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A Lorentz invariance respectful paradigm to explain the origin of ultra-high-energy cosmic ray energies

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Abstract

The issue of this article, whose approach consists in rigorously applying the principle of least action and the invariance of Minkowski space-time, is to explore and extend the equations of special relativity when velocities are greater than c while preserving their covariant nature. This approach, adapted to the study of special relativity, provides a privileged theoretical framework for probing the properties of the superluminal regime if it exists. Relativistic superluminal equations indicate that the speed of light is a singularity in the speeds' spectrum and that the evolution of these equations as a function of the velocity is reversed with respect to the equations of special relativity. This extension of special relativity makes it possible to formulate a credible hypothesis on the origin of ultra-high-energy cosmic ray energies.

keywords: Special relativity, Principle of least action, Ultra-high-energy cosmic ray (UHECR), GZK limit, Cosmic inflation

1 Introduction

Special relativity, the formal theory elaborated by Albert Einstein in 1905, is the theoretical consequence of Galilean relativity and the principle that the velocity of light in a vacuum has the same value in all Galilean reference frames (or inertial frame of reference) implicitly stated in Maxwell's equations. The two postulates of special relativity are the following:

- the laws of physics have the same form in all Galilean reference frames,
- the speed of light in a vacuum has the same value in all Galilean reference frames, which amounts to saying that space-time is homogeneous and isotropic.

In special relativity, the Lorentz transformations correspond to the law of changing Galilean referential in which the equations of physics must be preserved as well as the speed of light which is constant in any Galilean referential, while preserving the orientation of space and time. Maxwell's equations of classical electromagnetism are covariant within the group of Lorentz transformations, i.e. they keep the same mathematical form before and after application of a group operation. Lorentz transformations are linear transformations of the coordinates of a point in Minkowski's four-dimensional and relativistic space-time. The Metric of Minkowski $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ is an invariant quantity of Lorentz [1]. All the equations of special relativity can be found by applying the principle of least action where the invariance by change of Galilean referential imposes $S = -mc \int ds$ corresponding to the Lagrangian $L = -mc^2 \sqrt{(1 - \beta^2)}$ with $\beta = \frac{v}{c}$ [1]. In relativistic physics, and in the absence of an electromagnetic field, it is well known that the principle of least action consists in minimizing the function $-mc\tau$, where τ is the proper time along the

path, which is both the time flowing in the frame of reference of the body of mass m along the path and the length of the trajectory measured by the metric of the space: which amounts to maximizing the proper time [2]. The principle of least action is used in this paper to extend the theory of special relativity and to derive the laws of mechanics when the velocity is greater than c . The calculations developed in this paper are based on the invariance ds^2 and the maximization of the proper time.

2 Presentation of the theoretical model

2.1 Model assumptions

There are two relativistic regimes :

- subluminal regime ($v < c$) whose mechanical equations are those of special relativity,
- superluminal regime ($v > c$) whose equations the present study proposes to determine.

Like the special relativity, the superluminal regime is governed by the following principles:

- the speed of light in a vacuum is the same for all observers, regardless of the motion of the light source,
- the principle of least action consists in maximizing the proper time,
- $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$ is an always positive relativistic invariant.

2.2 Calculation steps

From the invariance ds^2 we will determine the law of change of a galilean referential i.e. the equivalent of the Lorentz transformations in superluminal regime, then we will deduce the equation of the superluminal doppler effect by applying the invariance of the wave phase. We will establish the definition of proper time and deduce the Lagrangian by applying the principle of least action. The last step of the calculation will determine the impulse and the energy of a massive body in superluminal regime.

3 Lorentz transformation in superluminal regime

We consider a reference frame \mathcal{R}' animated by a uniform velocity v with respect to the reference frame \mathcal{R} . The \mathcal{R}' axis coincides with the Ox axis of \mathcal{R} and the velocity direction v , the other two axes remaining parallel at all time. The origin of the times $t = 0$ is taken at the instant when the points O and O' coincide¹.

The conservation of intervals states:

$$c^2 t^2 - x^2 = c^2 t'^2 - x'^2. \quad (1)$$

Consequently x and ct can be considered as functions of x' and ct' , i.e. of the form:

$$\begin{cases} x = ax' + bct' \\ ct = dx' + ect', \end{cases} \quad (2)$$

where $a, b, d, e \in \mathbb{R}$

¹Notations in prime will refer to the \mathcal{R}' reference frame and simple notation to the \mathcal{R} reference frame

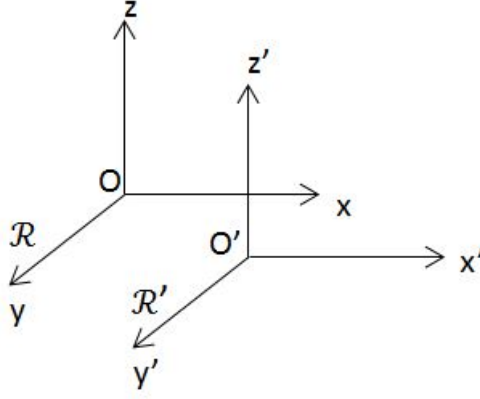


Figure 1: The two frames of reference \mathcal{R} et \mathcal{R}'

Using the expression of the equality of the intervals (1) and identifying term for term, we find that:

$$\begin{cases} a^2 - d^2 = 1 \\ e^2 - b^2 = 1 \\ de = ab. \end{cases} \quad (3)$$

If we consider a quantity θ such that $a = \cosh(\theta)$, we can deduce from the equations above that:

$$\begin{cases} a = \cosh(\theta) \\ b = \sinh(\theta) \\ e = a \\ d = b. \end{cases} \quad (4)$$

The equations (2) can be written in the same way as special relativity:

$$\begin{cases} x = x' \cosh(\theta) + ct' \sinh(\theta) \\ ct = x' \sinh(\theta) + ct' \cosh(\theta), \end{cases} \quad (5)$$

we put:

$$\tanh(\theta(v)) = \frac{ct}{x} = \frac{c}{v}, \quad (6)$$

where v is the velocity of the reference frame \mathcal{R}' with respect to \mathcal{R} by introducing the superluminal rapidity² $\theta(v) = \operatorname{arctanh}(\frac{c}{v})$:

$$\begin{cases} \cosh(\theta) = \frac{\frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}} = \frac{\beta}{\sqrt{\beta^2 - 1}} \\ \sinh(\theta) = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}} = \frac{1}{\sqrt{\beta^2 - 1}}. \end{cases} \quad (7)$$

²As with special relativity, the superluminal rapidity enables us to express Lorentz transformations as a hyperbolic rotation in Minkowski space-time. Due to its linear character, it preserves the classical mechanics' relation between speed and acceleration. This point will be analyzed in detail in chapter 4.

The expression of the superluminal Lorentz transformation is:

$$\begin{cases} x = \gamma_{>}(x' + \frac{c^2}{v}t') \\ t = \gamma_{>}(\frac{x'}{v} + t') \\ y = y' \\ z = z' \\ \gamma_{>} = \frac{\beta}{\sqrt{\beta^2 - 1}}. \end{cases} \quad (8)$$

In relativistic superluminal regime, we find that the Lorentz factor becomes $\frac{\beta}{\sqrt{\beta^2 - 1}}$. In the remainder of the document we will note the Lorentz factor in the superluminal regime $\gamma_{>} = \frac{\beta}{\sqrt{\beta^2 - 1}}$ and the Lorentz factor in subluminal regime $\gamma_{<} = \frac{1}{\sqrt{1 - \beta^2}}$.

The superluminal Lorentz factor indicates that:

$$\lim_{v \rightarrow +\infty} \gamma_{>} = 1 \text{ et } \lim_{v \rightarrow +c} \gamma_{>} = \infty. \quad (9)$$

The transformation (8) is noted $\mathcal{L}_{(>,x)}(v)$ and is seen to be easily reversed, and we have:

$$\begin{cases} x' = \gamma_{>}(x - \frac{c^2}{v}t) \\ t' = \gamma_{>}(t - \frac{x}{v}) \\ y' = y \\ z' = z \\ \gamma_{>} = \frac{\beta}{\sqrt{\beta^2 - 1}}. \end{cases} \quad (10)$$

Thus we have $\mathcal{L}_{(>,x)}^{-1}(v) = \mathcal{L}_{(>,x)}(-v)$. Furthermore, the composition of two transformations is another transformation. This set of transformations forms a group with $\mathcal{L}_{(>,x)}(\infty)$ as a neutral element. A four-vector position \tilde{E} of an event E is defined in a reference frame \mathcal{R} by its four coordinates (ct, x, y, z) , all homogeneous to lengths. A change of reference frame is associated to a matrix of change of base:

$$\tilde{E}' = \bar{L}_{(>,x)}(v)\tilde{E}. \quad (11)$$

4 Matrix representation

The linearity of these relations allows space-time matrix writing. The 4×4 matrix change of base is:

$$\bar{L}_{(>,x)}(v) = \begin{pmatrix} \gamma_{>} & -\frac{c^2}{v}\gamma_{>} & 0 & 0 \\ -\frac{\gamma_{>}}{v} & \gamma_{>} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (12)$$

It is possible to simplify the writing of the coefficients of the matrix by in fact using the superluminal rapidity θ (6) : $\gamma_{>} = \frac{\beta}{\sqrt{\beta^2 - 1}}$ allows us to write $\gamma_{>}^2 - \frac{\gamma_{>}^2}{\beta^2} = 1$.

By using the change of variable with hyperbolic functions:

$$\cosh(\theta) = \gamma_{>} \text{ and } \sinh(\theta) = \frac{\gamma_{>}}{\beta} \text{ with } \theta = \operatorname{arctanh}\left(\frac{c}{v}\right), \quad (13)$$

$$\bar{L}_{(>,x)}(\theta) = \begin{pmatrix} \cosh(\theta) & -c \sinh(\theta) & 0 & 0 \\ -\frac{\sinh(\theta)}{c} & \cosh(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (14)$$

It can be noted that with this setting:

$$\bar{L}_{(>,x)}(\theta) \circ \bar{L}_{(>,x)}(\theta') = \bar{L}_{(>,x)}(\theta + \theta'). \quad (15)$$

The set of superluminal Lorentz transformations $\mathcal{L}_{(>,x)}(v)$, parametrized by the relative speed v , forms a group³.

We can verify easily that s^2 is a relativistic invariant in all Galilean reference frames. We us the matrix change of base $\bar{L}_{(>,x)}(\theta)$ and we observe that:
 $s'^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2 = (x \cosh(\theta) - ct \sinh(\theta))^2 + y^2 + z^2 - (ct \cosh(\theta) - x \sinh(\theta))^2 = s^2$,
the quantity s^2 thus assumes the same value in all the reference frames.

4.1 Superluminal rapidity-addition law

Consider three reference frames: \mathcal{H} , \mathcal{H}' , moving at speed u relative to \mathcal{H} , and \mathcal{H}'' , moving at speed v relative to \mathcal{H}' and w relative to \mathcal{H} .

The rapidity velocity-addition formula is⁴ :

$$\theta(w) = \theta(u) + \theta(v). \quad (16)$$

We can easily derive the law of velocity-addition formula for superluminal velocities, in a simplified form, corresponding to velocities which are all collinear (we generalize in the following paragraph), by writing w as a function of u and v . To do this, we take the cosh and the sinh of (13), which results in⁵:

$$\begin{cases} \sinh(\theta(w)) = \frac{\gamma_{>}(w)}{\beta(w)} = \gamma_{>}(u)\gamma_{>}(v)\left(\frac{1}{\beta(u)} + \frac{1}{\beta(v)}\right) \\ \cosh(\theta(w)) = \gamma_{>}(w) = \gamma_{>}(u)\gamma_{>}(v)\left(1 + \frac{1}{\beta(u)\beta(v)}\right), \end{cases} \quad (17)$$

from which we immediately draw:

$$\beta(w) = \frac{1 + \beta(u)\beta(v)}{\beta(u) + \beta(v)}, \quad (18)$$

or

$$w = \frac{uv + c^2}{u + v}, \quad (19)$$

we notice that:

$$\text{if } u > c \text{ or } v > c \text{ then } w = \frac{uv + c^2}{u + v} \leq c \quad \forall u \leq c \text{ or } v \leq c. \quad (20)$$

If the addition of velocities cannot lead to a speed superior to c , rapidities accumulate without limit.

³It is possible to demonstrate that any group parametrized by a single parameter, provided that this parametrization is "sufficiently" continuous and differentiable, is isomorphic to the additive group of real numbers [3].

⁴The group of rotations around a point, parametrized by the angle of rotation, is directly parametrized in additive form [4].

⁵ $\cosh(a + b) = \cosh(a) \cosh(b) + \sinh(a) \sinh(b)$ and $\sinh(a + b) = \sinh(a) \cosh(b) + \cosh(a) \sinh(b)$

5 Superluminal velocity-addition formula

We consider a reference frame \mathcal{R} at rest and a reference frame \mathcal{R}' with a uniform v speed along the z axis of \mathcal{R} . We assume that the axes of the two reference frames \mathcal{R} and \mathcal{R}' are parallel to each other: x parallel to x' , y parallel to y' and z parallel to z' . A change of variable with hyperbolic functions is performed using the change of base defined in (5).

$$\begin{cases} x = x' \cosh(\theta) + ct' \sinh(\theta) = \frac{x' \frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}} + \frac{ct'}{\sqrt{\frac{v^2}{c^2} - 1}} = \frac{x' \frac{v}{c} + ct'}{\sqrt{\frac{v^2}{c^2} - 1}} \\ t = \frac{x'}{c} \sinh(\theta) + t' \cosh(\theta) = \frac{\frac{x'}{c}}{\sqrt{\frac{v^2}{c^2} - 1}} + \frac{t' \frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}} = \frac{\frac{x'}{c} + t' \frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}}, \end{cases} \quad (21)$$

from the equations (21) we can deduce:

$$\begin{cases} dx = \frac{dx' \frac{v}{c} + ct'}{\sqrt{\frac{v^2}{c^2} - 1}} \\ dy = dy \\ dz = dz \\ dt = \frac{\frac{x'}{c} + dt' \frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}}, \end{cases} \quad (22)$$

$$\begin{cases} v_x = \frac{dx}{dt} = \frac{\frac{v}{c} dx' + c dt'}{\frac{dx'}{c} + \frac{v}{c} dt'} = \frac{v v_{x'} + c^2}{v + v_{x'}} \\ v_y = \frac{dy}{dt} = \frac{dy' \sqrt{\frac{v^2}{c^2} - 1}}{\frac{dx'}{c} + \frac{v}{c} dt'} = \frac{v_{y'} \sqrt{\frac{v^2}{c^2} - 1}}{\frac{v_{x'}}{c} + \frac{v}{c}} \\ v_z = \frac{dz}{dt} = \frac{v_{z'} \sqrt{\frac{v^2}{c^2} - 1}}{\frac{v_{x'}}{c} + \frac{v}{c}}. \end{cases} \quad (23)$$

From the equation of v_x , which is identical to the equation (19), we can make the following remarks:

$$\text{if } v_{x'} = c \text{ or } v = c \text{ then } v_x = \frac{v v_{x'} + c^2}{v + v_{x'}} = c \quad \forall v_{x'} \text{ or } v, \quad (24)$$

As in special relativity, the speed of light is invariant and equal to c in all reference frames. Furthermore:

$$\text{if } v > c \text{ then } v_x = \frac{v v_{x'} + c^2}{v + v_{x'}} \leq c \quad \forall v_{x'} \leq c. \quad (25)$$

This result is also consistent with the hypothesis of the invariant ds^2 but raises a very important remark: the superluminal regime precludes, a priori, a traveller from moving at a speed higher than c . On the other hand, it does not prevent a free corpuscle (free particle or atomic nucleus) from moving at superluminal velocities. Indeed, if an isolated particle does not undergo any contact with another particle, it can continue in superluminal regime without being subjected to the constraints of the velocity-addition law. This point will be explain in detail in chapter (9.4).

6 Superluminal doppler effect

6.1 Superluminal wave phase invariance

The wave equation is also invariant by superluminal Lorentz transformation. Indeed if:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)A(x, y, z, t) = 0, \quad (26)$$

for a wave $A(x, y, z, t) = A_0 \cos(\omega t - \vec{k}\vec{r} + \Phi)$ where ω and \vec{k} are the angular frequency and the wave vector and with $\omega^2/c^2 - \vec{k}^2 = 0$ in the reference frame \mathcal{R} , $A'(x', y', z', t') = A_0 \cos(\omega' t' - \vec{k}'\vec{r}' + \Phi)$ verifies $\left(\frac{1}{c^2} \frac{\partial^2}{\partial t'^2} - \nabla'^2\right)A'(x', y', z', t') = 0$ in the reference frame \mathcal{R}' . The phase invariance can be written: $\omega t - \vec{k}\vec{r} = \omega' t' - \vec{k}'\vec{r}'$. Choose a direction of propagation of the waves parallel to the x-axis common to the two reference frames in which case: $\omega t - kx = \omega' t' - k' x'$. The Lorentz transformation (10) allows the substitution: $\omega t - kx = \omega' \gamma_{>}(t - \frac{x}{v}) - k' \gamma_{>}(x - \frac{c^2}{v} t) = \gamma_{>}(\omega' + k' \frac{c^2}{v})t - \gamma_{>}(\frac{\omega'}{v} + k')x$. By identifying term for term it becomes:

$$\begin{cases} k = \gamma_{>}(\frac{\omega'}{v} + k') \\ \omega = \gamma_{>}(\omega' + k' \frac{c^2}{v}), \end{cases} \quad (27)$$

This shows that ck and ω obey the superluminal Lorentz transformation. The superluminal equations are covariant.

6.2 Angular frequency transformation

The source now has a velocity greater than c and emits waves that move at the speed of light. The expression of the transformation of the angular frequency can also be written in a vectorial manner:

$$\omega = \gamma_{>}(\omega' + \vec{k}'v^{-1}c^2) = \gamma_{>}(1 + e_{\vec{k}'}v^{-1}c)\omega' = \gamma_{>}(1 + \frac{\cos(\theta)}{\beta})\omega', \quad (28)$$

where $e_{\vec{k}'}$ is the unit vector in the direction of the wave vector \vec{k}' and θ the angle between vectors $e_{\vec{k}'}$ and \vec{v} . There is a simple transformation between the two regimes: $\beta \rightarrow \frac{1}{\beta}$. If we apply this transformation to the relativistic doppler effect [5] we find the equation (28):

$$\omega = \frac{\beta}{\sqrt{\beta^2 - 1}}(1 + \frac{\cos(\theta)}{\beta})\omega' \text{ with } \beta = \frac{v}{c} \text{ and } k = \frac{\omega'}{c}. \quad (29)$$

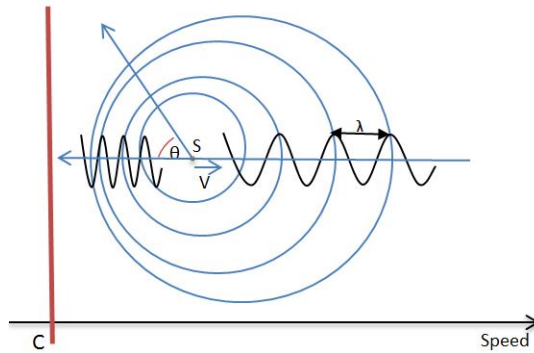


Figure 2: Superluminal doppler effect

7 Proper time and proper length

In superluminal regime the invariance of interval $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$ and the superluminal Lorentz transformation (8) allows us to determine the proper time and the proper length in the reference frame \mathcal{R}' relative to the reference frame \mathcal{R} .

Consider a length rule $l = x_2 - x_1$. Consider this rule observed from the reference frame \mathcal{R}' moving to the speed v relative to \mathcal{R} . According to the Lorentz transformations (22) we have in the reference frame \mathcal{R}' : $l' = x'_2 - x'_1$, with:

$$x_1 = \frac{\frac{v}{c}x'_1 + ct'}{\sqrt{\frac{v^2}{c^2} - 1}} \text{ et } x_2 = \frac{\frac{v}{c}x'_2 + ct'}{\sqrt{\frac{v^2}{c^2} - 1}}, \quad (30)$$

we immediately deduce that:

$$x_2 - x_1 = (x'_2 - x'_1) \frac{\frac{v}{c}}{\sqrt{\frac{v^2}{c^2} - 1}}, \quad (31)$$

thus $x'_2 - x'_1 = (x_2 - x_1) \frac{\sqrt{\frac{v^2}{c^2} - 1}}{\frac{v}{c}}$ which can be written:

$$dl' = \frac{\sqrt{\beta^2 - 1}}{\beta} dl, \quad (32)$$

we obtain a similar result with clocks and the equations (22) so we can write:

$$dt' = \frac{\sqrt{\beta^2 - 1}}{\beta} dt. \quad (33)$$

We can make several remarks:

- the higher the speed of the reference frame \mathcal{R}' , the more the effects of length contraction and duration expansion decrease,
- when the reference frame \mathcal{R}' tends towards an infinite speed, the clocks of the reference frames \mathcal{R}' and \mathcal{R} are synchronised and lengths are equal.

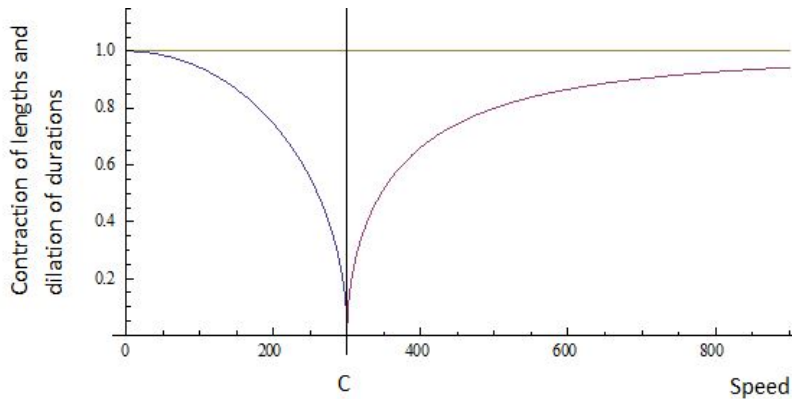


Figure 3: Spatiotemporal deformation in the two regimes

7.1 Twin paradox

In subluminal regime, the twin traveller ends up younger than the one remaining on Earth because the traveller changes Galilean reference frame while the other does not [6]. We have seen (25) that a traveller can not move at superluminal speeds so the twin paradox does not apply to this regime. Moreover, the equation of proper time (33) shows that, in superluminal regime, the expansion of the durations increases with the increase of the velocities (fig. 3) and converges towards zero dilation (equivalent to that of a resting traveller). Consequently, if the traveller could move at superluminal speeds, he would end up, just like the subluminal traveller, younger than the one on Earth. Furthermore, if the traveller could travel at speeds much greater than c and remained far from the singularity c throughout his journey, he would only be slightly affected by temporal dilatation and would return from his journey at almost the same age as his twin.

8 Impulse, mass and energy

In application of the principle of least action i.e. the maximization of proper time, the Lagrangian of the system in superluminal regime is:

$$L = \frac{\sqrt{\beta^2 - 1}}{\beta} mc^2. \quad (34)$$

Unlike the subluminal regime, and with L as a decreasing function of v , the Lagrangian in superluminal regime is always positive in order to maximize the proper time.

8.1 Superluminal impulse

We can now calculate the impulse:

$$\vec{P} = \frac{\partial L}{\partial \vec{v}} = \frac{\partial \sqrt{\frac{v^2}{c^2} - 1}}{\frac{v}{c}} mc^2 = \frac{1}{\frac{v^3}{c^3} \sqrt{\frac{v^2}{c^2} - 1}} m \vec{v} = \frac{1}{\beta^3 \sqrt{\beta^2 - 1}} m \vec{v}. \quad (35)$$

Two remarks about the impulse equation are worth making (35):

- contrary to special relativity, the superluminal impulse is not a momentum, and if we note the momentum $\vec{Q} = \gamma_{>} m \vec{v}$ then $\vec{P} = \frac{\vec{Q}}{\beta^4}$. The superluminal impulse decreases much faster than the momentum (fig. 4),
- the superluminal impulse has a non-trivial and unexpected formulation, it is no longer possible to apply the transformation $\beta \rightarrow \frac{1}{\beta}$ to the subluminal impulse in order to obtain the equation of the superluminal impulse.

8.2 Superluminal mass

The superluminal relativistic mass M of a free elementary particle of mass m at rest is:

$$M = \gamma_{>} m. \quad (36)$$

According to the calculation of the limits of $\gamma_{>}$ (9), the inertia tends towards infinity when the velocity tends towards c and it tends towards its mass at rest when the velocity

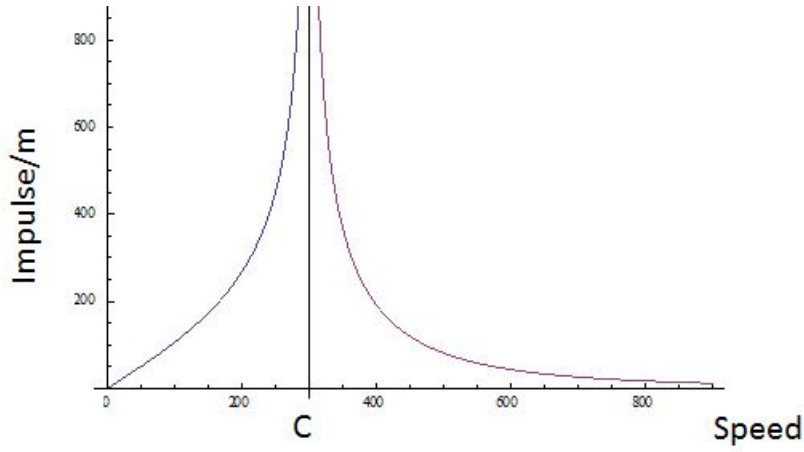


Figure 4: Superluminal impulse

tends to infinity. The work needed to accelerate a relativistic mass particle $\gamma > m$ from c to infinity is:

$$\int_c^\infty \vec{P} d\vec{v} = \int_c^\infty \frac{1}{\beta^3 \sqrt{\beta^2 - 1}} m \vec{v} d\vec{v} = mc^2. \quad (37)$$

This result is equivalent to that of special relativity because the work necessary to accelerate a particle of relativistic mass $\gamma < m$ from 0 to c is also equal to mc^2 .

8.3 Superluminal energy

The energy of a superluminal particle is $E = T + U$ with T as the kinetic energy and U the potential energy we know that $L = T - U$ thus:

$$E = 2T - L = \vec{p}\vec{v} - L, \quad (38)$$

which leads to:

$$\begin{aligned} E &= \frac{1}{\beta^3 \sqrt{\beta^2 - 1}} m v^2 - \frac{\sqrt{\beta^2 - 1}}{\beta} m c^2 \\ &= \frac{\beta}{\sqrt{\beta^2 - 1}} m (v^2 - c^2 (\beta^2 - 1)) \\ &= \gamma > m c^2. \end{aligned} \quad (39)$$

The equation (39) is the equivalent of the energy of special relativity in superluminal regime.

The equation of energy shows:

$$E > mc^2, \quad \lim_{v \rightarrow +\infty} E = mc^2 \text{ and } \lim_{v \rightarrow c} E = \infty. \quad (40)$$

Like the subluminal regime, the superluminal regime sweeps the entire spectrum of energies (i.e. from mc^2 to infinity). For a given energy the particle can have two speeds one in each regime (fig. 5).

8.4 Superluminal braking

The braking of a superluminal particle comes up against the singularity c (fig. 5). The more the superluminal particle approaches the singularity c the greater its inertia (36). Braking a superluminal particle to increase its energy is equivalent to extracting the energy contained in the quantum vacuum. This remarkable and inverse property of the subluminal regime will lead us to formulate a hypothesis on the origin of ultra-high-energy cosmic ray energies.

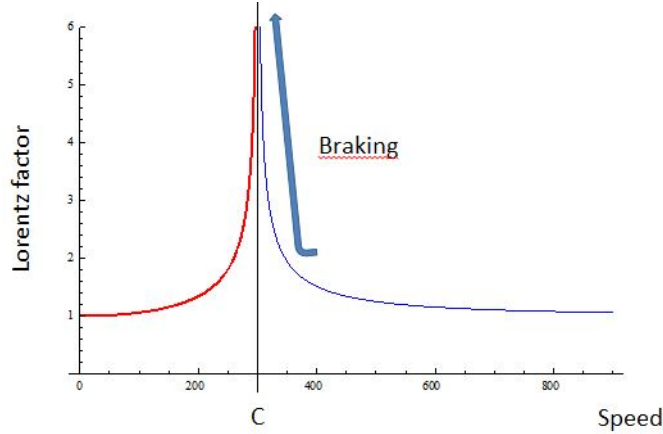


Figure 5: The inertia increases when approaching the singularity

9 “Permeability” of the singularity c

We have seen that for a given energy there are two possible theoretical velocities: a velocity for each regime. Is the singularity c “permeable”? Is it possible to cross the singularity c and move freely from one regime to another?

9.1 Superluminal \rightarrow subluminal

The velocity-addition formula (23) indicates that a massive body systematically passes through the singularity towards the subluminal regime. In this direction, the crossing, inscribed in the equations, seems natural. More precisely, the velocity-addition formula indicates that, following an interaction, a superluminal particle reverts to its subluminal regime. The supraluminal \rightarrow subluminal passage requires no energy and the new velocity of the particle following the crossing of the singularity can be calculated.

So a superluminal particle with energy E and mass m at rest:

$$E = \frac{\beta}{\sqrt{\beta^2 - 1}} mc^2 \text{ i.e. } \beta = \frac{1}{\sqrt{1 - (\frac{mc^2}{E})^2}}, \quad (41)$$

in subluminal regime it retains its energy (fig. 6):

$$E = \frac{1}{\sqrt{1 - \beta'^2}} mc^2 \text{ i.e. } \beta' = \sqrt{1 - (\frac{mc^2}{E})^2}, \quad (42)$$

thus:

$$\Delta v = (\frac{1}{\sqrt{1 - \alpha^2}} - \sqrt{1 - \alpha^2})c = \frac{\alpha^2}{\sqrt{1 - \alpha^2}}c \text{ avec } \alpha = \frac{mc^2}{E}. \quad (43)$$

The higher the energy of the particle, the greater the difference in velocity between the two regimes and, conversely, the more the energy of the particle tends towards its energy at rest, the higher the speed difference: $\lim_{E \rightarrow mc^2} \Delta v = \infty$.

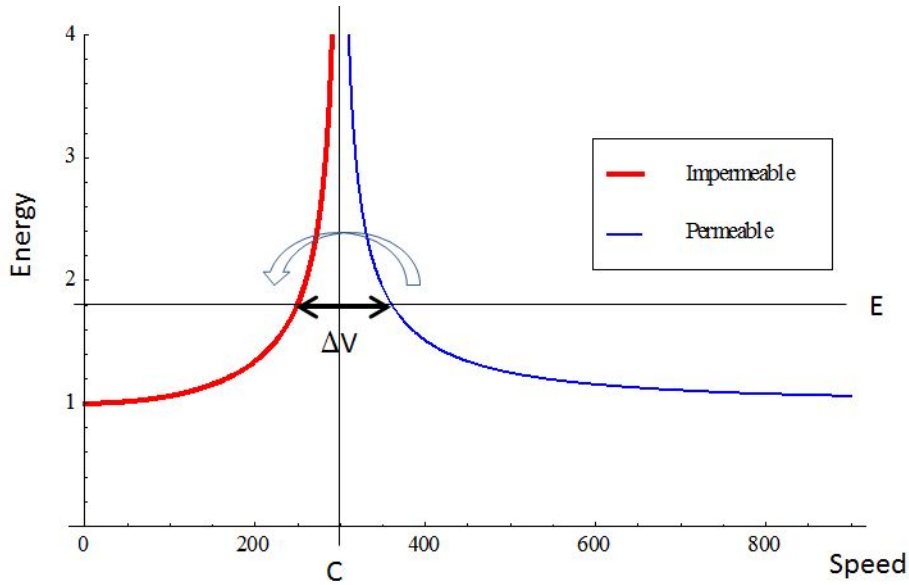


Figure 6: Superluminal \rightarrow subluminal permeability

9.2 Subluminal \rightarrow superluminal

Is this path natural? It clearly seems that the crossing of the singularity in this way is precluded i.e. it would be necessary to expend an infinite amount of energy to pass through this singularity and to reach the superluminal regime.

The principle of causality [7] is thus preserved because the transition to superluminal mode is precluded and the exchange of information between a massive superluminal particle and massive particle invariably results in a return to subluminal regime. The singularity c behaves in the manner of a one-way mirror, it allows the particles to pass through the singularity towards the subluminal regime but prohibits the return to the superluminal regime. If the transition to the superluminal regime is forbidden, the question of finding the origin of superluminal particles, if they exist, arises.

9.2.1 Lorentz invariance violation

We assume that there is an energy threshold E_L above which the singularity c no longer appears i.e. that the two regimes are unified. Above this level of energy E_L the Lorentz invariance would no longer apply, leaving elementary particles free to move at speeds much greater than c . It is reasonable to assume that the E_L threshold is less than Planck energy⁶. In this context, the epoch of cosmic inflation [8], [9], would be a period of Lorentz invariance violation [10], [11]. The universe would have known, before the Planck era, a brief period⁷ of violation of the Lorentz [12] invariance (of the order of 10^{-35} s) before the appearance of the singularity c and the universal application of the principle of invariance. From the emergence of the singularity c , and by virtue of the velocity-addition formula and interactions between particles, the great majority of elementary particles would have been in subluminal regime. This scenario has the advantage of providing a formal framework to

⁶Planck energy: $E_p \approx 10^{19}$ GeV

⁷Planck time: $t_p \approx 10^{-42}$ s

explain the sudden end of the inflation process. From this short period of cosmic inflation, only a handful of free elementary particles, not having undergone interactions, would have found themselves trapped in the superluminal regime and continued their journey at speeds $> c$.

9.3 Superluminal \rightarrow superluminal

What about an interaction between a free massive particle in superluminal regime and a photon? It seems that this interaction does not affect the regime of the particle which is braked by inverse Compton scattering. The particle transmits energy to the photon, it is interacting with, but despite slowing down its total energy increases following the formula $E = \gamma_{>} mc^2$ (chap. 8.4).

9.4 About the stability of the superluminal regime

These first results suggest that, unlike the subluminal regime, the superluminal regime is unstable by nature. The rare candidates for the superluminal regime are probably survivors of the post-Planckian era or of a very violent stellar explosion such as a superluminous supernova, exceptional events when the energy exceeds the E_L threshold beyond which the singularity c is suppressed and the two regimes merge. These superluminal particles are either braked by photons or forced to revert to a subluminal regime after an interaction with a massive particle. The probability of encountering superluminal particles decreases with the distance they cover. We will see in chapter 11 that the very low probabilities linked to the events of ultra-high-energy cosmic ray (fig.7) reinforce this idea.

9.5 Conclusions on the permeability of the singularity c

We can sum up the passage from one regime to the other in the table below:

Passage direction	Event
subluminal \rightarrow superluminal	Precluded (if $E < E_L$), \forall event
superluminal \rightarrow subluminal	After an interaction with a massive particle
superluminal braking	After an interaction with a photon by inverse Compton scattering

10 Synoptic table of equations

We have just shown that it is possible to construct a complete theory of relativistic superluminal mechanics which is covariant and compatible with the principles of special relativity. The speed of light is no longer merely an unbreakable speed limit but a singularity in the speeds' spectrum. In superluminal regime, the laws of relativity are reversed: mass and energy decrease with the increase of velocities until reaching the mass and the energy of an object at rest at infinity. Like the subluminal regime, the deformation of space-time increases as soon as one approaches the singularity c .

	SUBLUMINAL Special relativity $v < c$	SUPERLUMINAL $v > c$
	$\beta = \frac{v}{c}$	
c : speed of light in a vacuum	Invariant and equal to c	
Relativistic invariance	$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 $ always positive	
E: Energy	$\frac{1}{\sqrt{1 - \beta^2}} mc^2$	$\frac{\beta}{\sqrt{\beta^2 - 1}} mc^2$
L: Lagrangian	$-\sqrt{1 - \beta^2} mc^2$	$\frac{\sqrt{\beta^2 - 1}}{\beta} mc^2$
P: Impulse	$\frac{1}{\sqrt{1 - \beta^2}} m\vec{v}$	$\frac{1}{\beta^3 \sqrt{\beta^2 - 1}} m\vec{v}$
Proper time	$dt' = \sqrt{1 - \beta^2} dt$	$dt' = \frac{\sqrt{\beta^2 - 1}}{\beta} dt$
Proper length	$dl' = \sqrt{1 - \beta^2} dl$	$dl' = \frac{\sqrt{\beta^2 - 1}}{\beta} dl$
Lorentz factor	$\gamma_{<} = \frac{1}{\sqrt{1 - \beta^2}}$	$\gamma_{>} = \frac{\beta}{\sqrt{\beta^2 - 1}}$
Velocity-addition law	$v_x = \frac{v + v_{x'}}{1 + \frac{v}{c^2} v_{x'}}$ $v_y = \frac{v_{y'} \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} v_{x'}}$ $v_z = \frac{v_{z'} \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} v_{x'}}$	$v_x = \frac{v v_{x'} + c^2}{v + v_{x'}}$ $v_y = \frac{v_{y'} \sqrt{\frac{v^2}{c^2} - 1}}{\frac{v_{x'}}{c} + \frac{v}{c}}$ $v_z = \frac{v_{z'} \sqrt{\frac{v^2}{c^2} - 1}}{\frac{v_{x'}}{c} + \frac{v}{c}}$
Doppler effect	$\frac{1}{\sqrt{1 - \beta^2}} (1 + \beta \cos(\theta))$	$\frac{\beta}{\sqrt{\beta^2 - 1}} (1 + \frac{\cos(\theta)}{\beta})$

11 Ultra-high-energy cosmic ray (UHECR)

The limit of Greisen-Zatsepin-Kuzmin [13], [14] (or GZK limit) is a theoretical upper limit of cosmic ray energy from distant sources (beyond our galaxy). In other words, cosmic rays with an energy greater than this limit are not to be observed on Earth. In fact, excesses of this theoretical limit have been observed (fig. 7) [15].

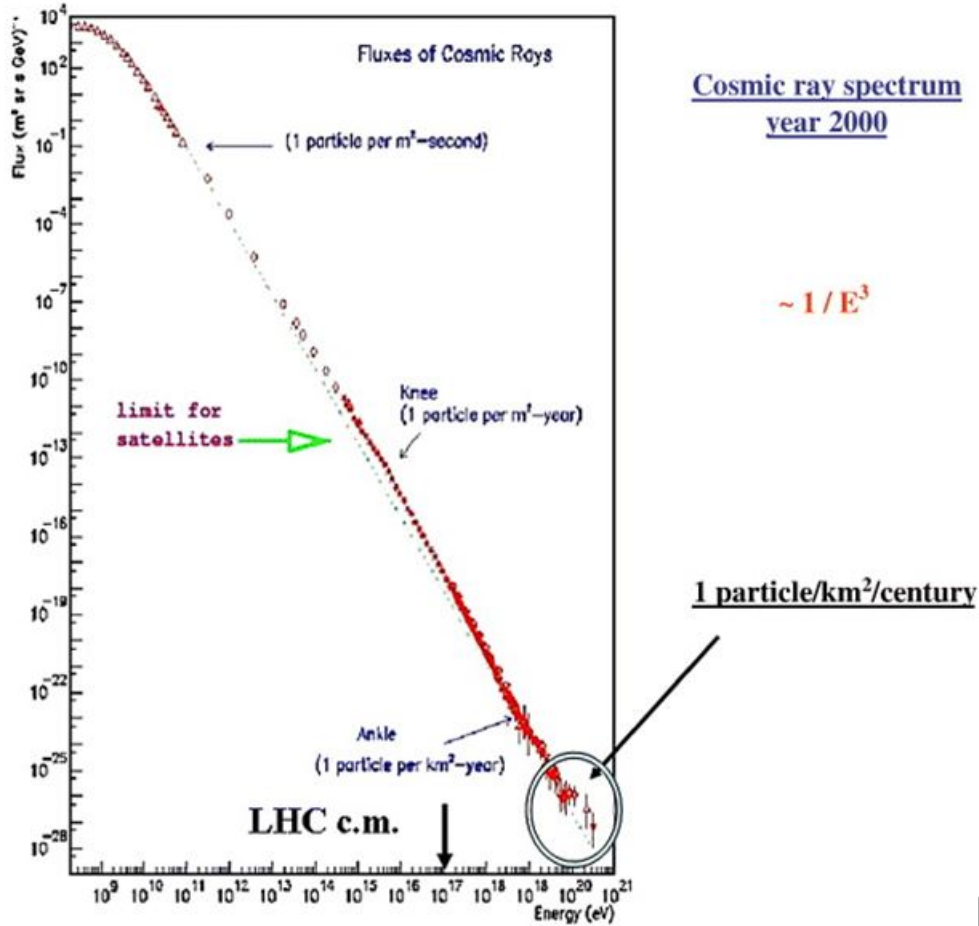


Figure 7: flux of primary cosmic rays as a function of energy [16]

11.1 UHECR

UHECR are particles whose estimated energy is in the order of $10^{19}eV$ and whose origin is unknown. The particles moving through space interact with the Cosmic Microwave Background (CMB) [17] and lose their energy gradually but rapidly (fig.8). Accordingly, UHECR should be formed at less than 100 Mpc from the Earth but no ultra-energetic phenomenon at the origin of these particles has been observed.

11.2 Hypothesis on the origin of UHECR energies

The physical mechanism behind UHECR energies is still not known and is the object of active theoretical research, in particular by testing the validity of Lorentz invariance and its hypothetical violation [18]. This study is in line with this logic but suggests developing physics with covariant equations, respectful of the Lorentz invariance. The idea that UHECR are relativistic superluminal particles braked by the CMB all along their path before reaching us is not to be excluded. Braked superluminal cosmic particles

can, gradually and over megaparsecs, acquire energies comparable to those detected on Earth as soon as they approach the singularity c . In this context, it would be interesting to recalculate the GZK limit in superluminal regime and compare the theoretical results with the measures delivered by cosmic ray observatories. This paradigm proposes a simple physical mechanism which does not require the violation of the Lorentz invariance principle to reach UHECR energies [18] [19]. It is now necessary to conjecture on the origin of these superluminal particles.

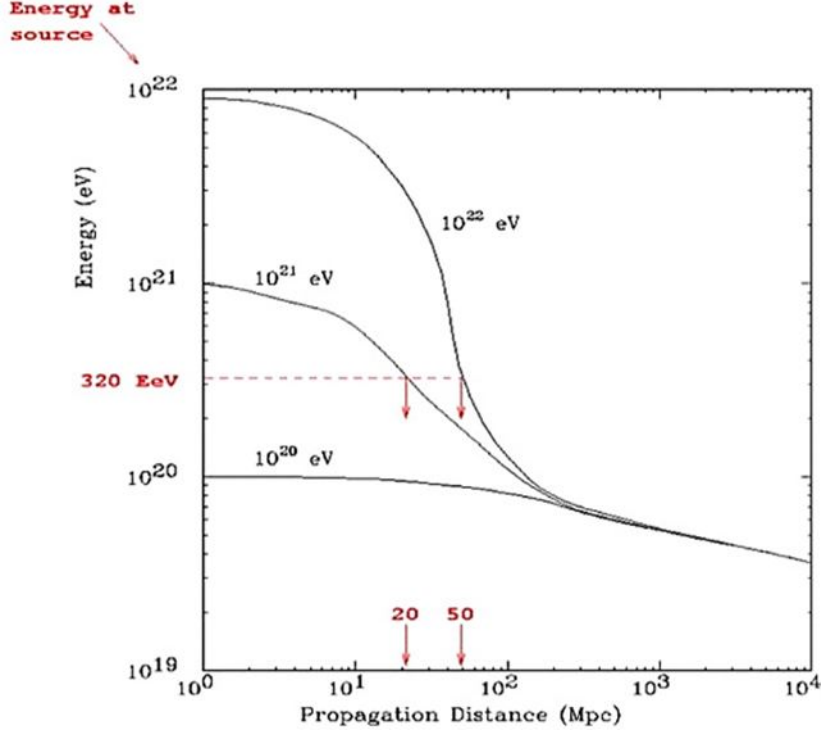


Figure 8: The energy of protons as a function of the propagation distance. As a consequence of the GZK effect, protons coming from a distance greater than ~ 100 Mpc have lost memory of their initial energy [20]

11.3 Hypothesis on the origin of UHECR

We have seen in chapter (9) that the instability of the superluminal regime suggests the violation of the Lorentz invariance when the energies exceed a threshold energy E_L inferior to Planck energy. This hypothesis suggests that during the brief period of cosmic inflation all the particles were superluminal and that after the numerous collisions between particles the great majority of the particles returned to subluminal regime. The rare superluminal particles surviving from this period of inflation have continued their path and those which have not encountered any obstacle will have been braked by the CMB until attaining the very high energies observed ($> 10^{19} eV$) before reaching us. We can also surmise that extremely violent stellar explosions (e.g. the explosion of superluminous supernova [21]) are able to reach the threshold E_L beyond which the two regimes are temporarily unified, thus suppressing the singularity c while propelling atomic nuclei into superluminal regime.

12 Conclusions

We have assumed the existence of a superluminal regime and we have calculated the equations of relativistic superluminal mechanics by applying the principle of least action and the invariance of Minkowski space-time. In this model, which respects Lorentz's invariance principle, the velocity of light is not just an unbreakable velocity limit, but a singularity that separates two distinct regimes with practically inverse behaviours: the subluminal regime governed by the laws of special relativity and the superluminal regime by equations whose evolution according to v is symmetrical with respect to the singularity c . The permeability of this singularity is not symmetrical: the transition to the superluminal regime is precluded by the equations, whereas the return to subluminal mode seems natural. The asymmetric behaviour of the singularity c , like a one-way mirror, guarantees the principle of causality. The braking of the superluminal particles, corollary of the singularity c , has the remarkable property of extracting the energy contained in the quantum vacuum. It provides an ideal theoretical framework for explaining the origin of UHECR energies $> 10^{19}$ eV. It is also probable that the singularity c is dissipated beyond an energy E_L which unifies the two regimes. This violation of the Lorentz invariance combined with the instability of the superluminal regime may provide a better understanding of the sudden end of the cosmic inflation epoch.

References

- [1] L. Landau et E. Lifshitz. *The Classical Theory of Fields*. Butterworth–Heinemann, 2002 (4th ed.).
- [2] J. A. Wheeler E. F. Taylor. *Exploring black holes : introduction to general relativity*. Addison Wesley Longman, 2000.
- [3] J.M. Lévy Leblond et al. Additivity, rapidity, relativity. *Am. Journal of Physics*, 47, 1979.
- [4] J.M. Raimond. Cours relativité restreinte. Laboratoire Kastler Brossel, Département de Physique, Ecole normale supérieure.
- [5] A. Einstein. Zur elektrodynamik bewegter körper. *Annalen der Physik*, 322:pp.891–921, 1905.
- [6] V. Ougarov. *Théorie de la relativité restreinte*. Edition MIR, Moscou, 1974.
- [7] M. Bunge. *The place of the causal principle in modern science*. Harvard University Press, 1959.
- [8] A. Guth. The inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23:347, 1981.
- [9] A. Linde. *Particle Physics and Inflationary Cosmology*. Harwood, 1990.
- [10] D. Mattingly. Modern tests of lorentz invariance. *Living reviews in Relativity*, 8, 2005.
- [11] A. R. Solomon. *Cosmology Beyond Einstein*. PhD thesis, University of Cambridge, 2015.
- [12] E. W. Kolb; M. S. Turner. *The Early Universe*. Basic Books, 1994.

- [13] G. Zatsepin et V. Kuzmin. Upper limit of the spectrum of cosmic rays. *Journal of Experimental and Theoretical Physics Letters*, 4:78, 1966.
- [14] K. Greisen. End to the cosmic-ray spectrum? *Physical Review Letters*, 16:748, 1966.
- [15] M. Takeda et al. Extension of the cosmic-ray energy spectrum beyond the predicted greisen-zatsepin-kuz'min cutoff. *Phys. Rev. Lett.*, 81:1163, 1998.
- [16] S.P. Swordy. *The Energy Spectra and Anisotropies of Cosmic Rays*. Space Science Reviews, 2001.
- [17] G. Matthiae. The cosmic ray energy spectrum as measured using the pierre auger observatory. *New Journal of Physics*, 2010.
- [18] W. Bietenholz. Cosmic rays and the search for a lorentz invariance violation. *Physics Reports*, 505:145, 2011.
- [19] S. Coleman et S.L. Glashow. High-energy tests of lorentz invariance. *Phys. Rev. D*, 59, 1999.
- [20] Todor Stanev et al. Propagation of ultrahigh energy protons in the nearby universe. *Phys. Rev. D*, 62, 2000.
- [21] B. Paczyński. Are gamma-ray bursts in star-forming regions? *The American Astronomical Society*, 494, 1998.