Constrained optimal control strategy for multimodal urban traffic network

Neila Bhouri

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N. BHOURI

INRETS / GRETIA, 2 Avenue du Gnral Malleret-Joinville, 94114 Arcueil Cedex, France (e-mail: neila.bhouri@inrets.fr).

Abstract: We propose in this paper a receding horizon optimal control strategy for the bimodal urban traffic, cars and public transport. The aim of this strategy is to control the traffic lights durations in order to relieve the traffic congestions in the hole of the network making more for the arcs where the public transport vehicles are present. The bimodal traffic is represented by a linear model. The constraints on the state and control variables are considered within the optimization problem. The numerical resolution is carried out thanks to the active set method. Simulations results are given for a small urban network.

Keywords: Optimal control, Receding horizon, Linear quadratic, Constrained control, Macroscopic traffic model, Urban traffic, Public Transport.

1. INTRODUCTION

Many efforts are carried out nowadays to improve public transport and to make their use competitive as compared to the cars use. Among the range of tools likely to improve the surface public transport (buses, tramways, etc.) performances, we can cote the control systems giving the priority to the public transport vehicles at the traffic lights.

Giving the priority to the public transport vehicles at traffic lights allows a good improvement of their quality of service. When giving them the priority one can improve, at the same time their speed, their frequency and their regularity and give by there a better comfort to the users. By reducing the number and the duration of their stops at the traffic lights, one can increase their speed and cause a drop in their total time of courses. The comfort of the users increases: faster and more regular vehicles, therefore also less charged, circulating with less accelerations and decelerations.

Several real-time urban traffic control systems are able to give the priority to public transport vehicles. We can cote for example CRONOS Boillot et al. (2000), PRODYN Henry and Farges (1994), SCATS Chen et al. (1998), SCOOT Hunt et al. (1982), TUC Diakaki et al. (2002) and UTOPIA Mauro and Tranto (1989). However, these systems can be applied to small network zones. A state of the art of theses systems is given in Bhouri et al. (2008). However, in urban areas where traffic is very dense and the number of public transport vehicles is very important, giving the priority according to local considerations of the intersection or of small zones is not sufficient. It can even imply twisted effects, since it can enable to feed road network sections or congested intersections, resulting in a deterioration of the general traffic conditions including buses traffic conditions themselves. That is why the objectif of our work is to build a global strategy for large scale networks. Its aim is to act on the intersection traffic lights in order to give the priority to the public transport vehicles and to regulate the trafic on the hole of the network.

A first strategy called NeTPrior (Network Transit Priority) has been developed (Bhouri and Lotito (2005) and Bhouri and Lotito (2006)). It consists in a Linear Quadratic (LQ) optimal control strategy. Its objective is to regulate the traffic lights with regards to the global traffic situation and to favor the arcs where and when the buses are present. The resolution of the LQ problem however does not take into account the traffic constraints when computing the optimal control solution. To mitigate this problem we proceeded in NeTPrior to a projection of the optimal control values given by the LQ control stage onto the set of feasible values.

We propose in this paper another methodology consisting in a receding horizon optimisation problem with constraints. We are still using, like in NeTPrior, a linear model and a quadratic criterion. By this way, we can use the advantages of the particular structure of the LQ problem and use one of the powerful available optimization tools. We choose an active set method to resolve this LQ problem. Active set methods are not very perform in terms of speed as compared of other methods such the interior-point methods but they are more robust and better suited for warm starts.

The rest of the paper is organized as follows. In section 2 we present the the linear bi-modal urban traffic model. Section 3 gives the quadratic objectif criterion and section 4 states the constraints. Section 5 is concerned with a brief recall of the NeTPrior strategy. In section 6 we give a short presentation of the receding horizon optimization. In section 7 we present the constrained LQ problem we are concerned with and in section 8 we give a very short description of the active set method which we use to get the numerical resolution of the LQ problem. Finally, the section 9 gives the main simulation results of this strategy.

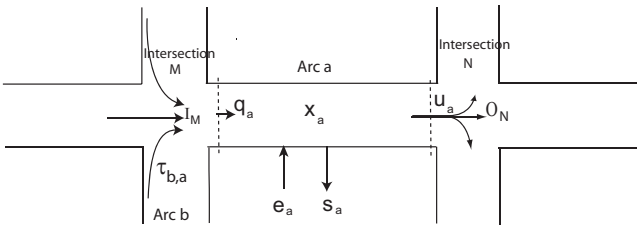

Fig. 1. Variables' definition

2. DYNAMIC MODEL

The network is represented by a directed graph composed of nodes and arcs. The nodes $j \in J$ represent intersections and the arcs $a \in A$ the unidirectional travel links. On every arc, the model consists in two equations, one of them modeling the progress of the total number of vehicles on the arc, expressed as private vehicle unit (PVU) (for example a bus equals 2,3 PVU). The second equation models the number of PT vehicles on the arc.

2.1 The general traffic dynamic equations

The traffic on each arc a is modeled using the continuity equation :

$$x_a(k+1) = x_a(k) + T[q_a(k) + d_a(k) - u_a(k) - s_a(k)], \quad (1)$$

where x_a is the number of cars on the link expressed in PVU, q_a and u_a are the inflow and the outflow of link a during $[kT, (k+1)T]$ where k is the discrete time step and T is the sampling time. d_a and s_a are respectively the demand and the exit flow within the link. d_a and s_a can be generated by the parking vehicles or due to non-controlled intersections situated between intersections M and N. See figure 1. We We will neglect thereafter this generated and consumed flow (d_a and s_a), it would be easy to include them without substantially changing the current development.

In order to explicit the equations for q and u we will consider the saturation flow of each link S_a, that represents the maximum traffic flow that can exit the link, expressed in PVU/s. The *Store and Forward* model assumes that the vehicles reaching the arc's end are stored there and exit with rate S_a during the green light. Hence, we can write :

$$u_a(k) = \frac{S_a.G_a(k)}{C}, \quad (2)$$

where C is the cycle time and $G_a(k)$ is the effective green time of link a, i.e., the green light duration attributed to arc a during the traffic light cycle C of the intersection situated at the arc exit, and will be the control variable in our approach.

If the green light periods are attributed to arc a during different phases (see figure 3), $G_a(k)$ is equal to the sum of all of these green light durations,

$$G_a(k) = \sum_{i \in P_N^a} G_{N,i}(k), \quad (3)$$

where $G_{N,i}(k)$ is the green light duration for the phase i on the junction N, P_N^a is the set of phases of the intersection N, during which arc a has the right of way (green light).

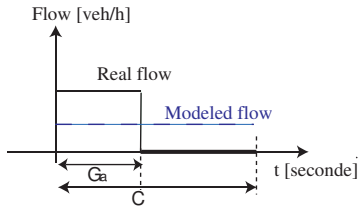

Fig. 2. The store and forward modeling

It also assumes that the outflow is distributed among the different following links according to the coefficients τ_{ab}, called turning rates, that represent the proportion of outflow from a entering in arc b.

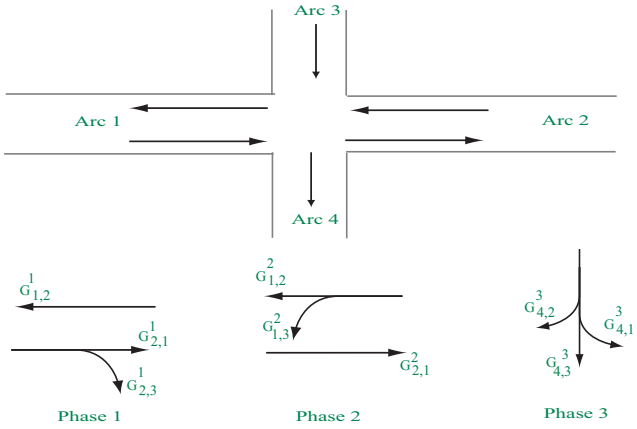

Fig. 3. Traffic light periods are attributed during different phases

$$G_1 = G^1_{1,2} + G^2_{1,2} + G^2_{1,3}; \qquad G_2 = G^1_{2,1} + G^1_{2,3} + G^2_{2,1};$$

$$G_3 = G^3_{4,1} + G^3_{4,2} + G^3_{4,3}$$

If the link a originates at the junction M, the inflow traffic rate entering arc a can be written as the sum of the outflow traffic rates coming from the arcs entering junction M (other than a). If the arc b precedes arc a, the corresponding flow is $\tau_{ba}u_b$, so the total flow entering arc a is

$$q_a(k) = \sum_{b \in I_M} \tau_{b,a} u_b(k), \quad (4)$$

where I_M is the set of arcs entering junction M.

Replacing all the previous definitions in the equation (1), we obtain the following model :

$$x_a(k+1) = x_a(k) + \frac{T}{C}[\sum_{b \in I_M} \tau_{b,a} S_b G_{M,i_b}(k) - S_a \sum_{j \in P_N^a} G_{N,j}(k)] \quad (5)$$

or in matrix form :

$$X(k+1) = X(k) + B.G(k), \quad (6)$$

where B is a matrix of dimension $N \times M$, N is the number of links and M is the total number of phases on

the network.This modeling is possible under the following assumptions :

- the sampling time interval T is at least equal to the duration of the light cycle C, we will use $T = C$. item the gaps between the intersections are not taken into account,
- variations in the queue are neglected, which means that the model considers that all of the input flows on the arc have the green phase at the same time.

2.2 The Public Transport Traffic Dynamic Equations

As we will be considering two kinds of traffic, the general one and the public transport one, we will distinguish the state variables as x^v for the number of vehicles and x^b for the number of PT vehicles (buses). Knowing the sequence of arcs which are used by each PT line, the progress of the PT vehicles is modeled by a delay equation :

$$x_a^{bi}(k) = x_{a'}^{bi}(k - \zeta_a^i) \tag{7}$$

where x_a^{bi} is the number of vehicles of the public transport line number i on arc a, a' is the arc preceding a for the line i and ζ_a^i is a parameter which expresses the mean travel time of the vehicles on line b_i to travel from arc a' to arc a. These values should be real ones, however, in order to be able to write the precedent equation, we take ζ_a^i as integer, meaning that the travel time is a multiple of the sampling interval T. Thus we consider that ζ_a^i is equal to 1 if the bus line has no station on arc a, otherwise ζ_a^i is equal to 2 (for example). Substituting these values in equation (7), the model of the public transport becomes the following :

$$x_a^{bi}(k+1) = \begin{cases} x_{a'}^{bi}(k-1), & \text{if line } i \text{ stops on } a \\ x_{a'}^{bi}(k), & \text{otherwise} \end{cases} \tag{8}$$

This simplification complies with the dynamical modeling of the PC, since it consists in assuming that both the private cars and public transport are "stored" during the red light period and then are "distributed" during the green light period, thus they spend a light cycle on the arc. However, the choice of the cycle duration should be done carefully.

The equation (8) written in vectorial form gives :

$$X^b(k+1) = A_0^b X^b(k) + A_1^b X^b(k-1), \tag{9}$$

where the matrix A_0^b is the adjacency matrix corresponding to the bus line for the arcs without stops. A_1^b is the adjacency matrix corresponding to the bus line for the arcs with a stop, and $X^b(k)$ is the vector of numbers of buses at each traversed arc. It can be further simplified as :

$$X^B(k+1) = A^b.X^B(k) \tag{10}$$

where X^B is the vector obtained stacking $X^b(k)$ and $X^b(k-1)$, and matrix A^b is given by :

$$A^b = \left(\begin{array}{c|c} A_0^b & A_1^b \\ \hline \mathrm{I} & 0 \end{array}\right). \tag{11}$$

2.3 The total bimodal Model

The state variable of the whole system consists is a vector of dimension $(N + 2N_b)$, where N is the number of arcs in the system, N_b is the number of arcs crossed by the PT lines. The dynamics of the system thus is represented by the following equation

$$X(k+1) = AX(k) + BG(k) \tag{12}$$

where A is a matrix of dimension $(N + 2N_b) \times (N + 2N_b)$. The matrix B is composed of two stacked blocks the upper one is defined by the topology of the road network, i.e., the coefficient B_{aj} when different from 0 means that phase j is found entering or leaving arc a and its value is defined according to (5). The lower block corresponds to the influence of the green lights on the bus, which, as it is neglected, has to be 0. We have then

$$A = \left(\begin{array}{c|c} \mathrm{I} & 0 \\ \hline 0 & A^b \end{array}\right), \qquad B = \left(\begin{array}{c} B \\ \hline 0 \end{array}\right). \tag{13}$$

With these matrices, it is clear that it will not be possible to command the public transport because of the null block of matrix B. However, it doesn't set any problem because in the definition of the model, we suppose that the travel times of the public transport are fixed. What we want is to act in such way that buses can comply with their schedules.

3. OPTIMISATION CRITERIA

From the viewpoint of the traffic regulation, our objective is to improve the traffic conditions of the public transport on the network, relative to the cars flow, without deteriorating the global traffic conditions. The objective function need to be quadratic in terms of the state and control variables to rest in the LQ case. We propose the following objective function :

$$J(G) = \sum_{k=0}^{\infty} (\alpha(X(k)'X^b(k)) + \beta\|X(k)\|^2 + \gamma\|G(k)\|^2) \tag{14}$$

where α, β and γ are non-negative weighting parameters and the X are given by the dynamic equations (5) and (8). The first term of the criteria, $(X(k)'X^b(k))$ puts forward the traffic conditions on the arcs crossed by the public transport at the time these public transport vehicles are present on it. The second member aims at reducing the number of vehicles on every arc on the network and thus to equalize the congestion on every arc. The role of this second term is mainly to not degrade too much the traffic in the other arcs. The last term is used in order to avoid large variations of the control (green light times).

This criteria can be written in the matrix form :

$$J(G) = \sum_{k=0}^{\infty} \|x\|_Q^2 + \|u\|_R^2 \tag{15}$$

where $Q = \left(\begin{array}{c|c} \beta * I_{NxN} & \alpha/2 * I_{N_bxN_b} \\ \hline \alpha/2 * I_{N_bxN_b} & \beta * I_{NxN} \end{array}\right)$,
$R = (\gamma * I_{MxM})$; I_{MxM} is the M-dimension identity matrix.

The choice of the values of these parameters enables to modify the objective of the regulation. For example, for $\alpha = 0$, $\beta = \gamma = 1$ the strategy is equivalent to TUC, which doesn't take into account the presence of the public transport. On the other hand a significant parameter α ($\alpha >> \beta$) will strongly penalize the arcs which don't support the public transport.

4. THE CONSTRAINTS

For operative needs, at every intersection j, the durations of green lights should comply with a certain number of constraints :

- the cycle duration (C),
- the phase diagram : all of phases P_j should have their green light within the cycle,
- the clearance times between phases R_j,

which implies :

$$\sum_{i \in P_j} G_{j,i} + R_j = C. \tag{16}$$

On the other hand, for safety reasons, the traffic lights have to respect some constraints. The duration of green lights is limited by a minimum and a maximum value. Indeed, a too short green light can not allow pedestrians to cross the road safely and a too long implies a long red light duration on the antagonistic road which can be interpreted by users as a malfunction of the intersection lights and imply their non-compliance :

$$G_{j,i,min} \leq G_{j,i} \leq G_{j,i,max}. \tag{17}$$

Also the state variable X, the number of vehicles on each arc must respect the capacity of the road and can not be negative :

$$0 \leq X_a \leq X_{a,max}. \tag{18}$$

5. THE NETPRIOR STRATEGY

In the already published strategy NeTprior Bhouri and Lotito (2005), we resolve a non-constrained LQ optimal control problem over an infinite horizon. The advantage of that relies in the fact that the optimal solution can be written as a linear feedback law and the matrix that defines this law is the solution of a matrix equation (Ricatti equation) stated in terms of the given data.

5.1 Control Law

The problem of optimal control consists in minimizing the criteria given by equation (14) respecting the dynamics of the system given by the equations (12).

Using the LQ optimisation method, the applied command law is given by the following equation

$$G(k) = G^N - F.X(k) \tag{19}$$

where F is the Feedback matrix defined as $F = (R + B^T PB)^{-1} B^T PA$ and the matrix P solves the Riccati matrix equation $P = Q + A^T PA - A^T PBF$ which depends on the coefficients α, β, and γ of the objective function through matrices Q ad R.

5.2 Constraints

The solution of the optimal control problem by the LQ methodology doesn't enable us to take the constraints into account because the Ricatti equation will no longer be valid. We solved this problem in NeTPrior, through a projection of the obtained control values onto the set of feasible values defined by the above constraints. It means to obtain the closest (for some distance) values to the optimal but not feasible ones.

6. RECEDING HORIZON OPTIMAL CONTROL WITH CONSTRAINTS

The Receding Horizon Optimal Control consists in starting with a fixed optimization horizon, of length K, using the current state of the plant as the initial state, we then optimize the objective function over this fixed interval accounting to constraints, obtain an optimal sequence of K control moves $\{u_i, ..., u_{i+K-1}\}$. Apply only the first control move$\{u_i\}$ to the plant. Time than advances one step and the same K-step optimization problem is considered using the new state of the plant as initial state as shown on the Figure 4. Thus one continuously revises the current control action based on the current state and accounting for the constraints over an optimisation horizon of length K (Goodwin et al., 2005).

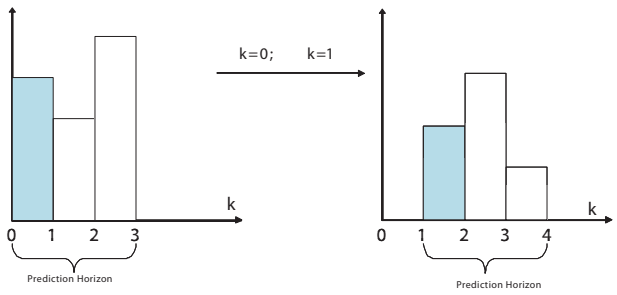

Fig. 4. Receding horizon optimisation principle

As the traffic conditions in the dense urban areas are very changeable it is important that control solution can be applied in a feedback loop, this is possible with the receding horizon optimal control as shown on the diagram 5.

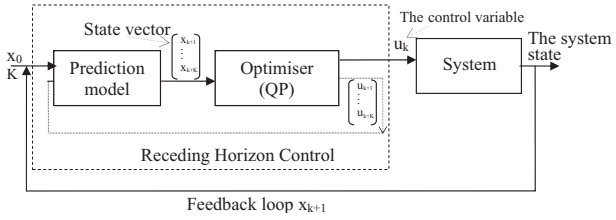

Fig. 5. Receding horizon optimal control operating in a feedback loop

In the general case the algorithms for the receding horizon optimal control remain relatively complex, however remarkable simplifications occur for the particular case of linear systems subject to linear inequality constraints. Thats why we will place our selves in this particular case. The fixed horizon optimal control problem for linear system with a quadratic objective function and linear constraints can be set up as a quadratic program, written for the general case as follows :

$$P_N(x): \begin{cases} J_{(x_k),(u_k)} = Min\frac{1}{2}x_K^T P x_K + \\ \qquad\qquad \frac{1}{2}\sum_{k=0}^{K-1}(x_k^T Q x_k + u_k^T R u_k) \\ x_{k+1} = Ax_k + Bu_k \\ x_0 = x \\ u_k \in U_k \forall k = 0, 1, \ldots, N-1, u \in R^m \\ x_k \in X_k \forall k = 0, \ldots, N,\ x \in R^n \end{cases}$$

The control and state variables u_k and x_k are subject to the inequality :

$$\begin{cases} U_k = \{u_k : Ku_k \leq v^u\} \\ X_k = \{x_k : Dx_k \leq v^x\} \end{cases}$$

7. OUR CONSTRAINED LQ OPTIMAL CONTROL

Given the time-invariant model of the system (12) the fixed horizon optimization problem (14) can be transformed into a Quadratic Problem (QP).

we denote by u_s and x_s the desired steady state values for u_k and x_k. We have then :

$$\begin{aligned} u_s &= [(I-A)^{-1}B]^{-1}x_s \\ x_s &= (I-A)^{-1}Bu_s \end{aligned} \tag{20}$$

our aim is to find, for the system (12) the K control sequence $\{u_0, \cdots, u_{K-1}\}$ and corresponding state sequence $\{x_0, \cdots, x_K\}$ that minimize the finite horizon objective function :

$$J_{(x_k),(u_k)} = Min\frac{1}{2}x_K^T P x_K + \frac{1}{2}\sum_{k=0}^{N-1}(x_k^T Q x_k + u_k^T R u_k) \tag{21}$$

where $P > 0$, $Q \geq 0$ and $R > 0$, K is the prediction horizon, $K \leq K$ is the control horizon, and u_k, x_k are the input and state setpoints given by (20). The control is set equal to its steady state setpoint after M steps, that is $u_k = u_s$ for all $k \geq K$. The above fixed horizon minimization problem is solved at each time step for the current state and disturbance values. Then the first move of the resulting control sequence is used as the current control, and procedure is repeated at the next time step in a RHC fashion, as described in section (6)

7.1 Objective function handling

Writing from (12)with $x_0 = x$, and using the constraint that $u_k = u_s$ for all $k \geq M$.

$$\begin{cases} x_1 = Ax_0 + Bu_0 \\ x_2 = Ax_1 + Bu_1 = A^2x_0 + ABu_0 + Bu_1 \\ x_3 = Ax_2 + Bu_2 = A^3x_0 + A^2Bu_0 + ABU_1 + Bu_2 \\ \vdots \\ x_N = A^Nx_0 + A^{N-1}Bu_0 + \cdots \\ \qquad\qquad \cdots + A^1Bu_{N-2} + Bu_{N-1} \end{cases}$$

let us note :

$$U' = (u_0', u_1', \cdots, u_{N-1}') \quad ; X' = (x_1', x_2', \cdots, x_N')$$

the state equation ((12) became :

$$X = \mathrm{S}.U + \mathbf{A}.x \tag{22}$$

where S is an $(N.n) * (N.m)$ and $\mathbf{A}$ an $N.n$ matrices given by :

$$\mathrm{S} = \begin{pmatrix} B & 0 & \cdots & \cdots & 0 \\ AB & B & 0 & \cdots & 0 \\ \vdots & \vdots & & & \\ A^{N-1}B & A^{N-2}B & \cdots & AB & B \end{pmatrix}; \qquad \mathbf{A} = \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix}$$

using this vector notation (22) and rewriting the objective function (21) to yield :

$$J = \underbrace{\frac{1}{2}x_0 Q x_0 + \frac{1}{2}(\mathbf{A}x)'\mathbf{Q}(\mathbf{A}x)}_{constante} + \frac{1}{2}U'\underbrace{(S'\mathbf{Q}S)}_{H_1}U + U'\underbrace{(S'\mathbf{Q}\mathbf{A}x)}_{F}$$

$$\Leftrightarrow$$

$$J = \frac{1}{2}U'\underbrace{(S'\mathbf{Q}S + \mathbf{R})}_{H}U + U'\underbrace{(S'\mathbf{Q}\mathbf{A}x)}_{F} \tag{23}$$

This criterion is a function only of the control vector U; matrices $\mathbf{Q}$ and $\mathbf{R}$ are given by:

$$\mathbf{Q} = \begin{pmatrix} Q & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & Q & \vdots \\ 0 & \cdots & 0 & P \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} R & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R \end{pmatrix}$$

We can notice that to have a positive definite matrix H it is sufficient to have matrices $\mathbf{Q}$ and $\mathbf{R}$ positive definite independently of the system dynamic. However, due to the non controllability of the model, this H matrix is independent of the parameter "α" of the Q matrix. Only the second term of the objective function (matrix F) is sensitive to transit vehicles.

7.2 Contraints Handling

In the same way, we use the vector notation to write the constraints. Let us note $V'^u = \underbrace{(v'^u, v'^u, \cdots, v'^u)}_{Ntimes}$; So

$U_k = \{u_k : Ku_k \leq v^u\} \quad \Leftrightarrow \quad \mathbf{K}U \leq V^u;$

and $V'^x = \underbrace{(v'^x, v'^x, \cdots, v'^x)}_{Ntimes}$; so $X_k = \{x_k : Dx_k \leq v^x\} \quad \Leftrightarrow \quad \mathbf{D}X \leq V^x;$

where $\mathbf{K} = \begin{pmatrix} K & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K \end{pmatrix}$

and $\mathbf{D} = \begin{pmatrix} D & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & D \end{pmatrix}$

State constraints can be expressed as a function of the only control variables :

$$\mathbf{D}X \leq V^x \Leftrightarrow \mathbf{D}\mathrm{S}.U \leq V^x - \mathbf{D}\mathbf{A}x \tag{24}$$

The QP problem is than formulated :

$$(QP)\begin{cases} J = \frac{1}{2}U'HU + U'F \\ \text{subject to :} \\ \mathbf{DS}.U \leq V^x - \mathbf{DA}x \\ \mathbf{K}U \leq V^u \end{cases} \quad (25)$$

8. NUMERICAL SOLUTION

There are several approaches for solving the constrained QP problem given by (25). One could use, for example, methods of feasible directions, which solve the problem by moving from a feasible point to an improved feasible point. Methods of primal-dual interior point algorithms have also been proposed, such the infeasible interior point methods proposed by Wright (1996). Another used algorithm is the active set method, in which at each step certain constraints, indexed by the active set, are regarded as equalities whilst the rest are temporarily disregarded. The method then sequentially solves an equality-constrained QP and adjusts the active set in order to identify the correct active constraints at the optimal solution. It is this method which is used in this work (for more details see Bonnans et al. (2006)).

8.1 active set method

To give a clear presentation of the method, let us simplifying the notation by writing the constraints in the form of $A_i'u \leq b_i \forall i \in I$ which means that $A = [\mathbf{DS}; K]$ and the constraints $b = [(V^x - \mathbf{DA}x); V^u]$. The problem can be written in a classical form :

$$\begin{cases} Minimize\{fu \ \ \frac{1}{2}u'Hu + u'b\} \\ \text{subject to :} \\ A_i'u \leq b_i \forall i \in I \end{cases} \quad (26)$$

where H est positive definite, and where the index set I correspond to inequality contraints. Let u_k be a feasible solution for (26) and let $I_k = \{i \in I : A_i^{'}u_k = b_i\}$ the set that records the active constraints at u_k. Because the constraints are linear, and since the the optimization problem is strictly convex, a necessary and sufficient condition for a feasible solution to be unique optimal solution of (26) is that there exist Lagrange multipliers λ_i for $i \in I_k$, such that the following Karush-Kuhn-Tucker (KKT) conditions holds :

$$Hu_k + c + \sum_{i \in I_k} \lambda_i A_i = 0 \quad (27)$$

$$\lambda_i \geq 0, i \in I_k$$

The active set method iterates to find the optimal solution of (26) in the following way. At the *kth* iteration we assume that a feasible solution u_k is available. The method now seeks to minimize the objective function subject only to equality constraints recorded in I_k and ignoring the remaining inequality constraints.
If we parameterize u as $u = u_k + d$, so :

$$f(u) = f(u_k + d) = f(u_k) + \ \frac{1}{2}d'Hd + d'(b + Hu_k)$$

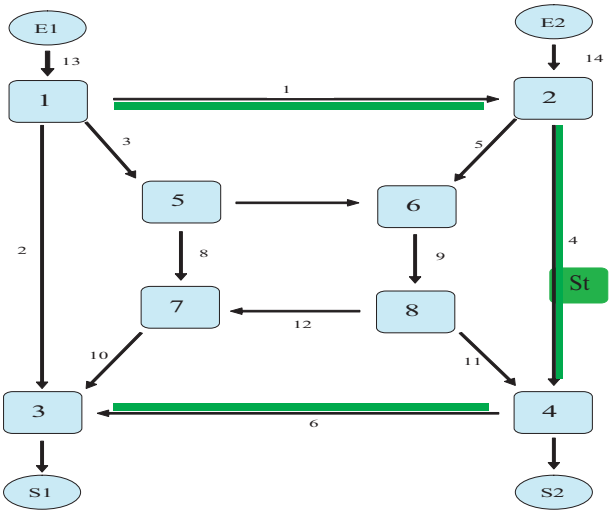

Fig. 6. The simulation urban network

Hence instead of solving (26), we solve the equivalent direction-finding problem :

$$\frac{1}{2}d'Hd + d'(b + Hu_k) \ \text{ subject to } A_i^{'}d = 0 \ \text{ for all } \ i \in I_k$$

We use a feasible direction method.

9. SIMULATION RESULTS

In order to evaluate the receding horizon strategy we performed several simulation tests on a small urban traffic network (Figure 6). This network has two inputs (E1) and (E2), and two outputs (S1) and (S2). It has seven intersections and fourteen arcs numerated according to the notation of section (6) : $J = \{1, 2, ...7\}$ and $A = \{1, 2, ...14\}$. Arcs related to exits are not considered. The transit vehicles follow a fixed line which is given by the arcs (1), (4) and (6) (the green lanes on (Figure 6). Vehicles has a commercial stop at arc (2).

For example we show here the traffic state on the arcs respectively 1 and 4 on which circulate the public transport vehicles (figures 7 and 8). The first parts of these figures give the state of the traffic optimized without favoring the public transport ($\alpha = 0$). In the second part, the blue curves give the traffic state when the public transport vehicles are given a favor ($\alpha = 10^4$) and the green color curves gives the moments of presence of the public transport vehicles on the arcs.

	Arc category	without priority ($\alpha = 0$)	with ($\alpha = 10^4$)	Gain
Arc 1	bus	536	18	97%
Arc 4	bus + bus-stop	437	27	94%
Arc 6	bus	214	28	87%
Arc 13	entree	579	1977	−241%
Arc 14	entree	891	2783	−212%
Network	total	3830	6006	−56%
Network	without the entrees	2360	1246	0, 47%

Table 1. Number of vehicles on the network

As we can see on the second curves: when the value of (α) is very large, the strategy manages to cancel the queue on

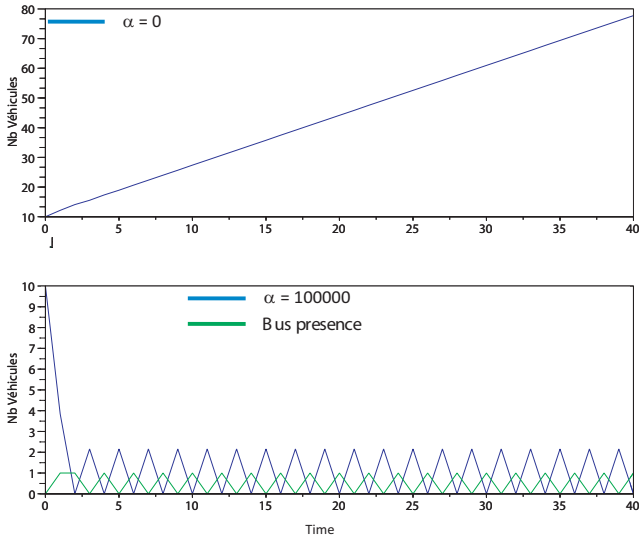

Fig. 7. Traffic state on the arc 1

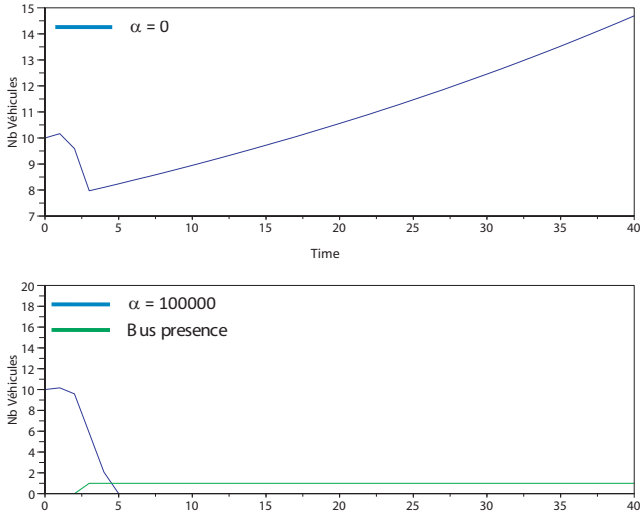

Fig. 8. Traffic state on the arc 4

the arcs when the buses are present. For the arc 4 (Fig. 8), where a bus vehicle is always present, there is no-vehicle on the queue since the fifth step of the regulation. On the other hand, as we can see on the table (1) we can notice that the total congestion is more important when the parameter alpha is important. A finer analysis of the results shows than the improvement of the arcs supporting public transport is done by retaining the traffic on the entries of the network and not on its totality. It is the price to pay to improve the trajectory of public transport.

10. CONCLUSION

The simulation shows that the strategy makes a good improvement to the arcs on which public transport circulates exactly when they are present but by retaining the traffic on the entries of the network Future work should include a nonlinear model of the traffic which will give a better precision on the progression of the buses and will thus allow a better precision at their time of presence in the arcs.

REFERENCES

Bhouri, N., Boillot, F., and Vinant, P. (2008). Rgulation multimodale du trafic routier et des transports en commun de surface. une classification des mthodes. *Recherche-Transports-Scurit*, (98), 53–72.

Bhouri, N. and Lotito, P. (2005). An intermodal traffic control strategy for private vehicle and public transport. In *10th Euro Working Group on Transportation*, 423–428. Poznan-Poland.

Bhouri, N. and Lotito, P. (2006). Regulation du trafic urbain multimodal avec priorite pour les transports en commun. In *Confrence Francophone de et SIMulation - MOSIM 06*. Maroc.

Boillot, F., Midenet, S., and Pierrele, J. (2000). Real-life cronos evaluation. In *Tenth International Conference on Road Traffic Information and Control*, 472, 182–186. IEE London.

Bonnans, J., Gilbert, J.C., Lemarchal, C., and Sagastizabal, C. (2006). *Numerical Optimization, theoretical and practical aspects*, volume 2nd Edition.

Chen, W., Jarjees, G., and Drane, C. (1998). A new approach for bus priority at signalised intesections. In *ARRB Transport Research LTD Conference, 19th*. Sydney-Australia.

Diakaki, C., Papageorgiou, M., and Aboudolas, K. (2002). A multivariable regulator approach to traffic-responsive network-wide signal control. *Control Engieneering Practice*, 10, 183–195.

Goodwin, G., Seron, M., and Dona, J.D. (2005). *Constrained Control and Estimation, an optimisation Approch*.

Henry, J. and Farges, J. (1994). P.t. priority and prodyn. In *Proceeding of the first World Congress On Applications of Transport Telematics and Intelligent Vehicle Highway Systems*, 3086–3093. Paris-France.

Hunt, P., Roberston, D., Bretherton, R., and M.C.Royle (1982). The scoot on-line traffic signal optimization technique. *Traffic Engineering & Control*, 23, 190–199.

Mauro, V. and Tranto, C.D. (1989). Utopia. In *Proceedings in the 6th IFAC/IFIP/IFORS symposium on control, computers, communications on Transportation*, 245–252. Paris-France.

Wright, S.G. (1996). A path-following interior-point algorithm for linear and quadratic problems. *Annals of Operations Research*, 62, 103–130.