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Interval observer design for Linear Parameter-Varying systems subject to component faults

R. LAMOUCI^{1,2} M. AMAIRI² T. RAÏSSI¹ and M. AOUN²

Abstract—In this paper an interval observer for Linear Parameter-Varying (LPV) systems is proposed. The considered systems are assumed to be subject to parameter uncertainties and component faults whose effect can be approximated by parameters deviations. Under some conditions, an interval observer with discrete-time Luenberger structure is developed to cope with uncertainties and faults ensuring guaranteed bounds on the estimated states and their stability. The interval observer design is based on assumption that the uncertainties and the faults magnitudes are considered as unknown but bounded. A numerical example shows the efficiency of the proposed technique.

Index Terms—LPV systems, Interval observers, Component faults, Parameter uncertainties, Stability

I. INTRODUCTION

Most of physical systems are nonlinear which leads to more complexity, especially when state estimation is required. In this case, observer design is usually based on the transformation of the system into canonical forms, which is difficult in practice [7]. This problem can be encountered in many industrial applications such as aircraft [1] [9], space vehicles [15] and wind turbine [19] [16]. To overcome this limitation, nonlinear systems can be represented by a Linear Parameter Varying (LPV) models. The main advantage of this representation is that the partial linearity of LPV models allows one to apply various frameworks developed for linear systems [10], [18], [17].

Systems are often affected by uncertainties (parameter, disturbances and noises), then the design of classical observers such as Luenberger observers [3], Kalman filter [4], is not easy to solve the estimation problem specially when the vector of scheduling parameters of LPV systems is not available for measurements. In such a case, set-membership approach can be considered as an alternative technique for robust estimation. Under some assumptions, interval observers can be used to compute the set of all the admissible values and provide certain lower and upper bounds for the estimate at each instant of time and in the presence of bounded uncertainties.

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Interval observers were introduced in [8] and extended and applied in many studies, such as [2], [11], [13], [14]. The case of LPV systems has been considered in several works. For instance, in [4] an interval observer design for discrete-time LPV systems has been developed assuming that the vector of scheduling parameters is not available for measurement. The case of known scheduling vector has been proposed in [6] using a static transformation of coordinates.

The methodology proposed in this paper consists in developing an interval observers for LPV systems subject to parameter uncertainties and component faults where the vector of scheduling parameter considered unknown but bounded. This so-called interval observer can be used for fault tolerant control scheme in order to handle faults effect.

This paper is organized as follows. Some preliminaries are given in Section II. Section III presents the problem statement. In Section IV, main results for designing the interval observer is developed. The efficiency of the proposed approach is illustrated through numerical examples in Section V. Finally, concluding remarks are given in Section VI.

II. PRELIMINARIES

A discrete-time dynamical system $x_{k+1} = f(x_k)$ is nonnegative if for any integer k_0 and any initial condition $x_{k_0} \geq 0$, the solution x satisfies $x_k \geq 0$ for all $k \geq k_0$.

A system described by

$$x_{k+1} = Ax_k + u_k,$$

with $x_k \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$, is nonnegative if and only if the matrix A is elementwise nonnegative, $u_k \geq 0$ and $x_{k_0} \geq 0$. In this case the system is also called cooperative.

A matrix $A \in \mathbb{R}^{n \times n}$ is Schur stable if all its eigenvalues have the modulus less than one and it is nonnegative if all its elements are nonnegative.

Given a matrix $A \in \mathbb{R}^{m \times n}$, define $A^+ = \max\{0, A\}$, $A^- = \max\{0, -A\}$ (similarly for vectors).

For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{R}^{n \times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood elementwise.

The symbol $|\cdot|$ denotes vector or corresponding induced matrix Euclidean norm. I and E_p denote respectively the $(n \times n)$ and the $(p \times 1)$ identity matrices.

For a measurable and locally essentially bounded input $u: \mathbb{N} \rightarrow \mathbb{R}$, the symbol $\|u\|_{[t_0, t_1]}$ denotes its \mathcal{L}_∞ -norm $\|u\|_{[t_0, t_1]} =$

$\sup\{|u_t|, t \in [t_0, t_1]\}$, $\|u\| = \|u\|_{[0, +\infty)}$. The set of all inputs u with the property $\|u\| < \infty$ is denoted by \mathcal{L}_∞ .

For a matrix $P = P^T$, the relation $P \succ 0$ means that P is positive definite. Let $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$, then, the inequality $|x+y|^2 \leq 2|x|^2 + 2|y|^2$ holds.

Lemma 1: [3]

Let $\underline{x}, x, \bar{x} \in \mathbb{R}^n$ if $\underline{x} \leq x \leq \bar{x}$ then

$$\underline{x}^+ \leq x^+ \leq \bar{x}^+ \text{ and } \bar{x}^- \leq x^- \leq \underline{x}^- \quad (1)$$

Similarly, let $\underline{A}, A, \bar{A} \in \mathbb{R}^{m \times n}$, if $\underline{A} \leq A \leq \bar{A}$ then

$$\underline{A}^+ \leq A^+ \leq \bar{A}^+ \text{ and } \bar{A}^- \leq A^- \leq \underline{A}^- \quad (2)$$

□

Lemma 2: [3] Let $x \in \mathbb{R}^n$ be a vector such that $\underline{x} \leq x \leq \bar{x}$ for some $\underline{x}, \bar{x} \in \mathbb{R}^n$.

1) If $A \in \mathbb{R}^{m \times n}$ is a constant matrix, then

$$A^+ \underline{x} - A^- \bar{x} \leq Ax \leq A^+ \bar{x} - A^- \underline{x} \quad (3)$$

2) If $A \in \mathbb{R}^{m \times n}$ is a matrix satisfying $\underline{A} \leq A \leq \bar{A}$ for some $\underline{A}, \bar{A} \in \mathbb{R}^{m \times n}$, then

$$\begin{aligned} \underline{A}^+ \underline{x}^+ - \bar{A}^- \underline{x}^- - \underline{A}^- \bar{x}^+ + \bar{A}^- \bar{x}^- &\leq Ax \\ &\leq \bar{A}^+ \bar{x}^+ - \underline{A}^+ \bar{x}^- - \bar{A}^- \underline{x}^+ + \underline{A}^- \underline{x}^- \end{aligned} \quad (4)$$

□

III. PROBLEM STATEMENT

Consider the following discrete LPV system:

$$\begin{cases} x_{k+1} = A(\rho, \eta)x_k + B(\eta)u_k \\ y_k = Cx_k + v_k \end{cases} \quad (5)$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^q$ is the input, $y_k \in \mathbb{R}^p$ is the output; v_k is bounded noise. $\eta \in \Xi$ denotes the vector of scheduling parameters considered unknown but bounded and only the set of admissible values Ξ is given. ρ is a component fault parameter vector, which is assumed to be in the set of admissible values Π .

In this paper it is assumed that the matrix $A(\rho, \eta)$ depends on η and ρ as

$$A(\rho, \eta) = A_0(\eta) + \rho_1 A_1(\eta) + \dots + \rho_r A_r(\eta) \quad (6)$$

where $\rho_i, i = 0, 1, \dots, r$ is the system fault parameter component and $A_i(\eta), i = 0, 1, \dots, r$ are affine matrices depending on η .

Two cases are considered: fault-free case ($\rho_i = 0$) and faulty case ($\exists i$ such that $\rho_i \neq 0$).

Equation (6) can be rewritten as

$$\begin{cases} A(\rho, \eta) = A_0(\eta) + \sum_{i=1}^r \rho_i A_i(\eta) & \text{if } \exists i, \rho_i \neq 0 \\ A(\rho, \eta) = A_0(\eta) & \text{if } \forall i, \rho_i = 0 \end{cases} \quad (7)$$

In the sequel, it is assumed that $A_0(\eta) = A_0 + \Delta A(\eta)$ and $B_0(\eta) = B_0 + \Delta B(\eta)$ with $\Delta A : \Xi \rightarrow \mathbb{R}^{n \times n}$ and $\Delta B : \Xi \rightarrow \mathbb{R}^{q \times q}$

are two known piecewise continuous matrix functions.

The following assumptions will be used in this work.

Assumption 1: $\underline{\Delta A} \leq \Delta A(\eta) \leq \bar{\Delta A}$, $\underline{A}_i \leq A_i(\eta) \leq \bar{A}_i \forall \eta \in \Xi$ for known $\underline{\Delta A}, \bar{\Delta A}, \underline{A}_i, \bar{A}_i \in \mathbb{R}^{n \times n}$. □

Assumption 2: $\underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i, \forall \rho \in \Pi$ for known $\underline{\rho}_i, \bar{\rho}_i \in \mathbb{R}^{n \times n}$. □

Assumption 3: $\underline{\Delta B} \leq \Delta B(\eta) \leq \bar{\Delta B} \forall \eta \in \Xi$ for known $\underline{\Delta B}, \bar{\Delta B} \in \mathbb{R}^{q \times q}$. □

Assumption 4: $\|v\| < V$, where V is a positive constant. □

Assumption 1 means that the matrix $\Delta A(\eta)$ belongs to the interval $[\underline{\Delta A}, \bar{\Delta A}]$ and the matrix $A_i(\eta)$ belongs to the interval $[\underline{A}_i, \bar{A}_i]$. The value of the scheduling vector η is not available for measurement but it is easy to compute $\underline{\Delta A}$ and $\bar{\Delta A}$ for a given set Ξ and a known function $\Delta A : \Xi \rightarrow \mathbb{R}^{n \times n}$. Assumption 2 states that the fault parameter magnitude ρ_i is unknown, but only its bounds $\underline{\rho}_i$ and $\bar{\rho}_i$ are given. Assumption 3 means that the matrix $\Delta B(\eta)$ belongs to the interval $[\underline{\Delta B}, \bar{\Delta B}]$. Finally, Assumption 4 means that the absolute value of the measurement noise v_k has a positive constant upper bound V .

In the faulty case, the system (5) can be written as

$$\begin{cases} x_{k+1} = [A_0 + \Delta A(\eta) + \sum_{i=1}^r \rho_i A_i(\eta)]x_k + [B_0 + \Delta B(\eta)]u_k \\ y_k = Cx_k + v_k \end{cases} \quad (8)$$

The objective of this paper is to design an interval observer for the LPV system (8) to cope with uncertainties and component faults ensuring guaranteed bounds on the estimated states and their stability. Since the system state is guaranteed to belong to the interval estimation, the interval observer stabilization yields the same property for the LPV system.

IV. INTERVAL OBSERVERS DESIGN

The LPV system (8) can be rewritten as

$$\begin{cases} x_{k+1} = A_0 x_k + \varphi(x_k) + \psi(x_k) + B_0 u_k + \phi(u_k) \\ y_k = Cx_k + v_k \end{cases} \quad (9)$$

with $\varphi(x_k) = \Delta A(\eta)x_k$, $\psi(x_k) = \sum_{i=1}^r \rho_i A_i(\eta)x_k$ and $\phi(u_k) = \Delta B(\eta)u_k$.

Interval observer design for (9), subject to uncertainties, requires the following assumption.

Assumption 5: The pair (A_0, C) is detectable and there exists a matrix gain $L \in \mathbb{R}^{n \times p}$ such that $A_0 - LC$ is nonnegative. □

Consider an observer structure for (9) given by

$$\begin{cases} \bar{x}_{k+1} = (A_0 - LC)\bar{x}_k + \bar{\varphi}(\bar{x}_k, \underline{x}_k) + \bar{\psi}(\bar{x}_k, \underline{x}_k) + B_0 u_k + \bar{\phi}(u_k) \\ \quad + Ly_k + |L|VE_p \\ \underline{x}_{k+1} = (A_0 - LC)\underline{x}_k + \underline{\varphi}(\bar{x}_k, \underline{x}_k) + \underline{\psi}(\bar{x}_k, \underline{x}_k) + B_0 u_k + \underline{\phi}(u_k) \\ \quad + Ly_k - |L|VE_p \end{cases} \quad (10)$$

where \bar{x}_k and \underline{x}_k are the upper and the lower bounds of the interval estimates of x_k and

$$\begin{cases} \bar{\varphi}(\bar{x}_k, \underline{x}_k) = \sum_{i=1}^r (\bar{H}_i^+ \bar{x}_k^+ - \underline{H}_i^+ \bar{x}_k^- - \bar{H}_i^- \underline{x}_k^+ + \underline{H}_i^- \underline{x}_k^-) \\ \underline{\varphi}(\bar{x}_k, \underline{x}_k) = \sum_{i=1}^r (\underline{H}_i^+ \underline{x}_k^+ - \bar{H}_i^+ \underline{x}_k^- - \underline{H}_i^- \bar{x}_k^+ + \bar{H}_i^- \bar{x}_k^-) \end{cases} \quad (11)$$

$$\begin{cases} \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \\ \underline{H}_i = \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \end{cases} \quad (12)$$

$$\begin{cases} \bar{\varphi}(\bar{x}_k, \underline{x}_k) = \bar{\Delta A}^+ \bar{x}^+ - \underline{\Delta A}^+ \bar{x}^- - \bar{\Delta A}^- \underline{x}^+ + \underline{\Delta A}^- \underline{x}^- \\ \underline{\varphi}(\bar{x}_k, \underline{x}_k) = \underline{\Delta A}^+ \underline{x}^+ - \bar{\Delta A}^+ \underline{x}^- - \underline{\Delta A}^- \bar{x}^+ + \bar{\Delta A}^- \bar{x}^- \end{cases} \quad (13)$$

$$\begin{cases} \bar{\phi}(u_k) = \bar{\Delta B} u_k^+ - \underline{\Delta B} u_k^- \\ \underline{\phi}(u_k) = \underline{\Delta B} u_k^+ - \bar{\Delta B} u_k^- \end{cases} \quad (14)$$

Introducing the estimation errors $\bar{e}_k = \bar{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$, it follows that

$$\begin{cases} \bar{e}_{k+1} = (A_0 - LC)\bar{e}_k + \bar{\Gamma}_k(\bar{x}_k, \underline{x}_k) \\ \underline{e}_{k+1} = (A_0 - LC)\underline{e}_k + \underline{\Gamma}_k(\bar{x}_k, \underline{x}_k) \end{cases} \quad (15)$$

with

$$\begin{cases} \bar{\Gamma}_k(\bar{x}_k, \underline{x}_k) = \bar{\psi}(\bar{x}_k, \underline{x}_k) - \psi(x_k) + \bar{\varphi}(\bar{x}_k, \underline{x}_k) - \varphi(x_k) \\ \quad + \bar{\phi}(u_k) - \phi(u_k) + |L|VE_p + Lv_k \\ \underline{\Gamma}_k(\bar{x}_k, \underline{x}_k) = \psi(x_k) - \underline{\psi}(\bar{x}_k, \underline{x}_k) + \varphi(x_k) - \underline{\varphi}(\bar{x}_k, \underline{x}_k) \\ \quad + \phi(u_k) - \underline{\phi}(u_k) + |L|VE_p - Lv_k \end{cases} \quad (16)$$

The functions $\bar{\Gamma}_k(\bar{x}_k, \underline{x}_k)$ and $\underline{\Gamma}_k(\bar{x}_k, \underline{x}_k)$ are globally Lipschitz, it follows that for $\underline{x}_k \leq x_k \leq \bar{x}_k$ and for a chosen submultiplicative norm $\|\cdot\|$, there exist positive constants a_1, a_2, a_3, b_1, b_2 and b_3 such that

$$\begin{cases} \|\bar{\Gamma}_k(\bar{x}_k, \underline{x}_k)\| \leq a_1 \|\bar{x}_k - x_k\| + a_2 \|\underline{x}_k - x_k\| + a_3 \\ \|\underline{\Gamma}_k(\bar{x}_k, \underline{x}_k)\| \leq b_1 \|\bar{x}_k - x_k\| + b_2 \|\underline{x}_k - x_k\| + b_3 \end{cases} \quad (17)$$

Theorem 1: Assume that Assumptions 1-4 are satisfied and $A_0 - LC$ is nonnegative and $x_k \in \mathcal{L}_\infty^n$. If the initial state x_0 verifies $\underline{x}_0 \leq x_0 \leq \bar{x}_0$, then the state x_k solution of (10) satisfies

$$\underline{x}_k \leq x_k \leq \bar{x}_k, \quad \forall k \in \mathbb{N} \quad (18)$$

In addition if there exist positive definite and symmetric matrices Q, P and W such that the Riccati matrix inequality

$$D^T P D - P + D^T P W^{-1} P D + \alpha(\|W + P\|)I + Q \leq 0 \quad (19)$$

is verified, where $D = A_0 - LC$ and $\alpha = 3 \max((a_1^2 + b_1^2), (a_2^2 + b_2^2))$, then $\underline{x}_k, \bar{x}_k \in \mathcal{L}_\infty^n$. \square

Proof:

According to Lemma 2 and Assumption 1, we have

$$\begin{aligned} \bar{\Delta A}^+ \underline{x}^+ - \bar{\Delta A}^+ \underline{x}^- - \underline{\Delta A}^- \bar{x}^+ + \bar{\Delta A}^- \bar{x}^- &\leq \varphi(x_k) \\ &\leq \bar{\Delta A}^+ \bar{x}^+ - \underline{\Delta A}^+ \bar{x}^- - \bar{\Delta A}^- \underline{x}^+ + \underline{\Delta A}^- \underline{x}^- \end{aligned} \quad (20)$$

According to Lemma 2, Assumption 1 and Assumption 2, we have

$$\begin{aligned} \underline{H}_i &= \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \leq H_i \\ &\leq \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \end{aligned} \quad (21)$$

It follows that

$$\begin{aligned} \sum_{i=1}^r (\underline{H}_i^+ \underline{x}_k^+ - \bar{H}_i^+ \underline{x}_k^- - \underline{H}_i^- \bar{x}_k^+ + \bar{H}_i^- \bar{x}_k^-) &\leq \psi(x_k) \\ &\leq \sum_{i=1}^r (\bar{H}_i^+ \bar{x}_k^+ - \underline{H}_i^+ \bar{x}_k^- - \bar{H}_i^- \underline{x}_k^+ + \underline{H}_i^- \underline{x}_k^-) \end{aligned} \quad (22)$$

According to Lemma 2 and Assumption 3, we have for any $u_k \in \mathbb{R}^q$

$$\underline{\Delta B} u_k^+ - \bar{\Delta B} u_k^- \leq \Delta B u_k \leq \bar{\Delta B} u_k^+ - \underline{\Delta B} u_k^- \quad (23)$$

Since $A_0 - LC$ is assumed to be nonnegative, and by construction $\bar{\Gamma}_k$ and $\underline{\Gamma}_k$ are positive, then the system (15) is cooperative. If \bar{x}_0 and \underline{x}_0 are chosen such that \bar{e}_0 and \underline{e}_0 are positive, the dynamics of interval estimation errors \bar{e}_k and \underline{e}_k stay positive for all $k \in \mathbb{N}$.

Let's show now that the variables $\bar{x}_k, \underline{x}_k$ stay bounded $\forall k \in \mathbb{N}$.

Consider the positive definite quadratic Lyapunov function:

$$V(\bar{e}_k, \underline{e}_k) = \bar{e}_k^T P \bar{e}_k + \underline{e}_k^T P \underline{e}_k \quad (24)$$

The increment of ΔV is given by

$$\begin{aligned} \Delta V &= V(\bar{e}_{k+1}, \underline{e}_{k+1}) - V(\bar{e}_k, \underline{e}_k) \\ &= \bar{e}_k^T (D^T P D - P) \bar{e}_k + 2\bar{e}_k^T D^T P \bar{\Gamma}_k + \bar{\Gamma}_k^T P \bar{\Gamma}_k \\ &\quad + \underline{e}_k^T (D^T P D - P) \underline{e}_k + 2\underline{e}_k^T D^T P \underline{\Gamma}_k + \underline{\Gamma}_k^T P \underline{\Gamma}_k \end{aligned} \quad (25)$$

From [20], we have the following inequalities:

$$\begin{aligned} 2\bar{e}_k^T D^T P \bar{\Gamma}_k &= 2\bar{e}_k^T D^T P W^{-0.5} W^{0.5} \bar{\Gamma}_k \\ &\leq \bar{e}_k^T D^T P W^{-1} P D \bar{e}_k + \bar{\Gamma}_k^T W \bar{\Gamma}_k \\ 2\underline{e}_k^T D^T P \underline{\Gamma}_k &= 2\underline{e}_k^T D^T P W^{-0.5} W^{0.5} \underline{\Gamma}_k \\ &\leq \underline{e}_k^T D^T P W^{-1} P D \underline{e}_k + \underline{\Gamma}_k^T W \underline{\Gamma}_k \end{aligned} \quad (26)$$

Using the inequalities (25) and (26), it yields

$$\begin{aligned}
\Delta V &\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + \bar{\Gamma}_k^T (W + P) \bar{\Gamma}_k + \underline{\Gamma}_k^T (W + P) \underline{\Gamma}_k \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + 3 \|W + P\| (a_1^2 \|\bar{e}_k\|^2 + a_2^2 \|\underline{e}_k\|^2 + a_3^2) \\
&+ 3 \|W + P\| (b_1^2 \|\bar{e}_k\|^2 + b_2^2 \|\underline{e}_k\|^2 + b_3^2) \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD) \bar{e}_k + \underline{e}_k^T (D^T PD - P \\
&+ D^T PW^{-1} PD) \underline{e}_k + \alpha \|W + P\| \bar{e}_k^T \bar{e}_k + \alpha \|W + P\| \underline{e}_k^T \underline{e}_k \\
&+ 3 \|W + P\| (a_3^2 + b_3^2) \\
&\leq \bar{e}_k^T (D^T PD - P + D^T PW^{-1} PD + \alpha \|W + P\| I) \bar{e}_k \\
&+ \underline{e}_k^T (D^T PD - P + D^T PW^{-1} PD + \alpha \|W + P\| I) \underline{e}_k \\
&+ 3 \|W + P\| (a_3^2 + b_3^2) \\
&\leq -\bar{e}_k^T Q \bar{e}_k - \underline{e}_k^T Q \underline{e}_k + 3 \|W + P\| (a_3^2 + b_3^2).
\end{aligned}$$

Using (19) we get $\Delta V \leq -\bar{e}_k^T Q \bar{e}_k - \underline{e}_k^T Q \underline{e}_k + \beta$ with $\beta = 3 \|W + P\| (a_3^2 + b_3^2)$ which provides the boundedness of the dynamics of estimation errors $\bar{e}_k, \underline{e}_k$, therefore the variables $\bar{x}_k, \underline{x}_k$ stay bounded $\forall k \in \mathbb{N}$. ■

It is worth to note that it is not always possible to compute a matrix L such that $A - LC$ is nonnegative. This restrictive condition can be relaxed by means of a change of coordinates $z_k = Rx_k$ with a nonsingular matrix R such that the matrix $E = R(A - LC)S$ is nonnegative where $S = R^{-1}$ [12],[5].

By introducing the change of coordinate $z_k = Rx_k$, the system (9) can be presented as

$$\begin{cases} z_{k+1} = Ez_k + \varphi(z_k) + \psi(z_k) + RB_0 u_k + \phi_z(u_k) \\ y_k = CSz_k + v_k \end{cases} \quad (27)$$

where $\varphi(z_k) = R\Delta A(\eta)S z_k$, $\psi(z_k) = \sum_{i=1}^r R\rho_i A_i(\eta)S z_k$ and $\phi_z(u_k) = R\Delta B u_k$.

An interval observer for the system (27) can be written in the new coordinates z as

$$\begin{cases} \bar{z}_{k+1} = E\bar{z}_k + RB_0 u_k + \bar{\varphi}(\bar{z}_k, \underline{z}_k) + \bar{\psi}(\bar{z}_k, \underline{z}_k) + \bar{\phi}_z(u_k) \\ \quad + RLY_k + |F|VE_p \\ \underline{z}_{k+1} = E\underline{z}_k + RB_0 u_k + \underline{\varphi}(\bar{z}_k, \underline{z}_k) + \underline{\psi}(\bar{z}_k, \underline{z}_k) + \underline{\phi}_z(u_k) \\ \quad + RLY_k - |F|VE_p \end{cases} \quad (28)$$

with

$$\begin{cases} \bar{\psi}(\bar{z}_k, \underline{z}_k) = (\bar{\sigma}^+ \bar{z}_k^+ - \underline{\sigma}^+ \bar{z}_k^- - \bar{\sigma}^- \underline{z}_k^+ + \underline{\sigma}^- \underline{z}_k^-) \\ \underline{\psi}(\bar{z}_k, \underline{z}_k) = (\underline{\sigma}^+ \bar{z}_k^+ - \bar{\sigma}^+ \bar{z}_k^- - \underline{\sigma}^- \underline{z}_k^+ + \bar{\sigma}^- \underline{z}_k^-) \end{cases} \quad (29)$$

$$\begin{cases} \underline{\sigma} = S^+(R^+ \underline{H}_i - R^- \bar{H}_i) - S^-(R^+ \bar{H}_i - R^- \underline{H}_i) \\ \bar{\sigma} = S^+(R^+ \bar{H}_i - R^- \underline{H}_i) - S^-(R^+ \underline{H}_i - R^- \bar{H}_i) \end{cases} \quad (30)$$

$$\begin{cases} \bar{H}_i = \bar{\rho}_i^+ \bar{A}_i^+ - \underline{\rho}_i^+ \bar{A}_i^- - \bar{\rho}_i^- \underline{A}_i^+ + \underline{\rho}_i^- \underline{A}_i^- \\ \underline{H}_i = \underline{\rho}_i^+ \underline{A}_i^+ - \bar{\rho}_i^+ \underline{A}_i^- - \underline{\rho}_i^- \bar{A}_i^+ + \bar{\rho}_i^- \bar{A}_i^- \end{cases} \quad (31)$$

$$\begin{cases} \bar{\varphi}(\bar{z}_k, \underline{z}_k) = (\bar{\Omega}^+ \bar{z}_k^+ - \underline{\Omega}^+ \bar{z}_k^- - \bar{\Omega}^- \underline{z}_k^+ + \underline{\Omega}^- \underline{z}_k^-) \\ \underline{\varphi}(\bar{z}_k, \underline{z}_k) = (\underline{\Omega}^+ \bar{z}_k^+ - \bar{\Omega}^+ \bar{z}_k^- - \underline{\Omega}^- \underline{z}_k^+ + \bar{\Omega}^- \underline{z}_k^-) \end{cases} \quad (32)$$

$$\begin{cases} \bar{\Omega} = S^+(R^+ \bar{\Delta A} - R^- \underline{\Delta A}) - S^-(R^+ \underline{\Delta A} - R^- \bar{\Delta A}) \\ \underline{\Omega} = S^+(R^+ \underline{\Delta A} - R^- \bar{\Delta A}) - S^-(R^+ \bar{\Delta A} - R^- \underline{\Delta A}) \end{cases} \quad (33)$$

$$\begin{cases} \bar{\phi}_z(u_k) = (R^+ \bar{\Delta B} - R^- \underline{\Delta B}) u_k^+ - (R^+ \underline{\Delta B} - R^- \bar{\Delta B}) u_k^- \\ \underline{\phi}_z(u_k) = (R^+ \underline{\Delta B} - R^- \bar{\Delta B}) u_k^+ - (R^+ \bar{\Delta B} - R^- \underline{\Delta B}) u_k^- \\ F = RL \end{cases} \quad (34)$$

Consider the dynamics of interval estimation errors $\bar{e}_k = \bar{z}_k - z_k$ and $\underline{e}_k = z_k - \underline{z}_k$, one can write

$$\begin{cases} \bar{e}_{k+1} = E\bar{e}_k + \bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) \\ \underline{e}_{k+1} = E\underline{e}_k + \underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) \end{cases} \quad (35)$$

with

$$\begin{cases} \bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) = \bar{\psi}(\bar{z}_k, \underline{z}_k) - \psi(z_k) + \bar{\varphi}(\bar{z}_k, \underline{z}_k) - R\varphi(z_k) \\ \quad + \bar{\phi}_z(u_k) - R\phi_z(u_k) + \bar{F}VE_p + RLv_k \\ \underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k) = \underline{\psi}(\bar{z}_k, \underline{z}_k) - \psi(z_k) + \underline{\varphi}(\bar{z}_k, \underline{z}_k) - \underline{\phi}_z(u_k) \\ \quad + R\phi_z(u_k) - \underline{\phi}_z(u_k) + \bar{F}VE_p - RLv_k. \end{cases} \quad (36)$$

$\bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)$ and $\underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)$ are globally Lipschitz, then for $\underline{z}_k \leq z_k \leq \bar{z}_k$ and for a chosen submultiplicative norm $\|\cdot\|$, there exist positive constants c_1, c_2, c_3, d_1, d_2 and d_3 such that

$$\begin{cases} \|\bar{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)\| \leq c_1 \|\bar{z}_k - z_k\| + c_2 \|\underline{z}_k - z_k\| + c_3 \\ \|\underline{\Gamma}_k^z(\bar{z}_k, \underline{z}_k)\| \leq d_1 \|\bar{z}_k - z_k\| + d_2 \|\underline{z}_k - z_k\| + d_3 \end{cases} \quad (37)$$

Theorem 2: Given a nonsingular matrix R such that the matrix $R(A - LC)S$ is nonnegative. Then, the solution of (28) satisfies

$$\underline{z}_k \leq z_k \leq \bar{z}_k, \quad \forall k \in \mathbb{N} \quad (38)$$

where $\underline{z}_0 \leq z_0 \leq \bar{z}_0$. In addition, if there exist positive definite and symmetric matrices Q, P and W such that the following Riccati matrix inequality is verified

$$E^T P E - P + E^T P W^{-1} P E + \alpha_z (\|W + P\|) I + Q \leq 0 \quad (39)$$

where $E = R(A_0 - LC)S$ and $\alpha_z = 3 \max((c_1^2 + d_1^2), (c_2^2 + d_2^2))$, then $\underline{z}_k, \bar{z}_k \in \mathcal{L}_\infty^n$. □

Proof:

The proof is similar to that of Theorem 1. ■

V. NUMERICAL SIMULATIONS

To illustrate the proposed methodology, let us consider the LPV system described by

$$\begin{cases} x_{k+1} = A(\eta, \rho)x_k + B_k(\eta) \\ y_k = Cx_k + v_k \end{cases} \quad (40)$$

For simulations, $A(\rho, \eta)$ is chosen as

$$A(\eta, \rho) = \begin{bmatrix} 0.3 + 2\eta\rho_1 & -0.7 + 0.5\eta + 0.5\eta\rho_2 \\ 0.6 + 0.2\eta & -0.5 + 0.1\rho_1 \end{bmatrix}$$

where $\rho = (\rho_1, \rho_2)^T$ is the fault parameter vector such that $|\rho_i| < 1, i = 1, 2$. The parameter η is considered unknown but bounded such that $\eta \in [0.04; 0.06]$, $C = [1 \ 0]$, $v_k = V \sin(k)$ and $V = 0.01$.

The system (40) can be rewritten as

$$\begin{cases} x_{k+1} = A_0x_k + \Delta A(\eta)x_k + \sum_{i=1}^2 \rho_i A_i(\eta)x_k + B_{0,k} + \Delta B_k(\eta) \\ y_k = Cx_k + v_k \end{cases} \quad (41)$$

$$\text{with } A_0 = \begin{bmatrix} 0.3 & -0.7 \\ 0.6 & -0.5 \end{bmatrix}, \Delta A(\eta) = \begin{bmatrix} 0 & 0.5\eta \\ 0.2\eta & 0 \end{bmatrix},$$

$$A_1(\eta) = \begin{bmatrix} 2\eta & 0 \\ 0 & 0.1 \end{bmatrix}, A_2(\eta) = \begin{bmatrix} 0 & 0.5\eta \\ 0 & 0 \end{bmatrix}$$

$$B_{0,k} = [\sin(0.1k) \quad \cos(0.2k)]^T$$

$$\Delta B_k(\eta) = \eta[\sin(0.5kx_{2,k}) \quad \sin(0.3k)]^T$$

where $\Delta B_k(\eta) \in [\underline{\Delta B}_k, \overline{\Delta B}_k]$.

For $L = [0.3 \ 0.6]^T$ the matrix $A_0 - LC$ is not nonnegative. Thus a transformation of coordinates,

$S = \begin{bmatrix} 0.609 & 0.814 \\ -1.162 & 0.581 \end{bmatrix}$ is used such that $E = R(A_0 - LC)S$, with $R = S^{-1}$, is nonnegative.

Consequently, the dynamic extension:

$$\begin{cases} \bar{z}_{k+1} = E\bar{z}_k + RB_{0,k} + \bar{\varphi}(\bar{z}_k, \underline{z}_k) + \bar{\psi}(\bar{z}_k, \underline{z}_k) + \bar{\phi}_k^z(\Delta B_k) \\ \quad + RLy_k + |F|VE_p \\ \underline{z}_{k+1} = E\underline{z}_k + RB_{0,k} + \underline{\varphi}(\bar{z}_k, \underline{z}_k) + \underline{\psi}(\bar{z}_k, \underline{z}_k) + \underline{\phi}_k^z(\Delta B_k) \\ \quad + RLy_k - |F|VE_p \end{cases} \quad (42)$$

with

$$\begin{cases} \bar{\phi}_k^z(\Delta B_k) = R^+\overline{\Delta B}_k - R^-\underline{\Delta B}_k \\ \underline{\phi}_k^z(\Delta B_k) = R^+\underline{\Delta B}_k - R^-\overline{\Delta B}_k \end{cases} \quad (43)$$

is an interval observer for the system (40).

The results of interval simulations are presented in Fig.1, Fig.2, Fig.3 and Fig.4, where the dashed lines correspond to the estimated lower and upper bounds and the continuous lines correspond to the actual state.

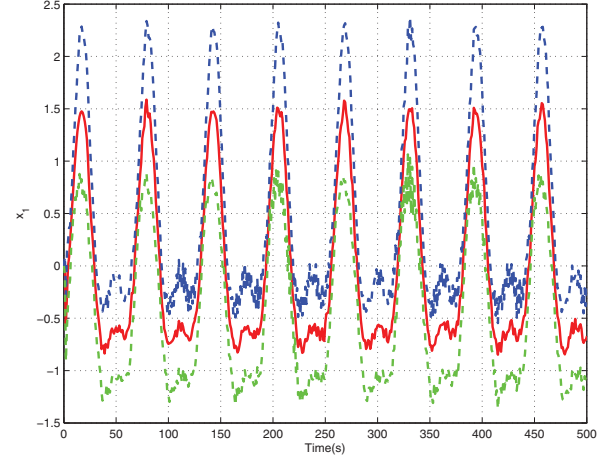


Fig. 1. Evolution of the first component of state x_1 without fault.

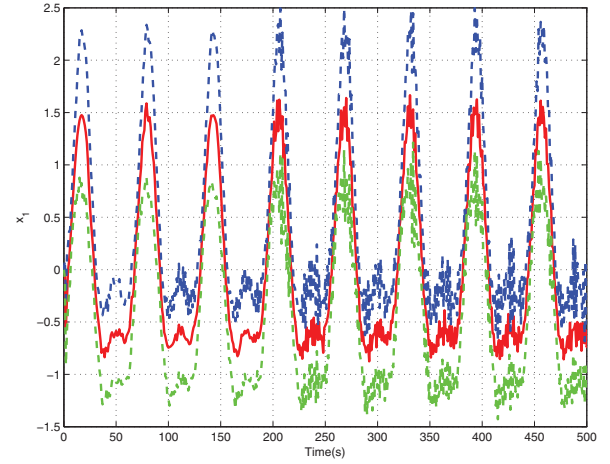


Fig. 2. Evolution of the first component of state x_1 with fault.

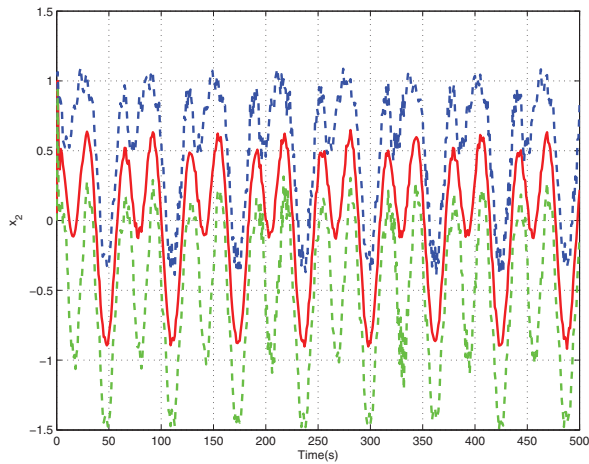


Fig. 3. Evolution of the second component of state x_2 without fault.

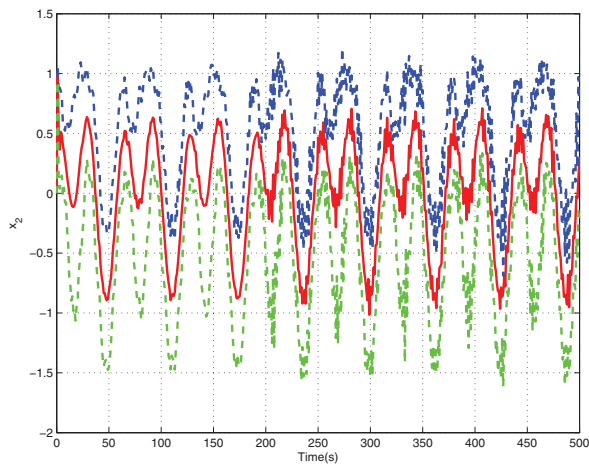


Fig. 4. Evolution of the second component of state x_2 with fault.

The faulty case is considered in the simulations, that is, before 200s, the system operates in a normal regime. At 200s, a fault occurs in the system.

The simulation results show that the upper and lower bounds of interval observer converge to a domain containing the actual state x_k .

VI. CONCLUSIONS

In this paper, an interval approach has been developed for the state estimation of LPV systems subject to uncertainties and component faults. An interval observer has been designed with a gain satisfying observation error positivity. A change of coordinates is used in order to make this methodology useful for a large class of LPV systems. A numerical example has been presented to illustrate the effectiveness of the approach. The proposed methodology will be used in the fields of Fault Tolerant Control in further works.

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