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# Aircraft Attitude Estimation Based on Central Difference Kalman Filter

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**Abstract**—When the extended Kalman filter (EKF) is applied in the aircraft attitude estimation, two defects exist: one is computational complexity; the other is large linearization error. Aiming at these problems, central difference Kalman filter (CDKF) based on Stirling interpolation formulation is applied to the low-cost aircraft attitude estimation system which is of less accurate and high noisy sensors. First, the nonlinear mathematical model of aircraft attitude based on quaternion is established, then CDKF is applied to attitude estimation. Experimental results with real flying data show that CDKF is superior to the commonly used EKF method and unscented Kalman filter (UKF). The algorithm not only improves the attitude estimation precision and stability effectively, but also avoids the computing burden of Jacobian matrices. In addition, it is more simple and easy to implement, because it has only one adjustable parameter instead of three in the UKF circumstances.

**Index Terms**—Attitude estimation; Quaternion; CDKF; UKF; EKF; SPKF

## I. INTRODUCTION

Aircraft attitude can be defined as the orientation relationship between the body frame and the earth frame, usually described by a set of Euler angles such as yaw, pitch and roll. It provides not only the pilot with the aircraft navigation information, but also a three-dimensional attitude reference for the autopilot, fire control system, the radar antenna and aerial camera and other airborne equipments. Therefore, it is a very important parameter for safety flying and in the above applications.

Aircraft attitude is usually measured by the strapdown inertial navigation system [1]. First, the nonlinear mathematical model of aircraft attitude based on quaternion is established, then the appropriate filter is utilized to estimate the attitude. For the nonlinear aircraft attitude estimation problem, extended Kalman filter (EKF) is probably the most widely used at present. However, it is only reliable for systems which are almost linear and the Jacobian matrices are hard to obtain. In fact, these difficulties arise from its use of linearization [2]. To address this issue, unscented Kalman filter (UKF) was proposed by Julier and Uhlmann [3]. UKF can be applied directly to nonlinear systems, and it uses a number of deterministic sampling points to capture the posterior mean and covariance of the pertinent Gaussian approximate densities, which can be accurate up to the second order of any nonlinearity [4]. However, three scalar scaling parameters

$(\alpha, \beta, \kappa)$  are employed by the UKF. These parameters are always selected based on the nonlinearities of the system models [5], and the choice of the parameters' values will affect the filter's estimate precision.

In this paper, Stirling interpolation in the state estimator is employed for aircraft attitude estimation problems. Stirling interpolation replaces the unscented transform in the UKF algorithm architecture. As proved in [5], [6], Stirling interpolation based central difference Kalman filter (CDKF) has the same or superior performance as the unscented transform based UKF, with one advantage over UKF: The CDKF uses only a single scalar scaling parameter, the central difference interval size  $h$ , as opposed to the three  $(\alpha, \beta, \kappa)$  that the UKF uses [3]. As shown in our experiment, the CDKF algorithm can get better performance on the estimate accuracy and error robustness.

The paper is organized as follows. The attitude estimation algorithm based on the CDKF is given in Section II. Section III gives the relationship between quaternion and kinematics, the gyroscope model and establishes the nonlinear system model of aircraft attitude. Section IV presents the experimental results of the EKF, UKF and CDKF, and their estimate performances are compared. Finally, Section V summarizes this paper.

## II. CENTRAL DIFFERENCE KALMAN FILTER

The CDKF is the core member of the sigma-point Kalman filters (SPKF) family of algorithms based on the sigma-point approach [5]. It uses a symmetric set of sigma points derived by Stirling interpolation to approximate nonlinear functions, whereas the EKF uses the Taylor series. The advantage of the Stirling interpolation is that the calculation of Jacobian matrices is not required, and can be accurate up to the second order of any nonlinearity. In case of Gaussian distributions of the system variables, the mean and covariance can be represented by these sigma points. More details about Stirling polynomial interpolation method for approximating nonlinear models can be found in [7].

Consider the nonlinear dynamic system composed of state equation and observation equation as follows.

$$\begin{cases} \mathbf{x}_k = F(\mathbf{x}_{k-1}) + \mathbf{w}_k \\ \mathbf{y}_k = H(\mathbf{x}_k) + \mathbf{v}_k \end{cases} \quad (1)$$

where  $\mathbf{x}_k \in \mathbf{R}^n$  is the state vector,  $\mathbf{y}_k \in \mathbf{R}^m$  is the observation vector,  $\mathbf{w}_k \in \mathbf{R}^n$  is the state noise vector, and  $\mathbf{v}_k \in \mathbf{R}^m$  is the measurement noise vector. It is assumed that  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are zero-mean Gaussian noise processes with covariances given by  $\mathbf{Q}_k$  and  $\mathbf{R}_k$ , respectively. The flow of the central difference Kalman filter is formulated as follows.

1) *Initialization:*

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0], \mathbf{P}_0 = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T] \quad (2)$$

2) *Calculate sigma-points for time-update:*

$$\chi_{k-1} = [\hat{\mathbf{x}}_{k-1} \quad \hat{\mathbf{x}}_{k-1} + h\sqrt{\mathbf{P}_{k-1}} \quad \hat{\mathbf{x}}_{k-1} - h\sqrt{\mathbf{P}_{k-1}}] \quad (3)$$

where  $h$  is the interval length, if the random variables obey a Gaussian distribution, the optimal value of  $h$  is  $\sqrt{3}$ .

3) *Time-update equations:*

These sigma points are further passed through the nonlinear function  $F(\bullet)$ , such that the predicted sigma points for the discrete time  $k$  are derived

$$\chi_{k/k-1}^* = F(\chi_{k-1}) \quad (4)$$

Finally, the first two moments of the predicted state vector are obtained by linear regression of the transformed sigma points

$$\hat{\mathbf{x}}_{k/k-1} = \sum_{i=0}^{2n} w_i^m \chi_{i,k/k-1}^* \quad (5)$$

$$\mathbf{P}_{k/k-1} = \sum_{i=1}^n [w_i^{c1} (\chi_{i,k/k-1}^* - \chi_{n+i,k/k-1}^*) (\chi_{i,k/k-1}^* - \chi_{n+i,k/k-1}^*)^T + w_i^{c2} (\chi_{i,k/k-1}^* + \chi_{n+i,k/k-1}^* - 2\chi_{0,k/k-1}^*) \times (\chi_{i,k/k-1}^* + \chi_{n+i,k/k-1}^* - 2\chi_{0,k/k-1}^*)^T] + \mathbf{Q}_{k-1} \quad (6)$$

where  $w_0^m = \frac{h^2-n}{h^2}, w_i^m = \frac{1}{2h^2} (i = 1, \dots, 2n), w_0^{c1} = \frac{1}{4h}, w_i^{c2} = \frac{h^2-1}{4h^4} (i = 1, \dots, n)$ .

4) *Calculate sigma-points for measurement-update:*

$$\chi_{k/k-1} = [\hat{\mathbf{x}}_{k/k-1} \quad \hat{\mathbf{x}}_{k/k-1} + h\sqrt{\mathbf{P}_{k/k-1}} \quad \hat{\mathbf{x}}_{k/k-1} - h\sqrt{\mathbf{P}_{k/k-1}}] \quad (7)$$

5) *Measurement-update equations:*

The predicted measurement points are obtained by transforming the sigma points through  $H(\bullet)$

$$\mathbf{y}_{k/k-1}^* = H(\chi_{k/k-1}) \quad (8)$$

Furthermore, the mean, covariance and cross-covariance are derived by

$$\mathbf{y}_{k/k-1} = \sum_{i=0}^{2n} w_i^m \mathbf{y}_{i,k/k-1}^* \quad (9)$$

$$\mathbf{P}_{y_k} = \sum_{i=1}^n [w_i^{c1} (\mathbf{y}_{i,k/k-1}^* - \mathbf{y}_{n+i,k/k-1}^*) (\mathbf{y}_{i,k/k-1}^* - \mathbf{y}_{n+i,k/k-1}^*)^T + w_i^{c2} (\mathbf{y}_{i,k/k-1}^* + \mathbf{y}_{n+i,k/k-1}^* - 2\mathbf{y}_{0,k/k-1}^*) \times (\mathbf{y}_{i,k/k-1}^* + \mathbf{y}_{n+i,k/k-1}^* - 2\mathbf{y}_{0,k/k-1}^*)^T] + \mathbf{R}_k \quad (10)$$



Fig. 1. Experimental aircraft equipped with the SBG and CHR sensor

$$\mathbf{P}_{x_k y_k} = \sqrt{w_i^{c1} \mathbf{P}_{k/k-1}} (\mathbf{y}_{1:n,k/k-1}^* - \mathbf{y}_{n+1:2n,k/k-1}^*)^T \quad (11)$$

Hence, the update of the estimated means and estimated error covariance at time  $k$  are given by:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k/k-1} + \mathbf{K}_k (\mathbf{y}_k - \mathbf{y}_{k/k-1}) \quad (12)$$

$$\mathbf{P}_k = \mathbf{P}_{k/k-1} - \mathbf{K}_k \mathbf{P}_{y_k} \mathbf{K}_k^T \quad (13)$$

where  $\mathbf{K}_k$  is the Kalman gain and can be defined as:

$$\mathbf{K}_k = \mathbf{P}_{x_k y_k} \mathbf{P}_{y_k}^{-1} \quad (14)$$

### III. AIRCRAFT ATTITUDE NONLINEAR MODEL

The CHR-6 dm AHRS sensor produced by the CH Robotics company in USA and the SBG AHRS sensor made by the French SBG System company are installed in the model plane which is applied for experimental flight, as shown in Fig. 1. Each of them contains a three-axis magnetometer, a three-axis angular rate sensor, and a three-axis accelerometer. The rate sensor, accelerometer and magnetometer measure angular rates, the gravity vector and local magnetic field vector respectively. The accelerometer and magnetometer are used for the rate gyro drift correction in order to improve the precision and stability of attitude estimation.

#### A. Quaternion and Kinematics

Quaternion [8] is widely used to describe the attitude of an aircraft, because it provides nonsingular attitude descriptions and expresses arbitrary and large rotations of the aircraft[9]. The quaternion is a four-dimensional vector defined as follows  $\mathbf{q}_u = [a \ b \ c \ d]^T$ . Note that the quaternion has the constraint

$$\mathbf{q}_u \mathbf{q}_u^T = \|\mathbf{q}_u\|^2 = a^2 + b^2 + c^2 + d^2 = 1 \quad (15)$$

The quaternion kinematic differential equation [10] can be derived as follows.

$$\dot{\mathbf{q}}_u = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}) \mathbf{q}_u \quad (16)$$

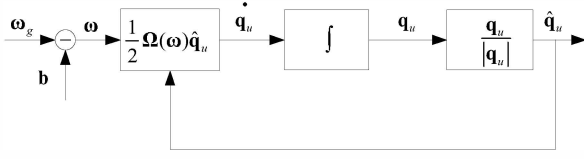


Fig. 2. Process Model for Angular Rates and Quaternion

where

$$\Omega(\omega) = \begin{pmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{pmatrix}$$

$\omega = [p \ q \ r]^T$ , and  $p, q$  and  $r$  are aircraft angular rate around  $x$  axis,  $y$  axis and  $z$  axis respectively in body frame.

To solve the equations of kinematic, we can get quaternion, then make use of the relationship between quaternion and attitude angle, we can obtain the yaw, pitch and roll angles as follows

$$\begin{cases} \varphi = \text{atan2}(2(ab + cd), 1 - 2(b^2 + c^2)) \\ \theta = \arcsin(2(ac - bd)) \\ \psi = \text{atan2}(2(ad + bc), 1 - 2(c^2 + d^2)) \end{cases} \quad (17)$$

where  $\varphi$ ,  $\theta$  and  $\psi$  are yaw, pitch and roll respectively.

### B. Sensor Modeling

In this study, a gyroscope is used as the attitude sensor of the aircraft. The gyroscope is a sensor that measures the angular rate of the aircraft. The gyroscope system can be expressed mathematically by modeling the measured angular velocity as the true angular velocity with an additive bias [11]. The bias dynamics are considered to be driven by a Gaussian white-noise process. The gyroscope model can be represented as

$$\omega = \omega_g - \mathbf{b}, \dot{\mathbf{b}}_u = \mathbf{n}_b \quad (18)$$

where  $\omega_g = [p_g \ q_g \ r_g]^T$  is the measured angular velocity,  $\omega = [p \ q \ r]^T$  is the true angular velocity,  $\mathbf{b}$  is the drift and  $\mathbf{b} = [b_p \ b_q \ b_r]^T$ ,  $\mathbf{n}_b = [n_{bp} \ n_{bq} \ n_{br}]^T$  is a zero-mean Gaussian white-noise process.

### C. State Equations

We choose the quaternion, angular rates and gyro biases as state vector, that is

$$\mathbf{x} = [\mathbf{q}_u^T \ \omega^T \ \mathbf{b}^T]^T = [a \ b \ c \ d \ p \ q \ r \ b_p \ b_q \ b_r]^T \quad (19)$$

The relationship among state vector is shown in Fig. 2.

We can get the nonlinear state equation from Fig. 2 as follows

$$\begin{cases} p = p_g - b_p \\ q = q_g - b_q \\ r = r_g - b_r \\ \dot{b}_p = n_{bp} \\ \dot{b}_q = n_{bq} \\ \dot{b}_r = n_{br} \\ \dot{\mathbf{q}}_u = \frac{1}{2} \Omega(\omega) \mathbf{q}_u \end{cases} \quad (20)$$

### D. Measurement Equation

We can get nine measurements which can be chosen as a measurement vector directly from the rate sensor, accelerometer and magnetometer. Therefore the measurement vector can be defined as

$$\mathbf{y} = [Acc_x \ Acc_y \ Acc_z \ Mag_x \ Mag_y \ Mag_z \ p_g \ q_g \ r_g] \quad (21)$$

where  $Acc_x, Acc_y$  and  $Acc_z$  are the gravity vector in the body frame, and  $Mag_x, Mag_y$  and  $Mag_z$  are the local magnetic field vector in the body frame.

According to the basic principle of strapdown inertial navigation, the relationship between what is measured in the body frame and the known values in the earth frame is:

$$\begin{pmatrix} Acc_x \\ Acc_y \\ Acc_z \\ Mag_x \\ Mag_y \\ Mag_z \\ p_g \\ q_g \\ r_g \end{pmatrix} = \begin{pmatrix} R_e^b & & \\ & R_e^b & \\ & & \mathbf{I}_{3 \times 3} \end{pmatrix} \begin{pmatrix} G_x \\ G_y \\ G_z \\ M_x \\ M_y \\ M_z \\ p_g \\ q_g \\ r_g \end{pmatrix} + \mathbf{n} \quad (22)$$

where

$$\mathbf{R}_e^b = \begin{pmatrix} a^2 + b^2 - c^2 - d^2 & 2bc + 2ad & 2bd - 2ac \\ 2bc - 2ad & a^2 - b^2 + c^2 - d^2 & 2cd + 2ab \\ 2bd + 2ac & 2cd - 2ab & a^2 - b^2 - c^2 + d^2 \end{pmatrix}$$

$[G_x \ G_y \ G_z]$  and  $[M_x \ M_y \ M_z]$  are the gravity vector and the magnetic field vector respectively whose values are known,  $\mathbf{I}_{3 \times 3}$  is a 3 order unit matrix,  $\mathbf{n}$  is the measurement noise vector which is a zero-mean Gaussian white-noise process.

## IV. EXPERIMENT RESULTS

In the experiment, the EKF algorithm which is internally integrated in the CHR-6 dm sensor and the SBG AHRS sensor combines data from onboard accelerometer, rate gyros, and magnetic sensors to produce yaw, pitch, and roll angle estimates, the untreated angular rates, gravity vector and magnetic field vector data. The performance, reliability and ease of use of the SBG AHRS sensor have been kept to the highest levels, but it is more expensive compared with CHR system. The price of CHR system is about 89 dollars [12], while SBG system is more than 2400 dollars. In order to verify CDKF algorithm's estimated effect, we utilize the CDKF method to treat the angular rates, gravity vector and magnetic field vector data obtained from the the CHR-6 dm sensor to get attitude estimation compared with the output of the SBG AHRS sensor. The

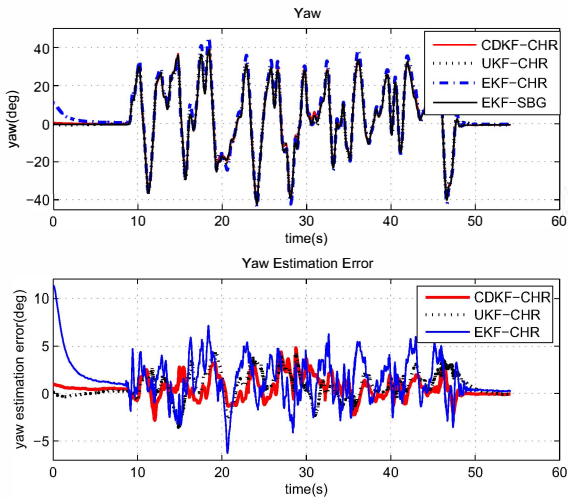


Fig. 3. Yaw and Yaw Estimation Error

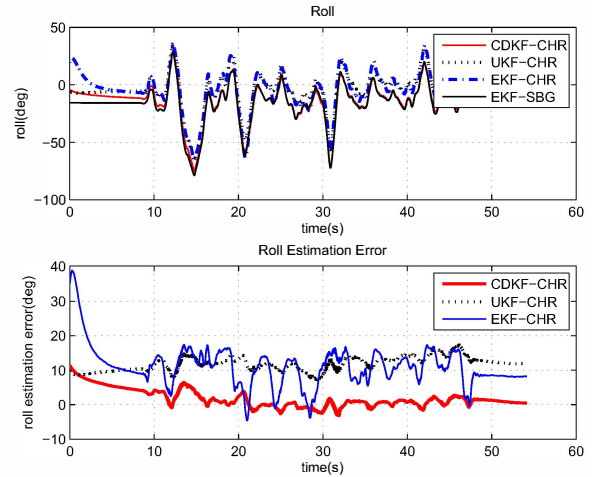


Fig. 5. Roll and Roll Estimation Error

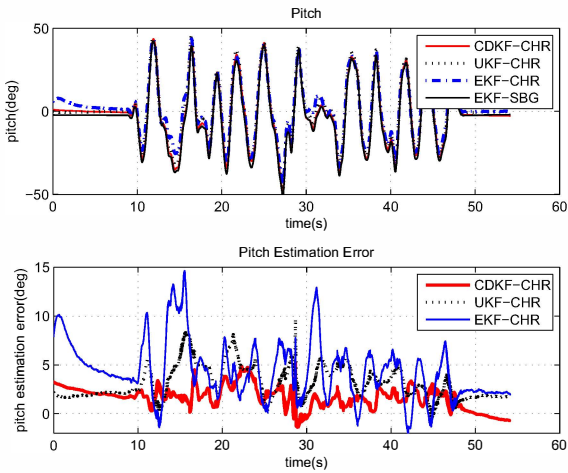


Fig. 4. Pitch and Pitch Estimation Error

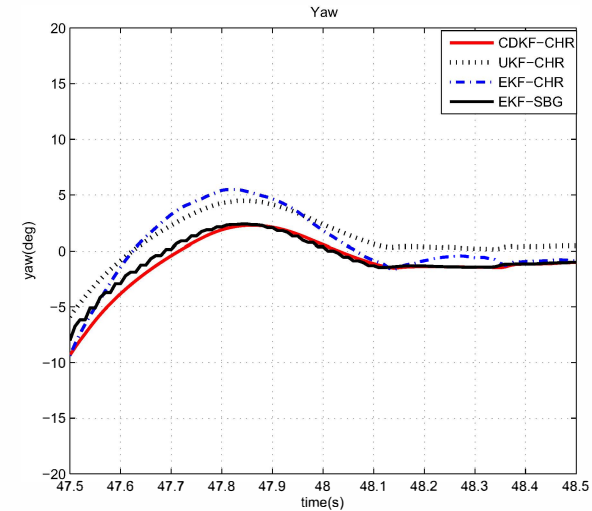


Fig. 6. Partial enlarged drawing of yaw

system parameters are as follows: The sampling frequency is 100 Hz, the initial state estimate and covariance matrix are chosen as  $\hat{\mathbf{x}}_0 = [0; 0; 0; 1; 0; 0; 0; 0; 0]^T$ ,  $\mathbf{P}_0 = 10^4 \times \mathbf{I}_{10 \times 10}$ ,  $\mathbf{I}_{10 \times 10}$  is a 10 order unit matrix, the interval length  $h = \sqrt{3}$ . The initial stage of the plane is in the ground starting state. We make use of the data collected this period to calculate the observation noise covariance matrix  $\mathbf{R}_k$  and process noise covariance matrix  $\mathbf{Q}_k$ . In order to compare the CDKF method with widely used attitude algorithms, we select the standard EKF algorithm and the UKF algorithm. Then the experimental results are given by Fig. 3 to Fig. 5.

In order to observe the estimated effect of CDKF, UKF and EKF method conveniently, we partially enlarge the attitude estimation results in Fig. 3 to Fig. 5, as shown in Fig. 6 to Fig. 8.

In Fig. 3 to Fig. 5, we regard the output of the SBG AHRS sensor as the benchmark, then calculate the difference between the estimation of each method and the output of the SBG AHRS sensor in order to obtain three attitude angle estimation errors.

The experimental results show the performance of the CDKF algorithm is better than EKF. The estimated attitude of CDKF coincides

very well with the output of the SBG AHRS sensor. Since the statistics of nonlinearly transformed Gaussian approximate random variables in CDKF are calculated by the deterministic sampling based sigma-point approach without using the Jacobian matrices and the linearization of the nonlinear model, the uncertainties are propagated well and the accuracy of the state estimation has been improved over the EKF approach. The same advantages also exist in UKF, however, it has only one adjustable parameter in contrast to three in the UKF circumstances, which affect the accuracy of the state estimation. Hence, the performance of CDKF is better than that of UKF.

## V. CONCLUSION

Aiming at the problem that low quality of the data collected from less accurate and high noisy sensors lead to the poor attitude estimation based on EKF, this paper presents an aircraft attitude estimation algorithm by introducing the CDKF method based on Stirling interpolation formulation. Experimental results show that the CDKF algorithm can get better performance on the estimate accuracy

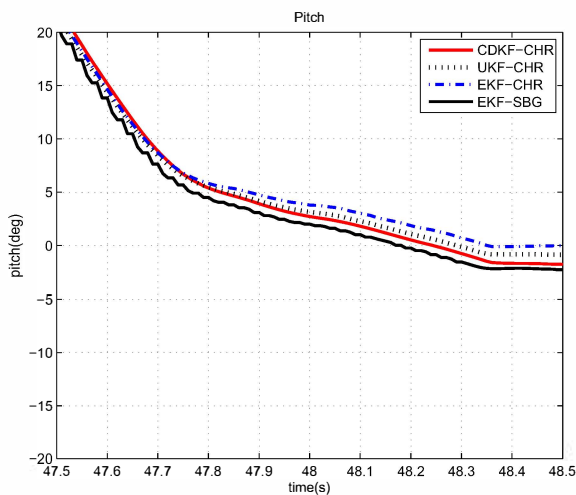


Fig. 7. Partial enlarged drawing of pitch

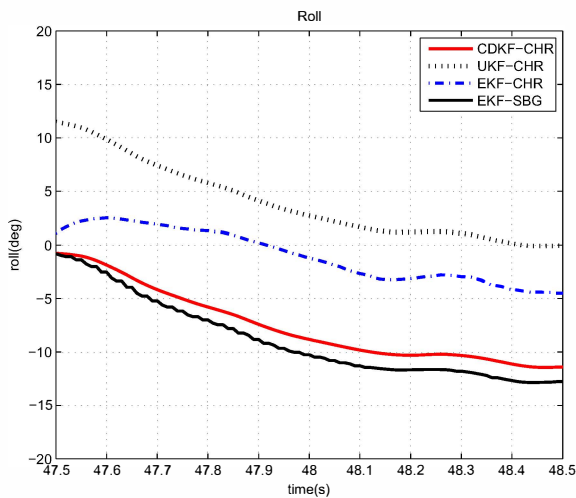


Fig. 8. Partial enlarged drawing of roll

and error robustness than that of EKF and UKF. Moreover, it does not employ the calculation of the Jacobian matrices and the linear approximations to the nonlinear models. So it provides a new way to improve the precision of attitude estimation.

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