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# Sharing rules for a Common-Pool Resource with private alternatives

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## Abstract

Agents with ex-ante claims over a common pool resource (CPR) of uncertain size face the risk of resource shortage if the sum of the claims they have over the common pool resource is incompatible with the actual resource size. In case of shortage, a sharing rule or bankruptcy rule is required to organize the restrictions and allocate the available resource among CPR users. However, in many situations, agents can diversify their resources and substitute an alternative safe but costly resource to the free but risky CPR. Optimal resource management requires to study the interaction between the sharing rule of the CPR and the resource diversification choices of agents. We find the optimal sharing rule under two assumptions concerning the regulator: i) the social planner defines simultaneously the sharing rule and the diversification choices in order to maximize social welfare; ii) the policy maker uses the sharing rule as an instrument to induce and each agent to make the optimal diversification decision. We interpret our results in the context of water management in France.

*Keywords: Diversification, Irrigation, Multiple resources, Sharing rules, Self-insurance, Water management*

*JEL Classification: Q25, Q58*

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# Introduction

Common pool resources (CPR) are often managed through the - formal or informal - allocation of access rights or use licenses to each member of the group entitled to claim a share of the resource. When the total availability of the resource is known with certainty, optimal allocation imposes that the marginal value of the resource to each user be equal. However, in many cases, CPR are natural resources and quantities available can vary stochastically due for example to climatic conditions. When the size of the CPR is uncertain, users are exposed to the risk of shortage. There is a need to establish a sharing rule, ie a rationing scheme defining the share of the total resource allocated to each user when the sum of claims is larger than the amount of resource that is available. Users therefore face the risk of getting a a lower quantity of the CPR than planned and claimed. When the possibility exists they may want to diversify their resource use. For example they can reduce their dependence on the free but risky CPR by investing into a costly but safer substitute resource which thereby provides an opportunity to self-insure against resource shortage from the CPR (Ehrlich and Becker (1972)). Investments in self-insurance are made ex-ante, before knowing the amount of the CPR. Since the sharing rule determines the share of resource allocated to each agent in case of shortage, it also affects agents' diversification decisions.<sup>1</sup> Each individual faces a trade-off between reducing the cost of his investment in the substitute resource and minimizing the risk of a CPR shortage. At the society level, the optimal management of a CPR involves two dimensions when an alternative resource is available: (i) the rule according to which the resource should be shared in case of shortage, and (ii) the level of shortage risk that should be taken at the society level, or equivalently, the level of investment in a secure substitute resource.

CPR shortage arises frequently in irrigated agriculture, when several farmers use the same water resource for irrigation. Typically, farmers need to choose their crops and decide about their production plans before knowing the amount of irrigation water that will be available for the coming season. They face two types of uncertainties : uncertainty about water availability and uncertainty about the needs of the other users of the CPR. In France for instance, each farmer has to fill a form for a water use license in October in order to be authorized to withdraw the corresponding volume from the river and cover his irrigation needs for the coming year. Volumetric licenses are granted on an annual basis by the Authority in charge

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<sup>1</sup>We call “diversification decision” or “resource portfolio” the choice of investment into the safe resource (the self-insurance level) and the dual decision of the claim to the CPR.

of water management. In times of shortage, when water flows are too low to fulfil licenses, irrigation water is shared through a system of temporary restriction roasters, managed and controlled by public authorities. But French farmers can also partially secure their access to irrigation water by investing into farm storage, by pumping equipment for groundwater, or by signing delivery contracts with water companies guaranteeing pressurized water from storage dams etc.<sup>2</sup> Such opportunities allow farmers to mitigate the risk of a water shortage in the river. Taking up those opportunities, they agree to forego part of their claims on the (uncertain) river water. The deal with public authorities is the following: their investments in substitute resources, such as reservoirs filled in winter, are subsidized but they must hand back a share of their pumping licenses corresponding to the safe volume obtained. Clearly, by relying more on private resources, they relieve the collective pressure on the river and reduce the probability that total demand exceeds total availability, mitigating as well the severity of shortage when it occurs. From a social welfare perspective, there is a need to find the right balance between economic losses due to the risk of water shortage and investment costs in alternative safe resources. Water managers must take into account the impact of the sharing rule they implement in times of drought on the self-insurance investment decisions of farmers in more secure resources.

Water allocation and river sharing problems are equivalent to “bankruptcy” problems when existing water rights cannot be fulfilled because of low available flows during drought times (Ansink and Weikard (2009)). The canonical example of bankruptcy involves the liquidation of a bankrupt firm, a situation where creditors’ entitlements exceed the worth of the firm. The bankruptcy literature provides well-behaved and acceptable sharing rules to award the remaining resources of the firm (the estate) between its creditors (Aumann and Maschler (1985), Herrero and Villar (2001), Moulin (2001)). Interestingly, the CPR and the bankruptcy literature are rarely referring to one-another whereas such bankruptcy rules have been observed in different contexts of CPR management. The best-known rule is the proportional rule, which recommends that awards be proportional to initial claims. The proportional rule is widely used in Australia: water users hold water rights which give them access to a proportion

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<sup>2</sup>French farmers show a growing interest in individual reservoirs in order to diversify their water resources. These reservoirs are called “*réserves de substitution*” or “*retenues collinaires*” in France. Some of these reservoirs are not individual but shared between a small number of neighbours. The reservoirs are filled during winter, when water is relatively abundant and when the reservoirs’ filling activity does not compete with irrigation. This resource is perceived as safe by the farmers because the quantities are known in advance (farmers can observe the quantity stored at the end of winter) and administrative restrictions do not apply to this resource. Reservoir building is assumed to have no effect on the probability of water shortage occurrence (Erdlenbruch and Montginoul (2009)).

of the overall amount of seasonal water available. The constrained equal-awards rule shares the estate equally amongst creditors, subject to the condition that no creditor receives more than her initial claim. A dual formulation of equality, focusing on the losses creditors incur as opposed to what they receive, underlies the constrained equal-losses rule. It proposes to share losses equally across creditors, subject to the condition that no creditor ends up with a negative award. In France, water is shared through an administrative procedure which often gives priority to pre-identified “priority” users, for example vegetable growers get the scarce water at the expenses of fodder producers, independently of their claim over the resource (their water licenses). It is akin to the constrained equal-awards rule but the equality condition is replaced by priority ranks. However, in cases when there is more homogeneity amongst farmers, the water is shared more practically by restricting allowed irrigation times. For example left bank farmers are allowed to irrigate on the first two days of the week, and right bank farmers are allocated the last two days. This rule is closer to the constrained losses rule: the missing water is shared among farmers but they can irrigate according to their pumping capacity, which is assumed to be calibrated on their volumetric licence.

In this paper we develop a model that allows us to study the interactions between the rule used to share the CPR in case of shortage and users’ resource diversification strategies. The paper aims at determining the optimal sharing rule of the CPR under two different set of assumptions concerning the regulator: i) the social planner defines simultaneously the optimal investment in a secure resource and the efficient allocation of the CPR in case of shortage; ii) the policy-maker can use the sharing rule as an instrument to influence CPR users diversification strategies. We find that the optimal sharing rule depends on social value from resource use when the only source of heterogeneity among agents is their valuation of the resource use and depends on relative risk tolerance when the only source of heterogeneity among agents is their risk tolerance. We also determine the optimal claim to the CPR as a function of agent’s valuation of the resource and her risk tolerance, the distribution of the CPR size and the cost of the safe resource. Finally, we find that agents choose an optimal diversification strategy when they know the sharing rule and when this rule does not depend on individual claims. The paper contributes to the empirical literature about the specific issue of water sharing rules in times of shortage and provides some advice targeted towards policy makers in charge of designing those rules.

The paper is organized as follows. The first section outlines the main assumptions of the model. In section 2 we consider the case of risk neutral agents with heterogeneous resource values. In section 3 we consider agents heterogeneous in their risk preferences through a

mean-variance model. Section 4 concludes and draws some general recommendations for resource management under uncertain resource size when a substitute resource is available. We illustrate and interpret our results in the specific context of water management in France. All the proofs are relegated in the appendix.

## 1 The model

The model is intended to provide answers to two questions:

(Q1) What is the optimal total claim to the CPR (or optimal diversification) and the optimal sharing of the resource when the total claim exceeds the CPR size? This first question is related to the solution that would be chosen by a perfectly informed benevolent social planner, who can decide both on the sharing rule and on each individual's diversification strategies.

(Q2) How well different sharing rules perform in decentralizing the optimal diversification solution? This second question considers the more realistic issue where the regulator can only choose the sharing rule. Indeed, for the type of secure resource we consider, the investment decision in the safe resource is taken at the farm level.<sup>3</sup>

### 1.1 Assumptions

For clarity, the assumptions are presented with the exemple of irrigation water management in France. It can easily be extended to any CPR of uncertain size with ex-ante claims over the resource, when a safe and costly alternative is also available for substitution.

We consider a population of  $n = 2$  farmers indexed by  $i = 1, 2$ . The results can be extended to the case of  $n$  agents but are more easily understandable with only two agents. Each farmer has access to a maximum quantity of water  $\bar{H}_i$  and is given the choice to diversify his water sources.<sup>4</sup> He has access to water from the river (CPR) and from a private reservoir (secure

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<sup>3</sup>The model does not capture other investments that can also securize the water resource such as big dams. Such solution would require public investment and thus regulatory involvement in the decision process.

<sup>4</sup>We assume that each farmer  $i$  historically request an annual water licence  $\bar{H}_i$  equal to his profit maximizing water input use. The  $\bar{H}_i$  can thus be interpreted as the historical allocation of pumping rights into the river, satisfying the historical needs of water input. Since pumping rights have been largely over allocated in the past, public authorities have decided that no additional quantities can be granted to individual farmers. Therefore,  $\bar{H}_i$  is not only the historical profit maximizing quantity of water for farmer  $i$ , it is also the maximum quantity that he is allowed to draw from the river.

resource). The river water is an uncertain resource: farmers are exposed to a risk on the size  $x$  of the CPR. The reservoir offers a safe alternative but it entails a variable cost of  $c > 0$  per unit of water (no fix cost). We assume that the two resources are perfect substitutes, allowing for any combination of claims to the two resources.

Note  $R_i$  farmer  $i$ 's ex-ante claim on the river water. His claim is constrained by  $0 \leq R_i \leq \bar{H}_i$  because it cannot exceed the historical water license  $\bar{H}_i$ . Once the resource size is known, the volume of water he can draw from the river is denoted  $Q_i$ . It may not be equal to the volume claimed if the resource size is lower than the sum of the claims over the CPR and restrictions are organized.  $\bar{H}_i - R_i$  is the complementary quantity claimed from the secure resource. All the units claimed in the secure resource are received by definition of a safe resource. In order to study the diversification choices of agents according to their individual characteristics such as value of the resource and risk preferences, we keep everything else equal and assume a total claim of water equal for all farmers:  $\bar{H}_i = \bar{H} \forall i$ .

Agent  $i$ 's profit from water use is denoted  $\Pi_i$ . It is set equal to the value of the water units drawn from both resources ( $H_i = (\bar{H} - R_i) + Q_i$  with  $0 \leq H_i \leq \bar{H}$ ). minus the cost of the units obtained from the secure resource. For simplicity, we assume that the value of each unit of resource is constant and equal to  $v_i$ . We assume it is always strictly larger than the cost of the alternative resource ( $v_i > c$ ).

$$\Pi_i = v_i [(\bar{H} - R_i) + Q_i(x, R_i, R_{-i})] - c(\bar{H} - R_i) \quad (1)$$

The volume of water  $Q_i$  that farmer  $i$  is allowed to draw from the river depends on the size of the resource  $x$  and on the total demand for river water  $R = R_1 + R_2$ . For simplicity, we assume that  $x$  follows a uniform distribution on the interval  $[a; b]$  ( $a \geq 0$ ):  $F(x) = \frac{x-a}{b-a}$  and  $f(x) = \frac{1}{b-a}$ . Nevertheless, the results of this section can easily be generalized (in implicit form) to any other distribution of  $x$  provided risk neutrality is assumed. If the size of the resource available is larger or equal to the total claim  $R$ , farmer  $i$  gets his claim  $R_i$  from the river. In the opposite case, a sharing rule is implemented. The sharing rule determines the quantity  $\theta_i$  that agent  $i$  gets in case of restriction. As a result,

$$\begin{cases} Q_i = R_i \text{ and } H_i = \bar{H} & \text{if } R \leq x \\ Q_i = \theta_i \text{ and } H_i < \bar{H} & \text{if } R > x \end{cases}$$

There are many possible ways to define the relation between  $\theta_i$  and  $x$ . The quantity received can eventually be function of claims:  $\theta_i(x, R_i, R_{-i})$ . Two obvious restrictions are necessary : (C1)  $\theta_1 + \theta_2 = x$  and (C2)  $0 \leq \theta_i \leq R_i$  for  $i = 1, 2$ . According to restriction (C1) the sharing rule must exhaust the total quantity available  $x$ . It means that any rule should be defined such that the available resource  $x$  is always fully allocated when  $R > x$ . This is of course a condition for optimality as a share of the resource not allocated will not contribute to social welfare.<sup>5</sup> Restriction (C2) states that no agent can get a negative quantity nor get a share of the CPR that exceeds her claim ( $0 \leq \theta_i \leq R_i$ ). This constraint rely on the assumption that an agent has no value for the units of a resource above  $\bar{H}$ . It is thus not optimal to allocate a share  $\theta_i > R_i$  to an agent as her total water allocation will be  $H_i > \bar{H}$ . The sharing rules should be specified to ensure these two conditions. Section 1.2 presents the different sharing rules mentioned in the bankruptcy literature that we shall consider in this paper.

## 1.2 Bankruptcy sharing rules

We follow the presentation of the bankruptcy rules of Thomson (2003) but slightly modify these rules in order to fullfill conditions (C1) and (C2) defined as the minimal requirements for an optimal rule in our problem. Moreover, we remove the constraint of equality and therefore modify the names: “constrained-equal awards rule” becomes “constrained awards rule” and “constrained-equal losses rule” is replaced by “constrained losses rule”. We can therefore define three sharing rules based on bankruptcy rules:

(i) The proportional rule divides the amount of resource available proportionally to the agents’ claims. In case of shortage an agent who claimed  $R_i$  receives  $\theta_i^P = x \frac{R_i}{R}$ . This rule automatically satisfies constraints (C1) and (C2).

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<sup>5</sup>In the field, when water is not fully allocated to irrigators, the water remains into the river and contributes to “environmental flows”. This is of course not a pure loss for the society. In the model, we consider that  $x$  is the quantity of resource that has been allocated to agricultural users such that they can use all of it without compromising the needs of other users (like the environment).

(ii) The CA (constrained awards) rule divides the estate among the creditors independently from their claims. In case of shortage, an agent who claimed  $R_i$  receives a share of the available resource  $\theta_i = \Omega_i x$  with  $\sum_{i=1}^n \Omega_i = 1$  and  $\Omega_i$  the “priority” rank of each agent independent from the vector of claims. In the case of two agents, in order to satisfy constraints (C1) and (C2), we rewrite the CA rule as follows for  $R_1 \geq R_2$ :  $\theta_2^{CA} = \min[\Omega_2 x; R_2]$  and  $\theta_1^{CA} = \min[x - \theta_2^{CA}; R_1]$ . It guarantees that no agent receives more than her claim and all the resource  $x$  is allocated when  $R > x$ . For example, if agent 2’s claim is lower than  $\Omega_2 x$ , she receives her claim  $R_2$  and the other agent is allocated all the resource left  $x - R_2$ .

(iii) The CL (constrained losses) rule allocates the missing amount of resource. The claim of each agent is restricted by a share of the difference between the aggregate claim and the total amount available, provided no agent ends up with a negative transfer. In case of shortage an agent who claimed  $R_i$  receives  $\theta_i = R_i - \beta_i (R - x)$  with  $\sum_{i=1}^n \beta_i = 1$ . For two agents, in order to satisfy constraints (C1) and (C2), we rewrite the CA rule as follows for  $R_1 \geq R_2$ :  $\theta_2^{CL} = \max[R_2 - \beta_2 (R - x); 0]$  and  $\theta_1^{CL} = x - \theta_2^{CL}$ . It guarantees that all the resource  $x$  is allocated when  $R > x$  and the awards cannot be negative.

These three rules will be compared on the basis of their capacity to participate to the efficient management of the CPR. We first study if one of these rules is an optimal sharing rule when the regulator both fix the sharing rule and decide on the optimal investment in the secure resource (Q1). Secondly, we determine which rule induces farmers to take the optimal decisions in terms of diversification choices (Q2). The answer to these questions will be given under different assumptions concerning agents preferences. In section 2, we first solve the model assuming risk neutral agents in order to isolate the effect of heterogeneous values of the resource. In section 3, we introduce a mean-variance objective function to observe the effect of heterogeneity in risk tolerance when values of the resource are identical.

## 2 Risk neutral agents with heterogenous values

In this section, we assume risk neutral agents with heterogeneous values of the resource. A risk neutral agent, with value  $v_i$  of the resource, chooses her portfolio of resources in order to maximize her expected profit defined as follows. We assume perfect knowledge of  $v_i$ ,  $\bar{H}$  and the distribution of  $x$  by the two agents and the regulator.

$$\underset{R_i}{Max} E [\Pi_i] = (v_i - c)(\bar{H} - R_i) + \frac{v_i}{b - a} \left[ \int_R^b R_i dx + \int_a^R \theta_i(x) dx \right] \quad \forall i = 1, 2 \quad (2)$$

We first solve the full optimization program of the regulator to answer Q1. Then we compare the rules on the basis of their capacity to decentralize the optimal claims to the CPR (Q2).

### 2.1 Social optimum: optimal sharing rule and diversification

We assume that the social planner is a perfectly informed benevolent regulator. Formally, he chooses both the vector of claims to the CPR  $\{R_i^*\}$  and the sharing rule  $\{\theta_i^*\}$  in order to maximize the weighted sum of the expected profits of the two agents.  $\lambda_i$  is the weight of agent  $i$  in the social welfare function, and  $\lambda_i v_i$  measures the “social value” ( $SV_i$ ) of the resource used by agent  $i$  (the private value  $v_i$  multiplied by the weight of agent  $i$   $\lambda_i$  in the social welfare function). Since the two resources are perfect substitutes, the investment in the safe resource is the dual decision of the claim to the CPR.

We consider a vast array of rules  $\theta_i(x, R_i, R_{-i})$  where the individual quantities allocated to agent  $i$  in case of shortage may depend on the amount of resource available  $x$  and the vector of claims  $(R_i, R_{-i})$ . The program of the regulator writes as follows:

$$\underset{\{R_i\}, \{\theta_i(x, R_i, R_{-i})\}}{max} SW = \sum_{i=1,2} \lambda_i \left[ (v_i - c)(\bar{H} - R_i) + \frac{v_i}{b - a} \left[ \int_R^b R_i dx + \int_a^R \theta_i(x, R_i, R_{-i}) dx \right] \right] \quad (3)$$

/c (1)  $\theta_1 + \theta_2 = x$   
(2)  $0 \leq \theta_i \leq R_i \quad \forall i = 1, 2$

**Proposition 1:** If  $SV_1 > SV_2$  ( $\lambda_1 v_1 > \lambda_2 v_2$ ), the optimal sharing rule is :

$$\begin{aligned}\theta_1^*(x) &= \min [x; R_1] \\ \theta_2^*(x) &= x - \theta_1^*(x)\end{aligned}\tag{4}$$

and the optimum claims to the CPR are given by :

$$\begin{aligned}R_1^* &= \max \left[ \min \left[ a + c(b-a) \frac{\lambda_1 - \lambda_2}{\lambda_1 v_1 - \lambda_2 v_2}; \bar{H} \right]; 0 \right] \\ R_2^* &= \max \left[ \min \left[ \frac{c}{v_2} (b-a) \frac{\lambda_1 (v_1 - v_2)}{\lambda_1 v_1 - \lambda_2 v_2}; \bar{H} \right]; 0 \right] \\ R^* &= \min \left[ a + \frac{c}{v_2} (b-a); 2\bar{H} \right]\end{aligned}\tag{5}$$

The optimal sharing rule is such that the agent with the higher social value of the resource gets all the resource, provided constraints (C1) and (C2) are satisfied. Intuitively, when agents are risk neutral, the uncertainty on resource size has no impact and the resource should be allocated according to social value of the resource.

The optimal total claim to the CPR is  $R^*$ : claiming more than  $R^*$  is sub-optimal because the probability of restriction becomes too high; claiming less than  $R^*$  is sub-optimal because of the cost of the alternative resource. This optimal total claim to the CPR is at least equal to the minimum quantity of CPR available (the lower bound of the distribution  $a$ ). It is increasing with the cost of the alternative resource  $c$  adjusted to the value of the resource by the lower value user  $v_2$ . The lower-value user is determinant as the alternative resource is relatively more costly to him. A decrease in the average quantity available of the CPR (a decrease in  $a$  and/or  $b$ ) leads to a lower optimal claim to the CPR. Note that the optimal claim is independent of the total demand  $\bar{H}$ , as long as  $R_i^* < \bar{H}$  for  $i = 1, 2$ . The optimal total claim is also independent of the weights  $\lambda_i$ . The weights only determine how this optimal total claim is shared between both agents. Whether the agent with the higher social value for the resource should withdraw more or less than the other from the CPR depends on their relative values.

**Lemma 1:** If agents are perfectly identical (equal social value), any sharing rule is optimal. The optimal total claim to the CPR remains equal to (5). Identically, when there is only one agent and thus no question of optimal sharing, the optimal claim to the CPR is equal to (5).

## 2.2 Optimal sharing rule when diversification is a private decision

We assume now that the regulator cannot decide in lieu of the agents. Agents choose their resource claims and the regulator use the sharing rule as an instrument. We identify sharing rules  $\{\theta_i\}$  such that agents' diversification choices and claims to the CPR lead to the optimal solution  $\{R_i^*\}$  found above. Formally, the agents and the regulator play a Stackelberg game where the regulator is the leader and the agents the followers. The timing of the game is as follows: step 1: The regulator announces the sharing rule; step 2: Each agent chooses his claim to the CPR so as to maximize her expected profit. Because the sharing rule introduces strategic interactions between agents, we calculate the best response of each agent and identify the unique Nash equilibrium. Knowing each agent's best response function, the regulator should choose the sharing rule announced in step 1 such that the Nash equilibrium found in step 2 coincides with the socially optimal total claim determined in (5). This will ensure that the chosen rule maximizes social welfare. The game is solved by backward induction.

**Proposition 2: The constrained-awards rule, where the regulator favors the agent with the highest social value for the resource, is the only rule that maximizes social welfare**

Indeed, the sharing rule  $\theta_1^*(x) = \min [x ; R_1]$  and  $\theta_2^*(x) = x - \theta_1^*(x)$  is equivalent to the CA rule with the parameters  $(\Omega_1 = 1; \Omega_2 = 0)$  for  $\lambda_1 v_1 > \lambda_2 v_2$ . It is the only rule (among the three rules described in 1.2) for which the sum of the private claims is equal to the optimal claim  $R^*$ . This rule maximizes social welfare when claims decisions are decentralized.

The CA rule depends on individual claims only through the constraint (C2). This rule thus does not create any strategic interaction.<sup>6</sup> On the contrary, the proportional and CL rules depend directly on individual claims. These two rules fail to yield a social optimum as they introduce strategic interactions between agents. They in fact create incentives to claim more from the CPR than what social optimality would require and lead to suboptimal pressure on the CPT. The social welfare is lower under the proportional and CL rule than under the CA rule defined in proposition 2.

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<sup>6</sup>It does not mean that individual claims are totally irrelevant to define the rule. Indeed, the CA sharing rule is independent of the claim only when the allocated share of the resource is lower than the individual claim for both agents.

With a numerical example, one can show the reduction in social welfare when the sharing rule imposed by the regulator to the agents is not the rule defined in proposition 2. *Figure 3* represents the social welfare as a function of the sharing rule. We only consider a sharing rule of the CA form and vary  $\Omega_i$ . It appears clearly that the social welfare maximizing sharing rule is  $(\Omega_1 = 1; \Omega_2 = 0)$  when  $v_2 < v_1$ .

In this second section, we have answered the two questions stated in section 1, when the two agents are risk neutral and heterogeneous in their social value of the resource (with  $\lambda_1 v_1 > \lambda_2 v_2$ ):

(Q1) The optimal claim to the CPR, equal to (5), is increasing in the cost of the secure resource and decreasing in the average quantity available from the CPR. The optimal sharing rule is such that the agent with the higher social value for the resource gets all the available resource, up to her claim (the other agent is allocated what is left of the CPR). When the two agents have equal social value, any sharing rule and any repartition of the total claim are optimal.

(Q2) The constrained-awards rule with  $(\Omega_1; \Omega_2) = (1; 0)$  is the only rule (among the bankruptcy rules considered) that maximizes social welfare when the claim to the CPR is a private decision, in response to the sharing rule fixed by the regulator.

The results of this first section are rather intuitive and constitute a benchmark. For an irrigation basin in which farmers would mainly differ by the marginal value of water input, the administrative rule adopted by France which consists in announcing priority rules according to the economic weight of crops leads to less strategic interactions (and therefore a better global use of water) than rules based on proportional sharing of available water. However, it is important to check whether such result also holds when farmers have identical marginal value of water but display different levels of risk aversion. The uncertainty on resource size becomes challenging as the way the resource is shared impacts the risk born by agents and their incentive to self-insure. Despite the large evidence of risk aversion amongst farmers (Binswanger (1980), Couture and Reynaud (2010)...), risk preferences are rarely taken into account in the design of mechanisms for efficient allocation of water between heterogeneous agents (see for rare exceptions Tarlock (2000), Fapchamps (2003), Calatrava and Garrido (2006)). The next section deals with heterogeneity in agents' risk tolerances.

### 3 Mean-Variance agents with homogeneous values

The model is modified to account for risk tolerance and heterogeneous risk preferences, by relying on the two-moment decision model first introduced by Markowitz (1952). Our focus is on the consequences of risk-tolerance heterogeneity on the optimal sharing of the resource when resource size is uncertain. Therefore, we assume equal value of the resource ( $v_1 = v_2 = 1$ ) in order to simplify the analysis. We showed that under risk neutrality the optimal sharing of the CPR in case of shortage is such that the agent with the higher social value for the resource receives the total available amount of resource. We can now study the impact of heterogeneity in risk tolerance on the optimal sharing rule and optimal and individual diversification choices.

In the two-moment decision model, the risk is approximated by the variance and agents rank choices according to their mean and their variance. For instance, a risk-averse agent will accept an increase in the variance of his profit only if he receives a compensation in terms of a higher mean profit. The two-moment decision model is compatible with any Von-Neumann Morgenstern utility function as long as the class of available choice options is restricted to distributions that differ from each other only by location and scale (Sinn (1983), Meyer (1987)). For tractability and computation of explicit solutions, we adopt the standard linear specification  $V(\mu, \sigma) = \gamma\mu - \alpha\sigma^2$ , where  $\mu$  and  $\sigma^2$  correspond to the mean and the variance respectively. The parameter  $\alpha$  captures the agent's risk-attitude or sensitiveness with respect to  $\sigma^2$  and  $\gamma$  measures her marginal utility for profit. A positive (negative) value for  $\alpha$  corresponds to risk-aversion (risk-seeking) and the null value to risk-neutrality. Assuming  $\alpha$  different of zero, under this linear specification, the marginal rate of substitution between profit and risk  $\frac{\gamma}{\alpha}$  is a constant.

Agents are assumed to maximize their objective function  $\Phi [\Pi_i]$  defined in (6). The first term is the expected profit and the second term captures the effect of the variance of profit on the objective function. Let  $T_i = \frac{1}{\alpha_i}$  be the agent's risk tolerance and set  $\gamma = 1$ , such that  $T_i$  is also the constant marginal rate of substitution between profit and risk. We call "relative risk tolerance" the ratio  $\frac{T_i}{T_i + T_{-i}}$ . We assume the two agents have different risk tolerance.

$$\Phi [\Pi_i] = \mu [\Pi_i] - \frac{1}{2T_i} \sigma^2 [\Pi_i] \tag{6}$$

$$\mu [\Pi_i] = (1 - c)(\bar{H} - R_i) + \frac{1}{b - a} \left[ \int_R^b R_i dx + \int_a^R \theta_i(x) dx \right]$$

$$\begin{aligned} \sigma^2 [\Pi_i] = & \frac{1}{b - a} \left[ R_i^2(b - R) + \int_a^R \theta_i^2(x, R_i, R_{-i}) dx \right] \\ & - \frac{1}{(b - a)^2} \left[ R_i^2(b - R)^2 + 2R_i(b - R) \int_a^R \theta_i(x, R_i, R_{-i}) dx + \left[ \int_a^R \theta_i(x, R_i, R_{-i}) dx \right]^2 \right] \end{aligned}$$

Note that all the agents with  $R_i > 0$  are subject to the risk on the CPR size. This risk is exogeneously given. However, the individual risk born by an agent is endogeneous: it depends on her own diversification choice. The more she chooses to rely on the CPR, the higher the risk she bears. The individual risk born by an agent is given by the distribution of her profit function defined in (1).

The above specification has several advantages. Firstly, risk preferences are captured by a single coefficient ( $T_i$ ). Secondly it takes into account three components of the cost of risk: (i) the risk premium that agents incur due to the variability of the resource received from the CPR; (ii) the cost  $c(\bar{H} - R_i)$  of relying on the secure resource; (iii) the opportunity cost of self insurance: under the substitution assumption, requiring a unit from the secure resource reduces the claim on the CPR and therefore decreases the quantity of free resource that one receive when there is no shortage. It's an opportunity cost in the sense that the opportunity to receive free units is reduced. Last but not least, we get some computable analytical results for this specification whereas a more general model is more difficult to solve.<sup>7</sup>

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<sup>7</sup>Samuelson (1970) writes: "Many writers have made valuable contributions the problem of optimal risk decisions by emphasizing to analyses of means and variances. These writers have realized that the results can be only approximate, but have also realized that approximate but computable results are better than none". Liu (2004) goes further: "the popularity of the mean-variance analysis is possibly not because of its precision of approximating the expected utility theory but because of its simplicity and the power of its implications".

### 3.1 Social optimum: optimal sharing rule and diversification

Under the assumption of a perfectly-informed benevolent social planner who can define both the vector of claims  $\{R_i\}$  and the sharing rule  $\{\theta_i\}$  in order to maximize social welfare, we identify the optimal  $\{R_i^*\}$  and  $\{\theta_i^*\}$ . The social welfare function writes as (7) for two agents. The same constraints (C1) and (C2) hold under risk aversion. We solve the model for equal weights and therefore equal social value ( $\lambda_1 v_1 = \lambda_2 v_2 = 1$ ).

$$\max_{\{R_i\}, \{\theta_i(x, R_i, R_{-i})\}} SW = \sum_{i=1,2} \left( \mu [\Pi_i] - \frac{1}{2T_i} \sigma^2 [\Pi_i] \right) \quad (7)$$

**Proposition 3: The optimal sharing rule depends on each agent's relative risk tolerance:**

$$\theta_i^{**} = \frac{T_i}{T_i + T_{-i}} x \quad \forall i = 1, 2 \quad (8)$$

**and the optimal individual claim is increasing in relative risk tolerance:**

$$R_i^* = \min \left[ \frac{T_i}{T_i + T_{-i}} Z^* ; \bar{H} \right] \quad \forall i = 1, 2 \quad (9)$$

with  $Z^*$  unique solution of  $g(Z) = 0$  in  $[0; b]$  and

$$g(Z) = Z^3 - Z^2 (b + 2a) + Z (2(T_i + T_{-i})(a - b) + 2ba + a^2) - a^2 b + 2(T_i + T_{-i})(b - a)(a + c(b - a)) \quad (10)$$

Contrarily to the risk neutral case, the optimal solution cannot be identified without specifying a functional form for the sharing rule. We are thus in a second-best world. We consider the three functional forms presented in 1.2 and find an optimal rule for each class of rules by optimizing on the parameter:  $\{\Omega_i\}$  for the CA rule and  $\{\beta_i\}$  for the CL rule. We show that the optimal rule defined for each form are all equivalent for the vector of optimal claims  $\{R_i^*\}$ :  $\Omega_i^* = \beta_i^* = \frac{T_i}{T_i + T_{-i}}$ . The social welfare is therefore equal for the three functional forms considered here.

We find that the total claim to the CPR ( $Z^*$ ) is increasing with the cost of the alternative resource  $c$  and decreasing with the average quantity available from the CPR (a decrease in  $a$  and/or  $b$ ). The more risk tolerant the agents, the higher the total claim to the CPR. The optimal sharing rule is a function of relative risk tolerance: the more risk tolerant agent gets a greater share of the CPR in case of shortage. This result can be surprising at first sight

as one could think that a risk tolerant agent is better able to bear risk and thus should be allocated less resource when it is scarce.<sup>8</sup> However, it is optimal that less risk tolerant agents invest more in the safe resource. We verify in (9) that the optimal claim to the CPR of less risk tolerant agent is effectively lower than the optimal claim of more risk tolerant agent. The less resource is promised to an agent through the application of the sharing rule in case of shortage, the lower her claims to the risky CPR at optimum.

### 3.2 Optimal sharing rule when diversification is a private decision

As in the risk neutral case, the objective is to identify the best sharing rule when it is used by the regulator as an instrument to influence the diversification decisions of the agents. The best sharing rule is such that decentralized Nash equilibrium claims of agents to the CPR coincide with the optimum determined in (9) and thus maximizes social welfare. We verify if the optimal sharing rule defined in the previous section (function of individual relative risk tolerances) is to be chosen by a regulator when he can only choose the rule and diversification decisions are decentralized.

**Proposition 4: The constrained-awards rule where resource is shared according to relative risk tolerance maximizes social welfare:**  $\theta_i^{**} = \frac{T_i}{T_i+T_{-i}}x \quad \forall i = 1, 2$

The Nash equilibrium claims to the CPR corresponds to the optimal claims defined in proposition 3 only for the constrained-awards rule with  $\Omega_i = \frac{T_i}{T_i+T_{-i}}$ . This rule allocates smaller volumes in case of shortage to the less risk tolerant such that they have an incentive to self-insure more. Any CA rule with another allocation criterion is suboptimal. For example, favorizing on the contrary the less risk tolerant agent would crowd out her incentive to self-insure and therefore reduce social welfare. The constrained-losses and proportional rules are also suboptimal as they introduce strategic interactions between agents. We find that they do not lead to optimal claims to the CPR and reduce social welfare compare to the CA rule defined in proposition 4.

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<sup>8</sup>Considering the optimal sharing rule of the CL form, one find that at optimum, the agent relatively more risk tolerant sees her claim to the CPR restricted by a higher proportion:  $\theta_i = R_i - \frac{T_i}{T_i+T_{-i}}(R - x)$ . This result is intuitive as it suggests that the more risk tolerant agent should bear more of the resource shortage at optimum (which is equivalent to bearing more of the risk). However, as the more risk tolerant takes more risk at optimum, she ends up with a higher share of the available resource. The rule is indeed equivalent with the CL and CA form and equal to  $\theta_i = \frac{T_i}{T_i+T_{-i}}x$ .

With a numerical example, one can show the reduction in social welfare when the sharing rule imposed by the regulator to the agents is not the rule defined in proposition 4. *Figure 5* shows the social welfare as a function of the sharing rule. We only consider a sharing rule of the CA form and vary  $\Omega_i$ . It appears clearly that the social welfare maximizing sharing rule of the CA type is such that  $\Omega_1 = \frac{T_1}{T_1+T_2} = 0.2$ .

In this third section, we have answered our two questions assuming agents have mean-variance preferences and have the same social value for the resource ( $\lambda_i v_i = 1 \forall i = 1, 2$ ):

(Q1) The optimal claim to the CPR, equal to (9), is increasing in the cost of the alternative resource and the risk tolerance of the agents, and decreasing in the average quantity available from the CPR. The optimal sharing rule is such that each agent receives a share of the CPR proportional to her relative risk tolerance, with the more risk tolerant agent receiving a greater share of the resource.

(Q2) The best choice of a regulator, in charge of implementing the social welfare maximizing sharing rule when agents are free to decide their level of diversification, is the constrained-awards rule with  $\Omega_i = \frac{T_i}{T_i+T_{-i}}$ . This sharing rule encourages the less risk tolerant agents to self-insure as they receive a low share of the CPR in case of shortage.

## 4 Conclusion and policy recommendations

In this paper we analyze different sharing rules of a risky common-pool resource when resource-users have both ex-ante claims and the possibility to invest into a safe substitute resource.

The first innovation of the paper is to address simultaneously the issue of optimal diversification choices and optimal allocation of the CPR in case of shortage. We demonstrate that the optimal decision of a social planner has the following characteristics: the optimal investment in the safe resource for each agent depends on his marginal social value of the resource and on his relative risk tolerance, as well as on the distribution of the CPR size and the cost of the safe resource. Relying too much on the CPR reduces self-insurance and therefore increases individual risk. It also increases the social risk as it increases the pressure on the CPR and therefore the frequency and severity of shortage. Alternatively, investing too much in the safe resource is suboptimal since it is costly. The social optimum is used as a benchmark to compare the performance of sharing rules when agents have the opportunity to diversify their resource use, reduce their reliance on the CPR and therefore decide the level of shortage risk they bear.

Our second contribution is to identify the sharing rule that a regulator should choose in order to induce CPR users to adopt the level of diversification leading to a social optimum. Our model confirms the well-known result that the rules decentralizing the optimum are rules which do not create strategic interactions between users (eg. the constrained-awards rule). Any rule sharing the resource according to individual claims introduces strategic interactions between agents and fails to encourage agents to diversify optimally their resource portfolio. For a population of risk-neutral agents with heterogeneous values for the resource, the priority parameter of the constrained-awards rule should be such that the agent with the higher social value for the resource gets all the resource available in case of shortage. When agents have heterogeneous risk preference but identical values of the resource, the priority is given to the agent with the higher individual relative risk tolerance. Further work could study how to design a contract such that CPR users reveal truthfully their risk tolerance to the regulator in order to be able to implement the optimal rule under risk aversion and asymmetric information. We also leave for further research numerical simulations to study the optimal sharing rule when agents are heterogeneous both in their risk tolerance and in their value for water. An other extension could include a three-moments model where agents have a mean-variance-skewness objective function. Recent findings in the finance literature showed

that most people react to skewness. This is relevant for examining the sharing rules when agents are sensitive to left-skewed distributions (i.e. there is a higher probability of a drought than an excess water supply, which is more and more likely with climate change).

Our work can help a practical implementation of a scarcity sharing rule, for example in the context of the water law reform that France is presently implementing in order to adjust to the increased frequency of drought. A first recommendation to policy makers is to take into account the impact of the restrictions rule that they implement in scarcity times on farmers choices of water resource use and investments in diversification. Water restriction rules which are defined as a proportion of existing pumping rights or licences are likely to induce sub-optimal investments in alternative safe resources (such as reservoirs). This is an important result since many countries do rely on such restriction rules. For example in Australia, water rights held by farmers determine their allocation as a proportion of total water available for agriculture. In some regions in Spain, volumes distributed in times of shortage are also calculated as proportions of the subscribed quotas. We recommend that restrictions rules be carefully revised in order to limit strategic interactions of farmers when claiming their annual water licenses. Ideally, the rule should take into account the relative marginal social value of water use by each irrigator and his relative risk tolerance. A second message concerns the common practice in France of defining priority users in times of water restriction. Farmers growing high value crops such as vegetables or fruit trees and contracted growers are usually exempted from water restrictions. Our result suggests that this practice is not inefficient as long as priority ranks are independent from volumetric licences. But with the increased variability of water availability, the economic value generated by irrigation should not be the only determinant of water allocation and the risk tolerance of the farmers should also be taken into account. When water available for irrigation is stochastic, contracted growers and fruit producers bear high risk on their profits. These farmers are therefore supposed to be less risk tolerant. As a result, the current practice of exemptions warranting relatively more water to the less risk tolerant farmers in case of drought is inefficient as it reduces their incentives to self-insure whereas it would be optimal from a social point of view that they do so. The social cost of drought can be reduced by using an efficient and transparent restriction rule, encouraging efficient risk management decisions at the farm level.

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## Figures

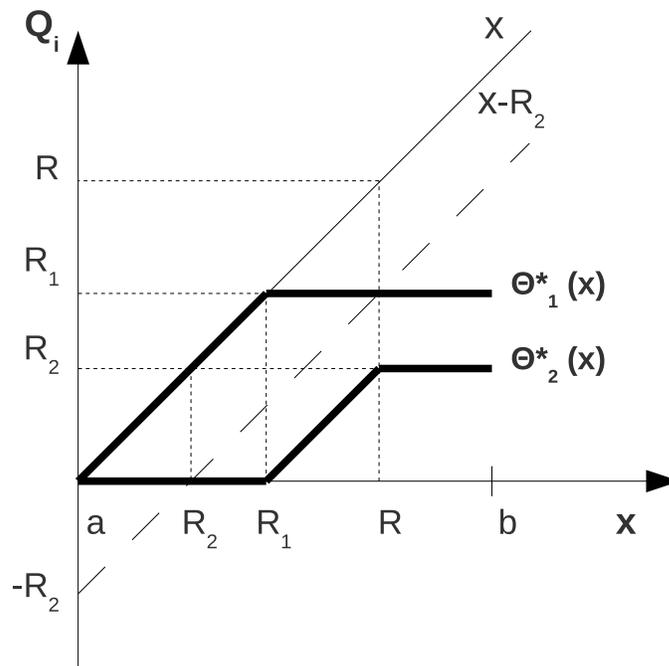


Figure 1: Optimal sharing rule under risk neutrality

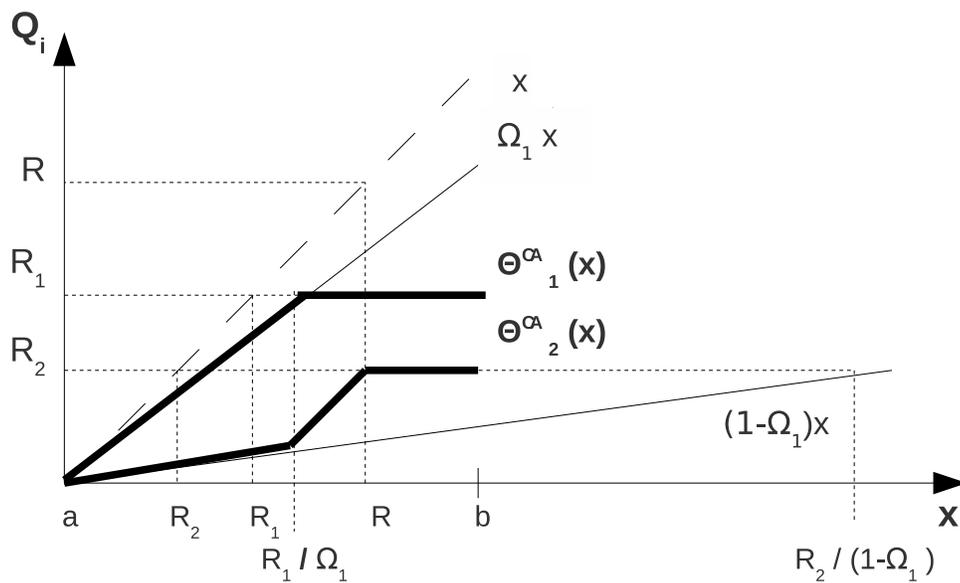


Figure 2: The CA rule does verify constraints C1 and C2 (case 2)

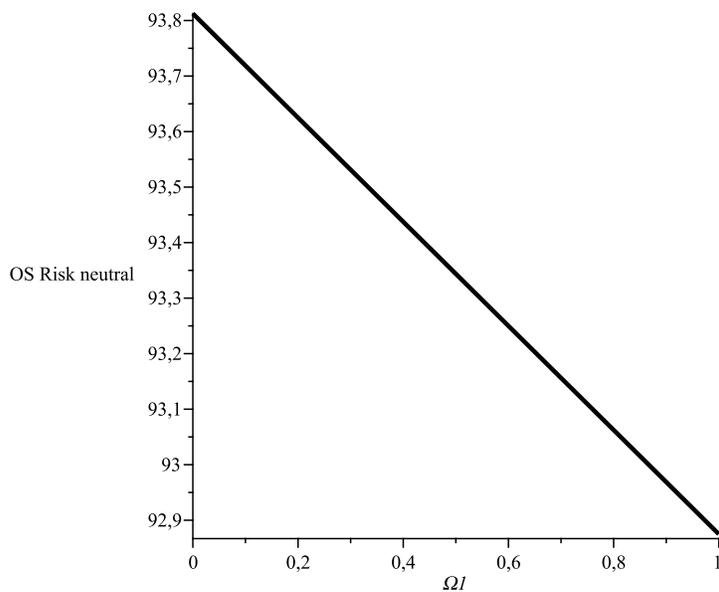


Figure 3: Social welfare according to the sharing rule (Risk neutral agents with  $v_1 > v_2$ )  
 $a = 0, b = 40, H = 20, c = 0.2, v_1 = 4, v_2 = 1$



## Proof Proposition 1

Under the constraint  $\theta_1 = x - \theta_2$ , the social welfare function simplifies in:

$$SW = \lambda_1(v_1 - c)(\bar{H} - R_1) + \lambda_2(v_2 - c)(\bar{H} - R_2) \\ + \frac{1}{b-a} \int_R^b [\lambda_2 v_2 R_2 + \lambda_1 v_1 R_1] dx + \frac{1}{b-a} \int_a^R [(\lambda_1 v_1 - \lambda_2 v_2) \cdot \theta_1(x, R_1, R_2) + \lambda_2 v_2 x] dx$$

For  $\lambda_1 v_1 - \lambda_2 v_2 > 0$ , the social welfare is increasing in  $\theta_1(x, R_1, R_2)$ . As a result, the social welfare maximizing sharing rule is such that  $\theta_1(x, R_1, R_2)$  is maximum:  $\theta_1^*(x) = \min[x; R_1]$ . The optimal sharing rule writes as (4) in order to satisfy constraints (C1) and (C2) (as illustrated in *Figure 1*).

Plugging back the optimal rule defined in (4) into the social welfare function, the social welfare function writes as:

$$SW = \lambda_1 \left[ (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_{R_1}^R R_1 dx + \int_a^{R_1} x dx \right] \right] \\ + \lambda_2 \left[ (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_{R_1}^R (x - R_2) dx + \int_a^{R_1} 0 dx \right] \right] \quad (11)$$

The first order conditions are:

$$\frac{\partial SW}{\partial R_1} = \frac{\lambda_1 c(b-a) + \lambda_1 v_1(a - R_1) - \lambda_2 v_2 R_2}{b-a} = 0 \\ \frac{\partial SW}{\partial R_2} = \frac{\lambda_2 c(b-a) + \lambda_2 v_2(a - R_1 - R_2)}{b-a} = 0$$

We find the optimal claims defined in (5) and verify that  $R_1^*$  and  $R_2^*$  are maxima.

$$\frac{\partial^2 SW}{\partial R_1^2}(R_1^*, R_2^*) = -\frac{\lambda_1 v_1}{b-a} < 0 \\ \frac{\partial^2 SW}{\partial R_2^2}(R_1^*, R_2^*) = -\frac{\lambda_2 v_2}{b-a} < 0 \\ \left( \frac{\partial^2 SW}{\partial R_1^2} \frac{\partial^2 SW}{\partial R_2^2} - \left( \frac{\partial^2 SW}{\partial R_1 \partial R_2} \right)^2 \right) (R_1^*, R_2^*) = -\frac{\lambda_2 v_2}{(b-a)^2} (\lambda_1 v_1 - \lambda_2 v_2) > 0$$

■

## Proof Lemma 1

For  $\lambda_i v_i = v \forall i = 1, 2$ , the social welfare function simplifies in:

$$SW = (v - c)(2\bar{H} - R) + \frac{v}{b - a} \left[ \int_R^b R dx + \int_a^R x dx \right]$$

The social welfare function does not depend on  $\{\theta_i\}$ . Therefore, any sharing rule such that  $\theta_1 + \theta_2 = x$ ,  $0 \leq \theta_i \leq R_i \forall i = 1, 2$  is optimal. Moreover, the social welfare is only function of  $R$ . Any repartition of the total claim between the two agents is optimal. The optimal claim  $R^*$  is such that  $\frac{\partial SW}{\partial R} = 0$  and equal to:

$$R^* = \min \left[ \frac{c}{v}(b - a) + a; 2\bar{H} \right].$$

We verify that  $R^*$  is a maximum  $\frac{\partial^2 SW}{\partial R^2}(R^*) = -\frac{v}{b-a} < 0$

The same result holds for one single agent. In that case, the social welfare function is:

$$SW = (v_i - c)(\bar{H} - R_i) + \frac{v_i}{b - a} \left[ \int_{R_i}^b R_i dx + \int_a^{R_i} x dx \right]$$

■

## Proof Proposition 2

We plug the different sharing rules defined in section 1.2 into the objective function of the agents and compute the corresponding Nash equilibrium. We then compare Nash equilibrium solutions and optimal claims to the CPR defined in proposition 1. When no explicit solution is available for Nash equilibrium, we verify if the FOC conditions of expected profit maximization vanish for  $R_i = R_i^* \forall i = 1, 2$ . If this is not the case, the Nash equilibrium solution do not correspond to optimum and the sharing rule does not maximize social welfare when diversification decisions are decentralized.

**CA rule:**  $\theta_1 = \min [\Omega_1 x; R_1]$  and  $\theta_2 = \min [x - \theta_1; R_2]$

Assuming constraints (C1) and (C2) need to be satisfied, three cases are possible. *Figure 2* represents one of them (case 2).

case 1: If  $\frac{R_1}{\Omega_1} > R$  and  $\frac{R_2}{(1-\Omega_1)} > R$ :

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_a^R \Omega_1 x dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_a^R (1 - \Omega_1)x dx \right]$$

case 2: If  $\frac{R_1}{\Omega_1} < R$  and  $\frac{R_2}{(1-\Omega_1)} < \frac{R_1}{\Omega_1}$  :

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_{R_1/\Omega_1}^R R_1 dx + \int_a^{R_1/\Omega_1} \Omega_1 x dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_{R_1/\Omega_1}^R (x - R_1) dx + \int_a^{R_1/\Omega_1} (1 - \Omega_1)x dx \right]$$

case 3: If  $\frac{R_2}{(1-\Omega_1)} < R$  and  $\frac{R_1}{\Omega_1} > \frac{R_2}{(1-\Omega_1)}$  :

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_{R_2/(1-\Omega_1)}^R x - R_2 dx + \int_a^{R_2/(1-\Omega_1)} \Omega_1 x dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_{R_2/(1-\Omega_1)}^R R_2 dx + \int_a^{R_2/(1-\Omega_1)} (1 - \Omega_1)x dx \right]$$

We compute the Nash equilibrium solution for the three cases and verify ex-post which case is relevant. We find that the Nash equilibrium solution verifies the condition of case 2. The Nash equilibrium solution is:

$$R_1 = \max \left[ \min \left[ \Omega_1 \left( a + \frac{c(b-a)}{v_1} \right); \bar{H} \right]; 0 \right]$$

$$R_2 = \max \left[ \min \left[ (1 - \Omega_1)a + \frac{c(b-a)}{v_2} - \frac{c(b-a)}{v_1} \Omega_1; \bar{H} \right]; 0 \right]$$

The total equilibrium claim is  $R = \max \left[ \min \left[ a + \frac{c}{v_2}(b-a); 2\bar{H} \right]; 0 \right]$ , which is exactly the optimal total claim  $R^*$ . In other words, any CA rule ( $\forall \Omega_i$ ) decentralizes any optimum ( $\forall \lambda_i$ ) at the group level. The equilibrium individual claims are optimal only for the specific parameters  $\lambda_1 = 1, \lambda_2 = 0, \Omega_1 = 1, \Omega_2 = 0$ .

The social welfare is equal to:

$$OS = \frac{c^2}{8}(b-a) \left( \frac{v_2}{v_1^2}(1 + \Omega_1) + \frac{v_1}{v_2^2}(2 - \Omega_1) + \frac{1 - \Omega_1}{v_2} + \frac{\Omega_1}{v_1} \right) + ca + \bar{H}(v_1 + v_2 - 2c) \quad (12)$$

**Proportional rule:**  $\theta_i = \frac{R_i}{R}x \forall i = 1, 2$

The objective function of each agent  $i = 1, 2$  is :

$$E[\Pi_i] = (v_i - c)(\bar{H} - R_i) + \frac{v_i}{b-a} \left[ \int_R^b R_i dx + \int_a^R \frac{R_i}{R} x dx \right]$$

The FOC conditions do not vanish for  $R_i = R_i^* \forall i = 1, 2$ . As a result, the proportional rule does not decentralize the optimal claims. The social welfare is thus necessarily lower than (12).

**CL rule:**  $\theta_1 = \min[R_1 - \beta_1(R - x); 0]$  and  $\theta_2 = \min[x - \theta_1; R_2]$

Assuming constraints (C1) and (C2) need to be satisfied, three cases are possible.

case 1: If  $R_1 - \beta_1(R - x)$  and  $R_2 - (1 - \beta_1)(R - x)$  do not vanish for  $x \in [a; b]$

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_a^R R_1 - \beta_1(R - x) dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_a^R R_2 - (1 - \beta_2)(R - x) dx \right]$$

case 2: If  $R_1 - \beta_1(R - x) = 0$  for  $x \in [a; b]$  and  $R_2 - (1 - \beta_1)(R - x)$  do not vanish for  $x \in [a; b]$

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_{R-R_1/\beta_1}^R R_1 - \beta_1(R - x) dx + \int_a^{R-R_1/\beta_1} 0 dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_{R-R_1/\beta_1}^R R_2 - (1 - \beta_1)(R - x) dx + \int_a^{R-R_1/\beta_1} x dx \right]$$

case 3: If  $R_2 - (1 - \beta_1)(R - x) = 0$  for  $x \in [a; b]$  and  $R_1 - \beta_1(R - x)$  do not vanish for  $x \in [a; b]$

$$E[\Pi_1] = (v_1 - c)(\bar{H} - R_1) + \frac{v_1}{b-a} \left[ \int_R^b R_1 dx + \int_{R-R_2/(1-\beta_1)}^R R_1 - \beta_1(R - x) dx + \int_a^{R-R_2/(1-\beta_1)} x dx \right]$$

$$E[\Pi_2] = (v_2 - c)(\bar{H} - R_2) + \frac{v_2}{b-a} \left[ \int_R^b R_2 dx + \int_{R-R_2/(1-\beta_1)}^R R_2 - (1 - \beta_1)(R - x) dx + \int_a^{R-R_2/(1-\beta_1)} 0 dx \right]$$

The FOC conditions do not vanish for  $R_i = R_i^* \forall i = 1, 2$  in none of the three cases. As a result, the CL rule does not decentralize the optimal claims and the social welfare is reduced compare to (12).

For the special case where  $v_1 = v_2$ , we show that the FOC evaluated at  $R_1^*$  and  $R_2^*$  are equal to zero in case 3. We can verify that  $R_1 = R_1^*$  and  $R_2 = R_2^*$  effectively respect conditions for case 3. Therefore, the Constrained Loss rule,  $\forall \beta_i$ , can lead to the optimal claims to the CPR when values are equal.

■

## Proof Proposition 3

In the case of mean-variance agents, the optimal solution cannot be identified without specifying a functional form for the sharing rule. We consider the three functional forms presented in 1.2. We find an optimal rule for each class of rules by optimizing on the parameter:  $\{\Omega_i\}$  for the CA rule and  $\{\beta_i\}$  for the CL rule. We do not take into account the full specification of the sharing rules as defined in 1.2 but verify ex-post that the optimal rules satisfies constraints (C1) and (C2) (see *figure 4* for the example of the CA rule).

### CA rule

We solve the problem under the constraint  $\theta_i = \Omega_i x$  with  $\sum_{i=1,2} \Omega_i = 1$ . The FOC of the problem are  $\frac{\partial SW}{\partial R_i} = 0$  and  $\frac{\partial SW}{\partial \Omega_i} = 0$  for  $i = 1, 2$ .

To solve the FOC we proceed as follows: From  $\frac{\partial SW}{\partial \Omega_1} = 0$ , we obtain  $\Omega_1^{int}$  as a function of  $R_1$  and  $R_2$ . Replacing  $\Omega_1^{int}$  in  $\frac{\partial SW}{\partial R_1} - \frac{\partial SW}{\partial R_2} = 0$ , we obtain three possible solutions:  $R_1 + R_2 = a$ ,  $R_1 + R_2 = b$ ,  $R_1 = \frac{T_1}{T_2} R_2$ . We can easily verify that  $R_1 + R_2 = a$  does not verify the FOC  $\frac{\partial SW}{\partial R_i} = 0$ , for  $i = 1, 2$ . Replacing  $R_1 = \frac{T_1}{T_2} R_2$  in  $\frac{\partial SW}{\partial R_i} = 0$  for  $i = 1, 2$ , we obtain that the solution  $\{R_i^*\}$  that maximizes social welfare is given by (10). The factor depends on relative risk tolerance defined as the ratio of individual risk tolerance over the sum of risk tolerances in the pair:  $\frac{T_i}{T_i + T_{-i}}$ . We can verify that the social welfare for this solution  $SW|_{R_1 = \frac{T_1}{T_2} R_2}$  is higher than  $SW|_{R_1 + R_2 = b}$ . We then replace  $\{R_i^*\}$  in  $\Omega_i^{int}$  to find  $\Omega_i^*$  defined in (8).

### Proportional rule

We solve the same problem under the constraint  $\theta_i = \frac{R_i}{R} x$ . There is no parameter to optimize because there is a unique rule of the form  $\theta_i = \frac{R_i}{R} x$  such that constraints (C1) and (C2) are verified.

The FOC of the problem are  $\frac{\partial SW}{\partial R_i} = 0$  for  $i = 1, 2$ . Solving  $\frac{\partial SW}{\partial R_1} - \frac{\partial SW}{\partial R_2} = 0$ , we obtain three possible solutions:  $R_1 + R_2 = a$ ,  $R_1 + R_2 = \frac{4}{3}b - \frac{1}{3}a$ ,  $R_1 = \frac{T_1}{T_2} R_2$ . We can easily verify that  $R_1 + R_2 = a$  and  $R_1 + R_2 = \frac{4}{3}b - \frac{1}{3}a$  does not verify the FOC  $\frac{\partial SW}{\partial R_i} = 0$  for  $i = 1, 2$ . Replacing  $R_1 = \frac{T_1}{T_2} R_2$  in  $\frac{\partial SW}{\partial R_i} = 0$ , we obtain that the solution  $\{R_i^*\}$  that maximizes social welfare is given by (10). If we replace  $\{R_i^*\}$  in  $\theta_i = \frac{R_i}{R} x$ , we find  $\theta_i^{**} = \frac{T_i}{T_i + T_{-i}} x$ . The social welfare is equal to the social welfare with the optimal CA rule.

## CL rule

We solve the problem under the constraint  $\theta_i = R_i - \beta_i (R - x)$  with  $\sum_{i=1,2} \beta_i = 1$ .

The FOC of the problem are  $\frac{\partial SW}{\partial R_i} = 0$  and  $\frac{\partial SW}{\partial \beta_i} = 0$  for  $i = 1, 2$ . From  $\frac{\partial SW}{\partial \beta_1} = 0$  and  $\frac{\partial SW}{\partial \beta_2} = 0$ , we obtain  $\beta_i^* = \frac{T_i}{T_i + T_{-i}}$  for  $i = 1, 2$ . Replacing  $\beta_i^*$  in  $\frac{\partial SW}{\partial R_i} = 0$  we obtain  $R^* = Z$  given by (10). For this rule individual claims are not determined. One can choose for example the repartition defined in (9). If we replace  $\{R_i^*\}$  in  $\theta_i^* = R_i - \beta_i^* (R - x)$ , we find  $\theta_i^{**} = \frac{T_i}{T_i + T_{-i}} x$ . The social welfare is equal to the social welfare with the optimal CA and proportional rules.

We verify that the total optimal claim to the CPR (9) tends to the risk neutrality optimal solution when risk tolerances tends to infinity. The total optimal claim is equal to  $Z^*$ , given by the solution of  $g(Z)$ . The roots of this polynomial in  $Z$  tend to the roots of the limit polynomial  $g^\infty(Z)$  given by:

$$g^\infty(Z) = Z(2(T_1 + T_2)(a - b)) + 2(T_1 + T_2)(b - a)(a + c(b - a)).$$

The solution to  $g^\infty(Z) = 0$  is  $Z = a + c(b - a)$ . Thus,  $\lim_{T_1, T_2 \rightarrow \infty} R^* = \lim_{T_1, T_2 \rightarrow \infty} Z = a + c(b - a)$ . This is the optimal total claim under risk neutrality with  $v_1 = v_2 = 1$ .

■

## Proof Proposition 4

The proof is similar to proposition 2. We verify if the FOC conditions of objective function maximization vanish for  $R_i = R_i^* \forall i = 1, 2$ . If this is the case, the Nash equilibrium solution corresponds to optimum and the sharing rule maximizes social welfare when diversification decisions are decentralized. We consider the different possible cases for  $(R_i, \Omega_i, \beta_i) \forall i = 1, 2$  and verify ex-post which case is relevant according to the equilibrium solution. We find that the FOC vanish for the optimal solutions  $\{R_i^*\}$  only for the CA rule with  $\Omega_i = \frac{T_i}{T_i + T_{-i}} x$ .

■

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