



**HAL**  
open science

## Interval observer design for Linear Parameter-Varying systems subject to component faults

Rihab Lamouchi, Messaoud Amairi, Tarek Raïssi, Mohamed Aoun

► **To cite this version:**

Rihab Lamouchi, Messaoud Amairi, Tarek Raïssi, Mohamed Aoun. Interval observer design for Linear Parameter-Varying systems subject to component faults. 24th Mediterranean Conference on Control and Automation (MED), 2016, Jun 2016, Athènes, Greece. pp.707 - 712, 10.1109/MED.2016.7536019. hal-01503483

**HAL Id: hal-01503483**

**<https://hal.science/hal-01503483>**

Submitted on 28 Jun 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Interval observer design for Linear Parameter-Varying systems subject to component faults

R. LAMOUCI<sup>1,2</sup> M. AMAIRI<sup>2</sup> T. RAÏSSI<sup>1</sup> and M. AOUN<sup>2</sup>

**Abstract**—In this paper an interval observer for Linear Parameter-Varying (LPV) systems is proposed. The considered systems are assumed to be subject to parameter uncertainties and component faults whose effect can be approximated by parameters deviations. Under some conditions, an interval observer with discrete-time Luenberger structure is developed to cope with uncertainties and faults ensuring guaranteed bounds on the estimated states and their stability. The interval observer design is based on assumption that the uncertainties and the faults magnitudes are considered as unknown but bounded. A numerical example shows the efficiency of the proposed technique.

**Index Terms**—LPV systems, Interval observers, Component faults, Parameter uncertainties, Stability

## I. INTRODUCTION

Most of physical systems are nonlinear which leads to more complexity, especially when state estimation is required. In this case, observer design is usually based on the transformation of the system into canonical forms, which is difficult in practice [7]. This problem can be encountered in many industrial applications such as aircraft [1] [9], space vehicles [15] and wind turbine [19] [16]. To overcome this limitation, nonlinear systems can be represented by a Linear Parameter Varying (LPV) models. The main advantage of this representation is that the partial linearity of LPV models allows one to apply various frameworks developed for linear systems [10], [18], [17].

Systems are often affected by uncertainties (parameter, disturbances and noises), then the design of classical observers such as Luenberger observers [3], Kalman filter [4], is not easy to solve the estimation problem specially when the vector of scheduling parameters of LPV systems is not available for measurements. In such a case, set-membership approach can be considered as an alternative technique for robust estimation. Under some assumptions, interval observers can be used to compute the set of all the admissible values and provide certain lower and upper bounds for the estimate at each instant of time and in the presence of bounded uncertainties.

<sup>1</sup>R. LAMOUCI and T. RAÏSSI are with Cedric-Lab, Conservatoire National des Arts et Mtiers, Paris, France, lamouchi.rihab@gmail.com, tarek.raïssi@cnam.fr

<sup>2</sup>R. LAMOUCI, M. AMAIRI and M. AOUN are with Research Unit Modeling, Analysis and Control of Systems (MACS) 06/UR/11-12, National Engineering School of Gabes, University of Gabes, Tunisia, lamouchi.rihab@gmail.com, amairi.messaoud@ieee.org, mohamed.aoun@gmail.com

Interval observers were introduced in [8] and extended and applied in many studies, such as [2], [11], [13], [14]. The case of LPV systems has been considered in several works. For instance, in [4] an interval observer design for discrete-time LPV systems has been developed assuming that the vector of scheduling parameters is not available for measurement. The case of known scheduling vector has been proposed in [6] using a static transformation of coordinates.

The methodology proposed in this paper consists in developing an interval observers for LPV systems subject to parameter uncertainties and component faults where the vector of scheduling parameter considered unknown but bounded. This so-called interval observer can be used for fault tolerant control scheme in order to handle faults effect.

This paper is organized as follows. Some preliminaries are given in Section II. Section III presents the problem statement. In Section IV, main results for designing the interval observer is developed. The efficiency of the proposed approach is illustrated through numerical examples in Section V. Finally, concluding remarks are given in Section VI.

## II. PRELIMINARIES

A discrete-time dynamical system  $x_{k+1} = f(x_k)$  is nonnegative if for any integer  $k_0$  and any initial condition  $x_{k_0} \geq 0$ , the solution  $x$  satisfies  $x_k \geq 0$  for all  $k \geq k_0$ .

A system described by

$$x_{k+1} = Ax_k + u_k,$$

with  $x_k \in \mathbb{R}^n$  and  $A \in \mathbb{R}^{n \times n}$ , is nonnegative if and only if the matrix  $A$  is elementwise nonnegative,  $u_k \geq 0$  and  $x_{k_0} \geq 0$ . In this case the system is also called cooperative.

A matrix  $A \in \mathbb{R}^{n \times n}$  is Schur stable if all its eigenvalues have the modulus less than one and it is nonnegative if all its elements are nonnegative.

Given a matrix  $A \in \mathbb{R}^{m \times n}$ , define  $A^+ = \max\{0, A\}$ ,  $A^- = \max\{0, -A\}$  (similarly for vectors).

For two vectors  $x_1, x_2 \in \mathbb{R}^n$  or matrices  $A_1, A_2 \in \mathbb{R}^{n \times n}$ , the relations  $x_1 \leq x_2$  and  $A_1 \leq A_2$  are understood elementwise.

The symbol  $|\cdot|$  denotes vector or corresponding induced matrix Euclidean norm.  $I$  and  $E_p$  denote respectively the  $(n \times n)$  and the  $(p \times 1)$  identity matrices.

For a measurable and locally essentially bounded input  $u: \mathbb{N} \rightarrow \mathbb{R}$ , the symbol  $\|u\|_{[t_0, t_1]}$  denotes its  $\mathcal{L}_\infty$ -norm  $\|u\|_{[t_0, t_1]} =$

$\sup\{|u_t|, t \in [t_0, t_1]\}, \|u\| = \|u\|_{[0, +\infty)}$ . The set of all inputs  $u$  with the property  $\|u\| < \infty$  is denoted by  $\mathcal{L}_\infty$ .

For a matrix  $P = P^T$ , the relation  $P \succ 0$  means that  $P$  is positive definite. Let  $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$ , then, the inequality  $|x+y|^2 \leq 2|x|^2 + 2|y|^2$  holds.

*Lemma 1:* [3]

Let  $\underline{x}, x, \bar{x} \in \mathbb{R}^n$  if  $\underline{x} \leq x \leq \bar{x}$  then

$$\underline{x}^+ \leq x^+ \leq \bar{x}^+ \text{ and } \bar{x}^- \leq x^- \leq \underline{x}^- \quad (1)$$

Similarly, let  $\underline{A}, A, \bar{A} \in \mathbb{R}^{m \times n}$ , if  $\underline{A} \leq A \leq \bar{A}$  then

$$\underline{A}^+ \leq A^+ \leq \bar{A}^+ \text{ and } \bar{A}^- \leq A^- \leq \underline{A}^- \quad (2)$$

□

*Lemma 2:* [3] Let  $x \in \mathbb{R}^n$  be a vector such that  $\underline{x} \leq x \leq \bar{x}$  for some  $\underline{x}, \bar{x} \in \mathbb{R}^n$ .

1) If  $A \in \mathbb{R}^{m \times n}$  is a constant matrix, then

$$A^+ \underline{x} - A^- \bar{x} \leq Ax \leq A^+ \bar{x} - A^- \underline{x} \quad (3)$$

2) If  $A \in \mathbb{R}^{m \times n}$  is a matrix satisfying  $\underline{A} \leq A \leq \bar{A}$  for some  $\underline{A}, \bar{A} \in \mathbb{R}^{m \times n}$ , then

$$\begin{aligned} \underline{A}^+ \underline{x}^+ - \bar{A}^- \underline{x}^- - \underline{A}^- \bar{x}^+ + \bar{A}^- \bar{x}^- &\leq Ax \\ &\leq \bar{A}^+ \bar{x}^+ - \underline{A}^+ \bar{x}^- - \bar{A}^- \underline{x}^+ + \underline{A}^- \underline{x}^- \end{aligned} \quad (4)$$

□

### III. PROBLEM STATEMENT

Consider the following discrete LPV system:

$$\begin{cases} x_{k+1} = A(\rho, \eta)x_k + B(\eta)u_k \\ y_k = Cx_k + v_k \end{cases} \quad (5)$$

where  $x_k \in \mathbb{R}^n$  is the state,  $u_k \in \mathbb{R}^q$  is the input,  $y_k \in \mathbb{R}^p$  is the output;  $v_k$  is bounded noise.  $\eta \in \Xi$  denotes the vector of scheduling parameters considered unknown but bounded and only the set of admissible values  $\Xi$  is given.  $\rho$  is a component fault parameter vector, which is assumed to be in the set of admissible values  $\Pi$ .

In this paper it is assumed that the matrix  $A(\rho, \eta)$  depends on  $\eta$  and  $\rho$  as

$$A(\rho, \eta) = A_0(\eta) + \rho_1 A_1(\eta) + \dots + \rho_r A_r(\eta) \quad (6)$$

where  $\rho_i, i = 0, 1, \dots, r$  is the system fault parameter component and  $A_i(\eta), i = 0, 1, \dots, r$  are affine matrices depending on  $\eta$ .

Two cases are considered: fault-free case ( $\rho_i = 0$ ) and faulty case ( $\exists i$  such that  $\rho_i \neq 0$ ).

Equation (6) can be rewritten as

$$\begin{cases} A(\rho, \eta) = A_0(\eta) + \sum_{i=1}^r \rho_i A_i(\eta) & \text{if } \exists i, \rho_i \neq 0 \\ A(\rho, \eta) = A_0(\eta) & \text{if } \forall i, \rho_i = 0 \end{cases} \quad (7)$$

In the sequel, it is assumed that  $A_0(\eta) = A_0 + \Delta A(\eta)$  and  $B_0(\eta) = B_0 + \Delta B(\eta)$  with  $\Delta A : \Xi \rightarrow \mathbb{R}^{n \times n}$  and  $\Delta B : \Xi \rightarrow \mathbb{R}^{q \times q}$

are two known piecewise continuous matrix functions.

The following assumptions will be used in this work.

*Assumption 1:*  $\underline{\Delta A} \leq \Delta A(\eta) \leq \bar{\Delta A}$ ,  $\underline{A}_i \leq A_i(\eta) \leq \bar{A}_i \forall \eta \in \Xi$  for known  $\underline{\Delta A}, \bar{\Delta A}, \underline{A}_i, \bar{A}_i \in \mathbb{R}^{n \times n}$ . □

*Assumption 2:*  $\underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i, \forall \rho \in \Pi$  for known  $\underline{\rho}_i, \bar{\rho}_i \in \mathbb{R}^{n \times n}$ . □

*Assumption 3:*  $\underline{\Delta B} \leq \Delta B(\eta) \leq \bar{\Delta B} \forall \eta \in \Xi$  for known  $\underline{\Delta B}, \bar{\Delta B} \in \mathbb{R}^{q \times q}$ . □

*Assumption 4:*  $\|v\| < V$ , where  $V$  is a positive constant. □

Assumption 1 means that the matrix  $\Delta A(\eta)$  belongs to the interval  $[\underline{\Delta A}, \bar{\Delta A}]$  and the matrix  $A_i(\eta)$  belongs to the interval  $[\underline{A}_i, \bar{A}_i]$ . The value of the scheduling vector  $\eta$  is not available for measurement but it is easy to compute  $\underline{\Delta A}$  and  $\bar{\Delta A}$  for a given set  $\Xi$  and a known function  $\Delta A : \Xi \rightarrow \mathbb{R}^{n \times n}$ . Assumption 2 states that the fault parameter magnitude  $\rho_i$  is unknown, but only its bounds  $\underline{\rho}_i$  and  $\bar{\rho}_i$  are given. Assumption 3 means that the matrix  $\Delta B(\eta)$  belongs to the interval  $[\underline{\Delta B}, \bar{\Delta B}]$ . Finally, Assumption 4 means that the absolute value of the measurement noise  $v_k$  has a positive constant upper bound  $V$ .

In the faulty case, the system (5) can be written as

$$\begin{cases} x_{k+1} = [A_0 + \Delta A(\eta) + \sum_{i=1}^r \rho_i A_i(\eta)]x_k + [B_0 + \Delta B(\eta)]u_k \\ y_k = Cx_k + v_k \end{cases} \quad (8)$$

The objective of this paper is to design an interval observer for the LPV system (8) to cope with uncertainties and component faults ensuring guaranteed bounds on the estimated states and their stability. Since the system state is guaranteed to belong to the interval estimation, the interval observer stabilization yields the same property for the LPV system.

### IV. INTERVAL OBSERVERS DESIGN

The LPV system (8) can be rewritten as

$$\begin{cases} x_{k+1} = A_0 x_k + \varphi(x_k) + \psi(x_k) + B_0 u_k + \phi(u_k) \\ y_k = Cx_k + v_k \end{cases} \quad (9)$$

with  $\varphi(x_k) = \Delta A(\eta)x_k$ ,  $\psi(x_k) = \sum_{i=1}^r \rho_i A_i(\eta)x_k$  and  $\phi(u_k) = \Delta B(\eta)u_k$ .

Interval observer design for (9), subject to uncertainties, requires the following assumption.

*Assumption 5:* The pair  $(A_0, C)$  is detectable and there exists a matrix gain  $L \in \mathbb{R}^{n \times p}$  such that  $A_0 - LC$  is nonnegative. □

Consider an observer structure for (9) given by

$$\begin{cases} \bar{x}_{k+1} = (A_0 - LC)\bar{x}_k + \bar{\varphi}(\bar{x}_k, \underline{x}_k) + \bar{\psi}(\bar{x}_k, \underline{x}_k) + B_0 u_k + \bar{\phi}(u_k) \\ \quad + L y_k + |L| V E_p \\ \underline{x}_{k+1} = (A_0 - LC)\underline{x}_k + \underline{\varphi}(\bar{x}_k, \underline{x}_k) + \underline{\psi}(\bar{x}_k, \underline{x}_k) + B_0 u_k + \underline{\phi}(u_k) \\ \quad + L y_k - |L| V E_p \end{cases} \quad (10)$$

where  $\bar{x}_k$  and  $\underline{x}_k$  are the upper and the lower bounds of the interval estimates of  $x_k$  and

$$\begin{cases} \bar{\varphi}(\bar{x}_k, \underline{x}_k) = \sum_{i=1}^r (\bar{H}_i^+ \bar{x}_k^+ - \underline{H}_i^+ \bar{x}_k^- - \bar{H}_i^- \underline{x}_k^+ + \underline{H}_i^- \underline{x}_k^-) \\ \underline{\varphi}(\bar{x}_k, \underline{x}_k) = \sum_{i=1}^r (\underline{H}_i^+ \underline{x}_k^+ - \bar{H}_i^+ \underline{x}_k^- - \underline{H}_i^- \bar{x}_k^+ + \bar{H}_i^- \bar{x}_k^-) \end{cases} \quad (11)$$

$$\begin{cases} \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \\ \underline{H}_i = \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \end{cases} \quad (12)$$

$$\begin{cases} \bar{\varphi}(\bar{x}_k, \underline{x}_k) = \bar{\Delta A}^+ \bar{x}^+ - \underline{\Delta A}^+ \bar{x}^- - \bar{\Delta A}^- \underline{x}^+ + \underline{\Delta A}^- \underline{x}^- \\ \underline{\varphi}(\bar{x}_k, \underline{x}_k) = \underline{\Delta A}^+ \underline{x}^+ - \bar{\Delta A}^+ \underline{x}^- - \underline{\Delta A}^- \bar{x}^+ + \bar{\Delta A}^- \bar{x}^- \end{cases} \quad (13)$$

$$\begin{cases} \bar{\phi}(u_k) = \bar{\Delta B} u_k^+ - \underline{\Delta B} u_k^- \\ \underline{\phi}(u_k) = \underline{\Delta B} u_k^+ - \bar{\Delta B} u_k^- \end{cases} \quad (14)$$

Introducing the estimation errors  $\bar{e}_k = \bar{x}_k - x_k$  and  $\underline{e}_k = x_k - \underline{x}_k$ , it follows that

$$\begin{cases} \bar{e}_{k+1} = (A_0 - LC)\bar{e}_k + \bar{\Gamma}_k(\bar{x}_k, \underline{x}_k) \\ \underline{e}_{k+1} = (A_0 - LC)\underline{e}_k + \underline{\Gamma}_k(\bar{x}_k, \underline{x}_k) \end{cases} \quad (15)$$

with

$$\begin{cases} \bar{\Gamma}_k(\bar{x}_k, \underline{x}_k) = \bar{\psi}(\bar{x}_k, \underline{x}_k) - \psi(x_k) + \bar{\varphi}(\bar{x}_k, \underline{x}_k) - \varphi(x_k) \\ \quad + \bar{\phi}(u_k) - \phi(u_k) + |L| V E_p + L v_k \\ \underline{\Gamma}_k(\bar{x}_k, \underline{x}_k) = \underline{\psi}(\bar{x}_k, \underline{x}_k) - \psi(x_k) + \underline{\varphi}(\bar{x}_k, \underline{x}_k) \\ \quad + \underline{\phi}(u_k) - \phi(u_k) + |L| V E_p - L v_k \end{cases} \quad (16)$$

The functions  $\bar{\Gamma}_k(\bar{x}_k, \underline{x}_k)$  and  $\underline{\Gamma}_k(\bar{x}_k, \underline{x}_k)$  are globally Lipschitz, it follows that for  $\underline{x}_k \leq x_k \leq \bar{x}_k$  and for a chosen submultiplicative norm  $\|\cdot\|$ , there exist positive constants  $a_1, a_2, a_3, b_1, b_2$  and  $b_3$  such that

$$\begin{cases} \|\bar{\Gamma}_k(\bar{x}_k, \underline{x}_k)\| \leq a_1 \|\bar{x}_k - x_k\| + a_2 \|\underline{x}_k - x_k\| + a_3 \\ \|\underline{\Gamma}_k(\bar{x}_k, \underline{x}_k)\| \leq b_1 \|\bar{x}_k - x_k\| + b_2 \|\underline{x}_k - x_k\| + b_3 \end{cases} \quad (17)$$

*Theorem 1:* Assume that Assumptions 1-4 are satisfied and  $A_0 - LC$  is nonnegative and  $x_k \in \mathcal{L}_\infty^n$ . If the initial state  $x_0$  verifies  $\underline{x}_0 \leq x_0 \leq \bar{x}_0$ , then the state  $x_k$  solution of (10) satisfies

$$\underline{x}_k \leq x_k \leq \bar{x}_k, \quad \forall k \in \mathbb{N} \quad (18)$$

In addition if there exist positive definite and symmetric matrices  $Q, P$  and  $W$  such that the Riccati matrix inequality

$$D^T P D - P + D^T P W^{-1} P D + \alpha(\|W + P\|)I + Q \leq 0 \quad (19)$$

is verified, where  $D = A_0 - LC$  and  $\alpha = 3 \max((a_1^2 + b_1^2), (a_2^2 + b_2^2))$ , then  $\underline{x}_k, \bar{x}_k \in \mathcal{L}_\infty^n$ .  $\square$

*Proof:*

According to Lemma 2 and Assumption 1, we have

$$\begin{aligned} \bar{\Delta A}^+ \underline{x}^+ - \bar{\Delta A}^+ \underline{x}^- - \underline{\Delta A}^- \bar{x}^+ + \bar{\Delta A}^- \bar{x}^- &\leq \varphi(x_k) \\ &\leq \bar{\Delta A}^+ \bar{x}^+ - \underline{\Delta A}^+ \bar{x}^- - \bar{\Delta A}^- \underline{x}^+ + \underline{\Delta A}^- \underline{x}^- \end{aligned} \quad (20)$$

According to Lemma 2, Assumption 1 and Assumption 2, we have

$$\begin{aligned} \underline{H}_i &= \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \leq H_i \\ &\leq \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \end{aligned} \quad (21)$$

It follows that

$$\begin{aligned} \sum_{i=1}^r (\underline{H}_i^+ \underline{x}_k^+ - \bar{H}_i^+ \underline{x}_k^- - \underline{H}_i^- \bar{x}_k^+ + \bar{H}_i^- \bar{x}_k^-) &\leq \psi(x_k) \\ &\leq \sum_{i=1}^r (\bar{H}_i^+ \bar{x}_k^+ - \underline{H}_i^+ \bar{x}_k^- - \bar{H}_i^- \underline{x}_k^+ + \underline{H}_i^- \underline{x}_k^-) \end{aligned} \quad (22)$$

According to Lemma 2 and Assumption 3, we have for any  $u_k \in \mathbb{R}^q$

$$\underline{\Delta B} u_k^+ - \bar{\Delta B} u_k^- \leq \Delta B u_k \leq \bar{\Delta B} u_k^+ - \underline{\Delta B} u_k^- \quad (23)$$

Since  $A_0 - LC$  is assumed to be nonnegative, and by construction  $\bar{\Gamma}_k$  and  $\underline{\Gamma}_k$  are positive, then the system (15) is cooperative. If  $\bar{x}_0$  and  $\underline{x}_0$  are chosen such that  $\bar{e}_0$  and  $\underline{e}_0$  are positive, the dynamics of interval estimation errors  $\bar{e}_k$  and  $\underline{e}_k$  stay positive for all  $k \in \mathbb{N}$ .

Let's show now that the variables  $\bar{x}_k, \underline{x}_k$  stay bounded  $\forall k \in \mathbb{N}$ .

Consider the positive definite quadratic Lyapunov function:

$$V(\bar{e}_k, \underline{e}_k) = \bar{e}_k^T P \bar{e}_k + \underline{e}_k^T P \underline{e}_k \quad (24)$$

The increment of  $\Delta V$  is given by

$$\begin{aligned} \Delta V &= V(\bar{e}_{k+1}, \underline{e}_{k+1}) - V(\bar{e}_k, \underline{e}_k) \\ &= \bar{e}_k^T (D^T P D - P) \bar{e}_k + 2\bar{e}_k^T D^T P \bar{\Gamma}_k + \bar{\Gamma}_k^T P \bar{\Gamma}_k \\ &\quad + \underline{e}_k^T (D^T P D - P) \underline{e}_k + 2\underline{e}_k^T D^T P \underline{\Gamma}_k + \underline{\Gamma}_k^T P \underline{\Gamma}_k \end{aligned} \quad (25)$$

From [20], we have the following inequalities:

$$\begin{aligned} 2\bar{e}_k^T D^T P \bar{\Gamma}_k &= 2\bar{e}_k^T D^T P W^{-0.5} W^{0.5} \bar{\Gamma}_k \\ &\leq \bar{e}_k^T D^T P W^{-1} P D \bar{e}_k + \bar{\Gamma}_k^T W \bar{\Gamma}_k \\ 2\underline{e}_k^T D^T P \underline{\Gamma}_k &= 2\underline{e}_k^T D^T P W^{-0.5} W^{0.5} \underline{\Gamma}_k \\ &\leq \underline{e}_k^T D^T P W^{-1} P D \underline{e}_k + \underline{\Gamma}_k^T W \underline{\Gamma}_k \end{aligned} \quad (26)$$

Using the inequalities (25) and (26), it yields

$$\begin{aligned}
\Delta V &\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + \bar{\Gamma}_k^T (W + P) \bar{\Gamma}_k + \underline{\Gamma}_k^T (W + P) \underline{\Gamma}_k \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + 3 \|W + P\| (a_1^2 \|\bar{e}_k\|^2 + a_2^2 \|\underline{e}_k\|^2 + a_3^2) \\
&+ 3 \|W + P\| (b_1^2 \|\bar{e}_k\|^2 + b_2^2 \|\underline{e}_k\|^2 + b_3^2) \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + \alpha \|W + P\| \bar{e}_k^T \bar{e}_k + \alpha \|W + P\| \underline{e}_k^T \underline{e}_k \\
&+ 3 \|W + P\| (a_3^2 + b_3^2) \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD + \alpha \|W + P\| I) \bar{e}_k \\
&+ \underline{e}_k^T (D^T PD - P + D^T PW^{-1} PD + \alpha \|W + P\| I) \underline{e}_k \\
&+ 3 \|W + P\| (a_3^2 + b_3^2) \\
&\leq -\bar{e}_k^T Q \bar{e}_k - \underline{e}_k^T Q \underline{e}_k + 3 \|W + P\| (a_3^2 + b_3^2).
\end{aligned}$$

Using (19) we get  $\Delta V \leq -\bar{e}_k^T Q \bar{e}_k - \underline{e}_k^T Q \underline{e}_k + \beta$  with  $\beta = 3 \|W + P\| (a_3^2 + b_3^2)$  which provides the boundedness of the dynamics of estimation errors  $\bar{e}_k, \underline{e}_k$ , therefore the variables  $\bar{x}_k, \underline{x}_k$  stay bounded  $\forall k \in \mathbb{N}$ . ■

It is worth to note that it is not always possible to compute a matrix  $L$  such that  $A - LC$  is nonnegative. This restrictive condition can be relaxed by means of a change of coordinates  $z_k = Rx_k$  with a nonsingular matrix  $R$  such that the matrix  $E = R(A - LC)S$  is nonnegative where  $S = R^{-1}$  [12],[5].

By introducing the change of coordinate  $z_k = Rx_k$ , the system (9) can be presented as

$$\begin{cases} z_{k+1} = Ez_k + \varphi(z_k) + \psi(z_k) + RB_0 u_k + \phi_z(u_k) \\ y_k = CSz_k + v_k \end{cases} \quad (27)$$

where  $\varphi(z_k) = R\Delta A(\eta)S z_k$ ,  $\psi(z_k) = \sum_{i=1}^r R\rho_i A_i(\eta)S z_k$  and  $\phi_z(u_k) = R\Delta B u_k$ .

An interval observer for the system (27) can be written in the new coordinates  $z$  as

$$\begin{cases} \bar{z}_{k+1} = E\bar{z}_k + RB_0 u_k + \bar{\varphi}(\bar{z}_k, \underline{z}_k) + \bar{\psi}(\bar{z}_k, \underline{z}_k) + \bar{\phi}_z(u_k) \\ \quad + RLY_k + |F|VE_p \\ \underline{z}_{k+1} = E\underline{z}_k + RB_0 u_k + \underline{\varphi}(\bar{z}_k, \underline{z}_k) + \underline{\psi}(\bar{z}_k, \underline{z}_k) + \underline{\phi}_z(u_k) \\ \quad + RLY_k - |F|VE_p \end{cases} \quad (28)$$

with

$$\begin{cases} \bar{\psi}(\bar{z}_k, \underline{z}_k) = (\bar{\sigma}^+ \bar{z}_k^+ - \underline{\sigma}^+ \bar{z}_k^- - \bar{\sigma}^- \underline{z}_k^+ + \underline{\sigma}^- \underline{z}_k^-) \\ \underline{\psi}(\bar{z}_k, \underline{z}_k) = (\underline{\sigma}^+ \bar{z}_k^+ - \bar{\sigma}^+ \underline{z}_k^- - \underline{\sigma}^- \bar{z}_k^+ + \bar{\sigma}^- \underline{z}_k^-) \end{cases} \quad (29)$$

$$\begin{cases} \underline{\sigma} = S^+(R^+ \underline{H}_i - R^- \bar{H}_i) - S^-(R^+ \bar{H}_i - R^- \underline{H}_i) \\ \bar{\sigma} = S^+(R^+ \bar{H}_i - R^- \underline{H}_i) - S^-(R^+ \underline{H}_i - R^- \bar{H}_i) \end{cases} \quad (30)$$

$$\begin{cases} \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \\ \underline{H}_i = \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \end{cases} \quad (31)$$

$$\begin{cases} \bar{\varphi}(\bar{z}_k, \underline{z}_k) = (\bar{\Omega}^+ \bar{z}_k^+ - \underline{\Omega}^+ \bar{z}_k^- - \bar{\Omega}^- \underline{z}_k^+ + \underline{\Omega}^- \underline{z}_k^-) \\ \underline{\varphi}(\bar{z}_k, \underline{z}_k) = (\underline{\Omega}^+ \bar{z}_k^+ - \bar{\Omega}^+ \underline{z}_k^- - \underline{\Omega}^- \bar{z}_k^+ + \bar{\Omega}^- \underline{z}_k^-) \end{cases} \quad (32)$$

$$\begin{cases} \bar{\Omega} = S^+(R^+ \bar{\Delta A} - R^- \underline{\Delta A}) - S^-(R^+ \underline{\Delta A} - R^- \bar{\Delta A}) \\ \underline{\Omega} = S^+(R^+ \underline{\Delta A} - R^- \bar{\Delta A}) - S^-(R^+ \bar{\Delta A} - R^- \underline{\Delta A}) \end{cases} \quad (33)$$

$$\begin{cases} \bar{\phi}_z(u_k) = (R^+ \bar{\Delta B} - R^- \underline{\Delta B}) u_k^+ - (R^+ \underline{\Delta B} - R^- \bar{\Delta B}) u_k^- \\ \underline{\phi}_z(u_k) = (R^+ \underline{\Delta B} - R^- \bar{\Delta B}) u_k^+ - (R^+ \bar{\Delta B} - R^- \underline{\Delta B}) u_k^- \\ F = RL \end{cases} \quad (34)$$

Consider the dynamics of interval estimation errors  $\bar{e}_k = \bar{z}_k - z_k$  and  $\underline{e}_k = z_k - \underline{z}_k$ , one can write

$$\begin{cases} \bar{e}_{k+1} = E\bar{e}_k + \bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) \\ \underline{e}_{k+1} = E\underline{e}_k + \underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) \end{cases} \quad (35)$$

with

$$\begin{cases} \bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) = \bar{\psi}(\bar{z}_k, \underline{z}_k) - \psi(z_k) + \bar{\varphi}(\bar{z}_k, \underline{z}_k) - R\varphi(z_k) \\ \quad + \bar{\phi}_z(u_k) - R\phi_z(u_k) + \bar{F}VE_p + RLv_k \\ \underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) = \underline{\psi}(\bar{z}_k, \underline{z}_k) - \psi(z_k) + \underline{\varphi}(\bar{z}_k, \underline{z}_k) - \underline{\phi}_z(u_k) \\ \quad + R\phi_z(u_k) - \underline{\phi}_z(u_k) + \bar{F}VE_p - RLv_k. \end{cases} \quad (36)$$

$\bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)$  and  $\underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)$  are globally Lipschitz, then for  $\underline{z}_k \leq z_k \leq \bar{z}_k$  and for a chosen submultiplicative norm  $\|\cdot\|$ , there exist positive constants  $c_1, c_2, c_3, d_1, d_2$  and  $d_3$  such that

$$\begin{cases} \|\bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)\| \leq c_1 \|\bar{z}_k - z_k\| + c_2 \|\underline{z}_k - z_k\| + c_3 \\ \|\underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)\| \leq d_1 \|\bar{z}_k - z_k\| + d_2 \|\underline{z}_k - z_k\| + d_3 \end{cases} \quad (37)$$

*Theorem 2:* Given a nonsingular matrix  $R$  such that the matrix  $R(A - LC)S$  is nonnegative. Then, the solution of (28) satisfies

$$\underline{z}_k \leq z_k \leq \bar{z}_k, \quad \forall k \in \mathbb{N} \quad (38)$$

where  $\underline{z}_0 \leq z_0 \leq \bar{z}_0$ . In addition, if there exist positive definite and symmetric matrices  $Q, P$  and  $W$  such that the following Riccati matrix inequality is verified

$$E^T P E - P + E^T P W^{-1} P E + \alpha_z (\|W + P\|) I + Q \leq 0 \quad (39)$$

where  $E = R(A_0 - LC)S$  and  $\alpha_z = 3 \max((c_1^2 + d_1^2), (c_2^2 + d_2^2))$ , then  $\underline{z}_k, \bar{z}_k \in \mathcal{L}_\infty^n$ . □

*Proof:*

The proof is similar to that of Theorem 1. ■

## V. NUMERICAL SIMULATIONS

To illustrate the proposed methodology, let us consider the LPV system described by

$$\begin{cases} x_{k+1} = A(\eta, \rho)x_k + B_k(\eta) \\ y_k = Cx_k + v_k \end{cases} \quad (40)$$

For simulations,  $A(\rho, \eta)$  is chosen as

$$A(\eta, \rho) = \begin{bmatrix} 0.3 + 2\eta\rho_1 & -0.7 + 0.5\eta + 0.5\eta\rho_2 \\ 0.6 + 0.2\eta & -0.5 + 0.1\rho_1 \end{bmatrix}$$

where  $\rho = (\rho_1, \rho_2)^T$  is the fault parameter vector such that  $|\rho_i| < 1, i = 1, 2$ . The parameter  $\eta$  is considered unknown but bounded such that  $\eta \in [0.04; 0.06]$ ,  $C = [1 \ 0]$ ,  $v_k = V \sin(k)$  and  $V = 0.01$ .

The system (40) can be rewritten as

$$\begin{cases} x_{k+1} = A_0x_k + \Delta A(\eta)x_k + \sum_{i=1}^2 \rho_i A_i(\eta)x_k + B_{0,k} + \Delta B_k(\eta) \\ y_k = Cx_k + v_k \end{cases} \quad (41)$$

with  $A_0 = \begin{bmatrix} 0.3 & -0.7 \\ 0.6 & -0.5 \end{bmatrix}$ ,  $\Delta A(\eta) = \begin{bmatrix} 0 & 0.5\eta \\ 0.2\eta & 0 \end{bmatrix}$ ,

$$A_1(\eta) = \begin{bmatrix} 2\eta & 0 \\ 0 & 0.1 \end{bmatrix}, A_2(\eta) = \begin{bmatrix} 0 & 0.5\eta \\ 0 & 0 \end{bmatrix}$$

$$B_{0,k} = [\sin(0.1k) \ \cos(0.2k)]^T$$

$$\Delta B_k(\eta) = \eta[\sin(0.5kx_{2,k}) \ \sin(0.3k)]^T$$

where  $\Delta B_k(\eta) \in [\underline{\Delta B}_k, \overline{\Delta B}_k]$ .

For  $L = [0.3 \ 0.6]^T$  the matrix  $A_0 - LC$  is not nonnegative. Thus a transformation of coordinates,

$S = \begin{bmatrix} 0.609 & 0.814 \\ -1.162 & 0.581 \end{bmatrix}$  is used such that  $E = R(A_0 - LC)S$ , with  $R = S^{-1}$ , is nonnegative.

Consequently, the dynamic extension:

$$\begin{cases} \bar{z}_{k+1} = E\bar{z}_k + RB_{0,k} + \bar{\varphi}(\bar{z}_k, \underline{z}_k) + \bar{\psi}(\bar{z}_k, \underline{z}_k) + \bar{\phi}_k^z(\Delta B_k) \\ \quad + RLy_k + |F|VE_p \\ \underline{z}_{k+1} = E\underline{z}_k + RB_{0,k} + \underline{\varphi}(\bar{z}_k, \underline{z}_k) + \underline{\psi}(\bar{z}_k, \underline{z}_k) + \underline{\phi}_k^z(\Delta B_k) \\ \quad + RLy_k - |F|VE_p \end{cases} \quad (42)$$

with

$$\begin{cases} \bar{\phi}_k^z(\Delta B_k) = R^+\overline{\Delta B}_k - R^-\underline{\Delta B}_k \\ \underline{\phi}_k^z(\Delta B_k) = R^+\underline{\Delta B}_k - R^-\overline{\Delta B}_k \end{cases} \quad (43)$$

is an interval observer for the system (40).

The results of interval simulations are presented in Fig.1, Fig.2, Fig.3 and Fig.4, where the dashed lines correspond to the estimated lower and upper bounds and the continuous lines correspond to the actual state.

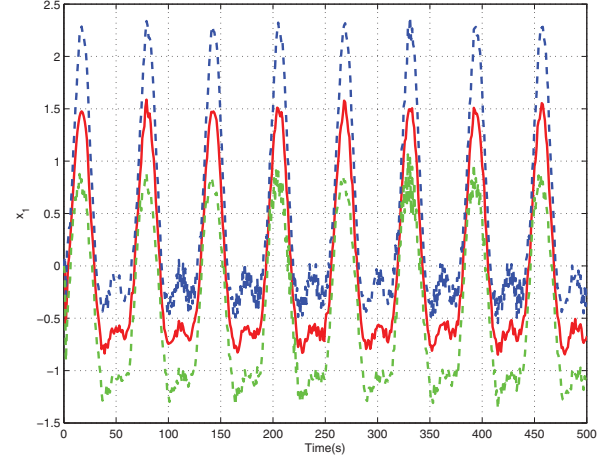


Fig. 1. Evolution of the first component of state  $x_1$  without fault.

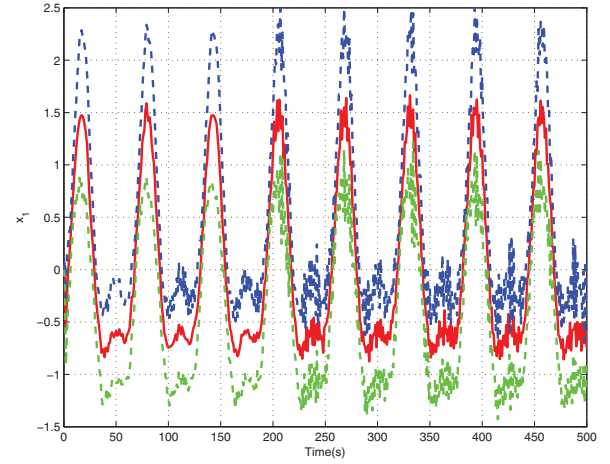


Fig. 2. Evolution of the first component of state  $x_1$  with fault.

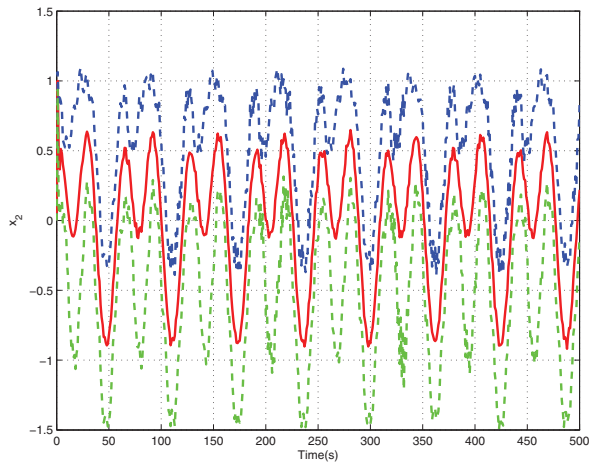


Fig. 3. Evolution of the second component of state  $x_2$  without fault.

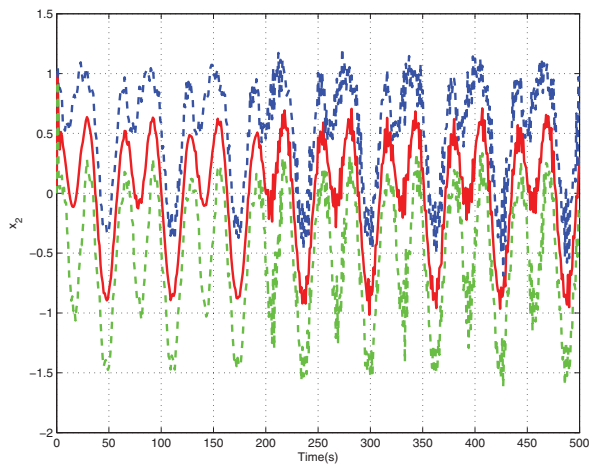


Fig. 4. Evolution of the second component of state  $x_2$  with fault.

The faulty case is considered in the simulations, that is, before 200s, the system operates in a normal regime. At 200s, a fault occurs in the system.

The simulation results show that the upper and lower bounds of interval observer converge to a domain containing the actual state  $x_k$ .

## VI. CONCLUSIONS

In this paper, an interval approach has been developed for the state estimation of LPV systems subject to uncertainties and component faults. An interval observer has been designed with a gain satisfying observation error positivity. A change of coordinates is used in order to make this methodology useful for a large class of LPV systems. A numerical example has been presented to illustrate the effectiveness of the approach. The proposed methodology will be used in the fields of Fault Tolerant Control in further works.

## REFERENCES

- [1] Halim Alwi, Christopher Edwards, and Chee Pin Tan. Sliding mode estimation schemes for incipient sensor faults. *Automatica*, 45(7):1679–1685, 2009.
- [2] Olivier Bernard and Jean-Luc Gouzé. Closed loop observers bundle for uncertain biotechnological models. *Journal of process control*, 14(7):765–774, 2004.
- [3] Stanislav Chebotarev, Denis Efimov, Tarek Raïssi, and Ali Zolghadri. On interval observer design for a class of continuous-time lpv systems. In *IFAC Ncolcos 2013*, 2013.
- [4] Stanislav Chebotarev, Denis Efimov, Tarek Raïssi, and Ali Zolghadri. Interval observers for continuous-time lpv systems with  $l_1/l_2$  performance. *Automatica*, 2015.
- [5] Denis Efimov, Wilfrid Perruquetti, Tarek Raïssi, and Ali Zolghadri. Interval observers for time-varying discrete-time systems. *IEEE Transactions on Automatic Control*, 2013.
- [6] Denis Efimov, Tarek Raïssi, Stanislav Chebotarev, and Ali Zolghadri. Interval state observer for nonlinear time varying systems. *Automatica*, 49(1):200–205, 2013.
- [7] Denis V Efimov, Tarek Raïssi, Wilfrid Perruquetti, and Ali Zolghadri. Estimation and control of discrete-time lpv systems using interval observers. In *CDC*, pages 5036–5041, 2013.
- [8] Jean-Luc Gouzé, A Rapaport, and Mohamed Zakaria Hadj-Sadok. Interval observers for uncertain biological systems. *Ecological modelling*, 133(1):45–56, 2000.
- [9] Jin Jiang and Youmin Zhang. Accepting performance degradation in fault-tolerant control system design. *Control Systems Technology, IEEE Transactions on*, 14(2):284–292, 2006.
- [10] Andrés Marcos and Gary J Balas. Development of linear-parameter-varying models for aircraft. *Journal of Guidance, Control, and Dynamics*, 27(2):218–228, 2004.
- [11] Frédéric Mazenc and Olivier Bernard. Asymptotically stable interval observers for planar systems with complex poles. *Automatic Control, IEEE Transactions on*, 55(2):523–527, 2010.
- [12] Frédéric Mazenc, Thach Ngoc Dinh, and Silviu-Iulian Niculescu. Robust interval observers and stabilization design for discrete-time systems with input and output. *Automatica*, 49(11):3490–3497, 2013.
- [13] Marcelo Moisan, Olivier Bernard, and Jean-Luc Gouzé. Near optimal interval observers bundle for uncertain bioreactors. *Automatica*, 45(1):291–295, 2009.
- [14] Fatiha Nejari, Vicenç Puig, Saúl Montes de Oca, and Atefeh Sadeghzadeh. Robust fault detection for lpv systems using interval observers and zonotopes. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 1002–1007. IEEE, 2009.
- [15] FN Pirmoradi, F Sassani, and CW De Silva. Fault detection and diagnosis in a spacecraft attitude determination system. *Acta Astronautica*, 65(5):710–729, 2009.
- [16] Montadher Sami Shaker and Ron J Patton. Active sensor fault tolerant output feedback tracking control for wind turbine systems via t-s model. *Engineering Applications of Artificial Intelligence*, 34:1–12, 2014.
- [17] Jeff S Shamma. An overview of lpv systems. In *Control of linear parameter varying systems with applications*, pages 3–26. Springer, 2012.
- [18] Jeff S Shamma and James R Cloutier. Gain-scheduled missile autopilot design using linear parameter varying transformations. *Journal of Guidance, Control, and Dynamics*, 16(2):256–263, 1993.
- [19] Fengming Shi and Ron Patton. An active fault tolerant control approach to an offshore wind turbine model. *Renewable Energy*, 75:788–798, 2015.
- [20] Gang Zheng, Denis Efimov, and Wilfrid Perruquetti. Design of interval observer for a class of uncertain unobservable nonlinear systems. *Automatica*, 63:167–174, 2016.