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JOINT INTEGRATED PRODUCTION-MAINTENANCE POLICY WITH PRODUCTION PLAN SMOOTHING THROUGH PRODUCTION RATE CONTROL

Sofiene Dellagi

LGIPM, Université de Lorraine, Metz, France

Email: Sofiene.dellagi@univ-lorraine.fr

Anis Chelbi

University of Tunis, Ecole Nationale Supérieure d'Ingénieurs de Tunis, CEREP, Tunis, Tunisia

Email: anis.chelbi@planet.tn

Wajdi Trabelsi

LGIPM, ICN Business School, Nancy Metz, France

Email: wajdi.trabelsi@icn-groupe.fr

Abstract

We consider a production system that has to satisfy a random demand during a finite planning horizon under a required service level. This study consists in developing an analytical model in order to determine a near-optimal integrated maintenance-production plan which takes into consideration the influence of the production rate on the system's failure rate while attempting in the same time to smooth the production plan through the production rate control between periods of the planning horizon. A numerical example is presented in order to illustrate the contribution of the proposed modelling approach and discuss the different trade-offs that are considered.

Key Words: Integrated production-maintenance strategies, Optimization, Variable production rates, Degradation, and Smoothing production rate.

1. INTRODUCTION AND LITERATURE REVIEW

Manufacturing companies must manage several functional areas successfully, such as production, maintenance, quality and marketing. One of the keys to success consists in treating all these services simultaneously. In order to ensure an efficient coordination between them, managers and decision makers have to consider a global systemic approach integrating the interactions between parts or all of those complementary functions. In this perspective, several researchers have investigated ways of integrating maintenance and production after decades during which those two functions had been studied separately. Works on maintenance policies started with Barlow *et al.* [1] and continued with a huge number of contributions as it can be seen in a survey on maintenance models by Wang [2]. During the last two decades, several companies have realized that strategies dissociating maintenance and production were ineffective. The need for developing new integrated maintenance-production strategies became evident. Brandolese *et al.* [3] proposed a strategy for maintaining a multiple machines production system. The planning consists in programming the execution date of each task and the machine that should perform it. They integrated preventive maintenance tasks in the planning placing them as close as possible to the optimal maintenance periods.

Chelbi *et al.* [4] developed a mathematical model in order to determine both the buffer stock size and the preventive maintenance period for an unreliable production unit which is subject to regular preventive maintenance of random duration. The optimization of the preventive maintenance period and the buffer size was also the problem addressed by Pal *et al.* [5] in the context of an imperfect production system that may shift to an ‘out-of-control’ state after a random time. Non-conforming products are reworked incurring an additional cost. They also considered the situation where both the buffer size and the production rate are the decision variables. Rezg *et al.* [6] developed an analytical model and a numerical procedure which allow determining a joint optimal inventory control and age-based preventive maintenance policy for a randomly failing production system. Recently, Tambe and Kulkarni [7] developed a selective maintenance and quality control decision optimization framework considering the production schedule of the machine. They derive an optimal maintenance decision, consisting in one of three actions (repair, replace or do-nothing) for the system components along with the optimal sample size, the acceptance number and the time between samples, taking into account the optimal production schedule. The authors used a genetic algorithm in order to solve the problem.

In the same framework, some researchers have taken into account several external constraints. In fact, integrated maintenance-production strategies which take into consideration subcontracting have been studied by Dellagi *et al.* [8]. They developed and optimized a maintenance policy incorporating subcontractor constraints. They demonstrated through a case study, the influence of the subcontractor constraints on the optimal integrated maintenance-production strategy. Dealing also with the same subject, Dahane *et al.* [9] studied analytically the problem of the integration of subcontracting activities and determined the optimal number of subcontracting tasks to be performed during a maintenance cycle. Recently, Nourelfath *et al.* [10] dealt also with the same problem of integrating preventive maintenance and production planning, for a production system composed of a set of parallel components. A subsequent work by Nourelfath and Châtelet [11] assumed the presence of economic dependence and common cause failures in the production system. Mifdal *et al.* [12] considered the same problem in the case of multiple-products manufacturing systems.

Looking at the literature on integrated maintenance-production policies, we noticed that the influence of the production rate on the system degradation in presence of a random demand over a finite planning horizon was rarely addressed in depth. Very few works dealt with this issue. For example, the study of Hajej *et al.* [13] which presents an analytical stochastic optimization model based on the operational age concept, revealed the significant influence of the production rate on the deterioration of the manufacturing system and consequently on the integrated production-maintenance policy. Later, Hajej *et al.* [14] dealt with the same problem by integrating a subcontracting constraint in order to recycle a certain quantity of returned products. In these two studies, the authors started by establishing an optimal production plan which minimizes the total inventory and production cost taking into consideration the subcontractor constraint. Then, using this optimal production plan, they derived an optimal maintenance schedule which minimizes the total maintenance cost. Their approach is sequential. Moreover, they do not consider the influence of having differences between quantities produced in successive periods and the impact of such differences on labour and set-up related costs.

The approach adopted in this study favours the fact of smoothing the production rate between periods in order to avoid the disadvantages of a significant variation (mainly in terms of set-up costs).

This paper is organized as follows: In the next section we specify the targeted contributions of this work. Then the problem and the working assumptions are described in section 3. Section 4 is dedicated to the working assumptions and the development of the analytical model. A numerical example is presented in section 5 to illustrate the proposed modelling approach and discuss different trade-offs that are considered. Finally, conclusions and potential future work are provided in Section 6.

2. TARGETED CONTRIBUTIONS

Several industries such as steel production exhibit fluctuation of the production rate between production periods. This yields a great deal of set-up and preparation work like unloading and loading raw materials using special equipment and human resources. Moreover, in the case of production lines, generally buffers with finite capacities are present between machines. The variation of the production rate between periods brings the necessity of changing the buffers' levels involving a certain handling and logistics cost. Hence, it is clear that the production rate fluctuation between periods may have a significant impact on the set-up cost incurred at the beginning of each period. To the best of our knowledge, this issue has not been considered in the literature. The adopted approach in this study favors the fact of smoothing the production rate between periods in order to avoid the disadvantages of a significant variation (mainly in terms of set-up costs). We also propose an integrated (and not sequential) optimization of the maintenance schedule and the production plan in order to derive a near-optimal integrated maintenance-production strategy taking into account the penalty induced by excessive variation of the production rate between periods of the planning horizon. To do so, we express the set-up cost as being proportional to the production rate variation between successive periods of the production plan. Besides, we propose new modelling features in the total integrated cost modelling, particularly when dealing with the shortage and inventory holding costs by considering them separately contrarily to the previous studies mentioned above.

The proposed model allows the investigation of different trade-offs. The first one is between the smoothing penalty and the production plan, the inventory, and the preventive maintenance schedule. We also consider the effect of the demand variability on the smoothing penalty, the production and maintenance plans, and on the total expected cost.

3. PROBLEM STATEMENT

We consider a single machine M subject to degradation and random failures. It produces one type of product whose demand is random and characterized by a Normal probability distribution with known average and standard deviation. The demand is satisfied from a stock (Fig. 1). Production is planned over a finite horizon divided in H production periods of equal length Δt . It is assumed that the quantity produced at every period is added to the stock at the end of the period and the demand of every period is satisfied at the end of the period. The machine failure rate increases with both time and the production rate. Periodic preventive maintenance actions have to be scheduled to reduce the probability of failure.

We are in presence of a production control problem with a state variable, namely the inventory level, together with the following control variables: the production rates for every period and the number of preventive maintenance actions to be performed over the time horizon $H\Delta t$.

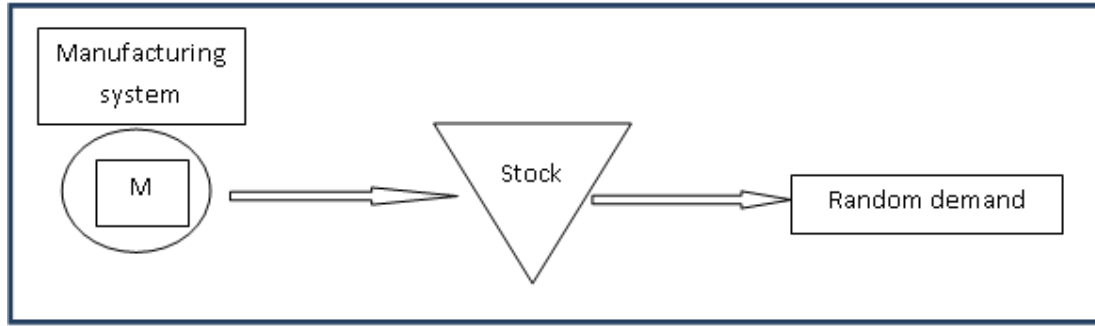


Figure 1: Problem statement

The objective is to minimize the total expected cost over the production planning horizon made of H periods. This cost integrates production costs, inventory and shortage costs, preventive and corrective maintenance costs and an additional cost related to a penalty due to the production rate variation between successive periods.

As it will be shown in next section, compared to previous works dealing with the same subject, the first contribution of this study consists in the total integrated cost modelling. On one hand, the shortage and inventory holding costs are formulated separately, and on the other hand, the average number of failures is modelled differently considering an increasing failure rate according to both time and production rate.

The second feature consists in the integrated (and not sequential) optimization of the production plan and the maintenance schedule taking into account the influence of the production rate variation on the system degradation. Finally, the third main contribution is the control of the production rate variation between successive periods in order to smooth the production plan by reducing the penalty induced by excessive variation of the production rate over the planning horizon.

4. ANALYTICAL STUDY

4.1 Used notation

The following parameters are used in the mathematical formulation of the model:

Δt : length of a production period

H : number of production periods in the planning horizon

$H.\Delta t$: length of the finite planning horizon

$z(p)$: produced quantity during period p ($p=1, \dots, H$)

z_{max} : maximum quantity that can be produced during period p

$\mathbf{Z}=\{z(1), \dots, z(p), \dots, z(H)\}$: Production plan over the horizon $H.\Delta t$

$u(p)$: production rate of the system during period p ($p=1, \dots, H$); ($u(p)=z(p)/\Delta t$)

u_{max} : maximum production rate of the system ($u_{max}=z_{max}/\Delta t$)

$\mathbf{U}=\{u(1), \dots, u(p), \dots, u(H)\}$: Set of production rates adopted for each period

$d(p)$: average demand during period p ($p=1, \dots, H$)

V : variance of demand for all periods p ($p=1, \dots, H$)

σ : standard deviation of demand for all periods

$S(p)$: inventory level at the end of period p

S_0 : the inventory level at the start of period 1

C_{pr} : unit production cost

C_s : inventory holding cost of one product unit per time unit.
 C_l : shortage cost of one product unit.
 CTS : expected total cost of production, shortage and inventory holding over the time horizon $H\Delta t$
 C_v : unit set-up cost
 CM : total average maintenance cost
 C_{pm} : preventive maintenance action cost
 C_{cm} : corrective maintenance action cost
 SC : penalty cost relative to production rate variation
 mu : monetary units
 θ : probability to satisfy the demand (service level).
 $f(t)$: probability density function associated with the time to failure of M
 $F(t)$: probability distribution function associated with the time to failure of M
 $R(t)$: reliability function: $1 - F(t)$
 $\lambda_n(t)$: nominal failure rate corresponding to the maximum production rate (depends only on time)
 $\lambda(t, u(p))$: machine failure rate function during period p ($p = 1, \dots, H$) (depends on time and on the production rate during period p)
 tu : time units

The cost model to be developed corresponds to a production/maintenance planning problem with constraints related to inventory, production and preventive maintenance.

4.2 Working Assumptions

The following assumptions are considered:

- The machine failure rate increases with both time and production rate
- Preventive maintenance actions restore the machine to an “as-good-as-new” state.
- The planning time horizon starts with a new or as good as new machine. However, it doesn't end necessarily with a perfect preventive maintenance action. The equipment could be replaced by a different one or maintained differently after the end of the planning period.
- The quantity $z(p)$ produced in period p is added to the inventory at the end of the period and then the demand $d(p)$ is satisfied.
- Preventive maintenance and repair actions durations are negligible.
- The units short are lost (no backorders) and a shortage cost is incurred.

4.3 Formulation of the integrated production-maintenance total expected cost

We develop in what follows the expressions of the three major components of the total expected cost incurred over the planning horizon. These costs are: the production and inventory cost, the maintenance cost, and the set-up cost (smoothing penalty). Recall that the decision variables are the production rates for every period and the number of preventive maintenance actions to be performed over the planning horizon.

4.3.1 Production, shortage and inventory costs formulation

The constraints related to inventory balance, service level satisfaction, and maximum production quantity per period, are expressed as follows:

Inventory balance:

$$S(p+1) = S(p) + z(p+1) - d(p+1) \quad \forall p = 0, \dots, H-1 \quad (1)$$

$$\text{With: } S(1) = S_0 + z(1) - d(1)$$

Service level satisfaction:

$$\text{Prob}[S(p+1) \geq 0] \geq \theta \quad \forall p = 0, \dots, H-1 \quad (2)$$

Production capacity:

$$0 \leq z(p) \leq z_{\max} \quad \forall p = 1, \dots, H \quad (3)$$

Accordingly, the inventory evolves as shown in the example below in Fig. 2:

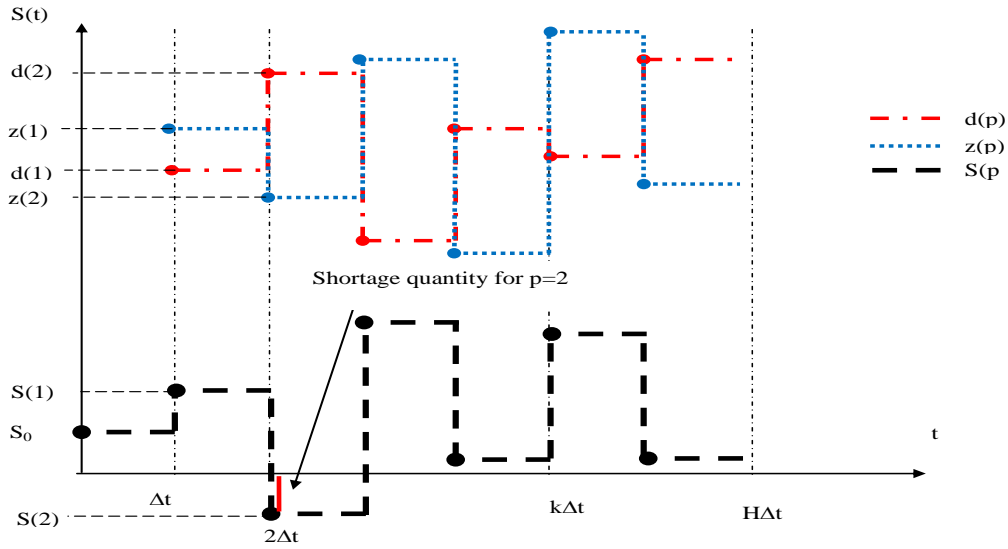


Figure 2: Inventory evolution over the planning horizon $H\Delta t$

Thus, the expression of the total cost of production, shortage and inventory over the finite horizon $H\Delta t$ is given by:

$$CTS = \left(C_s \times \Delta t \times \sum_{p=1}^H (S(p) \times 1_{S(p)>0}) \right) + \left(Cl \times \sum_{p=1}^H (|S(p)| \times 1_{S(p)<0}) \right) + \left(C_{pr} \times \sum_{p=1}^H z(p) \right) \quad (4)$$

With: $1_{S(p)>0}$ is equal to 1 if $S(p)>0$ and equal to zero otherwise

$1_{S(p)<0}$ is equal to 1 if $S(p)<0$ and equal to zero otherwise

It is worth noticing here that most of the papers dealing with the same subject such as Hajej *et al.* [13,14,15] and Ayed *et al.* [16] consider a quadratic model (known as HMMS (Holt, Madigliani, Muth, and Simon) [17]) integrating shortage and excess of stock considering however the same unitary cost for shortage and for inventory holding.

Our approach (Eq. 4) makes this important distinction between these unitary costs that are not the same in practice.

The constraint relative to the service level is stochastic. It is possible to transform this constraint to a deterministic one (Hajej *et al.* [13]). In fact, it provides the minimal quantities to produce at each period p in order to satisfy the service level.

$$\text{Prob}[S(p+1) \geq 0] \geq \theta \Rightarrow z(p+1) \geq z_{(S(p), \theta)} \quad \forall p \in \{0, \dots, H-1\} \quad (5)$$

Where $z_{(S(p), \theta)}$ represents a minimum cumulative production quantity expressed as follows:

$$z_{(S(p), \theta)} = \left((\Phi_{p+1})^{-1}(\theta) \right) \times V_{p+1} + d(p+1) - S(p) \quad \forall p \in \{0, \dots, H-1\} \quad (6)$$

With:

V_{p+1} : variance of demand at period $p+1$;

Φ_{p+1} : cumulative Gaussian distribution function with mean $d(p+1)$ and finite variance V_{p+1} .

Φ_{p+1}^{-1} : inverse Gaussian distribution function

4.3.2. The maintenance cost formulation

We consider a periodic preventive maintenance (PM) policy with minimal repair at failure. The interval $[0, H\Delta t]$ is partitioned into N equal parts each of length $k\Delta t$ plus a possible remaining duration shorter than $k\Delta t$. Perfect preventive maintenance actions are performed periodically at times $i \times k\Delta t$, $i=1, \dots, N$.

For a given production plan \mathbf{Z} ($\mathbf{Z}=\{z(1), \dots, z(p), \dots, z(H)\}$), and a given PM period $k\Delta t$, the total maintenance cost is given by:

$$CM(U, k) = C_{pm} \times N(k) + C_{cm} \times \varphi(U, k) \quad (7)$$

$N(k)$ represents the integer part of (H/k) , which corresponds to the number of PM actions. k is the decision variable through which the maintenance policy can be defined. $\varphi(U, k)$ stands for the average number of failures (minimal repairs) over the horizon $H\Delta t$.

The lemma below provides the expression of the average number of failures:

Lemma

$$\begin{aligned} \varphi(U, k) = & \sum_{j=0}^{N(k)-1} \left[\int_0^{\Delta t} \frac{u_{1j}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^k \left[\int_0^{\Delta t} \frac{u_{ij}}{u_{\max}} \lambda_n(t) dt + (k-i-1) \left(\frac{u_{(i-1)j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \right] \\ & + \left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt \right] \times (\mathbf{1}_{(H-N(k) \times k) \geq 1}) \\ & + \left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt \right] + \sum_{i=2}^{H-N(k) \times k} \left[\int_0^{\Delta t} \frac{u_{iN(k)}}{u_{\max}} \lambda_n(t) dt + (k-i-1) \left(\frac{u_{(i-1)N(k)}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \times (\mathbf{1}_{(H-N(k) \times k) > 1}) \end{aligned} \quad (8)$$

u_{ij} represents the production rate during period i after the j^{th} PM action.

Proof

Recall that *PM* actions are performed every k production periods, which means every $k\Delta t$ restoring the system to as good as new state.

The planning horizon $H\Delta t$ is divided into $(N(k) + 1)$ periods, each one is noted j , $j \in \{0, 1, 2, \dots, (N(k))\}$. Hence, $j=0$ corresponds to the first *PM* interval $[0, k\Delta t]$, the second *PM* period $[k\Delta t, 2k\Delta t]$ corresponds to period $j=1$, etc. Note that the last *PM* action is performed at instant $(N(k).k.\Delta t)$, and the interval $[(N(k)-1).k.\Delta t, (N(k).k.\Delta t)]$ corresponds to $j = (N(k)-1)$.

To illustrate this decomposition, Fig. 3 below shows an example with the following values: $H = 13$ periods; $k = 3$ periods, $N = 4$, $\Delta t = 1$ *tu*. Hence, $N(k) = 4$. $(N(k).k.\Delta t) = 12$ *tu*.

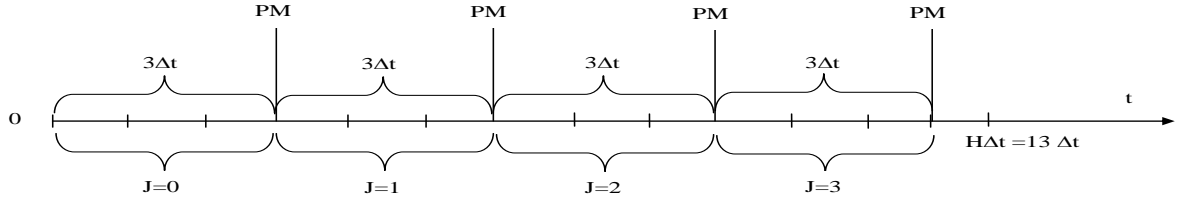


Figure 3: Example of *PM* periods over a planning horizon $H\Delta t = 13$ time units

The average number of failures over the planning horizon $H.\Delta t$ is the sum of the average number of failures during periods $j=0$ to $(N(k) - 1)$, to which should be added the average number of failures during the last interval $[(N(k).k.\Delta t, H\Delta t)]$ corresponding to $j = (N(k))$.

Average number of failures during a *PM* period j

Recall that the system failure rate evolves according to time and to the production rate which varies from one period to another. In order to take into account the influence of the production rate variation on the system degradation, we use Cox model [18]. Formally, the failure rate is expressed according to the production rate during each period and the nominal failure rate. The following expression is proposed (Hajej *et al.* [13]):

$$\lambda_p(t, u(p)) = \lambda_n(t) \times g(u(p))$$

Where $\lambda_p(t, u(p))$ represents the instantaneous failure rate function at period p as a function of the production rate $u(p)$. $\lambda_n(t)$ is the failure rate for nominal conditions which corresponds to the failure rate when the production system is operating with its maximum production rate,

$$\text{and } g(u(p)) = \frac{u(p)}{u_{\max}}.$$

Although this linear relationship between the production rate and the failure rate does not necessarily hold for any type of systems, it is adopted here like in (Hajej *et al.* [13], [14], [15]) as an approximation.

Let u_{ij} represent the production rate during period i after the j^{th} *PM* action.

$$u_{ij} \quad \text{with} \quad (i, j) \in (\{1, \dots, k\}, \{0, \dots, N(k)\})$$

Hence, the failure rate during period i after the j^{th} *PM* action is given by:

$$\lambda_{ij}(t, u_{ij}) = \lambda_n(t) \frac{u_{ij}}{u_{\max}} \quad \forall \quad (i, j) \in (\{1, \dots, k\}, \{0, \dots, N(k)\})$$

The evolution of the production system's failure rate within any preventive maintenance cycle period $j = h$ is presented in Fig. 4 below:

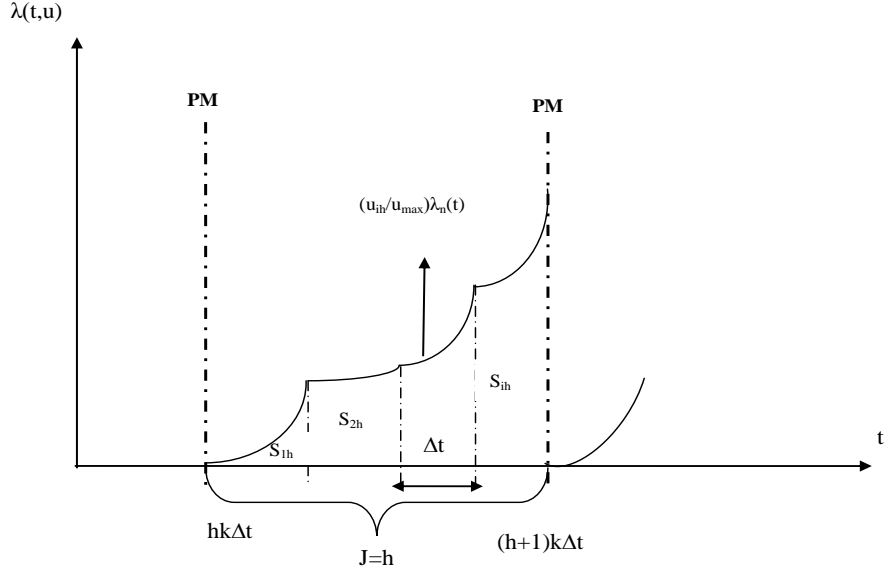


Figure 4: Evolution of the production system failure rate within a preventive maintenance cycle (period $j = h$)

Let S_{1j} be the average number of failures during the first production period after the j^{th} PM action:

$$S_{1j} = \int_0^{\Delta t} \frac{u_{1j}}{u_{\max}} \lambda_n(t) dt$$

In the same way, S_{2j} , corresponding to the average number of failures during the second production period after the j^{th} PM action, is given by:

$$S_{2j} = \int_0^{\Delta t} \frac{u_{2j}}{u_{\max}} \lambda_n(t) dt + \left(\frac{u_{1j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right)$$

Generalizing this to all production periods after the j^{th} PM actions, we obtain:

$$S_{ij} = \int_0^{\Delta t} \frac{u_{ij}}{u_{\max}} \lambda_n(t) dt + (k - (i - 1)) \left(\frac{u_{(i-1)j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \quad \text{for } i \in \{2, 3, \dots, k\}$$

Hence, the average number of failures during any period j is expressed as follows:

$$\int_0^{\Delta t} \frac{u_{1j}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^k \left[\int_0^{\Delta t} \frac{u_{ij}}{u_{\max}} \lambda_n(t) dt + (k-i-1) \left(\frac{u_{(i-1)j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \quad \forall \quad j \in \{0, \dots, N(k)\}$$

Average number of failures over the period $[0, (N(k).k.\Delta t]$

Considering all periods j ($j=0$ to $N(k)-1$), the total average number of failures over the interval $[0, (N(k).k.\Delta t]$ is expressed as follows:

$$\sum_{j=0}^{N(k)-1} \left[\int_0^{\Delta t} \frac{u_{1j}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^k \left[\int_0^{\Delta t} \frac{u_{ij}}{u_{\max}} \lambda_n(t) dt + (k-i-1) \left(\frac{u_{(i-1)j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \right]$$

Average number of failures over the interval $[(N(k).k.\Delta t, H\Delta t]$

Looking at the remaining period $[N(k).k.\Delta t, H\Delta t]$ after the last *PM* action is performed during the planning horizon, one can distinguish three possible cases:

- **Case 1:** this period doesn't exist. This means that the last preventive maintenance action coincides with $H.\Delta t$, the end of the planning horizon.
Formally: $H-(N(k).k) = 0$
- **Case 2:** we have only one production period of length Δt , in the interval $[N(k).k.\Delta t, H\Delta t]$.
Formally: $H-(N(k).k) = 1$
- **Case 3:** we have more than one production period in the interval $[N(k).k.\Delta t, H\Delta t]$.
Formally: $H-(N(k).k) > 1$

Taking into account these three possible situations, the average number of failures in the interval $[N(k).k.\Delta t, H\Delta t]$ is given by:

$$\left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt \right] \times (1_{(H-N(k).k)=1}) + \left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^{H-N(k).k} \left[\int_0^{\Delta t} \frac{u_{iN(k)}}{u_{\max}} \lambda_n(t) dt + (k-i-1) \left(\frac{u_{(i-1)N(k)}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \right] \times (1_{(H-N(k).k)>1})$$

The first term of this sum corresponds to case 2, and the second and third terms correspond to case 3. Obviously, in the first case (case 1), the number of failures is equal to zero.

Average number of failures over the planning horizon $H.\Delta t$

Summing up the obtained average number of failures for all periods, we obtain:

$$\begin{aligned}
\varphi(\mathbf{U}, k) = & \sum_{j=0}^{N(k)-1} \left[\int_0^{\Delta t} \frac{u_{1j}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^k \left[\int_0^{\Delta t} \frac{u_{ij}}{u_{\max}} \lambda_n(t) dt + (k - (i - 1)) \left(\frac{u_{(i-1)j}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \right] \\
& + \left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt \right] \times (1_{(H-N(k) \times k)=1}) \\
& + \left[\int_0^{\Delta t} \frac{u_{1N(k)}}{u_{\max}} \lambda_n(t) dt + \sum_{i=2}^{H-N(k) \times k} \left[\int_0^{\Delta t} \frac{u_{iN(k)}}{u_{\max}} \lambda_n(t) dt + (k - (i - 1)) \left(\frac{u_{(i-1)N(k)}}{u_{\max}} \lambda_n(\Delta t) \times \Delta t \right) \right] \right] \times (1_{(H-N(k) \times k) > 1})
\end{aligned}$$

End of the proof

4.4 Penalty cost relative to the production rate variation: variable set-up cost

As mentioned earlier, one of the objectives of this study is to smooth the production rate between periods. To do so, we consider a variable set-up cost, which is proportional to the difference of the quantity produced from one period to the next one. Indeed, reducing or increasing the quantity to produce may induce in many situations labour related costs (overtime, or hiring/firing costs) and more or less complexity of the set-up.

We define C_v as unit set-up cost. It corresponds to the direct cost of resources needed to adapt the production unit to the production quantity change between successive periods.

Hence, the penalty cost relative to production rate variation is given by:

$$SC = C_v \times \sum_{p=1}^{H-1} |z(p+1) - z(p)| \quad (9)$$

4.5 The total integrated maintenance/ production/inventory cost taking into account the set up cost

Using the developed mathematical expressions above in this section, the total cost optimization problem can be formulated as follows:

$$\underset{(\mathbf{Z}, k)}{\text{Min}}(CTS + CM + SC) \quad (10)$$

Subject to:

$$\begin{aligned}
S(p+1) &= S(p) + z(p+1) - d(p+1) \quad \forall p = 0, \dots, H-1 \\
\text{Prob}[S(p+1) \geq 0] &\geq \theta \quad \forall p = 0, \dots, H-1 \\
0 \leq z(p) &\leq z_{\max} \quad \forall p = 1, \dots, H
\end{aligned}$$

4.6 Solution method

Formally, solving the constrained minimization problem stated above (Eq. 10), yields the determination of optimal production quantity for every period $\{z(1)^*, z(2)^*, \dots, z(p)^*, \dots, z(H)^*\}$ and the optimal period $(k^* \Delta t)$ of PM actions to be performed over the finite horizon $H \Delta t$. Due to the obvious complexity of the model, we developed an iterative search procedure to find a near-optimal solution. It is presented below in four steps.

- **Step 1:** Use Eqs. (5) and (6) to determine the minimum cumulative production quantity to produce in each period in order to satisfy the service level constraint given by Eq. (2).
- **Step 2:** Perform a simple enumeration by varying the production quantity for each period p (using a relatively small step) between the minimum value obtained in step 1 and the maximum one z_{max} given in constraint (3) in order to consider all possible production plans.
- **Step 3:** For every production plan established in step 2, vary k , number of production periods between preventive maintenance actions, between 1 and H , and calculate each time the corresponding total cost (Eq. 10).
- **Step 4:** Determine the values of the decision variables ($(z(1)^*, z(2)^*, \dots, z(p)^*, \dots, z(H)^*), k^*$) which yielded the minimal total cost.

MATHEMATICA[®] software was used to perform the calculations and obtain a near-optimal solution for all instances of the problem considered in this paper.

5. NUMERICAL EXAMPLE AND TRADE-OFFS ANALYSIS

5.1 Reference example

A numerical example is presented in this section in order to illustrate the use of the analytical model developed in the previous sections. We assume that the finite planning horizon is made of 36 periods of one month each ($\Delta t = 1$ month). The other input data are summarized in Table 1 below (' mu ' stands for 'monetary units):

Reliability data	
System time to failure distribution	$F(t)$: Weibull distribution with : Scale parameter : $\beta = 16$ & Shape parameter : $\alpha = 3$
Corrective maintenance cost	$C_{cm} = 3000 mu$
Preventive maintenance cost	$C_{pm} = 500 mu$
Inventory data	
Shortage cost per unit	$Cl = 1 mu/unit$ short
Inventory holding cost per unit per time unit	$Cs = 0.65 mu/unit/tu$
Initial inventory level	$S(0) = 0$
Service level	$\theta = 0.9$
Production data	
Unit production cost	$C_{pr} = 7 mu/unit$
Maximum production rate Maximum production quantity per period	$u_{max} = 500$ units per month $z_{max} = 500$ units
Nominal set-up cost	$C_v = 5 mu$

Table 1: Numerical data

As for the demand, the variance is supposed to be equal for all periods: $V = 4.52$. The average demand per period is presented in Table 2 below:

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
350	420	340	392	431	444	442	340	392	375	392	400
350	370	395	415	431	444	442	340	392	375	400	420
350	340	392	370	431	392	500	350	320	420	365	480

Table 2: Monthly mean demand

Solving the problem using the procedure described in section 3, we obtain the following near-optimal maintenance/production plan presented in Table 3 below:

Near-optimal production plan over the finite horizon $H\Delta t = 36$ months												N^*	k^*
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	6	6
500	500	439	500	500	269	434	352	500	500	395	470		
500	438	485	400	400	127	400	500	465	193	400	370		
65	438	500	200	400	400	499	323	407	351	166	400		

N^* : Optimal number of preventive maintenance actions over $H\Delta t$

k^* : Optimal number of production periods between preventive maintenance actions

Table 3: Near-optimal maintenance/production plan

The total cost related to this plan is equal to : 5.24429×10^7 mu.

5.2 Sensitivity analyses

- Effect of C_v on the production plan, the inventory, and the preventive maintenance schedule

In the perspective of smoothing the production plan through the production rate control between periods of the planning horizon, we studied the impact of the variation of the unit set-up cost between successive periods, C_v , on the obtained solution.

Considering the same input data, we obtained the following results presented in Table 4:

C_v : Unit set-up cost between successive periods	Average production quantity variation between successive periods	N^*	k^*	Average end of month inventory
2	143.71	4	9	156.94
5	126.17	6	6	464.56
7	75.31	7	5	955.58
8	74.57	7	5	1200.53
10	58.34	9	4	1221.47

Table 4: Effect of the variation of the unit set-up cost

Fig. 5 below shows the trade-off between the unit-setup cost and the average production quantity variation between successive periods, the optimal number of preventive maintenance actions, and the average end of month inventory.

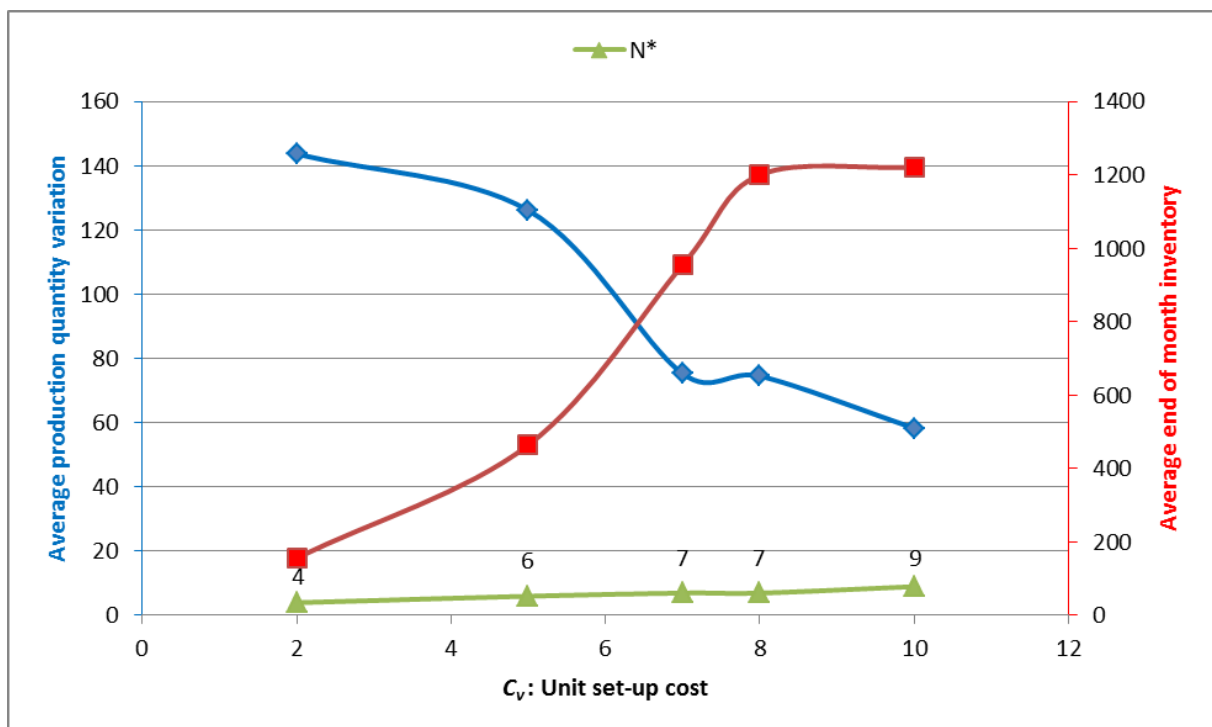


Figure 5: Evolution of the average production quantity variation between successive periods, and the average end of month inventory according to the unit set-up cost C_v

First, one can easily notice that, as expected, when the unit set-up cost between successive periods increases, the average variation of the production quantity, between successive periods, decreases. This clearly illustrates the effect of production smoothing.

Furthermore, it is interesting to notice that the decrease of the average variation of the production quantity between successive periods yields an increase of the quantity to be produced in some periods, and consequently a higher average end of month inventory. Indeed, we know that the quantity to be produced in each period should be between a certain minimum given by the service level constraint (Eqs. 5 and 6) and a given maximum corresponding to the capacity of the machine (Eq. 3). These limits are considered in the first two steps of the solving procedure. Hence, as the smoothing penalty increases, reducing the average variation of the production quantity between successive periods can be achieved either by increasing or by decreasing successive production quantities between the two limits. This is achieved using the model considering simultaneously the trade-off between inventory and shortage costs. The near-optimal solutions, for this specific example, display an increase of the quantities to be produced and therefore an increase of the average end of month inventory. In the same time, the increase of the production rate through the production periods affects the system degradation and induces the need for a more frequent preventive maintenance as it can also be seen in Fig. 5.

The above observations illustrate the fact that the developed model allows the simultaneous consideration of the different trade-offs between the smoothing penalty, the production plan, the inventory, and the preventive maintenance schedule.

We performed several other numerical experiments with different input parameters settings to check the smoothing effect. We present below a sample of the obtained results corresponding to a situation involving a higher shortage cost. While keeping the original combination of input parameters, we increased the shortage cost Cl from 1 mu to 5 mu . A priori, for such a situation, the quantities to be produced should be higher throughout the production periods since shortages are more costly while production and inventory holding costs remain the same.

The following near-optimal maintenance/production plan has been obtained (Table 5):

Near-optimal production plan over the finite horizon $H\Delta t = 36$ months												N^*	k^*
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	7	5
500	500	392	422	481	500	500	500	500	444	481	404		
388	500	322	321	443	132	434	480	467	500	438	500		
437	330	500	500	479	377	0	500	450	500	500	200		

Table 5: Near-optimal maintenance/production plan with $Cl = 5 mu$ and $Cv = 5 mu$

The total cost related to this near-optimal plan is equal to : $7.38 \times 10^8 mu$.

Compared to the situation with a lower shortage cost ($Cl = 1 mu$), one can easily observe comparing Tables 3 and 5 that, in total, more products are produced in the second situation (a total of 15782, compared to a total of 14186 items). Moreover, this increase of total production over the same horizon yielded an increase of the number of preventive maintenance actions that have to be performed (from 6 to 7 actions). This is due to the fact that the machine failure rate increases with the production rate.

Again, in this situation, we studied the impact of the smoothing penalty on the optimal strategy and on the inventory.

The following results were obtained (Table 6):

C_v : Unit set-up cost between successive periods	Average production quantity variation between successive periods	N^*	k^*	Average end of month inventory
2	116.74	4	9	592.22
5	75.14	7	5	950.42
8	63.97	7	5	967.5
10	60	9	4	1296.22

Table 6: Effect of the variation of the unit-setup cost with $Cl = 5 mu$

One can easily notice the same trade-offs as in the case of ($Cl=1 mu$) discussed above. The increase of C_v induces a decrease of the average production quantity variation between successive periods, the need for more preventive maintenance (higher N^*) and the increase of inventory.

- Effect of demand variability on the production and maintenance plans, and on the total expected cost

While keeping the original set of input parameters (Table 1) as well as the monthly average demand (Table 2), we ran experiments varying the demand variance. The following near-optimal maintenance/production plans have been obtained (Tables 7, 8 and 9):

Near-optimal production plan over the finite horizon $H\Delta t = 36$ months												N^*	k^*
Jan	Feb	Mar	Apr	Ma	Jun	Jul	Aug	Sep	Oct	Nov	Dec	7	5
500	500	439	500	500	369	434	352	500	500	395	470		
500	438	485	500	500	127	490	500	465	393	500	370		
365	438	500	200	500	500	499	323	407	351	366	500		

Table 7: Near-optimal maintenance/production plan with demand variance $V=8$

Near-optimal production plan over the finite horizon $H\Delta t = 36$ months												N^*	k^*
Jan	Feb	Mar	Ap	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	7	5
500	500	500	500	500	500	500	500	449	500	305	500		
500	500	500	500	500	500	500	500	500	500	500	500		
500	500	500	500	500	500	500	377	269	318	500	500		

Table 8: Near-optimal maintenance/production plan with demand variance $V=10$

Near-optimal production plan over the finite horizon $H\Delta t = 36$ months												N^*	k^*
Jan	Feb	Mar	Ap	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	12	3
500	500	500	500	500	500	500	500	500	500	500	500		
500	500	500	500	500	500	500	500	500	500	500	500		
500	500	500	500	500	500	500	500	500	500	500	500		

Table 9: Near-optimal maintenance/production plan with demand variance $V=20$

We summarize the obtained results V in table 10 below:

V : Demand Variance	N^*	k^*	Average production quantity variation between successive periods	Average end of month inventory	Total Cost (mu)
4.52	6	6	126.17	464.56	5.25×10^7
8	7	5	90.8	987.33	5.77×10^7
10	7	5	27.25	1741.83	6.07×10^7
20	12	3	0	1980.25	6.57×10^7

Table 10: Effect of the variation of the demand

One can clearly see that a demand with a higher variability causes the system to produce more and generate higher levels of inventory. Hence, because of higher production rates yielding a faster degradation of the machine, more frequent preventive maintenance is needed (N^* increases). We also note that higher variability of demand yields an increase of the total expected cost.

In fact, as the demand variance increases, obviously, the minimum quantity determined in Step 1 increases in order to satisfy the required service level. The system is then forced to

produce more per period without exceeding the machine capacity. Here comes the trade-off between the smoothing penalty, the inventory, and the maintenance policy. Going further with a relatively high variance ($V= 20$), the obtained near-optimal integrated policy consists in producing at the maximum production rate for all periods. In this situation, smoothing is perfectly achieved (set-up cost, SC , is null).

6. CONCLUSION

In this paper we considered a manufacturing system made of a single machine that is subject to random failures. In order to reduce the failure frequency, a periodic preventive maintenance policy with minimal repair at failure is adopted. The production system must satisfy a random product demand over a finite planning horizon with a given required service level. The finite horizon is divided into equal periods. A mathematical model has been developed in order to derive a joint near-optimal production plan and preventive maintenance schedule minimizing the total cost of production, maintenance and inventory, taking into account constraints related to the production system capacity, the service level to be satisfied and the inventory balance.

Compared to previous works, a different cost modelling approach has been proposed. Furthermore, an integrated (not sequential) optimization of the production plan and the maintenance schedule has been proposed. It takes into account the influence of the production rate variation on the production system failure rate. Finally, expressing the set-up cost as being proportional to the production rate variation between successive periods, allowed reducing the penalty induced by excessive variation of the production rate over the planning horizon.

The proposed modelling approach has been illustrated through a numerical example. A sensitivity study with respect to the set-up cost has been performed in order to show the impact of the production plan smoothing on the obtained solution.

The trade-off between the smoothing penalty and the production plan, the inventory, and the preventive maintenance schedule has also been discussed. We finally studied the effect of the demand variability on the smoothing penalty, the production and maintenance plans, and on the total expected cost.

Extensions to this work are under consideration. One of them consists in the investigation of more realistic situations in which preventive maintenance actions are imperfect and have non-negligible durations. Another extension of this work would consist in going beyond the limitation imposed by the linear relationship between the production rate and the failure rate of the system. More general expressions should be investigated empirically and analytically.

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