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# Discretization of space and time: a slight modification to the Newtonian gravitation which implies the existence of black holes

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## Abstract

Assuming that space and time can only have discrete values, it is shown how deformed space and time cause gravitational attraction, whose law in a discrete context is slightly different from the Newtonian, but to it exactly coincident at large distance. This difference is directly connected to the existence of black holes, which result to have the structure of a hollow sphere.

## 1. Introduction

Let's assume, as work hypothesis, the existence of both discrete space and discrete time, namely spatial and temporal intervals not further divisible; this assumption leads to some interesting consequences. Here we find how deformed space and time cause an attractive force between two material bodies.

So, if we suppose that neither space nor time are continuous, but that instead both are discrete, and following the terminology used in a previous document<sup>[1]</sup>, we call  $l_0$  the fundamental length and  $t_0$  the fundamental time.

## 2. Determining the attraction force between two material bodies

In a continuous context the acceleration is given by  $a(r)=d^2r/dt^2$ , but in a discrete context it's not correct to use derivatives or integrals, because we can not handle infinitesimal quantities nor for space nor for time, whose lower limits, in presence of matter, are  $l_0(r)$  and  $t_0(r)$  as shown in a previous document<sup>[2]</sup>. We must then use the finite differences  $\Delta r$ ,  $\Delta t$ ,  $\Delta^2 r$ ,  $\Delta^2 t$ , being  $\Delta r=l_0(r)$  and  $\Delta t=t_0(r)$ .

For convenience we quote the expressions of  $l_0(r)$  and  $t_0(r)$  obtained in the above mentioned document<sup>[2]</sup>.

In case of  $d < r$  (see here<sup>[2]</sup> for the discussion about  $d$ ) we have:

$$l_0(r) = l_0 \frac{r-d}{r} \quad (1)$$

and

$$t_0(r) = \frac{l_0}{c} \frac{r-d}{r} \quad (2)$$

In case of  $d > r$  we have:

$$l_0(r) = l_0 \frac{d-r}{r} \quad (3)$$

and

$$t_0(r) = \frac{l_0}{c} \frac{d-r}{r} \quad (4)$$

In the cited document<sup>[2]</sup> no assumption has been made about the physical meaning of  $d$  and we cannot exclude at the moment that it could be  $d > r$  or  $d < r$ , so we have to proceed examining both cases to obtain the correct expression for  $a(r) = \Delta^2 r / \Delta t^2$ .

For simplicity, in this discussion we assume that  $d$  corresponds exactly to the actual size of A (for the discussion about this physical system, please see this document<sup>[2]</sup>).

### 2.1 Case $d < r$ (B is outside of A)

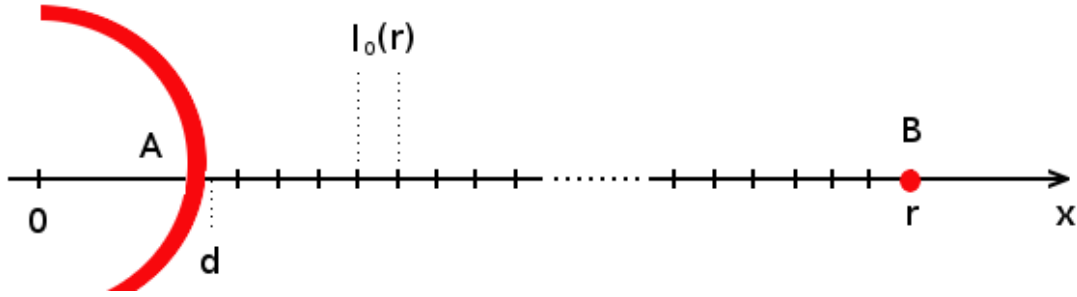


Fig. 1

In this case, using Eq. (1), we have:

$$\Delta r = l_0(r) = l_0 \frac{r-d}{r} \quad (5)$$

and

$$\Delta^2 r = \Delta l_0(r) \quad (6)$$

Using Eq. (2) we have:

$$\Delta t = t_0(r) = \frac{l_0}{c} \frac{r-d}{r} \quad (7)$$

and

$$\Delta t^2 = t_0(r)^2 = \frac{l_0^2}{c^2} \frac{(r-d)^2}{r^2} \quad (8)$$

Again, having made no assumption on this physical system, we can not exclude any possibilities; so we have to examine two distinct sub-cases, depending on the behavior of the body B (for the discussion about this physical system, please see this document<sup>[2]</sup>).

#### Sub-case 1: the body B moves away from the origin and from the border of A

In this sub-case Eq. (6) becomes:

$$\Delta^2 r = \Delta l_0(r) = l_0(r+l_0) - l_0(r) \quad (9)$$

Applying Eq. (1) to  $l_0(r+l_0)$  and  $l_0(r)$  we have:

$$\Delta^2 r = l_0^2 \frac{d}{r(r+l_0)} \quad (10)$$

At this point it is possible to calculate the acceleration  $a(r)$  of the body B.

It is:

$$a(r) = \frac{\Delta^2 r}{\Delta t^2} = c^2 d \frac{r}{(r+l_0)(r-d)^2} \quad (11)$$

If the mass of the body B is  $m$ , we can obtain the force acting on B:

$$f(r) = ma(r) = mc^2 d \frac{r}{(r+l_0)(r-d)^2} \quad (12)$$

Being  $f(r) > 0$ , the force acting on B in this sub-case is **repulsive**.

### Sub-case 2: the body B moves toward the origin and toward the border of A

In this sub-case Eq. (6) becomes:

$$\Delta^2 r = \Delta l_0(r) = l_0(r-l_0) - l_0(r) \quad (13)$$

Applying Eq. (1) to  $l_0(r-l_0)$  and  $l_0(r)$  we have:

$$\Delta^2 r = -l_0^2 \frac{d}{r(r-l_0)} \quad (14)$$

At this point it is possible to calculate the acceleration  $a(r)$  of the body B.

It is:

$$a(r) = \frac{\Delta^2 r}{\Delta t^2} = -c^2 d \frac{r}{(r-l_0)(r-d)^2} \quad (15)$$

If the mass of the body B is  $m$ , we can obtain the force acting on B:

$$f(r) = ma(r) = -mc^2 d \frac{r}{(r-l_0)(r-d)^2} \quad (16)$$

Being  $f(r) < 0$ , the force acting on B in this sub-case is **attractive**.

### 2.2 Case $d > r$ (B is inside of A)

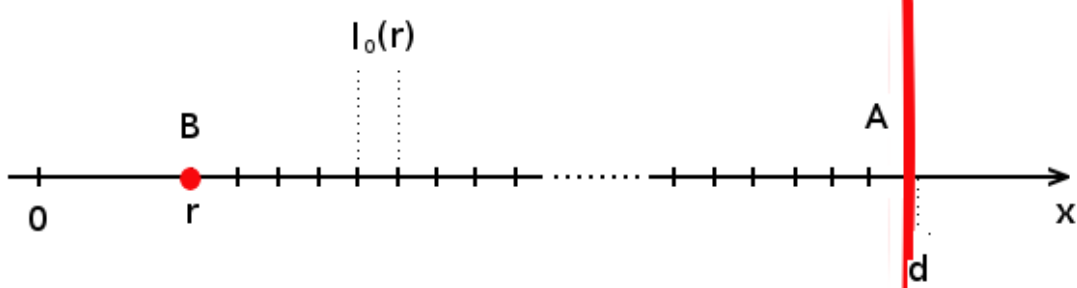


Fig. 2

In this case, using Eq. (3), we have:

$$\Delta r = l_0(r) = l_0 \frac{d-r}{r} \quad (17)$$

and

$$\Delta^2 r = \Delta l_0(r) \quad (18)$$

Using Eq. (4) we have:

$$\Delta t = t_0(r) = \frac{l_0}{c} \frac{d-r}{r} \quad (19)$$

and

$$\Delta t^2 = t_0(r)^2 = \frac{l_0^2}{c^2} \frac{(d-r)^2}{r^2} \quad (20)$$

Again, having made no assumption on this physical system, we can not exclude any possibilities; so we have to examine two distinct sub-cases, depending on the behavior of the body B (for the discussion about this physical system, please see this document<sup>[2]</sup>).

### Sub-case 1: the body B moves away from the origin and toward the border of A

In this sub-case Eq. (18) becomes:

$$\Delta^2 r = \Delta l_0(r) = l_0(r+l_0) - l_0(r) \quad (21)$$

Applying Eq. (3) to  $l_0(r+l_0)$  and  $l_0(r)$  we have:

$$\Delta^2 r = -l_0^2 \frac{d}{r(r+l_0)} \quad (22)$$

At this point it is possible to calculate the acceleration  $a(r)$  of the body B.

It is:

$$a(r) = \frac{\Delta^2 r}{\Delta t^2} = -c^2 d \frac{r}{(r+l_0)(d-r)^2} \quad (23)$$

If the mass of the body B is  $m$ , we can obtain the force acting on B:

$$f(r) = ma(r) = -mc^2 d \frac{r}{(r+l_0)(d-r)^2} \quad (24)$$

Being  $f(r) < 0$ , the force acting on B in this sub-case is **attractive**.

### Sub-case 2: the body B moves toward the origin and away from the border of A

In this sub-case Eq. (18) becomes:

$$\Delta^2 r = \Delta l_0(r) = l_0(r-l_0) - l_0(r) \quad (25)$$

Applying Eq. (3) to  $l_0(r-l_0)$  and  $l_0(r)$  we have:

$$\Delta^2 r = l_0^2 \frac{d}{r(r-l_0)} \quad (26)$$

At this point it is possible to calculate the acceleration  $a(r)$  of the body B.

It is:

$$a(r) = \frac{\Delta^2 r}{\Delta t^2} = c^2 d \frac{r}{(r-l_0)(d-r)^2} \quad (27)$$

If the mass of the body B is  $m$ , we can obtain the force acting on B:

$$f(r) = ma(r) = mc^2 d \frac{r}{(r-l_0)(d-r)^2} \quad (28)$$

Being  $f(r) > 0$ , the force acting on B in this sub-case is **repulsive**.

## 3. Summary

The force acting between A and B is gravitational, so it is always attractive: we must consider only the attractive forces among the ones we obtained.

Resuming:

### Case $d < r$

The gravitational force that A exerts on B is:

$$f(r) = -mc^2 \frac{r}{(r-l_0)} \frac{d}{(r-d)^2} \quad (29)$$

### Case $d > r$

The gravitational force that A exerts on B is:

$$f(r) = -mc^2 \frac{r}{(r+l_0)} \frac{d}{(d-r)^2} \quad (30)$$

It should be noted that when  $d < r$  (the body B is *external* to the body A), the gravitational force pushes the body B towards the origin and therefore also toward the

edge of A, in analogy with what occurs in a continuous context. However when  $d > r$  (the body B is *internal* to the body A), the gravitational force pushes the body B toward the edge of A, but away from the origin, in contrast with what is expected in a continuous context.

This means that in both cases the body B is pushed toward the edge of the body A, which then becomes the point of attraction for the body B.

#### 4. Modification to the law of gravitation

Eq. (29) and Eq. (30) represent the expression of the law of gravitation in a discrete context. The main problem is that they do not depend exactly on the inverse square of the distance, as stated by the Newtonian theory. However, if we consider very large distances, that is when  $r \gg d$  (and by consequence also  $r \gg l_0$ ), Eq. (29) becomes:

$$f(r) = -mc^2 \frac{d}{r^2} \quad (31)$$

Assuming that the body A has mass  $M$ , Eq. (31) coincides with the Newtonian expression  $f(r) = -GMm/r^2$  if

$$d = \frac{GM}{c^2} \quad (32)$$

If the body A is the Earth, we have  $d \sim 5 \cdot 10^{-3} m$ . Already on the surface of the Earth we can consider completely lawful the approximation  $r \gg d$ .

From what has been achieved, it is clear that  $d$  has no relation with the actual size of the body A, which in the case of the Earth is about  $10^9$  times greater than  $d$ .

In a discrete context Eq. (29) then becomes the new expression of the gravitational attraction between two bodies, which flows into the Newtonian one at sufficiently large distances if compared to the quantity  $d$  indicated by Eq. (32).

#### 5. What is $d$ ?

The expression  $GM/c^2$  is widely<sup>[3][4][5]</sup> indicated as the *gravitational radius of a black hole*. There are other definitions<sup>[6]</sup> for the radius of the black hole depending on its characteristics (static, spinning, charged and so on). For instance the radius of a black hole, as determined by Schwarzschild<sup>[5][7]</sup>, is  $R_s = 2GM/c^2$ , the double of the gravitational radius.

In this discrete context Eq. (29) gives an infinite value for the gravitational force exerted by A on B when  $r = d$ , at the boundary of the black hole. This means that there is no way to move away B from this position. So, in this discrete context, Eq. (32) is the expression of the *actual radius* of a black hole having mass  $M$ .

We can also obtain information about the internal structure of a black hole. In fact, the attractive force tends to become larger as the body gets closer to the surface of the black hole, at a distance  $d$  from its center; and it becomes infinite when  $r = d$ . But we can note from Eq. (29) and Eq. (30) that this is true whether the body approaches  $d$  from outside, whether the body approaches  $d$  from inside; if a body could penetrate inside the black hole, it would be inevitably attracted toward the edge. The black hole, then, tends to have the structure of a hollow sphere, of a shell.

In a previous document<sup>[2]</sup> we supposed that the actual size of A could not be less than  $d$ , even without knowing anything about  $d$ ; now we know that  $d$  is the radius of the black hole, so the above statement means that the actual size of a body can not be minor than

the radius of the black hole with the same mass. For a given mass, the black hole is the most compact object. This is reasonable, and strengthens the correctness of the chosen model<sup>[2]</sup>.

## 6. Conclusion

Assuming that space and time can only have discrete values, it has been shown how deformed space and time cause gravitational attraction. The law obtained in this discrete context is slightly different from the Newtonian, but they are exactly coincident at large distance, where  $r \gg d$ . This difference is directly connected to the existence of black holes, having radius equal to  $GM/c^2$ . Also, black holes result to have the structure of a hollow sphere.

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