Supplemental Material for "Strong thermal and electrostatic manipulation of the Casimir force in graphene multilayers"

Chahine Abbas,¹ Brahim Guizal,¹ and Mauro Antezza^{1,2}

¹Laboratoire Charles Coulomb (L2C), UMR 5221 CNRS-Université de Montpellier, F- 34095 Montpellier, France

²Institut Universitaire de France - 1 rue Descartes, F-75231 Paris, France

(Dated: February 24, 2017)

We detail the procedure used to calculate the TE and TM reflection coefficients for the graphene multilayer structures employed in the main paper to study the Casimir-Lifshitz pressure.

PACS numbers: 12.20.-m,78.67.Wj, 81.07.Oj,42.50.Ct

I. THE S-MATRIX ALGORITHM FOR A MULTILAYERED STRUCTURE WITH EMBEDDED GRAPHENE

In order to calculate the multilayer reflection coefficient $R(Q, \omega)$ we use the so called *S*-matrix algorithm, well know for its effectiveness and stability. We present the algorithm for real frequencies ω , but it remains valid also at imaginary Matsubara frequencies (needed in Eq.(2) of [1]) simply by setting $\omega = i\xi_n$.

Let's consider the general multilayered structure, shown in figure 1, made of N dielectric layers and N_q graphene sheets at their interfaces $(N_g = N + 1)$. Each layer is characterized by its width h_p , by its relative dielectric permittivity ε_p and its relative magnetic permeability μ_p (in [1] we set $\mu_p = 1$), while each graphene sheet is characterized by its conductivity σ_p . We label each layer by its position number in the stack p = 1, ..., Nand label the lower and upper half spaces by 0 and N+1respectively. The whole structure is invariant in the ydirection and thus one can distinguish the two cases of polarization TE/TM according to this axis. Under the TE polarization, the electromagnetic field is such that $\mathbf{E} = (0, E_u, 0)$ and $\mathbf{H} = (H_x, 0, H_z)$ while for the TM case it is such that $\mathbf{E} = (E_x, 0, E_z)$ and $\mathbf{H} = (0, H_y, 0)$. Thus, for each polarization, the fields can be expressed through their non null y component only; the other components being deduced from this latter through Maxwell's equations. With these notations, we express the fields in the p^{th} medium in terms of plane waves solutions:

$$U_p(x,z) = e^{iQx} \left(a_p e^{iq_p(z-z_{p-1})} + a_p e^{iq_p(z-z_{p-1})} \right) \quad (1)$$

With $z_{-1} = 0$ (by convention) and where $U_p(x, z)$ stands for $E_{py}(x, z)$ (respectively $H_{py}(x, z)$) in the TE (respectively TM) polarization case. Here Q is the parallel component of the wave-vector and $q_p = \sqrt{k_0^2 \varepsilon_p - Q^2}$ is its normal one, $k_0 = \omega/c$ being the vacuum wave-number of the incoming plane wave. For a propagating incident wave, Q can be related to the angle of incidence θ trough $Q = k_0 \sqrt{\varepsilon_0} \sin \theta$.

In order to compute the outgoing amplitudes $b_0 = R$ and $a_{N+1} = T$ in terms of the incoming ones $a_0 = I$ and $b_{N+1} = 0$, we must take into account the boundary conditions at the different interfaces. These depend on the TE and TM polarization cases, and thus will be treated separately.



Figure 1. (color online). Graphene-based multilayers scheme.

1. TE polarization

For the *TE* polarization case the boundary conditions can be expressed for each interface $z = z_p$ as follows:

$$\forall x \in \mathbb{R} : \begin{cases} E_{py}(x, z_p) = E_{(p+1)y}(x, z_p) \\ H_{(p+1)x}(x, z_p) - H_{px}(x, z_p) = \sigma_p E_{py}(x, z_p). \end{cases}$$
(2)

Then, using equation (1) and the Maxwell equation $H_x = (-i\omega\mu_0\mu_p)^{-1}\partial_z E_y$, we obtain:

$$\begin{cases} \phi_p a_p + \phi_p^{-1} b_p = a_{p+1} + b_{p+1} \\ q'_{p+1} (a_{p+1} - b_{p+1}) - q'_p (\phi_p a_p - \phi_p^{-1} b_p) = \\ -k_0 \eta_p (\phi_p a_p + \phi_p^{-1} b_p) , \end{cases}$$
(3)

where $q'_p = q_p/\mu_p$, $\phi_p = e^{iq_ph_p}$ ($h_p = z_{p+1} - z_p$ and $\phi_0 = 1$ by convention) and $\eta_p = Z_0\sigma_p$, Z_0 being the electromagnetic impedance of vacuum. These boundary conditions constitute an algebraic set of 2N + 2 equations for the 2N + 2 unknowns a_p, b_p . One of the most efficient and stable ways to solve this latter system is to use the

S-matrix algorithm. By definition, the S-matrix relates the outgoing amplitudes to the incoming ones:

$$\begin{pmatrix} b_p \\ a_{p+1} \end{pmatrix} = S_p^{\text{TE}} \begin{pmatrix} a_p \\ b_{p+1} \end{pmatrix}.$$
 (4)

Therefore, we can deduce its expression easily from equations (2):

$$S_p^{\text{TE}} = \frac{1}{q'_{p+1} + q'_p + k_0 \eta_p} \times \begin{pmatrix} \phi_p^2 (q'_p - q'_{p+1} - k_0 \eta_p) & 2\phi_p q'_{p+1} \\ 2\phi_p q'_p & q'_{p+1} - q'_p - k_0 \eta_p \end{pmatrix}.$$
 (5)

Then chaining the successive S-matrices leads to the overall scattering matrix of the structure:

$$S^{\rm TE} = S_0^{\rm TE} \star \dots \star S_p^{\rm TE} \star \dots \star S_N^{\rm TE}$$
(6)

where the \star product $S=S^a\star S^b$ between two S-matrices S^a and S^b is

$$\begin{cases} S_{11} = S_{11}^{a} + S_{12}^{a} (1 - S_{11}^{b} S_{22}^{a})^{-1} S_{11}^{b} S_{21}^{a} \\ S_{12} = S_{12}^{a} (1 - S_{11}^{b} S_{22}^{a})^{-1} S_{12}^{b} \\ S_{21} = S_{21}^{b} (1 - S_{22}^{a} S_{11}^{b})^{-1} S_{21}^{a} \\ S_{22} = S_{22}^{b} + S_{21}^{b} (1 - S_{22}^{a} S_{11}^{b})^{-1} S_{22}^{a} S_{12}^{b}. \end{cases}$$
(7)

Finally, the reflection and transmission coefficients are

readily obtained from the global S-matrix:

$$\begin{pmatrix} R \\ T \end{pmatrix} = S^{\text{TE}} \begin{pmatrix} I \\ 0 \end{pmatrix} \tag{8}$$

so that the TE reflection coefficient we need for the Casimir-Lifshitz pressure calculation is simply the (1,1) element of S^{TE} : $R_{\text{TE}}^{(1)}(Q,\omega) = R_{\text{TE}}^{(2)}(Q,\omega) = R/I = S_{11}^{\text{TE}}$. For completeness, the TE transmission coefficient will be $T_{\text{TE}}^{(1)}(Q,\omega) = Te^{-iq_{N+1}L}/I = S_{21}^{\text{TE}}e^{-iq_{N+1}L}$, where L is the size of the total multilayer structure, and where the phase factor $e^{-iq_{N+1}L}$ is introduced to have the transmission coefficient defined with respect to the z_0 plane, as for the reflection coefficient.

2. TM polarization

For the TM polarization case we follow the same procedure used for the TE case. We express the boundary conditions for each interface $z = z_p$ as follows:

$$\forall x \in \mathbb{R} : \begin{cases} E_{px}(x, z_p) = E_{(p+1)x}(x, z_p) \\ H_{(p+1)y}(x, z_p) - H_{py}(x, z_p) = -\sigma_p E_{px}(x, z_p). \end{cases}$$
(9)

And now, by using equation (1) and the Maxwell equation $E_x = (i\omega\varepsilon_0\varepsilon)^{-1}\partial_z H_y$ we obtain:

$$\begin{cases} q'_{p}(\phi_{p}a_{p} - \phi_{p}^{-1}b_{p}) = q'_{p+1}(a_{p+1} - b_{p+1}) \\ k_{0}(a_{p+1} + b_{p+1}) - k_{0}(\phi_{p}a_{p} + \phi_{p}^{-1}b_{p}) = \\ -\eta_{p}q'_{p}(\phi_{p}a_{p} - \phi_{p}^{-1}b_{p}), \end{cases}$$
(10)

where $q_p' = q_p / \varepsilon_p$. The S-matrix can then be obtained:

$$S_p^{\text{TM}} = \frac{1}{k_0 q'_p + k_0 q'_{p+1} + \eta_p q'_p q'_{p+1}} \begin{pmatrix} \phi_p^2 (k_0 q'_p - k_0 q'_{p+1} + \eta_p q'_p q'_{p+1}) & 2k_0 \phi_p q'_{p+1} \\ 2k_0 \phi_p q'_p & -k_0 q'_p + k_0 q'_{p+1} + \eta_p q'_p q'_{p+1} \end{pmatrix}.$$
 (11)

The global TM S-matrix of the structure is then obtained by a chaining analogous to Eq. (6), and the TM reflection coefficient we need for the Casimir-Lifshitz pressure calculation is $R_{\text{TM}}^{(1)}(Q,\omega) = R_{\text{TM}}^{(2)}(Q,\omega) = R/I = S_{11}^{\text{TM}}$.

[1] C. Abbas, B. Guizal, and M. Antezza, Phys. Rev. main text