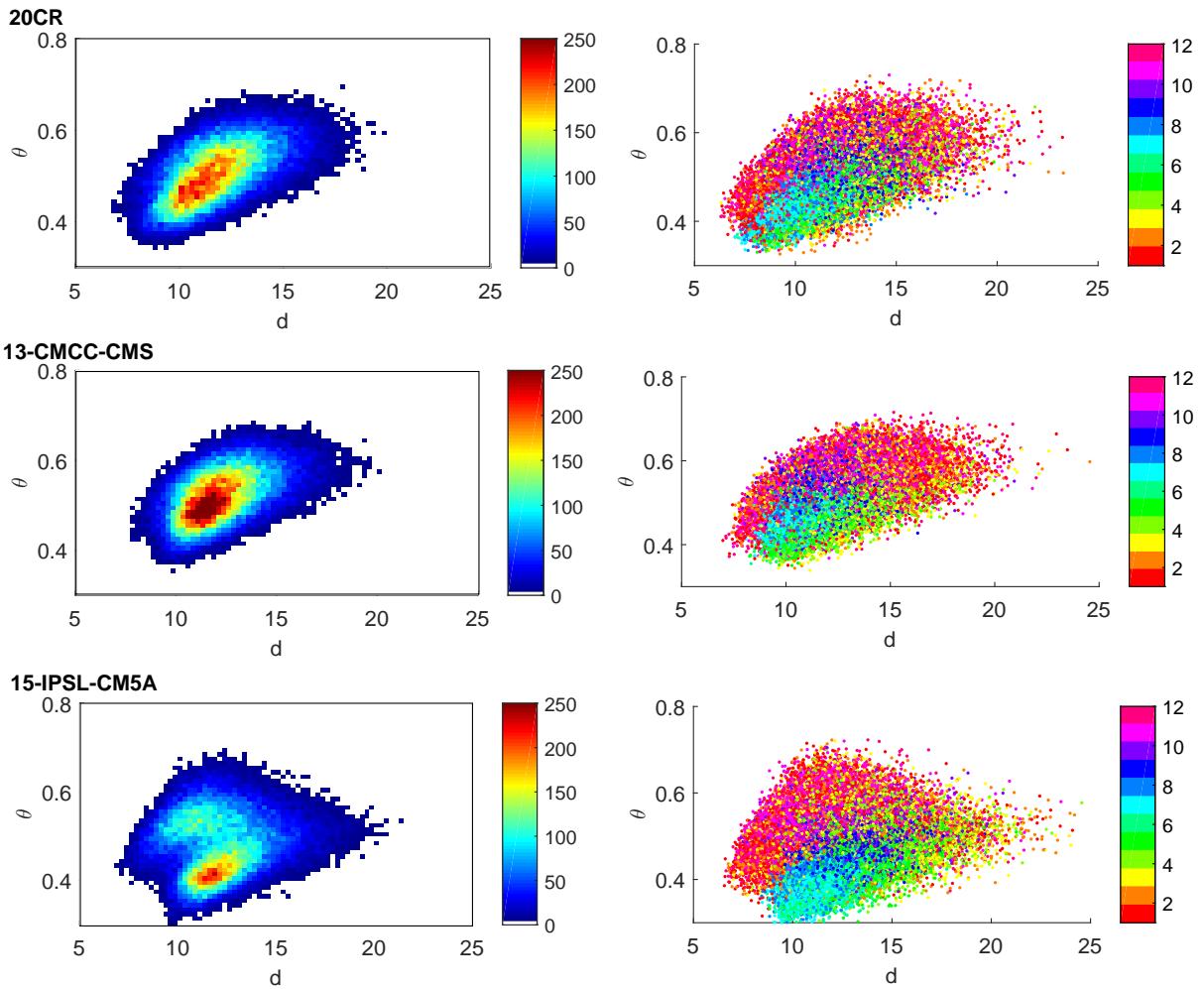
459 FIG. 2. Schematic representation of the dynamical indicators: the local dimension d is proportional to the
 460 number of possible configurations originating that of the day analyzed and resulting from it. θ , is the inverse of
 461 the persistence time of a certain configuration.



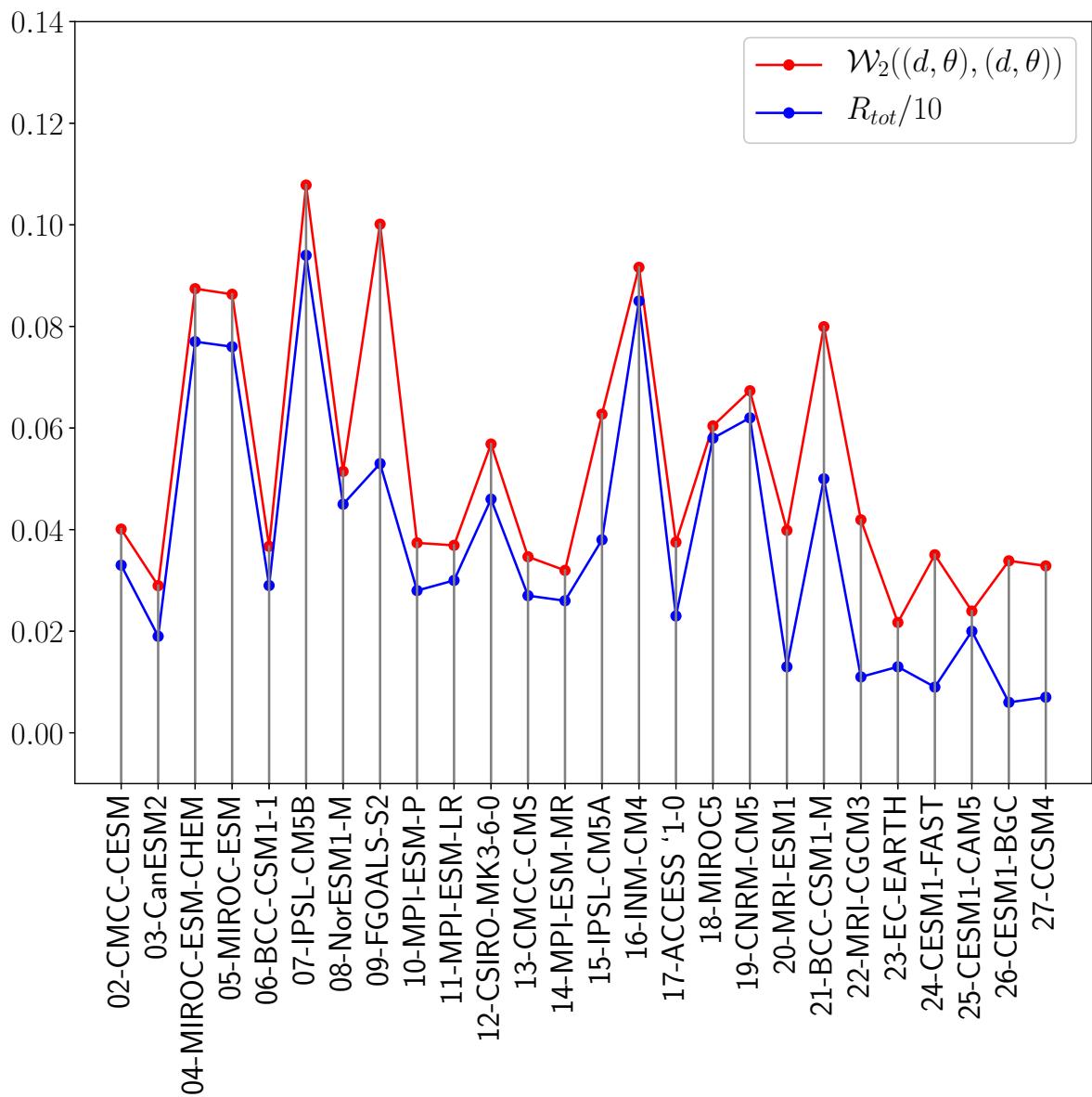
462 FIG. 3. Top: Medians of d and θ for 30 realizations of the Lorenz attractor with $\sigma = 28$ (blue) and $\sigma = 28.5$
 463 (red); the crosses display the median of the ensemble of realizations. Bottom: Empirical probability density
 464 functions (pdf) of the pairwise distances among d and θ medians in $\sigma = 28$ realizations ($\Lambda(\sigma_{28})$), $\sigma = 28.5$
 465 realizations ($\Lambda(\sigma_{28})$) and mixed realizations ($\Lambda(\sigma_{28}/\sigma_{28.5})$).



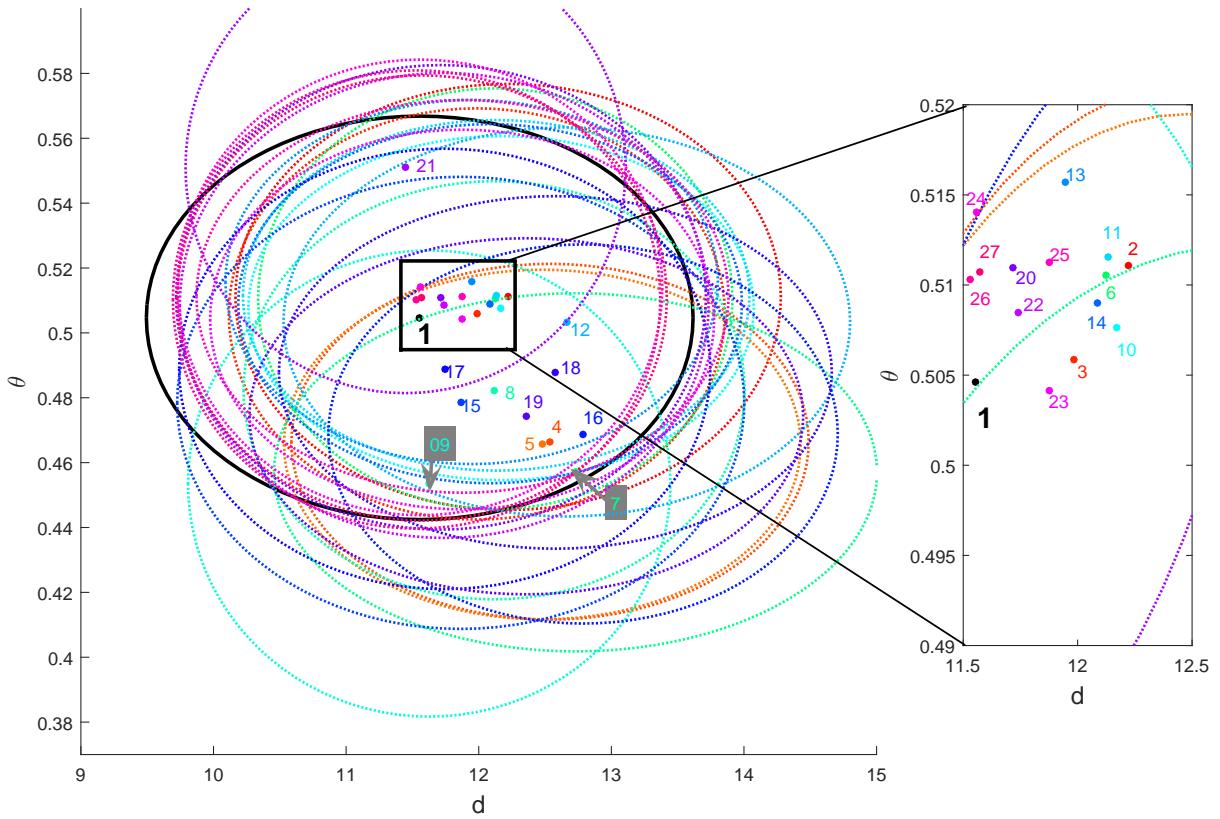
466 FIG. 4. The scatter plot displays the daily values of the instantaneous dimension d and the persistence θ of
 467 the field. The straight black lines mark the 0.02 and 0.98 quantiles of d and θ . The composite anomalies in SLP
 468 for the four regions delimited by the black lines are plotted as side panels and can be associated with known
 469 weather regimes: Atlantic Ridge (maxima of θ), NAO- (minima of θ), Blocking (maxima of d), NAO+ (minima
 470 of d). The black lines indicates regions where at least the 2/3 of the composite members display the same sign.



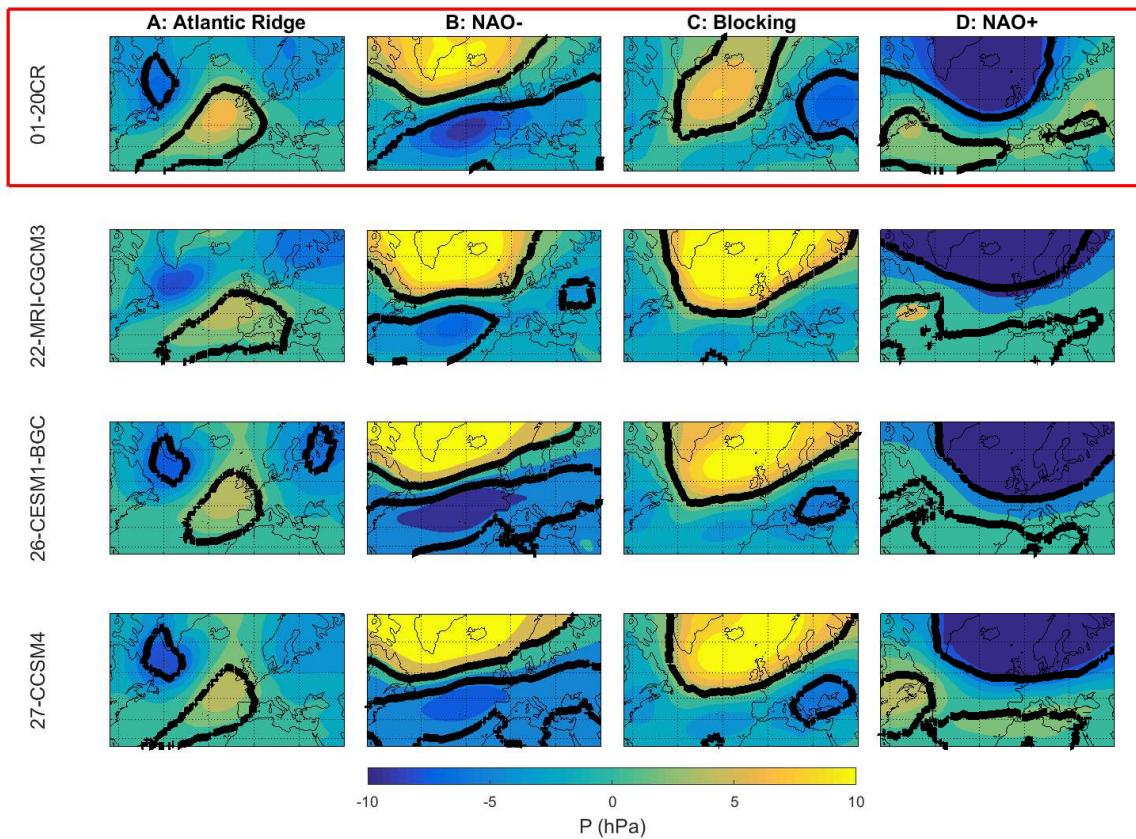
471 FIG. 5. (d, θ) bivariate histograms (left) and scatter-plots (right) for the 20 CR (top), the CMCC-CMS (center)
 472 and the IPSL-CM5A models(bottom). The color-scales on the left indicate the frequency of observations in
 473 number of days. The colorscales on the right indicate the month of the observation and show the dependence of
 474 the (d, θ) diagrams on the seasonal cycle.



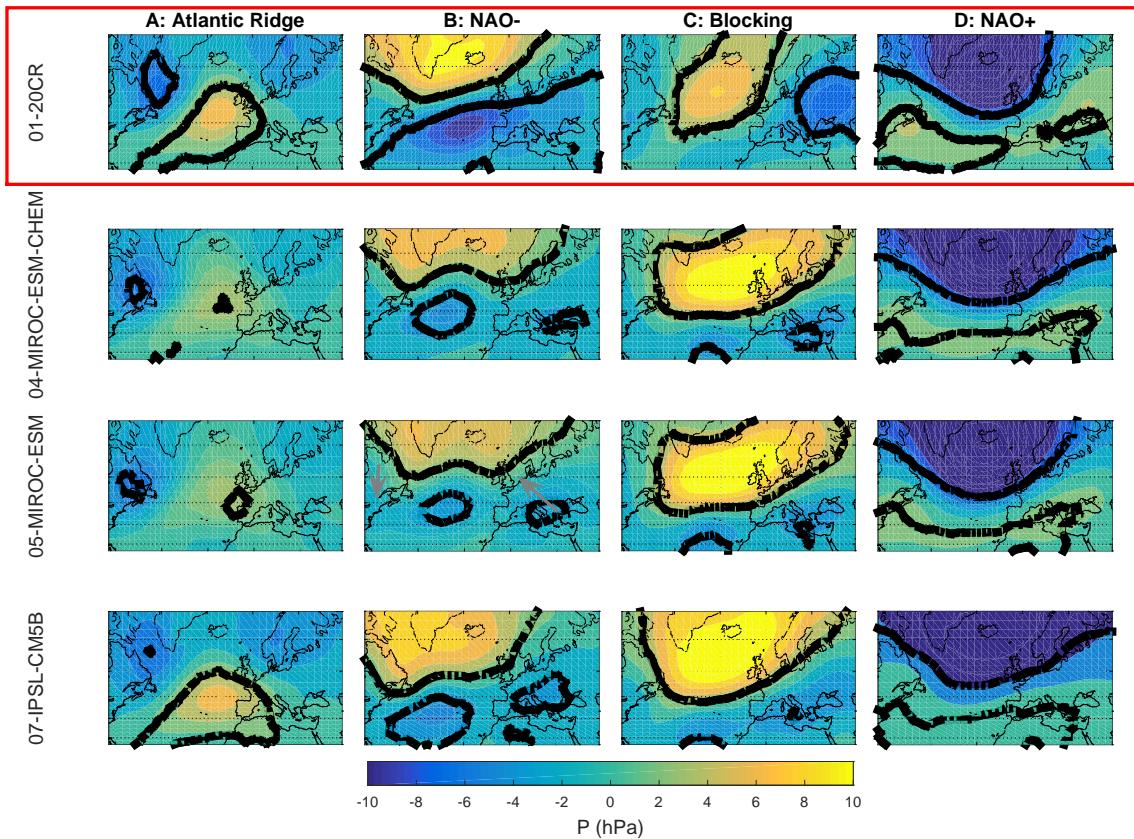
475 FIG. 6. Comparison between R_{tot} values (blue) and Wasserstein distances \mathcal{W}_2 (red) computed with the proce-
 476 dure described in Robin et al. (2017) between the (d, θ) 20CR and CMIP5 distributions.



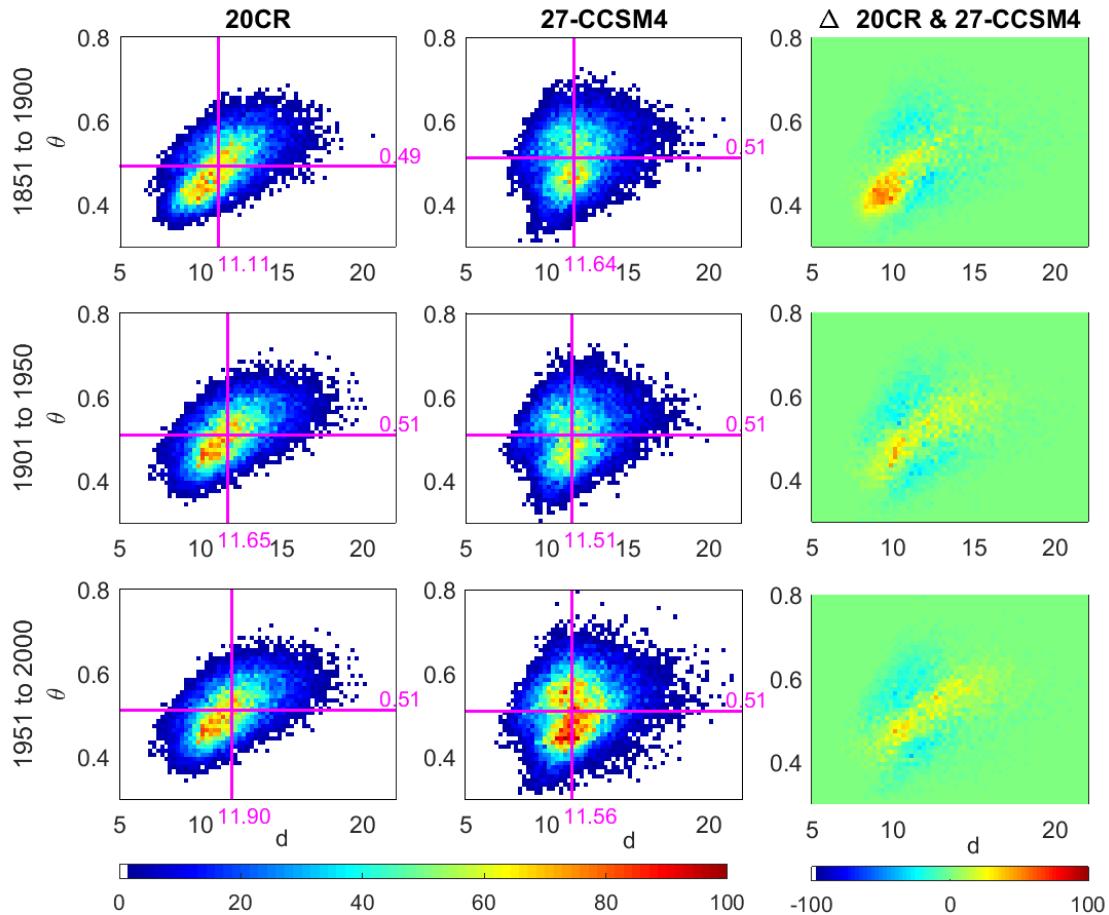
477 FIG. 7. Comparison between 20 CR median values of (d, θ) (denoted by 1) and all the other CMIP5 models
 478 (progressive numbers 2-27, see table 1 for the details). The semiaxes of each ellipse represent one standard
 479 deviation of d and θ . Models 07 and 09 are highlighted by arrows and a grey background to increase their
 480 visibility.



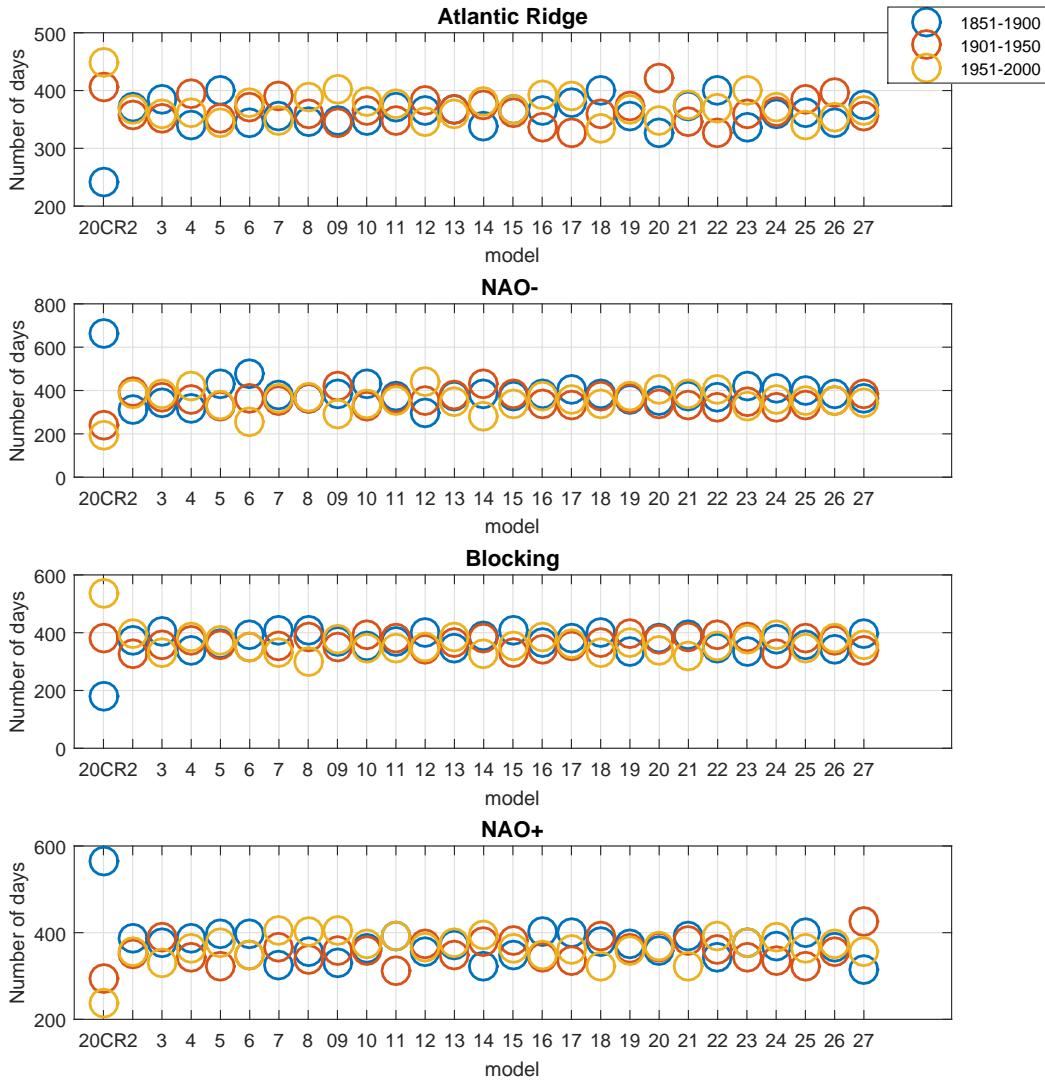
481 FIG. 8. Comparison between 20CR and the best 3 models (according to the metric R_{tot} , a measure of distance
 482 from the median values of d, θ) of the composite anomalies in SLP for the four regions of the (d, θ) diagram,
 483 highlighted in Fig. 4. The black lines indicate regions where at least the $2/3$ of the composite members display
 484 the same sign.



485 FIG. 9. Comparison between 20CR and the worst 3 models (according to the metric R_{tot} , a measure of distance
 486 from the median values of d, θ) of the composite anomalies in SLP for the four regions of the (d, θ) diagram,
 487 highlighted in Fig. 4. The black lines indicate regions where at least the $2/3$ of the composite members display
 488 the same sign.



489 FIG. 10. Bivariate histograms of (d, θ) for 20CR (left), the model CCSM4 (centre) and their difference Δ
 490 (right) for three periods: 1851-1900 (top), 1901-1950 (centre), 1951-2000 (bottom). The violet lines indicate
 491 the median values. The colorbars represent the frequency of joint d, θ observations in number of days.



492 FIG. 11. Change in the number of extreme days, computed using the 0.98 and 0.02 quantiles of the full time
 493 series for each model, for three different sub-periods. From top to bottom different regimes: Atlantic Ridge
 494 (maxima of θ), NAO- (minima of θ), Blocking (maxima of d), NAO+ (minima of d)