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# Time is not Fundamental - unlike Distance and Velocity

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# Abstract

Distance and time are considered fundamental attributes of the reality and the result of their division is velocity. In this paper we suggest, in contrast, to consider distance and velocity as fundamental and time as merely a practicality. This approach yields a new understanding of physics.

Key Words: Time; Distance; Light velocity; General relativity; Special relativity

# **1** Introduction

#### **1.1 Time**

We do not know what time is, we only know what motion is.

We can build devices called clocks; whose hand movement (or whatever is analogous to the hand movement) describes to us the "passing of time". The time of an event would be the position of the hands at the moment, and at the place of the event. The hands of the clock must move in a rhythmic circular motion so that each cycle is exact. In the case of a non-circular, but rhythmic motion the physical conditions must be preserved so that every motion is like the one preceding it. Thus we can talk about the clock's rhythm, or frequency of its motions, where each cycle represents a unit of time.

Note that all the equations in physics are invariant to time reversal.

#### 1.2 Distance

In a space lattice [1] the distance between two points is simply the number of space cells on the straight line between the two points for a Euclidian space and on the geodesic for a Riemannian (deformed) space.

#### 1.3 On Two Fundamental Velocities

The Greek philosopher Zeno contended that there is **no motion**. In contrast, in our theory - the GDM [1], there is **no rest**, see Section 4.1(this paper), and **only** two fundamental velocities exist.

The GDM considers the space lattice to be elastic with only two real fundamental velocities; that of transversal waves, and that of longitudinal waves. Transversal waves are shown to be the EM waves [1], [2] (and references there). Longitudinal waves are shown to be related to the inner motion in elementary particles (not discussed here) [1], [3]. The Navier equation

governs elastic media. Its solution for elastic transversal waves gives the expression for light velocity [1]:

$$c' = \sqrt{(\mu/m)} \tag{1}$$

where  $\mu$  is a Lamé coefficient and m is the mass density of the media. Since space is massless we take m as:

$$m = \epsilon_s / c^2 \tag{2}$$

where  $\epsilon_s$  is the standard space energy density, and c is the relevant light velocity. Inserting (2) into (1) gives:

$$c' = c\sqrt{(\mu/\epsilon_s)}$$
(3)

Thus  $[\mu] = [\epsilon_s]$ , and we can rename the numerator and use  $\epsilon$  instead of  $\mu$ . Thus (3) becomes:

$$c' = c\sqrt{(\epsilon/\epsilon_s)}$$
(4)

By using (2) we have turned (1), an equation that determines c', into an equation (4) that determines the ratio c'/c. But light velocity is determined by the permittivity and permeability of space and hence by the tension in the space cells. This tension is dependent on the energy per cell; hence the relation:

$$\epsilon/\epsilon_{\rm s} = \rho_0/\rho \tag{5}$$

where  $\rho$  is space density and  $\rho_0$  is the standard space density - no deformations. Substituting (5) in (4) gives:

$$c' = c\sqrt{(\rho_0/\rho)}$$
 (6)

Every space disturbance, a transversal or longitudinal wavepacket, must move at the velocity  $c_T$  or  $c_L$ , respectively. We distinguish between  $c_T$  and  $c_L$  when necessary. In most cases we use the symbol c instead of  $c_T$ .

## 2 The Cosmic Microwave Background (CMB)

At large, the CMB is isotropic and homogenous blackbody radiation, with a peak temperature of 2.7°k. The CMB was predicted theoretically by Gamow and discovered accidentally by Penzias and Wilson [4]. In 1989 the CMB was again measured, this time by the Cosmic Background Explorer (COBE) satellite [5], see Fig. (2).

An observer's motion relative to the background radiation is accompanied by a Doppler shift. This shift enables the measurement of the observer's velocity relative to the radiation bath "attached" (i.e., space's vibrations) to space.

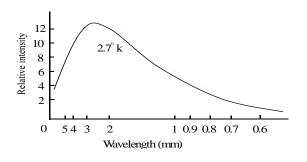


Fig. (2) The Cosmic Background

# **3** Velocity Relative to Space

A CMB anisotropy was first observed by Smoot et al [6], and interpreted as the result of the above Doppler shift [7].

Fig. (3) shows the vector of the velocity of planet Earth relative to the universe [8]. Recently, the velocity of the Earth around the Sun and its rotational velocity have been derived from the Doppler shift.

Earth's velocity relative to space is:  $v = 371.0 \pm 0.5$  km sec<sup>-1</sup>. This velocity is towards a point with equatorial coordinates:  $(\alpha, \delta) = (11.20^{h} \pm 0.01^{h}, -7.22^{0} \pm 0.08^{0})$ , [9]. This direction points from the cluster of galaxies, Aquarius, towards the cluster Leo-Virgo.

Say the peak of the background radiation is green. An observer, seeing green in all directions, knows they are at rest relative to space. An observer, seeing blue in one direction, red in the

opposite direction, and green on the sides, knows they are moving in the direction of the blue.

Similarly, we can also make a distinction regarding acceleration.

An observer moving in a circle notices that, tangentially to the circle, there is no symmetry; the horizon in one direction looks red, and in the other, blue.

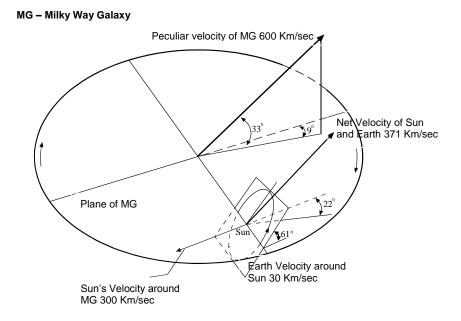


Fig. (3) Earth Velocity Relative to the Space

Coleman and Glashow [10], [11], also adopted this idea that space is a special frame.

The CMB necessarily "resides" in the same "stationary system", which is the space lattice.

Note that for observers on Earth, with velocity v relative to the space frame, v << c, and

 $\gamma \sim 1+1/2 \cdot v^2/c^2 \sim 1+3 \cdot 10^{-7}$ . Thus, ignoring  $3 \cdot 10^{-7}$  and taking  $\gamma = 1$ , we are left with the SR notion that only relative velocity matters, as if there is no special frame.

## 4 Rest and Motion and Length Contraction

#### 4.1 "Rest"

In the GDM "rest" is related to the situation in which a wavepacket moves in a closed loop. In this case the geometrical center of the circulating wavepacket is at rest. The particle, which is this wavepacket, can be regarded as staying at "rest".

**The electron (positron)** at rest is a circulating longitudinal dilational (contractional) wavepacket with a tangential velocity of propagation c and radius of circulation  $R_0$ . In [3] we elaborate on this circulation which is actually more complex. Thus, from a distance, the electron (positron) is observed as if at rest, and from close, as a volumetric dilation (contraction), which is its negative (positive) charge that moves in a circle [12].

Fig. (1) is a simplistic description of the electron (positron). The gray sphere (circle) represents the zone of dilated (contracted) space, which is the electric charge. In this discussion we omit, for simplicity, the circulation of the charge, with radius  $r_e$  around its surface, as shown in the detailed model in [3]. We also do not show in Fig. (1) that the circulation plane is **not** perpendicular to the velocity vector **v**. The projection of the angular momentum **L**, of this circulation, on **v** is  $L_z = 1/2\hbar$  whereas  $L = \hbar\sqrt{j(j+1)} = \hbar\sqrt{1/2(1/2+1)} = 0.866\hbar$ , hence the inclination of the circulation plane.

"Rest" is described on the left of Fig. (1). The zone of dilated space (charge) of our electron revolves, in the xy plane, around the origin with a radius  $R_0$ . In this case:  $c = \omega R_0$ . From now on, when we relate to the electron we also relate to the positron, unless otherwise mentioned.

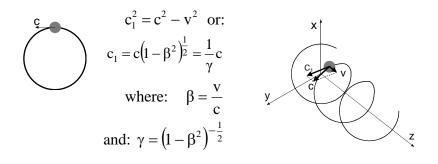
The electron model, presented here, is a simplified version of a much more detailed model that appears in [3] and yields accurately the leptons' masses and some other features.

#### 4.2 Motion

Fig. (1), on the right, describes motion. Motion is the situation in which the circle of revolution of a wavepacket becomes a spiral. The GDM considers the **length of the wavepacket** to be **retained**. From this conjecture alone we derive the results of the Theory of Special Relativity (SR). Since the length of the wavepacket is retained, the spiral radius R is smaller than  $R_0$ . This is analogous to a stretched spring. As Fig. (1) shows:

## $\mathbf{R} = 1/\gamma \ \mathbf{R}_0 \qquad \qquad \mathbf{Microscopic \ Contraction} \tag{7}$

The resultant electron motion is always at the wave velocity c. Thus a translatory motion at constant velocity, v, does not involve any exertion of force and necessarily  $v \le c$ .



"Rest"

Motion

#### Fig. (1) Rest and Motion

**Newton's first law,** and the **first postulate of the theory of relativity**, which implies that no particle or signal can move at a speed that exceeds the light/wave velocity c, are thus a natural result. The helical motion, which is an electric current formally in the direction -z, is related to the magnetic field **B** and the vector potential **A**. The Circular Track is stable since the centrifugal force is balanced by an equal but opposite centripetal force. The Lorentz force, created by the magnetic field of the circulating charge acting on itself, is the required centripetal force, see [3].

#### 4.3 The Elementary Particle Inertia and Length Contraction

At rest, the energy of the electron is  $U_0$ , linear momentum is  $P_o = U_0/c$  and the angular momentum is  $L_0 = P_o R_o$ .

In motion, energy, linear momentum and angular momentum are U, P = U/c and L = PR respectively.

The conservation of angular momentum  $L = L_0$  (which is  $1/2\hbar$  for the electron) implies that:

 $PR = P_0R_0$ . Hence  $UR = U_0R_0$ , but  $R = 1/\gamma \cdot R_0$ , (7), thus:

$$\mathbf{U} = \gamma \mathbf{U}_0 \tag{8}$$

Considering the energy as purely electromagnetic, where the elementary charge Q is a sphere of radius  $r_0$  [3], the relations  $U_0 = Q^2/2r_0$ ,  $U = Q^2/2r$  and (2) give:

 $r = 1/\gamma \cdot r_0$  Microscopic Contraction (9) For v << c (9) is:

$$\mathbf{r} = (1 - \mathbf{v}^2 / 2c^2) \mathbf{r}_0 \tag{10}$$

Acceleration is accompanied by an increase of energy U, and a reduction in the radii R and r. The applied force, needed to accelerate the particle, is doing work to curve space more strongly and thus to reduce r. This force is needed to increase stress and cause more strain in order to enlarge the curvature. Thus we arrive at **Newton's Second Law** and understand where **Inertia** comes from.

Note that we have replaced Newton's axiomatic laws by our postulated model of an elementary particle. Some of the merits in this replacement, as we show, are:

• It leads to the proof of the equivalence of gravitational mass and inertial mass [12].

- It proves that mass is not a fundamental attribute of matter (Section 4.4).
- It proves that inertia is an intrinsic attribute of matter, with no need for an additional field to induce it (Higgs Field).

Note that the relations  $L=1/2 \hbar$ ,  $R=1/2 \hbar c/U$ ,  $U_0=Q^2/2r_0$  and  $r=Q^2/2U$  give:

$$\alpha = r/R = Q^2 / \hbar c$$
 Fine Structure Constant. (11)

Note also that at "rest" the angular momentum can point in any direction, whereas in motion it can point, generally (see Section 3.1), in the direction of motion or opposite to it. This attribute is related to Space Quantization.

#### 4.4 Mass

Our "relativistic" relation (2) gives:

$$U = \gamma U_0 = U_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \simeq U_0 + \frac{\frac{1}{2}U_0v^2}{c^2} \qquad \text{The second term:} \qquad \frac{\frac{1}{2}U_0v^2}{c^2} = \frac{1}{2}(U_0/c^2)v^2 ,$$

is identified as the **kinetic energy** where:

$$M_0 = U_0 / c^2$$
 (12)

Thus inertial mass becomes merely a practical term. We also get:

$$\mathbf{M} = \gamma \mathbf{M}_0 \tag{13}$$

The literature distinguishes between the **kinetic energy**  $\frac{1}{2}M_0v^2$ , which a particle possesses by virtue of its motion, and its **internal energy**  $M_0c^2$ . But here, the kinetic energy is also an internal energy of deformation (the reduction in size of r and R) that cannot be distinguished from the rest energy.

Note again that the electron model, presented here, is a simplified version of a much more detailed model that appears in [3] and yields the masses of the leptons and quarks.

## **5** The Contraction of Macroscopic Bodies

Fitzgerald and Lorentz (1904) explained the then lack of ability to discover motion relative to the aether (Michelson and Morley 1887) assuming that the lengths of bodies are contracted in the direction of their motion:

$$L = L_0 \left( 1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} = \frac{1}{\gamma} L_0 \qquad \text{Lorentz Length Contraction}$$
(14)

The translational velocity v is with respect to space, as we see it. An observer moving in space would not notice any contraction, since not only the yardstick but also they are contracted. However, they can always measure their velocity relative to space and calculate their own contraction. At this stage we can only speculate as to why and how a macroscopic body in motion is contracted.

We consider a possible explanation for macroscopic contraction.

According to Bell [14] pp. 69-78, and our exploration [2], the electric field of a moving charge is:

$$E' = \frac{HQ}{r'^2} \frac{1 - \beta^2}{\left(1 - \beta^2 \sin^2 \theta'\right)^{3/2}}$$
(15)

We denote the distance from the charge Q, momentarily at the origin, to the point (x',z')where the field is measured:  $r' = (x'^2 + z'^2)^{1/2}$ .  $\theta'$  denotes the angle between this radius vector and the velocity of the charge Q, which is moving in the positive x' direction in the frame k'. Then, since  $z' = r' \sin \theta'$ , (15) is the magnitude of the field.

For low speeds the field reduces simply to E'  $\approx$  HQ/r'<sup>2</sup>. But if  $\beta^2$  is not negligible, at the same distance from the charge, the field is stronger at right angles to the motion than in the

direction of the motion. Fig. (2) shows a simple representation of the field indicating field lines in the x'z' plane. For the field in the x'y' plane we get an identical representation.

Fig. (2) shows that the field perpendicular to the direction of motion gets stronger whereas in the direction of motion - weaker. This phenomenon [14] explains **macroscopic contraction**, since the orbits of electrons in atoms are distorted to become ellipses with their shorter axis in the direction of motion.

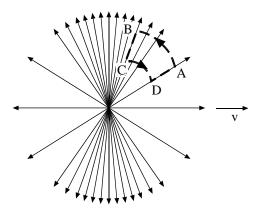


Fig. (2) Representation of the Field of a Uniformly Moving Charge

# 6 Light Deflection

Fig. (3) shows a projector consisting of a point radiator located at 0, the focal point of a parabolic mirror. When the projector does not move, v = 0, the radiation reaches both edges of the reflector at the same time, and the outgoing beam from the reflector moves toward the ceiling with its wave front parallel to the ceiling. When the projector is on the move,  $v \neq 0$ , the radiation from 0 reaches the left side of the parabolic mirror before it reaches the right side, thus tilting the wave front and deflecting the beam.

$$\mathbf{t}_{_{\mathrm{L}}} = \frac{l}{c+v} \qquad \mathbf{t}_{_{\mathrm{R}}} = \frac{l}{c-v} \qquad \Delta \mathbf{t} = \mathbf{t}_{_{\mathrm{R}}} - \mathbf{t}_{_{\mathrm{L}}} = \frac{2lv}{c^2}\gamma^2 \qquad \alpha_{_{\mathrm{rad}}} = \mathbf{t}_{_{\mathrm{g}}}\alpha = \mathrm{Sin} \ \alpha \quad \text{for small } \alpha.$$

$$\sin \alpha = \frac{\Delta t \cdot c}{2l_0} = \frac{\frac{2lv}{c^2}\gamma^2 \cdot c}{2l\gamma} = \gamma\beta$$
(16)

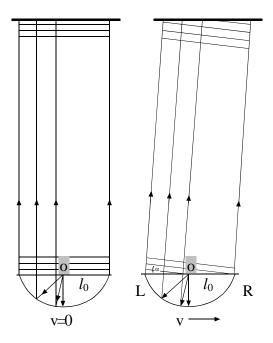


Fig. (3) A Projector

For an observer moving with the radiator, the beam of light appears as if it is moving perpendicularly to the direction of motion.

Note that the same phenomenon takes place, on a microscopic level, in Synchrotron Radiation.

# 7 Larmor Time Dilation

## 7.1 The Cell

Fig. (4) and Fig. (5) show a cell with sides  $L_0$ . The cell is equipped with a source S and detector D, on one side, and a mirror on the opposite side. If the cell is at rest relative to space, the time it takes for the light beam to reach the mirror from source S and back to detector D is  $\Delta T_0$ :

$$\Delta T_0 = \frac{2L_0}{c} \tag{17}$$

## 7.2 A Moving Cell with a Beam Perpendicular to its Motion

Fig. (4) shows the source S and detector D on the "floor", and the mirror on the "ceiling". If the cell moves in a direction perpendicular to the direction of the beam of light, the observer resting in space sees a deflected beam:

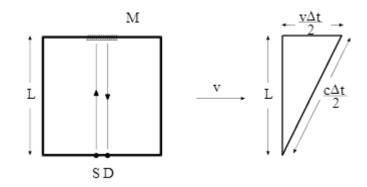


Fig. (4) A Moving Cell

 $\left(\frac{c\Delta T}{2}\right)^2 = L^2 + \left(\frac{v\Delta T}{2}\right)^2$ 

 $L = L_0$ , since there is **no contraction** in the direction perpendicular to the direction of motion.

Hence: 
$$4L_0^2 = (c^2 - v^2)\Delta T^2$$
 and  $\Delta T = \frac{2L_0}{c} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$  or

 $\Delta T = \gamma \Delta T_0$  Larmor Time Dilation

(18)

### 7.3 A Moving Cell with a Beam Parallel to its Motion

Fig. (5) shows the source S and detector D on one side, and the mirror on the other side. When the cell and the light beam move at speed v relative to space:

$$\Delta T = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2L}{c} \frac{1}{1 - \frac{v^2}{c^2}} = \frac{2L}{c} \gamma^2$$
(19)

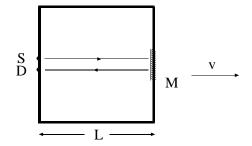


Fig. (5) A Moving Cell

According to equation (14):

$$L = \frac{L_0}{\gamma} \qquad \text{Lorentz Length Contraction}$$
(14)

and again

$$\Delta T = \gamma \Delta T_0$$
 Larmor Time Dilation

(18)

Based on (17) and (19) we conclude time to be a scalar.

## 7.4 Conclusion

We consider both length contraction, and light deflection to be real. Thus we have dispelled the need to relate to time as a fundamental attribute of reality. Time appears as simply the division of Distance by Velocity. By virtue of motion, or deformed space, objects and light velocity change in a way that keeps light velocity constant, hence the term constant of nature [12], but time by itself has no meaning.

This conclusion has implications as to how we relate to the **first term** in the Schwarzschild metric:

$$ds^{2} = -g_{00}c^{2}dt^{2} + 2g_{0r}dr dt + g_{rr}dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$

Some physicists wrongly consider c to be constant whereas it is time that changes. We, in

contrast, consider c to be a variable, but a constant of nature [12].

Note that a mass contracts space around it (the GDM interpretation of curving [15]), hence according to Section 1.3 light velocity c, at the mass, is slower than that in inter-galactic space. Space is contracted, but so are the local observer close to the mass and his yardstick. As [12] shows, the value for c, measured by the local observer, is the same as for the far-away observer. This is the reason for considering c a constant of nature, despite being a variable.

## 8 GDM versus Special Relativity (SR)

In SR, length contraction and time dilation are **not real**. In SR, neither space nor time is objective. Only **spacetime is objective and absolute**, since the interval  $\Delta S^2$  is invariant under Lorentz Transformation.

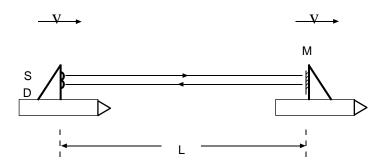
In the GDM, physical objects get shorter in the direction of motion relative to space. Hence, the spatial distance between two disconnected objects, moving in the same direction and at the same velocity, is **retained.** 

SR considers these two objects to reside in **the same** inertial frame. Thus, an observer in another inertial frame concludes that **not only** the objects get shorter, but also the spatial distance between them.

# 9 Case Study

Consider two rockets, at rest, a distance  $L_0$  from each other. The rockets start to move till they both acquire velocity v, relative to the space frame k. We assume a very slow and identical acceleration for both rockets until they reach this velocity.

In Fig. (6) a beam of light travels from a source S, on the left rocket, to a mirror M, on the right rocket, and back to a detector D. Since  $L = L_0$  for an observer at rest in the space frame k, the time taken to travel according to the GDM is:



#### Fig. (6) Two Rockets

$$\Delta T = \frac{L_0}{c+v} + \frac{L_0}{c-v} = \frac{2L_0}{c} \frac{1}{1 - \frac{v^2}{c^2}} = \gamma^2 \Delta T_0$$
(19)

where  $\Delta T_0 = 2L_0/c$ .

For the observer on the left rocket  $\Delta T = \gamma \Delta T_0$ , since the unit of time of his clock is dilated.

In SR the two rockets reside in the same reference frame k', hence relative to k the distance  $L = \frac{L_0}{\gamma}$  is contracted and the travel time is:  $\Delta T = \gamma \Delta T_0$ , equation (18).

This difference, between (19) and (18), is the difference between the predictions of the GDM and SR.

## **10 On the Lifetime of the Muon**

#### **10.1 SR Interpretation**

The muon particle decays via the weak interaction into an electron and a pair of neutrinos after 2.2 microseconds. In the late 1990s, scientists at Brookhaven National Laboratory, used the Alternative Gradient Synchrotron (AGS) to produce a beam of muons circulating around a 14m-diameter ring at a speed of 0.9994·c. Muons lifetime, at rest, is 2.2 microseconds.

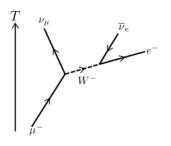


Fig. (7) Feynman Diagram of the Muon Decay

If this is also the lifetime at this speed, they would manage no more than 15 turns of the ring before they disintegrate.

But the muons managed around 433 turns, which means their lifetime was extended by a factor of 28.8 to just over 60 microseconds.

And indeed, for a speed of 0.9994·c the Lorentz factor of time-dilation is:  $\gamma = 28.87$ .

SR predicts, as observed, that these speedy muons should live 28.87 times longer than at rest.

According to SR, we can also argue that space in the frame of the muon, which moves with a tangential velocity 0.9994·c relative to the Lab frame, is contracted, a Lorentz contraction, by the same  $\gamma = 28.87$ . And as a result the muon, with its lifetime of 2.2 microseconds, moves 433 times around this contracted circumference. The fact that there are two alternate SR explanations is confusing.

#### **10.2 GDM Interpretation**

Our model, Section 4.1 Fig. (1), of the elementary particle, and equations (7) and (9), show that when it moves it gets smaller:  $R = 1/\gamma R_0$ ,  $r = 1/\gamma r_0$ . Fig. (7) shows that the muon disintegrates via its interaction with the vibrating space around it. Hence the smaller it becomes the weaker is the interaction. The interaction dependence on  $\gamma$  is, therefore, reflected in the dependence of the particle's life time on  $\gamma$ :  $\tau = \gamma \tau_0$ 

# **11 Summary**

Real motion is only that of the transvers or longitudinal waves. Space is a special frame and motion relative to space causes real contraction and deflection. These phenomena dispel the need for relating to time as a fundamental attribute of reality. The saying that "Time is Money", albeit with a different connotation, expresses our understanding that Time is indeed like Money.

## Acknowledgements

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