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► **To cite this version:**

Stéphanie Bidon, Olivier Besson, Jean-Yves Tournet, François Le Chevalier. Bayesian sparse estimation of migrating targets in autoregressive noise for wideband radar. IEEE International Radar Conference (RadarCon 2014), May 2014, Cincinnati, OH, United States. pp. 579-584. hal-01493782

**HAL Id: hal-01493782**

**<https://hal.science/hal-01493782>**

Submitted on 22 Mar 2017

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**To cite this version** : Bidon, Stéphanie and Besson, Olivier and Tournet, Jean-Yves and Le Chevalier, François *Bayesian sparse estimation of migrating targets in autoregressive noise for wideband radar*. (2014) In: IEEE International Radar Conference (RadarCon 2014), 19 May 2014 - 23 May 2014 (Cincinnati, OH, United States).

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# Bayesian Sparse Estimation of Migrating Targets in Autoregressive Noise for Wideband Radar

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**Abstract**—In recent work we showed the interest of using sparse representation techniques to estimate a target scene observed by wideband radar systems. However the principle was demonstrated in a white noise background only. In this paper, we present an extended version of our sparse estimation technique that attempts to take into account the (possible) presence of diffuse clutter. More specifically, an autoregressive model is considered for the noise vector. Performance of the technique is studied on synthetic and experimental data. Pertinence of the noise model is discussed.

## I. INTRODUCTION

The discrimination capability of a radar system is of utmost importance when it comes to detect and classify targets in challenging scenarios. Due to their high range resolution (HRR), wideband radar systems have thus attracted much attention. Nevertheless processing the data returns of a wideband waveform requires considering new phenomena. Particularly, moving targets may migrate along the range during the coherent processing interval (CPI) [1].

In recent work, we proposed a new model [2] able to give a satisfying sparse representation of the target scene observed by a wideband radar with low pulse repetition frequency (PRF). The algorithm used to estimate the parameters of this model was based on a hierarchical Bayesian approach where the migration of the moving targets was considered while sparsity on the target amplitude vector was enforced. As a result, each scatterer was estimated as a single peak without sidelobes and was located unambiguously in the range-velocity map (range migration allows removal of velocity ambiguities).

Nonetheless the algorithm of [2] may not entirely be suited for a realistic scenario since only a white noise background is modeled. Particularly, if clutter entails a diffuse component, the method is not designed to handle it properly. As a consequence residual sidelobes located at blind velocities (arising from the diffuse component) can be interpreted as targets. This may lead to false alarms as well as preventing from detecting true targets located at the ambiguous velocities.

The work of S. Bidon and O. Besson is supported by DGA/MRIS under grant 2012.60.0012.00.470.75.01.

In this paper, we assume that a diffuse component in the clutter is possibly present in the data returns. Accordingly, the hierarchical model of [2] is modified to take into account this component. In search of simplicity and to keep the number of parameters to estimate as low as possible, the following assumptions are made: i) the disturbance vector models both thermal noise plus clutter and is centered Gaussian ii) it is decorrelated from subband to subband iii) it is correlated in the slow-time according to a stationary auto-regressive (AR) process with finite order iv) the clutter is locally homogeneous (i.e., the AR coefficients are subband-independent). Note that modeling the disturbance vector via an AR process has been used with success in several radar applications, e.g., [3]. It can be seen as an efficient regularization technique for estimating covariance matrices [4], [5]. In our case, the AR approach requires to include an unknown vector (containing the AR parameters) to the estimation problem.

The remaining of the paper is organized as follows. The augmented hierarchical Bayesian model is introduced in Section II. Section III describes shortly the estimation technique associated with this model. More time is spent in Section IV to describe the performance of the proposed algorithm on synthetic and experimental data collected from the PARSAX radar [6]. Section V concludes with a discussion on upcoming work.

## II. BAYESIAN MODEL

The hierarchical Bayesian model proposed in this paper is represented graphically in Fig. 1 and detailed herein.

### A. System description

In what follows, a radar with a low PRF  $1/T_r$ , sending a series of  $M$  wideband pulses is considered. The carrier frequency and the bandwidth are denoted  $f_c$  and  $B$  respectively (typical values are  $f_c \approx 10$  GHz and  $B \approx 1$  GHz). After down-conversion and range matched filtering,  $K$  range gates are selected and define a low range resolution (LRR) segment. A range transform (a simple fast Fourier transform, FFT) is applied on the fast-time dimension so that the data

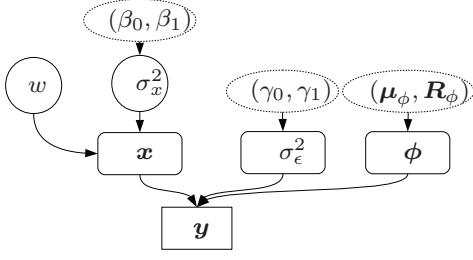


Fig. 1. Graphical representation of the proposed Bayesian model. Parameters circled by a dotted line have to be set by the radar operator.

are observed in the fast-frequency<sup>1</sup>/slow-time domain. The corresponding observation vector  $\mathbf{y}$  is of length  $KM$  and is built by concatenating the  $M$  pulse returns subband-by-subband.

### B. Likelihood

1) *Linear model*: It was shown in [2] that a convenient data model enforcing sparsity for the target representation can be expressed as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

where

- $\mathbf{H}$  is an interpolation-Fourier transform matrix;
- $\mathbf{x}$  is the target amplitude vector in the fast-time/slow-frequency domain;
- $\mathbf{n}$  is the disturbance vector.

Note that the  $KM \times K\bar{M}$  matrix  $\mathbf{H}$  allows the signal from each subband to be resampled into  $\bar{M}$  samples at the rate  $\bar{f}_r$ . The virtual number of pulses  $\bar{M}$  and the virtual PRF  $\bar{f}_r$  have to be chosen by the radar operator (see [2] for more details).

2) *Noise modeling*: As stated in the introduction,  $\mathbf{n}$  is assumed to be centered Gaussian, i.e.,

$$\mathbf{n}|\mathbf{R} \sim \mathcal{CN}_{KM}(\mathbf{0}, \mathbf{R})$$

where  $\mathbf{R}$  is a  $KM$ -by- $KM$  covariance matrix. Using the other assumptions ii), iii) and iv) implies that the covariance matrix  $\mathbf{R}$  has a specific structure

$$\mathbf{R} = \mathbf{I}_K \otimes \mathbf{\Gamma} \quad (1)$$

where  $\otimes$  is the Kronecker product,  $\mathbf{I}_K$  is the identity matrix of size  $K$ , and  $\mathbf{\Gamma}$  is an  $M \times M$  matrix whose inverse is  $P$ -banded [7]. ( $P$  is the order of the AR model.) More precisely, the Cholesky factorization of  $\mathbf{\Gamma}^{-1}$  can be expressed as

$$\mathbf{\Gamma}^{-1} = \sigma_\epsilon^{-2}(\mathbf{I} - \mathbf{\Phi})^H(\mathbf{I} - \mathbf{\Phi})$$

where

- $\mathbf{\Phi}$  is a lower triangular Toeplitz matrix with zero diagonal elements

$$\mathbf{\Phi} = \text{Toeplitz}\{[0, \phi^T, 0, \dots, 0]\}$$

with  $\phi = [\phi_1, \dots, \phi_P]^T$  the  $P$ -length vector containing the AR parameters;

- $\sigma_\epsilon^2$  is the variance of the white input to the AR model.

<sup>1</sup>The fast-frequency will be denoted also as ‘‘subband’’.

### C. Parameters and hyperparameters

Since a Bayesian approach is chosen in this paper, a prior probability density function (pdf) is assigned to each unknown parameter of the model. Choosing a prior distribution requires a compromise between physical considerations and mathematical tractability.

1) *Target amplitude vector*: The target amplitude vector  $\mathbf{x}$  is modeled as in [2]. More precisely, a Bernoulli-Gaussian distribution is assigned to each (presumably independent) element  $x_i$  of  $\mathbf{x}$ , i.e., for  $i = 0, \dots, K\bar{M} - 1$

$$x_i|w, \sigma_x^2 \sim \text{Ber} \mathcal{CN}(w, 0, \sigma_x^2)$$

where  $w$  is the probability that at the  $i$ th range-velocity bin of analysis a scatterer is present and  $\sigma_x^2$  is the power of the possible scatterer. Furthermore, a uniform probability is assigned to  $w$  while an inverse gamma pdf is assumed for  $\sigma_x^2$

$$w \sim \mathcal{U}_{[0,1]} \quad (2a)$$

$$\sigma_x^2|\beta_0, \beta_1 \sim \mathcal{IG}(\beta_0, \beta_1) \quad (2b)$$

where  $\beta_0, \beta_1$  are respectively the shape and scale parameters of (2b). They are chosen later in Section IV to ensure a wide range of possible target powers in the radar scene.

2) *Disturbance vector*: The model novelty concerns the description of the disturbance parameters, namely  $\sigma_\epsilon^2$  and  $\phi$ . Conjugate priors are selected in this work which leads to an inverse-gamma prior for  $\sigma_\epsilon^2$  and a Gaussian prior for  $\phi$

$$\sigma_\epsilon^2|\gamma_0, \gamma_1 \sim \mathcal{IG}(\gamma_0, \gamma_1) \quad (3a)$$

$$\phi|\mu_\phi, \mathbf{R}_\phi \sim \mathcal{CN}_P(\mu_\phi, \mathbf{R}_\phi) \quad (3b)$$

with  $\gamma_0, \gamma_1$  the shape and scale parameters of (3) and  $\mu_\phi, \mathbf{R}_\phi$  the mean vector and the covariance matrix of  $\phi$ . Additionally to the concurrent mathematical convenience, the priors (3) can be made very, moderately or non-informative according to the values of their hyperparameters  $\gamma_0, \gamma_1$  and  $\mu_\phi, \mathbf{R}_\phi$ . Later in Section IV, non-informative priors are favored to express our absence of knowledge about the clutter component. Practically, a degenerate case of (3) is considered and yields the following flat priors [7]

$$f(\sigma_\epsilon^2) \propto \frac{1}{\sigma_\epsilon^2} \mathbb{I}_{[0,+\infty)}(\sigma_\epsilon^2) \quad (4a)$$

$$f(\phi) \propto 1 \quad (4b)$$

where the hyperparameters of (3) are set to  $\gamma_0 = \gamma_1 = 0$  and the variance of the AR-vector  $\phi$  is assumed to be infinite.

*Remark 1*: Note that if  $P = 0$ , the proposed model reduces to that of [2] and  $\sigma_\epsilon^2$  describes the thermal noise power which is usually well known. Otherwise if  $P > 0$ , having precise information about  $\sigma_\epsilon^2$  may not be so straightforward.

### III. BAYESIAN ESTIMATION

According to the hierarchical model presented in Section II, Bayesian estimators can now be derived for the unknown parameter of interest  $\mathbf{x}$ . The keystone to design Bayesian estimators is the posterior distribution that merges the information brought by the observations and the priors. However, in our

case, the posterior pdf  $f(\mathbf{x}|\mathbf{y})$  cannot be easily manipulated so that, for instance, neither the minimum mean square error (MMSE) nor the maximum a posteriori (MAP) estimator can be obtained analytically. Instead a numerical approach is undertaken. More precisely, a Monte Carlo Markov chain (MCMC) method is investigated in this paper. It can be summarized as iteratively generating according to the conditional distributions of the parameters  $\mathbf{x}$ ,  $w$ ,  $\sigma_x^2$ ,  $\sigma_\epsilon^2$  and  $\phi$  [8]. The latter can be easily obtained via the use of the joint posterior distribution

$$\begin{aligned} f(\mathbf{x}, w, \sigma_x^2, \sigma_\epsilon^2, \phi|\mathbf{y}) &\propto f(\mathbf{y}|\mathbf{x}, \sigma_\epsilon^2, \phi) \\ &\times f(\mathbf{x}|w, \sigma_x^2) f(w) f(\sigma_x^2) \\ &\times f(\sigma_\epsilon^2) f(\phi). \end{aligned}$$

After a burn-in time  $N_{bi}$ , the Markov chain generates samples that are asymptotically distributed according to the posterior distribution of interest. MMSE estimates can then be obtained for each parameter as an empirical mean

$$\hat{\theta}_{\text{MMSE}} = N_r^{-1} \sum_{n_r=1}^{N_r} \theta^{(n_r)}$$

where  $N_r$  is the number of samples  $\theta^{(n_r)}$  used to approximate the MMSE estimate and  $\theta$  designates successively the parameters  $\mathbf{x}$ ,  $w$ ,  $\sigma_x^2$ ,  $\sigma_\epsilon^2$  and  $\phi$ . Note that the proposed algorithm provides other estimators than  $\mathbf{x}$  and thus can offer additional information about the radar scene.

#### IV. NUMERICAL SIMULATIONS

##### A. Synthetic wideband radar data

The performance of the proposed Bayesian estimation is firstly assessed on synthetic data. The observation vector  $\mathbf{y}$  is generated according to the linear model

$$\mathbf{y} = \sum_{n=1}^N \alpha_n \mathbf{a}_n + \mathbf{n}$$

where

- $N$  is the number of scatterers in the scene;
- $\alpha_n, \mathbf{a}_n$  are the amplitude and steering vector of the  $n$ th scatterer (see [2] for more details);
- $\mathbf{n}$  is generated independently subband-by-subband according to an AR model of order 1 driven by a white Gaussian noise.

Numerical values of the simulation parameters can be found in Table I. Note that the AR coefficients  $\phi$  are chosen slightly differently from one subband to another in order to test the robustness of the estimation technique towards slight AR-coefficient fluctuation.

Fig. 2 compares the true target scene with the range-velocity maps of the amplitudes estimated from the Bayesian technique proposed in [2] and the augmented algorithm proposed herein (i.e.,  $\hat{\mathbf{x}}_{\text{MMSE}}/\sqrt{KM}$ ). The output of the matched filter defined in [1] is also depicted in Fig. 2(b). As expected, due to the high sidelobes of the wideband ambiguity function, the matched filter allows neither the blind velocities nor the target sidelobes to be removed. Moreover the algorithm of [2] which assumes

TABLE I  
PARAMETERS FOR THE SYNTHETIC SCENARIO

| Data                      |  |                              |
|---------------------------|--|------------------------------|
| carrier                   | $f_c = 10$ GHz                               |                              |
| bandwidth                 | $B = 1$ GHz                                  |                              |
| PRF                       | $f_r = 1$ kHz                                |                              |
| # pulses                  | $M = 32$                                     |                              |
| LRR segment               | $K = 6$                                      |                              |
| noise power               | $\sigma^2 = 1$                               |                              |
| AR-order                  | $P = 1$                                      |                              |
| AR-coefficients           |  |                              |
| $k = 0$                   | $\sigma_\epsilon^2 = 1.26$                   | $\phi = [0.997e^{+j0.0010}]$ |
| $k = 1$                   | $\sigma_\epsilon^2 = 1.71$                   | $\phi = [0.998e^{-j0.0035}]$ |
| $k = 2$                   | $\sigma_\epsilon^2 = 1.44$                   | $\phi = [0.999e^{+j0.0027}]$ |
| $k = 3$                   | $\sigma_\epsilon^2 = 1.94$                   | $\phi = [0.996e^{-j0.0076}]$ |
| $k = 4$                   | $\sigma_\epsilon^2 = 1.65$                   | $\phi = [0.999e^{+j0.0030}]$ |
| $k = 5$                   | $\sigma_\epsilon^2 = 1.51$                   | $\phi = [0.998e^{+j0.0012}]$ |
| Processing                |  |                              |
| AR-order                  | $P = 1$                                      |                              |
| $\sigma_\epsilon^2$ prior | $(\gamma_0, \gamma_1) = (0, 0)$              |                              |
| $\sigma_x^2$ prior        | $(\beta_0, \beta_1)/(KM) \approx (2.2, 1.2)$ |                              |
| virtual PRF               | $\bar{f}_r \approx 2.4$ kHz                  |                              |
| virtual # pulses          | $\bar{M} = 86$                               |                              |

a white noise background is unable to deal correctly with the diffuse component. Numerous false detections are observable at the location of the usual blind velocities. Note that the actual target located at the first blind velocity seems to be identified but is surrounded by false detections. On the other hand, the augmented algorithm based on an AR noise model is able to remove clutter sufficiently enough to estimate each scatterer (even the one located in the first blind velocity). The clutter spectrum associated with the estimated AR-coefficients is depicted in Fig. 3, i.e.,

$$S_{AR}(f) = \frac{\hat{\sigma}_\epsilon^2_{\text{MMSE}}}{\left| 1 - \sum_{p=1}^P [\hat{\phi}_{\text{MMSE}}]_p e^{-j2\pi f p} \right|^2}$$

where  $f$  is the slow-frequency. It is compared with the true AR-spectrum averaged over the  $K$  range bins. As can be seen, the spectrum obtained with the MMSE estimators  $\hat{\sigma}_\epsilon^2_{\text{MMSE}}, \hat{\phi}_{\text{MMSE}}$  is very near from the true spectrum which tends to show that the proposed algorithm is not sensitive to a slight fluctuation of the AR-coefficients along the range. Interestingly, in presence of an AR-based diffuse clutter, the burn-in time of the proposed MCMC algorithm is dramatically decreased compared to that of [2] owing certainly to the better fit between the model and the data.

##### B. Experimental PARSAX data

In this section, the proposed algorithm is tested on experimental data collected from the PARSAX radar of the Delft University of Technology [6]. The bandwidth is not as high as thought previously, nonetheless moving targets still migrate of a few range gates during the CPI. For better results interpretation, a presumably free-target region is chosen and synthetic

TABLE II  
PARAMETERS FOR THE PARSAX SCENARIO

| Data                      |  |
|---------------------------|--|
| carrier                   | $f_c = 3.315$ GHz                              |
| bandwidth                 | $B = 100$ MHz                                  |
| PRF                       | $f_r = 1$ kHz                                  |
| # pulses                  | $M = 32$                                       |
| LRR segment               | $K = 6$  |
| noise power               | $\sigma^2 \approx 1$                           |
| Processing                |  |
| AR-order                  | $P = 1$  |
| $\sigma_\epsilon^2$ prior | $(\gamma_0, \gamma_1) = (0, 0)$                |
| $\sigma_x^2$ prior        | $(\beta_0, \beta_1 / (KM)) \approx (2.2, 1.2)$ |
| virtual PRF               | $\bar{f}_r \approx 2.4$ kHz                    |
| virtual # pulses          | $\bar{M} = 81$                                 |

targets are injected in the PARSAX data. Results are depicted in Fig. 4. Similar observations can be made as for the synthetic data. However, one can notice that the proposed algorithm identifies “targets” at the zero velocity of each range gate and estimate a moderate power AR component. The spectrum of the latter is represented in Fig. 5. A possible interpretation is that the clutter here may be better described by the sum of a diffuse component plus discretely (corresponding to the so-called coherent component [1]). Accordingly, the proposed algorithm is designed to filter only the diffuse component.

## V. CONCLUSION AND PERSPECTIVES

A Bayesian sparse representation technique for migrating targets in diffuse clutter has been presented in case of wide-band radar signals. The algorithm performs well on synthetic data and tends to show the dual nature of the clutter (diffuse plus discrete) on experimental data. Further analysis should be performed on other wideband data set to assess the pertinence of the clutter model. Finally, a robustification towards grid mismatch should be added to the proposed algorithm for an application on fully experimental data.

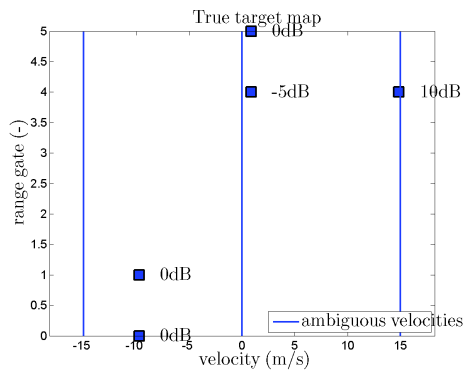
## ACKNOWLEDGMENT

The authors would like to thank Alain Minisini for the simulations conducted during his internship. They are also very grateful to Oleg Krasnov at TU-Delft for kindly providing the PARSAX experimental data.

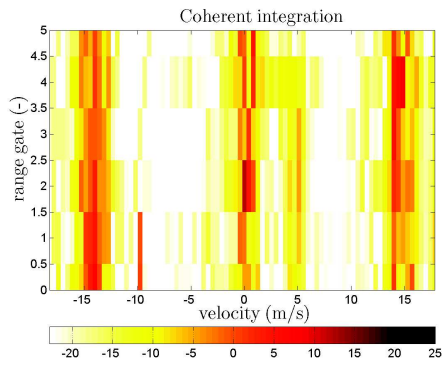
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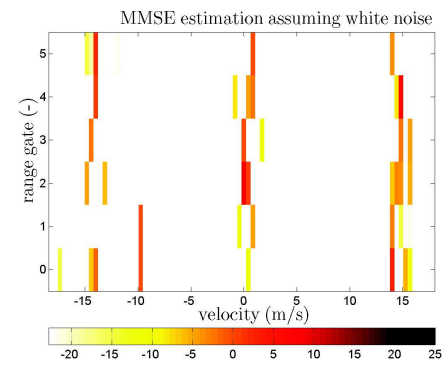
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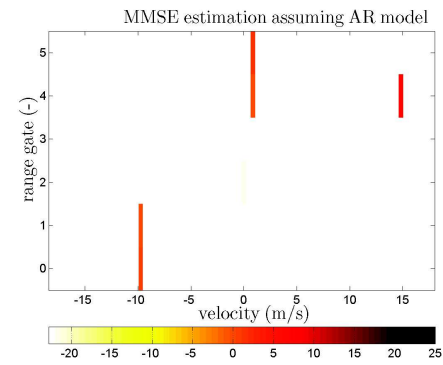
(a)



(b)



(c)



(d)

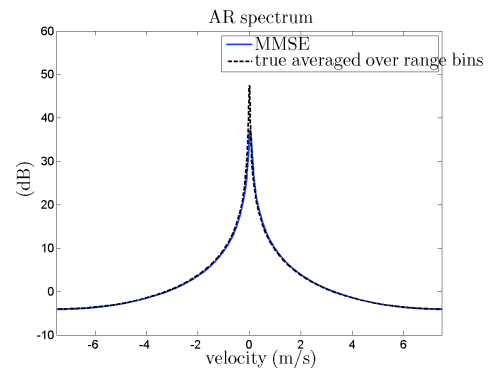
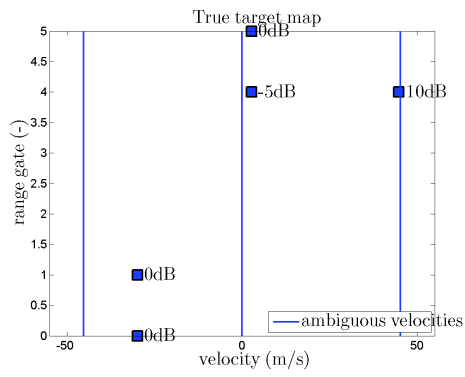
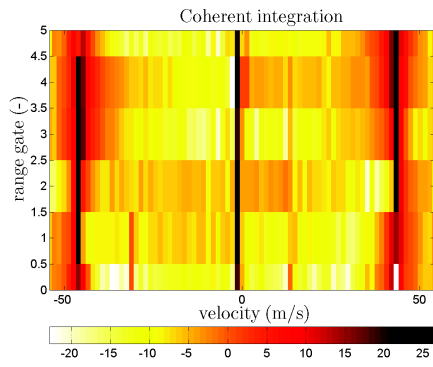


Fig. 3. AR spectrum.

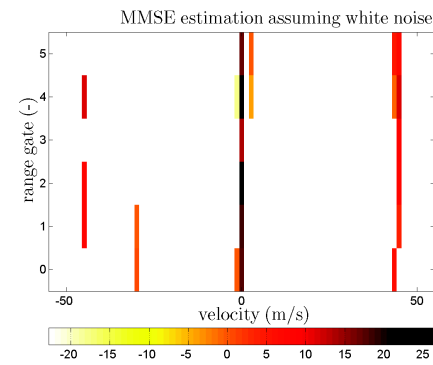
Fig. 2. Range-velocity map (modulus of the complex amplitude only). The range resolution is  $\delta_R = 15$  cm. The ambiguous velocity is  $v_a = 15$  m/s. (a) Location and amplitude of the synthetic targets. (b) Coherent integration. (c) *Data*: Synthetic data with targets and AR-model based clutter. *Processing*: White noise model. (d) *Data*: Synthetic data with targets and AR-model based clutter. *Processing*: AR noise model.



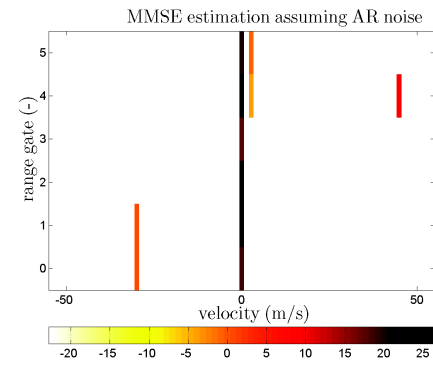
(a)



(b)



(c)



(d)

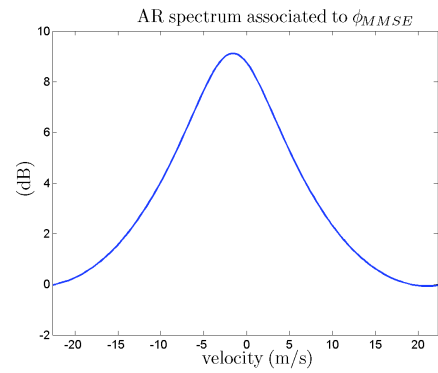


Fig. 5. AR spectrum.

Fig. 4. Range-velocity map (modulus of the complex amplitude only). The range resolution is  $\delta_R = 1.5$  m. The ambiguous velocity is  $v_a = 45.25$  m/s. (a) Location and amplitude of the synthetic targets. (b) Coherent integration. (c) Data: PARSAX data injected with synthetic targets. Processing: White noise model. (d) Data: PARSAX data injected with synthetic targets. Processing: AR noise model.