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# A Statistical Analysis of Multipath Interference for Impulse Radio UWB Systems

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## Abstract

In this paper, we develop a statistical characterization of the multipath interference in an Impulse Radio (IR)-UWB system, considering the standardized IEEE 802.15.4a channel model. In such systems, the chip length has to be carefully tuned as all the propagation paths located beyond this limit can cause interframe/intersymbol interferences (IFI/ISI). Our approach aims at computing the probability density function (PDF) of the power of all multipath components with delays larger than the chip time, so as to prevent such interferences. Exact analytical expressions are derived first for the probability that the chip length falls into a particular cluster of the multipath propagation model and for the statistics of the number of paths spread over several contiguous clusters. A power delay profile (PDP) approximation is then used to evaluate the total interference power as the problem appears to be mathematically intractable. Using the proposed closed-form expressions, and assuming minimal prior information on the channel state, a rapid update of the chip time value is enabled so as to control the signal to interference plus noise ratio.

## Index Terms

Impulse Radio, Time-Hopping, Chip duration, Multipath Interference, Statistical Analysis, IEEE 802.15.4a, Channel Model.

## I. INTRODUCTION

Since its approval by the U.S. Federal Communications Commission (FCC) in 2002, UWB has given rise to considerable interest in wireless communications research community, due to its many attractive properties [1]. Today, IR is considered as a promising technique from an industrial perspective [2]; in particular, it is a main candidate solution for applications such as wireless sensor networks [3] due to its ability to provide joint data transmission and precise positioning [4]. However, a number of IR-UWB system design challenges remain to be solved to ensure a broader use of that technology in practice [5]. The very fine time resolution in IR transmission coupled with the rich multipath diversity of the UWB channels need a careful

signal and architecture design to achieve good performances at reasonable complexity. The large path delay spread may require sophisticated signal processing algorithms at the receiver to cope with IFI/ISI. This can be avoided if we increase the minimum delay between two consecutive time hopping code values, that is to say if we decrease the chip rate (and thus the bit rate), or if we don't make use of consecutive code elements (corresponding to a time lag of one chip). The multiuser interference (MUI) has also to be taken into account in system design and some schemes limiting its impact on performances may be required. Considering the chips as adjacent channels available for multiple users, the main cause of interference between channels is the time spread of the channel, if it exceeds the chip duration. In particular, it is shown in [6] that performance of correlator-based TH-PPM and TH-BPSK receivers is severely degraded by the MUI. Thus, given a particular channel profile, any information on the power of interfering multipath components (MPCs) can provide valuable assistance either for evaluating the signal to interference plus noise ratio (SINR) and the performance in terms of bit error rate, or for adapting the system parameters to the propagation conditions. This problem has been investigated only in a limited number of papers. Approximate analytical expressions for Signal to ISI power ratio was first derived in [12] for various UWB transmission formats over two paths channels. In [13], a closed-form expression of the ISI/IFI energy at the output of a rake receiver is stated by considering the IEEE 802.15.3a statistical channel model [7] with one cluster. The influence of various parameters (time-hopping code, rake number of fingers, guard-time size) on the performance is conducted, hence enabling a pertinent design of the system. Even if IFI is an important issue, especially for high data rate UWB systems, [14] considers that its influence can eventually be neglected in BPSK or PPM time-hopping systems under certain conditions : a large number of frames per symbol is required and the considered scenario neglects ISI and multiple access (transmission of a single symbol for a single user). Gezici *et al.* established in [15] that there is a tradeoff between the pulse combining gain  $N_f$  and the pulse spreading gain  $N_c = T_f/T_c$  to get a minimal bit error probability in presence of timing jitter, for frequency-selective environments, as IFI is mitigated for larger values of  $N_c$ , the effect of timing jitter being mitigated by increasing  $N_f$ . A different approach is proposed in [16] to analyse the influence of channel and system parameters : by computing the first and second order moments of the received UWB pulses, the authors can express the intra-burst interference resulting from multipath propagation. In [17], the authors derive a few analytical expressions for IFI/ISI in pulsed direct sequence (DS) and hybrid DS/TH UWB communications, considering a frequency selective Nakagami fading channel. With standard Gaussian approximation of ISI/IFI, an expression of the error probability is given for a MRC rake combiner and the effects of various system parameters on the performances are studied.

In this paper, we seek to express the probability density function (PDF) of the power of the whole set of MPCs having delays exceeding the chip length, given a particular channel profile stemming from a prior identification step [11]. We consider in particular the statistical channel model adopted by the IEEE 802.15.4a Task Group<sup>1</sup> [9] to derive an analytical form of the PDF with some approximations. Proposed method may facilitate the characterization of a radio link in terms of SINR. Our theoretical analysis involves the following intermediate steps to achieve the interference power characterization : first, we derive the probability that the chip length falls into a particular cluster of the multipath propagation model; then, the

<sup>1</sup>The interest to consider this model is that it is widely studied by the research community, due to its ability to reflect propagation phenomena with acceptable statistical precision. However, there is still ongoing researches to improve this model such as [10] where the frequency dependence of the multipath components is investigated. Extensions of the present work could then be considered for future research, depending on the evolution of standards.

statistics of the number of paths in a cluster is expressed, which yields the statistics of the number of paths spread over several contiguous clusters; finally, we obtain an approximate relationship of the PDF of the power of all interfering MPCs using a simplified power delay profile expression. Due to dense multipath propagation and the non-stationary nature typical to UWB channels, the level of interference can change very quickly and an adaptation of some key parameters of the modulation may be required so as to control IFI/ISI (or multiuser interference) according to the environment in which the system is operating. Such possibility has been recently investigated in [18], where the frame duration is periodically updated depending on the channel state information and SINR, which are measured using training sequences. The development proposed here can serve as a tool for a rapid performance assessment of a TH UWB radio link; as only a LOS/NLOS detection is required as prior information, our result could possibly be applied in a non-data aided scenario.

The paper is organized as follows. Section II briefly recalls some properties of the IEEE 802.15.4a propagation channel and specifies the problem of statistical analysis of the interference power. Then, in section III, we derive the statistics of the first cluster matching the chip length and the number of MPCs is investigated in section IV. A power delay approximation then follows in section V and a procedure for interference power estimation is developed in section VI. A few numerical results are then discussed before some conclusions to confirm the pertinence of our theoretical analysis.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider here a Time-Hopping Binary Pulse Amplitude Modulation (BPAM) format, with the following typical expression of the transmitted signal :

$$s_{tx}(t) = \sqrt{\frac{E_s}{N_f}} \sum_{p=-\infty}^{\infty} d_{\lfloor p/N_f \rfloor} w_{tx}(t - pT_f - c_p T_c), \quad (1)$$

where  $E_s$  denotes the symbol energy and  $d_{\lfloor p/N_f \rfloor} \in \{-1, 1\}$  are the binary transmitted symbols,  $T_f$  stands for the average period of the pulse train (also known as the frame time), the  $c_p \in \{0, 1, \dots, N_c - 1\}$  represent pseudo-random code elements required both for code division multiple access and spectral shaping purposes, the quantification of temporal hops being controlled by the chip length  $T_c$ . The system is designed so that the unit energy pulse  $w_{tx}(t)$  is confined within duration  $T_c$  and we have  $T_f = N_c T_c$ . The indoor propagation channel usually involves multiple reflections due to the objects in the vicinity of receiver and transmitter; a classical way to characterize such propagation conditions is to make use of the Saleh-Valenzuela (SV) channel model [7] whose basic assumption is that MPCs arrive in clusters, with complex variations of the received power due to path loss, large scale fading and small scale fading. The corresponding discrete-time impulse response is usually expressed as

$$h(t) = \sum_{l=0}^L \sum_{k=0}^K a_{k,l} \exp(j\phi_{k,l}) \delta(t - T_l - \tau_{k,l}) \quad (2)$$

where  $a_{k,l}$  denotes the tap weight of the  $k$ -th component in the  $l$ -th cluster,  $\tau_{k,l}$  is the delay of the  $k$ -th MPC relative to the  $l$ -th cluster arrival time  $T_l$  and the phases  $\phi_{k,l}$  being uniformly distributed in the range  $[0, 2\pi]$ ;  $\delta(t)$  is the Dirac delta function.

In case the maximum delay spread of the channel is larger than the chip time, the transmitted signal corresponding to one pulse may overlap with signals in some next frames, as shown in Fig. 1, thus causing IFI/ISI. Our objective is to derive a closed

form expression of the power lying in time bins with delays beyond the chip length, with the aim of controlling the SINR. As will be seen in the sequel, the problem rapidly becomes intractable due to the complexity of the mathematical relations involved; a few approximations will then be proposed, in such a way that the SINR is overestimated.

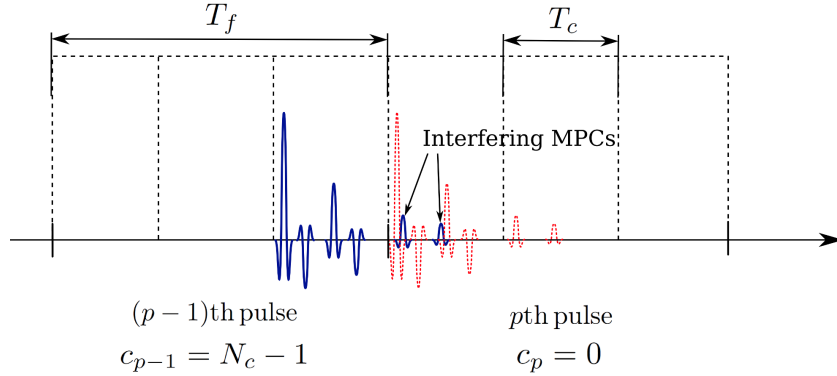


Figure 1. Interframe interference from the  $(p-1)$ th frame to the  $p$ th frame; in this example  $N_c = 3$  and the two consecutive pulses have a relative delay equal to  $T_c$ .

Many variations on the SV model have been proposed in the literature according to the considered environment, frequency band and transmission range. The popular IEEE 802.15.4a statistical channel model will be considered in the following, as it is close to a realistic channel; moreover, it is valid for UWB systems irrespective of their data rate and their modulation format. Based on measurements and simulations in various environments, the 802.15.4a model includes several improvements on previously proposed statistical models : a frequency dependent path gain is used, the number of clusters  $L$  is assumed to be Poisson distributed, ray arrival times are modeled via a mixture of Poisson processes and different shapes of power delay profiles (PDP) are assumed to better reflect the line-of-sight (LOS) or non-line-of-sight (NLOS) configurations.

### III. STATISTICS OF THE FIRST CLUSTER MATCHING THE CHIP LENGTH

In this section, we turn our attention to the probability  $P(T_c \in \mathcal{C}_\ell)$  that the chip length “falls into the  $\ell$ -th cluster”<sup>2</sup>  $\mathcal{C}_\ell = [T_{\ell-1}, T_\ell)$ , that is the probability that  $T_{\ell-1} \leq T_c \leq T_\ell$ . To achieve this goal, it is required to compute the PDF  $f_\ell(x) = \int_0^x f_\ell(x|T_0) f(T_0) dT_0$  of the cluster arrival time  $T_\ell$ , supposing that  $f(T_0) = \Lambda_0 \exp(-\Lambda_0 T_0)$ ,  $T_0 \geq 0$ . Using the expression  $T_\ell = \left(\sum_{j=1}^{\ell} \Delta T_j + T_0\right)$  and considering that distributions of the cluster arrival times are given by a Poisson processes with cluster arrival rate  $\Lambda$ , i.e.  $f_{\Delta T_\ell}(x) = \Lambda \exp(-\Lambda x)$  where  $\Delta T_\ell = T_\ell - T_{\ell-1}$ , we get

$$f_\ell(x) = \frac{\Lambda_0 \Lambda^\ell \exp(-\Lambda x)}{(\ell-1)!} \int_0^x (x-t)^{\ell-1} \exp(-(\Lambda_0 - \Lambda)t) dt. \quad (3)$$

Then, by observing that integral of the form  $I_{l,b}(x) = \int_0^x (x-t)^l \exp(-bt) dt$  satisfies the recursive relation<sup>3</sup>  $I_{l,b}(x) = \frac{x^l}{b} - \frac{l}{b} I_{l-1,b}(x)$  with  $b = \Lambda_0 - \Lambda$ , we can easily show that

$$f_\ell(x) = \frac{\Lambda_0 \Lambda^\ell \exp(-\Lambda x)}{(\ell-1)!} \frac{(-1)^{\ell-1} (\ell-1)!}{(\Lambda_0 - \Lambda)^\ell} \quad (4)$$

<sup>2</sup>In the sequel, the index  $\ell$  will refer to the cluster containing the chip length value whereas index  $l$  will be used for any other cluster.

<sup>3</sup>This relation results from the integration by parts  $\int_0^x u'v = [uv]_0^x - \int_0^x uv'$ , with  $u' = \exp(-bt)$  and  $v = (x-t)^l$ .

$$\times \left( \sum_{i=0}^{\ell-1} \frac{(-bx)^{\ell-1-i}}{(\ell-1-i)!} - \exp(-(\Lambda_0 - \Lambda)x) \right)$$

Before computing the probability of interest  $P(T_c \in \mathcal{C}_\ell)$ , let us define  $z = T_c$ ,  $x = T_{\ell-1}$  and  $y = \Delta T_\ell$  to simplify the mathematical notations. Then, we can rewrite  $P(T_c \in \mathcal{C}_\ell)$  as

$$\begin{aligned} P(z \in \mathcal{C}_\ell) &= \mathbf{P}(z - y < x \leq z) \\ &= \int_0^z \left( \int_{z-y}^z f_\ell(x) dx \right) \Lambda \exp(-\Lambda y) dy \\ &\quad + \int_z^\infty \left( \int_0^z f_\ell(x) dx \right) \Lambda \exp(-\Lambda y) dy \end{aligned} \quad (5)$$

To proceed further, it is required to express integral of the type  $J_{m,\Lambda}(T_c) = \int_{T_c}^\infty x^m \exp(-\Lambda x) dx$  and  $\bar{J}_{m,\Lambda}(T_c) = \int_0^{T_c} x^m \exp(-\Lambda x) dx$ , where  $\Lambda \in \mathbb{R}_+^*$  and  $m \in \mathbb{N}$ . It can be easily verified that  $J_{m,\Lambda}(T_c) + \bar{J}_{m,\Lambda}(T_c) = \frac{m!}{\Lambda^{m+1}}$  and  $J_{m,\Lambda}(T_c) = \frac{\exp(-\Lambda T_c)}{\Lambda^{m+1}} \sum_{p=0}^m \frac{m!(T_c \Lambda)^p}{p!}$ .

Finally, from the relations above we obtain the following result after a few mathematical calculations :

$$\mathbf{P}(T_c \in \mathcal{C}_\ell) = \frac{\Lambda_0 \Lambda^\ell}{(\Lambda - \Lambda_0)^{\ell+1}} e^{-\Lambda T_c} R_\ell(-(\Lambda_0 - \Lambda) T_c) \quad (6)$$

where  $R_\ell(u)$  stands for the rest of the  $\ell$ -th order Taylor series expansion of the function  $e^u$ , that is

$$R_\ell(u) = e^u - \sum_{i=0}^{\ell} \frac{u^i}{i!} \quad (7)$$

Note that we assumed throughout the previous development that  $T_c$  is larger than the first cluster delay  $T_0$ . In case we have  $T_c < T_0$ , the interference corresponds to all the received pulse power. This event has the probability  $\mathbf{P}(\{T_c < T_0\}) = e^{-\Lambda_0 T_c}$ .

#### IV. ON THE NUMBER OF INTERFERING MPCs

This section is devoted to the estimation of the number of MPCs that can cause interference, that is the overall number of components located beyond the chip length. So, for any cluster index  $l \in \mathbb{N}$ , we need to compute first the probability  $P(n_i)$  for each subsequent cluster  $\mathcal{C}_i$ ,  $l+1 \leq i \leq n$ , where  $n_i$  denotes the number of MPCs belonging to the  $i$ -th cluster. The distribution associated to the whole set of interfering clusters will then be easily obtained in a second step.

In order to simplify our development, we will consider the approximation that there is no inter-cluster interference; hence the delay  $\tau_{m,i}$  of the  $m$ -th MPC relative to the  $i$ -th cluster arrival time  $T_{i-1}$  belongs to the interval  $[T_{i-1}, T_i]$ . A second simplification is assumed for the ray arrival times; from [9] we know that they can be modelled with mixtures of two Poisson processes with mixture probability  $\beta$  and arrival rates  $\{\lambda_1, \lambda_2\}$  being determined experimentally for various environments. As in the classical SV model, we will adopt a Poisson process for the ray arrival times, so that the distribution of the delay difference  $\tau_{l+1,i} - \tau_{l,i}$  of any two adjacent MPCs in cluster  $\mathcal{C}_i$  takes the form  $f(\tau_{l+1,i} - \tau_{l,i}) = \lambda \exp(-\lambda(\tau_{l+1,i} - \tau_{l,i}))$ , where the value of the parameter  $\lambda$  is computed by minimizing the mean squared error between simplified and original models, for a given radio environment (known values for  $\beta$ ,  $\lambda_1$ , and  $\lambda_2$ ).

For any cluster  $\mathcal{C}_i$ , it can be shown (see Appendix) that the probability  $P(n_i)$  that the number of MPCs in the cluster is  $n_i$ , is

$$P(n_i) = \frac{\lambda^{n_i} \Lambda}{(\lambda + \Lambda)^{n_i+1}}, \quad n_i \geq 0 \quad (8)$$

Then, we can characterize the number of MPCs within clusters  $\{\mathcal{C}_i; i = \ell, \ell + 1, \dots, L\}$ , where  $L$  is the total number of clusters and supposing that  $T_c \in \mathcal{C}_\ell$ . This can be achieved by summing  $r + 1$  independent and identically discrete random variables, each with probability (8), where  $r = L - \ell$ . The resulting probability  $P_r(n)$  can be computed by recursion<sup>4</sup>, starting with  $P_1(n) = \sum_{k=0}^n P_0(k) P_0(n-k)$  and  $P_0(n)$  being expressed as in (8) :

$$P_r(n) = \frac{\lambda^n}{(\lambda + \Lambda)^n} \left( \frac{\Lambda}{(\lambda + \Lambda)} \right)^{r+1} \frac{(n+r)!}{n!r!}. \quad (9)$$

## V. POWER DELAY PROFILE APPROXIMATION

The Power Delay Profile (PDP) is defined as the squared magnitude of the channel impulse response, averaged over the small-scale fading. In the frame of IEEE 802.15.4a it is expressed as

$$E \left\{ |a_{k,l}|^2 \right\} \propto \Omega_l \exp(-\tau_{k,l}/\gamma_l) \quad (10)$$

where the integrated energy  $\Omega_l$  over the  $l$ th cluster follows an exponential decay and the intra-cluster decay time constant  $\gamma_l$  depends linearly on the arrival time of the cluster. This model makes the closed-form derivation of the interfering power estimate intractable; so we will adopt the following first approximation for the PDP :

$$\Omega(t) = \Omega_c \exp\left(-\frac{t}{\Gamma}\right) \quad (11)$$

where  $t$ ,  $\Omega_c$  and  $\Gamma$  denote the path delay, the integrated energy of the cluster and the intra-cluster decay time constant, respectively.

Concerning the small-scale fading, it can be shown that the  $i$ -th path relative to the  $l$ -th cluster has its power distributed as

$$f(x|\Omega_{il}) = \frac{1}{\Gamma(m)} \left( \frac{m}{\Omega_{il}} \right)^m x^{m-1} \exp\left(-\frac{mx}{\Omega_{il}}\right), \quad (12)$$

where  $\Omega_{il}$  stands for the PDP of the considered path,  $m$  is the Nakagami  $m$ -factor of the small-scale amplitude distribution and  $\Gamma(m)$  is the gamma function.

For CM1/CM2 channel models, the above relation can be simplified by considering a mean value<sup>5</sup>  $m \cong 2$  for the parameter  $m$ ; in this case  $\Gamma(m) = 1$  and we get

$$f(x|\Omega_{il}) = \left( \frac{2}{\Omega_{il}} \right)^2 x \cdot \exp\left(-\frac{2x}{\Omega_{il}}\right), \quad (13)$$

<sup>4</sup>Note that this probability depends on the number  $(r + 1)$  of clusters beyond  $\mathcal{C}_\ell$ , with  $T_c \in \mathcal{C}_\ell$ .

<sup>5</sup>As the random variable  $m$  follows a lognormal distribution, the mean value is expressed as  $\mu_m = \exp(m_0 + \hat{m}_0^2/2)$ , where the parameters  $(m_0, \hat{m}_0)$  have specified values depending on the considered environment [9].

which represents a  $\Gamma(\alpha, \theta)$  distribution with  $\alpha = 2$  and  $\theta = \frac{\Omega_{ii}}{2}$ .

Now, we need to express the mean value of the PDP corresponding to the whole set of MPCs located beyond the chip length

:

$$\Omega_0 = E[\Omega_{T_c}(\tau_i)] \quad (14)$$

where  $\Omega_{T_c}(\tau_i) = \Omega(\tau_i)$ , for  $\tau_i \geq T_c$  and zero elsewhere,  $\tau_i$  denoting the path delay. We propose to derive an approximate expression of  $\Omega_0$  by picking  $n$  uniformly spaced samples of the original distribution (11) in the interval  $[T_c, T_L]$ , where  $T_L$  corresponds to the upper bound of the  $L$ -th cluster :

$$\Omega_0(L) = \exp\left(-\frac{T_c}{\Gamma}\right) \frac{1}{n} \sum_{i=0}^{n-1} \left[ \exp\left(\frac{T_c}{\Gamma}\right) \exp\left(-\frac{T_N}{\Gamma}\right) \right]^{i/n}, \quad (15)$$

which has the equivalent form

$$\Omega_0(L) = \frac{1}{n} \frac{g_L - g_0}{(g_L/g_0)^{1/n} - 1}, \quad (16)$$

where  $g_0 = \exp(-T_c/\Gamma)$  and  $g_L = \exp(-T_L/\Gamma)$ .

An additional simplification can be achieved for sufficiently large value of  $n$  by considering that

$$\left[ (g_L/g_0)^{1/n} - 1 \right] / (1/n) \simeq \ln(g_L/g_0) \quad (17)$$

Hence, we finally obtain the following approximation for the PDP :

$$\Omega_0(L) \cong \begin{cases} \Gamma [\exp(-T_c/\Gamma) - \exp(-T_L/\Gamma)] \\ \quad / (T_L - T_c), & T_L > T_c, \\ \exp(-T_c/\Gamma), & T_L = T_c. \end{cases} \quad (18)$$

As can be seen in Fig. 2, considering (17) generally gives a good approximation of (16), for various values of  $L$  and  $T_c$ , with an overestimation of the true value. Concerning the number of clusters, it can be assumed a Poisson distribution [9]

$$P_L(L) = \frac{\bar{L}^L e^{-\bar{L}}}{L!}, \quad (19)$$

where  $\bar{L}$  denotes the average number of clusters.

We know also that the last cluster delay has its PDF given by  $f(T_L) = \frac{T_L^{L-1} \Lambda^L \exp(-\Lambda T_L)}{(L-1)!}$ , hence

$$f_{T_L}(T_L | T_L \geq T_c) = \begin{cases} \frac{T_L^{L-1} \exp(-\Lambda T_L)}{\left( \int_{T_c}^{\infty} t^{L-1} \exp(-\Lambda t) dt \right)}, & T_L \geq T_c, \\ 0, & T_L < T_c. \end{cases} \quad (20)$$

From this last equation, we can then derive a closed form expression of the mean PDP, that will be used in the next section, devoted to interference power estimation :

$$\bar{\Omega}_0(L) = E[\Omega_0(L)]$$



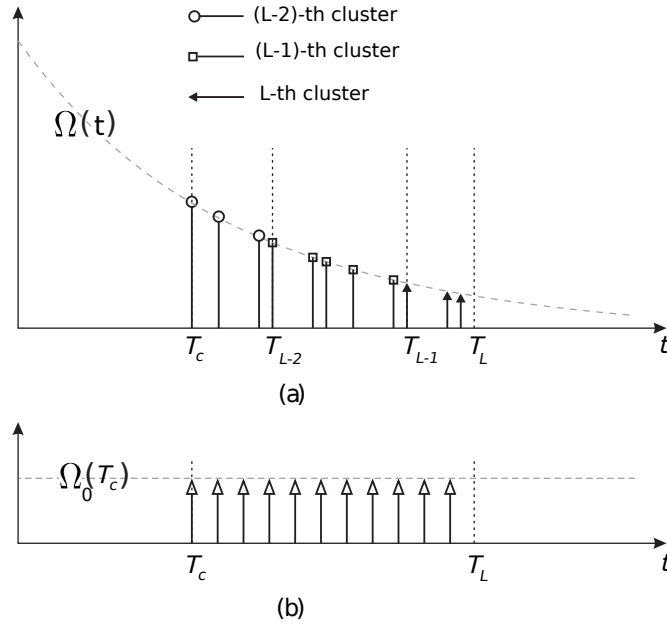


FIGURE 2. Proposed approximation of the PDP

$$\cong \left( \Gamma \int_{T_c}^{\infty} \frac{\exp(-T_c/\Gamma) - \exp(-x/\Gamma)}{x - T_c} x^{L-1} \exp(-\Lambda x) dx \right) \quad (21)$$

$$/ \left( \int_{T_c}^{\infty} t^{L-1} \exp(-\Lambda t) dt \right)$$

## VI. INTERFERENCE POWER ESTIMATION

To begin with, let us recapitulate below the intermediate results derived until now :

- first, we derived in section III the probability that the chip length “falls into a particular cluster” of the multipath propagation model;
- then, the statistics of the number of paths in any cluster was expressed in section IV, together with the statistics of the total number of paths located beyond the chip duration;
- a simplified power delay profile expression has then be derived in section V.

We are now going to derive an approximate relationship of the PDF of the power of all interfering MPCs. Our approach relies on the summation of a number  $n$  of r.v. each following the PDF (13), with the factor  $\theta = \Omega_0(L)$  computed as (21). Hence, for a given number of clusters  $N$ , the resulting PDF is expressed as, for  $x > 0$ ,

$$g(x | \Omega_0(L)) = \sum_{n=1}^{\infty} g(x | \Omega_0(L), n) \cdot \mathbf{P}(n|L) \quad (22)$$

where

$$g(x | \Omega_0(L), n) = \left( \frac{2}{\Omega_0(L)} \right)^{2n} \frac{x^{2n-1}}{(2n-1)!} \cdot \exp\left(-\frac{2x}{\Omega_0(L)}\right) \quad (23)$$

and the number of paths with delays exceeding  $T_c$  being determined as (under the assumption that each cluster contains at least one path)

$$P(n|L) = \sum_{k=0}^{L-1} P(n|k, L) \cdot P(k), \quad (24)$$

The above expression can be computed by taking advantage of previous developments; for example, in case of a LOS environment (and considering that  $T_0 = 0$ ), we get

$$P(n|k, L) = \left( \frac{\lambda}{\lambda + \Lambda} \right)^{n-L+k} \left( \frac{\Lambda}{\lambda + \Lambda} \right)^{L-k} \times \frac{(n-1)!}{(n-L+k)!(L-k-1)!}, \quad \forall n \geq 1 \quad (25)$$

and

$$P(k) = \frac{(\Lambda T_c)^k \exp(-\Lambda T_c)}{k!}. \quad (26)$$

To complete our analysis, we need also to consider the case  $x = 0$ , which results in a discrete part of the PDF :

$$\begin{aligned} g(x=0 | \Omega_0(L)) &= P(n=0|L) \\ &= \sum_{k=L}^{\infty} P(k) \end{aligned} \quad (27)$$

And finally, the interference power is obtained as

$$g(x) = \sum_{L=1}^{\infty} g(x | \Omega_0(L)) P_L(L). \quad (28)$$

No compact analytical expression of this PDF can be obtained due to the high complexity of the terms involved; however, we can observe that the proposed result can easily be implemented to get an estimate at very limited computational cost. Note also that our development remains valid for both LOS and NLOS environments; in the latter case, the probability that the chip length falls into the  $k$ th cluster has the alternative expression

$$P(k) = \frac{(\Lambda T_c)^{k+1} \exp(-\Lambda T_c)}{(k+1)!}. \quad (29)$$

The proposed algorithm for interference power estimation is summarized in the form of a block diagram in Fig. (3).

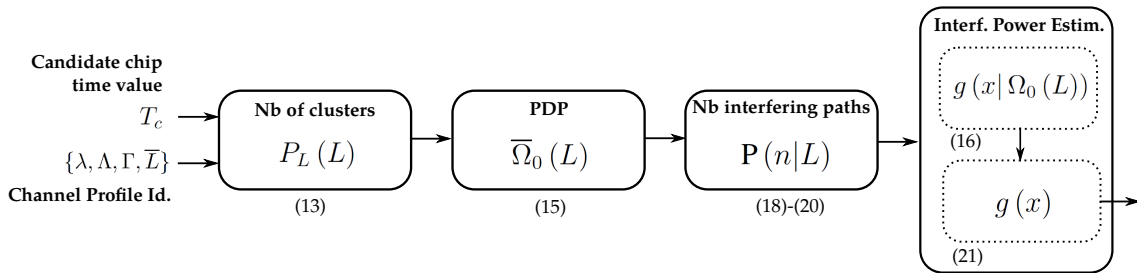


FIGURE 3. Block diagram of the proposed algorithm for interference power estimation

## VII. SIMULATION RESULTS

As mentioned before, our theoretical analysis involves a few approximations to get the statistics of the interference power, due to the high complexity of some mathematical relations :

- 1) A Poisson process is adopted for modelling the ray inter-arrival times, instead of a mixture of two Poisson processes as recommended in the frame of the IEEE 802.15.4a channel model;
- 2) A simplified exponential expression (11) is considered for the mean power of the different paths (PDP); we do not take into account the possible different values of the integrated energy and decay time constant for distinct clusters;
- 3) The computation of the mean PDP associated to MPCs located beyond the chip duration is achieved owing to a simple sampling scheme.

A great number of Monte-Carlo simulations have been conducted using the Matlab implementation [8] of the IEEE 802.15.4a channel model to verify the pertinence of our approach. A few illustrations are given hereafter to show the impact of various approximations on the statistics. The case of CM1 channel model is considered here, but it should be noticed that the same procedure could be applied in another radio environment. Firstly, we can examine the error resulting from the proposed uniform sampling for the computation of the mean PDP (17). As can be seen in Fig. 4, the computed value of  $\bar{\Omega}_0(L)$  tends to match the true value when the number of clusters  $L$  increases and the error decreases for larger chip length. Also, it can be clearly observed that our approximation yields an overestimation of the mean PDP, which is particularly important from the point of view of applications.

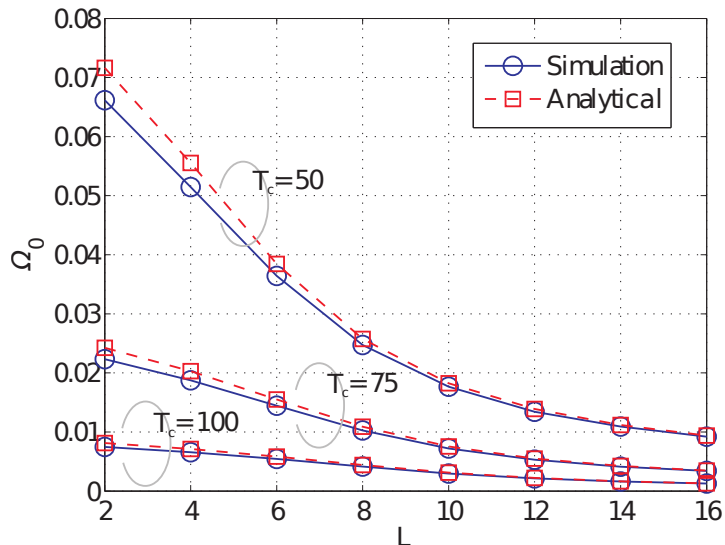


FIGURE 4. Mean value of  $\Omega_0(L)$  for various chip length values (CM1 channel model;  $10^4$  MC simulations)

We can now consider the PDF (24) of the number of paths with delays larger than the chip length, which plays a central role in estimating the interference power. As shown in Fig. 5, there is almost a perfect match between the values obtained through simulations and the values obtained by numerical evaluation of analytical expressions. Evidently, it can be seen that the numbers of MPCs increases with the number of clusters. In order to assess the goodness of fit of our statistical model of the number of paths with delays exceeding  $T_c$ , we applied the one-sample Kolmogorov-Smirnov (K-S) test [20] which consists in

measuring the deviation of the observed Cumulative Distribution Function (CDF) from the hypothesized CDF (corresponding to our model); the case of CM1 with  $T_c = 50$  was chosen as it corresponds to the most difficult case that we considered in our study. As can be clearly observed in Fig. 6, plotting the p-values of the K-S test against the number of clusters  $L$  (for a significance level  $\alpha$  of 0.05), the hypothesis that the simulated data are drawn from the proposed model is adopted.

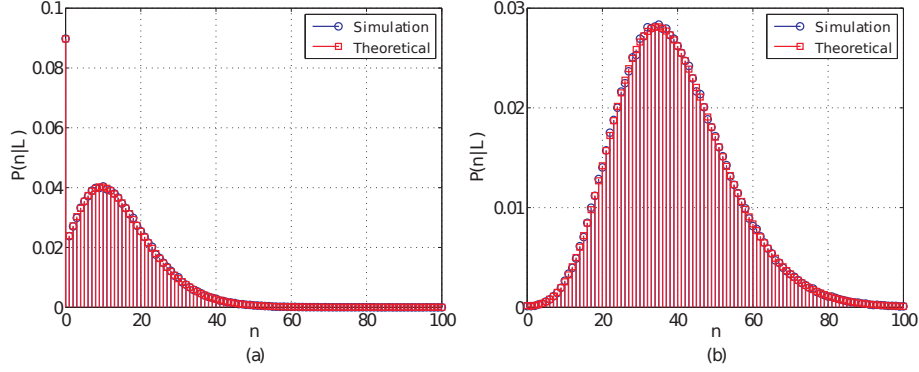


FIGURE 5. PDF of the number of paths with delays exceeding the chip length  $T_c = 50$  (CM1 channel model;  $10^5$  MC runs) : (a)  $L = 5$  clusters (b)  $L = 10$  clusters

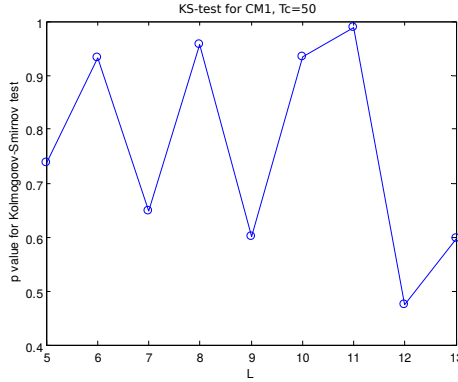


FIGURE 6. Kolmogorov-Smirnov test for evaluating the distance between our statistical model (24) and the distribution resulting from simulations.

Figure (7) illustrates the PDF  $g(x|\Omega_0(L))$  resulting from summing  $n$  r.v., each one following (13), once the mean PDP associated to interfering MPCs has been computed. Then, Fig. 8 shows the PDF of the interference power for two distinct chip time values. The difference between the “true” distribution (estimated through MC simulations) and the PDF derived from our method comes from the approximations 2) and 3) explained above. However, this limited statistical precision is not an obstacle for practical applications since a low error is noticed for the two first moments of the distribution (as will be shown in a next figure).

Additional simulations have been conducted in NLOS radio environments to evaluate the pertinence of the proposed algorithm. For CM2/CM4 channels (NLOS residential/office), we obtained the PDFs depicted in Fig. 9 & 10, for the same candidate values of the chip time.

First and second order moments have been computed for various channel models and different  $T_c$  (Fig. 11). Overall, the agreement is rather good between our model and the simulated data (provided that  $T_c$  is not too small) and we can note that the two moments are in all cases overestimated; hence, the proposed approach can be very helpful for decreasing the SINR.

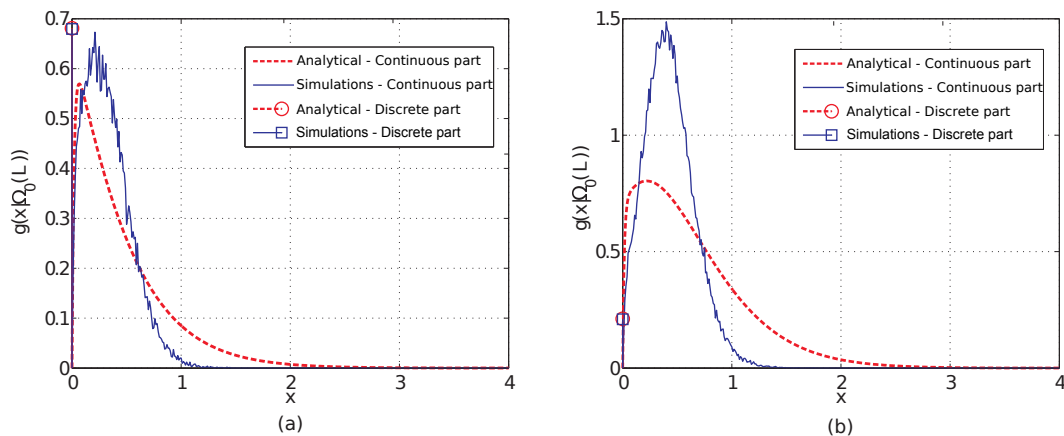


FIGURE 7. PDF  $g(x|\Omega_0(L))$  for CM1 channel model,  $T_c = 50$  : (a)  $L = 2$  clusters (b)  $L = 4$  clusters

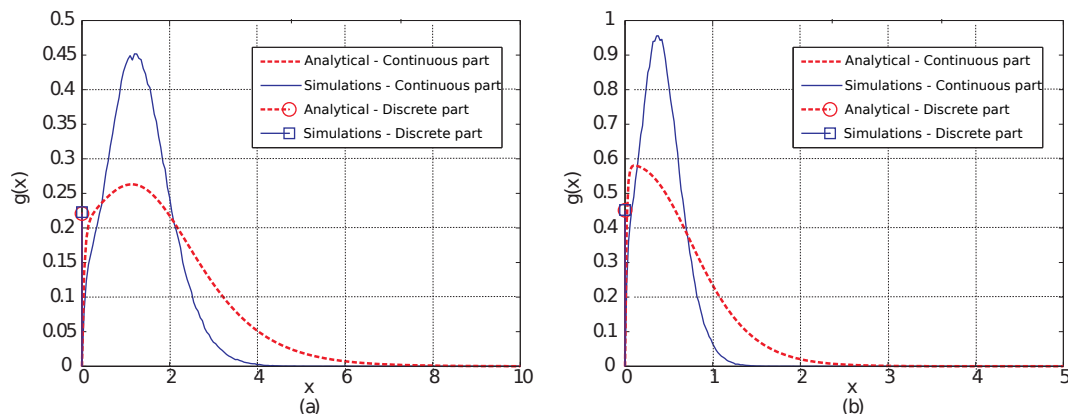


FIGURE 8. PDF of the interference power  $g(x)$  for CM1 channel model ( $10^5$  MC runs) : (a)  $T_c = 25$  (b)  $T_c = 50$

## VIII. CONCLUSIONS

A novel approach for the statistical characterization of the multipath interference in IR-UWB systems has been developed in this paper. In the frame of the IEEE 802.15.4a channel model, we derived a theoretical analysis which aims to compute the power corresponding to all multipath components located beyond the chip length. Since the proposed approach requires minimal knowledge on the channel state, it can be helpful for real time adaptation of modulation parameters so as to limit the signal to interference plus noise ratio. Our method relies on the following key developments : first, we derived the probability that the chip length falls into a particular cluster of the multipath propagation model; then, the statistics of the number of paths spread over several contiguous clusters have been computed in closed-form; finally, we obtained an approximate relationship of the PDF of the power of all interfering MPCs using a simplified power delay profile expression. Numerous Monte-Carlo simulations have been carried out to verify the pertinence of our results; although these simulations revealed a significant gap between the true interference power distribution and that obtained through our approach, the first two computed moments are very close and upper-bound the true ones. This is a very useful result because they represent an information that helps us to control the SINR level and to ensure an effective functioning of the IR-UWB system in real-world scenarios. The proposed approach is rather general and could actually be implemented for another channel model (however, a specific study is required depending on the channel model considered). Different application scenarios of the proposed algorithm could be imagined; for

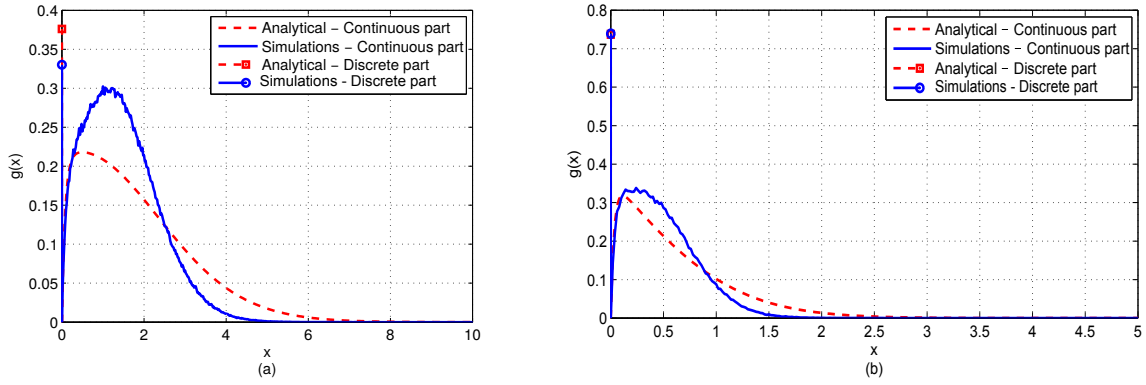


FIGURE 9. PDF of the interference power  $g(x)$  for CM2 channel model ( $10^5$ MC runs) : (a)  $T_c = 25$  (b)  $T_c = 50$

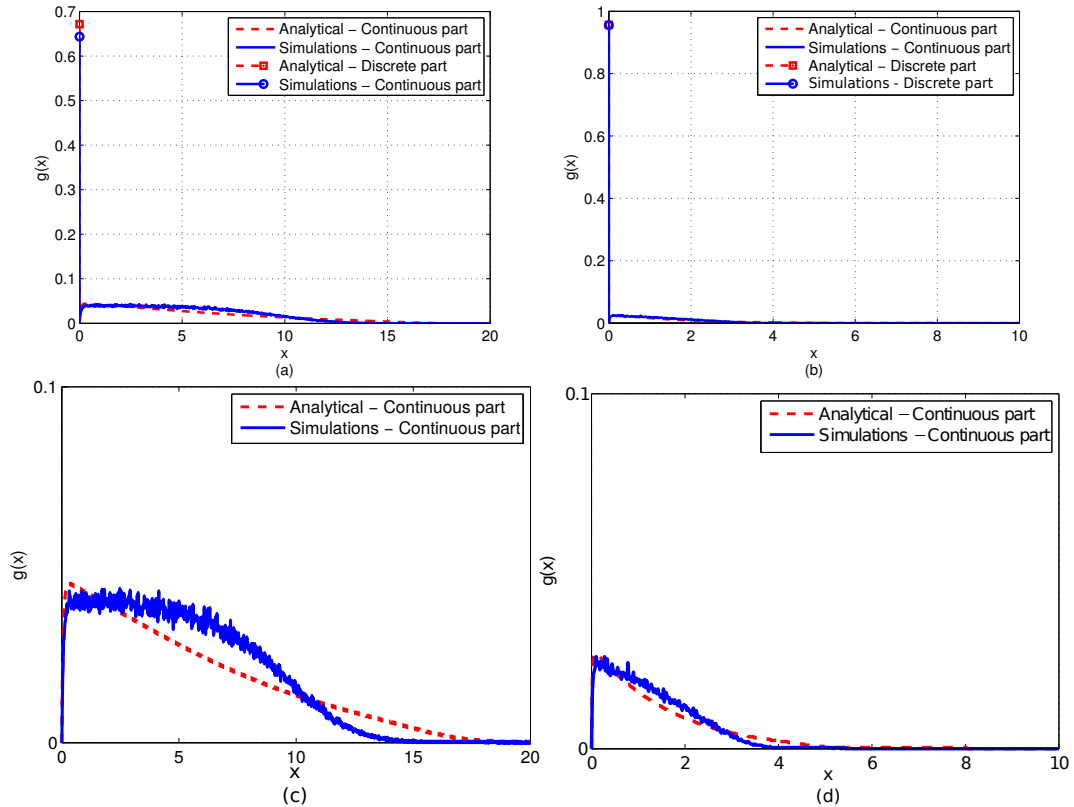


FIGURE 10. PDF of the interference power  $g(x)$  for CM4 channel model ( $10^5$ MC runs) : (a)  $T_c = 25$  (b)  $T_c = 50$ ; closeups of continuous part of the PDFs are shown in (c) and (d) for  $T_c = 25$  and  $T_c = 50$ , respectively.

example, key parameters associated to the channel could be identified based on some received pilot symbols statistics with a default chip time value known by the receiver. Data symbols could then be transmitted with an “optimized” chip time duration depending on the channel state information sent to the transmitter; a mapping between channel profiles and chip time value could be designed and implemented using look-up-tables.

## APPENDIX

### Appendix A - Proof of (8)

For any cluster  $\mathcal{C}_i$ , let us define the following notations:  $x = \tau_{n_i,i} - \tau_{0,i}$ ,  $y = \tau_{(n_i+1),i} - \tau_{n_i,i}$  and  $z = \Delta T_i$ . The probability

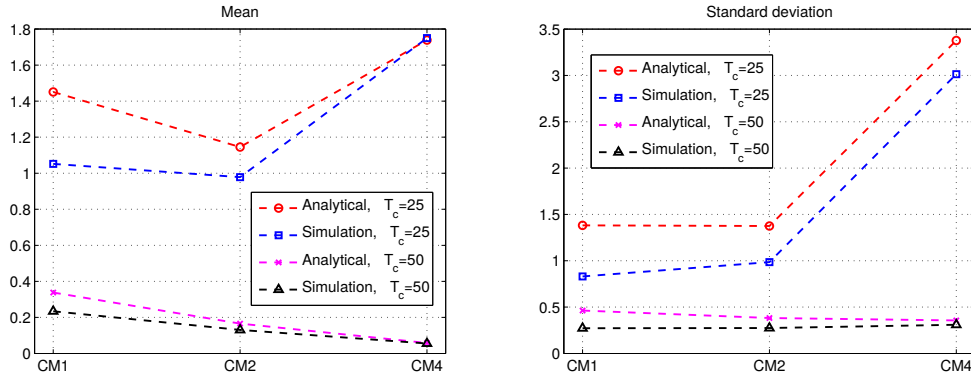


FIGURE 11. Theoretical first and second order moments compared to simulated data, for different CM and  $T_c$ .

that the number of MPCs in the cluster is  $n_i$  can then be written as

$$P(n_i) = \int_0^{\infty} P(n_i|z) f(z) dz \quad (30)$$

where  $f(z) = \lambda \exp(-\lambda z)$  is the PDF of the cluster arrival times and with the conditional probability

$$\begin{aligned} P(n_i|z) &= P(z - y < x \leq z) \\ &= \int_0^z \left( \int_{z-y}^z f(x) dx \right) \lambda \exp(-\lambda y) dy \\ &\quad + \int_z^{\infty} \left( \int_0^z f(x) dx \right) \lambda \exp(-\lambda y) dy \end{aligned} \quad (31)$$

Considering the simplified model of the ray arrival times, it can be easily seen that

$$f(x) = \frac{x^{n_i-1}}{(n_i-1)!} \lambda^{n_i} \exp(-\lambda x) \quad (32)$$

Therefore we get, after a few algebra steps,

$$P(n_i|z) = \frac{(\lambda z)^{n_i}}{n_i!} \exp(-\lambda z) \quad (33)$$

which finally yields (8).

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