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► **To cite this version:**

Sophie Soury-Lavergne. DUOS OF ARTEFACTS, CONNECTING TECHNOLOGY AND MANIPULATIVES TO ENHANCE MATHEMATICAL LEARNING. ICME13, Jun 2016, Hamburg, Germany. pp.24 - 31. hal-01492990

**HAL Id: hal-01492990**

**<https://hal.science/hal-01492990>**

Submitted on 20 Mar 2017

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## **DUOS OF ARTEFACTS, CONNECTING TECHNOLOGY AND MANIPULATIVES TO ENHANCE MATHEMATICAL LEARNING**

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*We present the idea of duo of artefacts and an example: the pascaline, a mechanical arithmetic tool and the e-pascaline, its digital counterpart. The duo of artefacts enables us to create learning situations supporting the transfer of procedures, thus toning down the physical properties of the concrete manipulative irrelevant to mathematics. We analyze the added value of this duo to learning situations in terms of feedback. We finally present a new kind of learning situation involving directly-connected concrete and virtual manipulatives.*

### **CONNECTING MANIPULATIVES AND TECHNOLOGICAL TOOLS**

The use of information and communication technology by French primary school teachers is still not as developed could be expected considering the level of equipment and the learning attainments achieved by technology (Ravenstein & Ladage, 2014). One of the reasons may be the teachers' poor understanding of technology as an added value to learn mathematical concepts. They are not convinced of its usefulness. When considering manipulatives, like base-ten kits, sets of cards, coins, dice and so on, the situation is not similar (Moyer-Packenham, Slakind, & Bolyard, 2008). Teachers are aware of the role of manipulation providing physical and perceptual experience as well as solid mathematical conceptualization as regards long-standing research on education. Even though the use of concrete manipulatives may also raise difficulties for students, because they embed perceptual and mechanical elements irrelevant to mathematical knowledge or limit knowledge transfer from one situation to another, teachers actually use such teaching resources.

Therefore, our proposal is to design duos of artefacts, associating a concrete manipulative tool to a technological tool to combine the advantages of both types of learning tools and to overcome some of their limits. It may be an invitation to use both tools and especially technology at primary school level. Thus, our question is about the characteristics of a duo of artefacts in order to improve the learning experience of students. The use of concrete tools implies physical engagement. Many works from different research fields have established the necessity for mathematical conceptualization ((Lakoff & Nunez, 2000) (Edwards, Radford, & Arzarello, 2009) (Kalenine, Pinet, & Gentaz, 2011)). The technological tool of the duo must present some evolution in comparison to the physical one in order to help students overcome some of the manipulatives limits. It may offer students a new opportunity to identify the mathematical properties embedded in the artefact behavior. Moreover, virtual parts of the duo may evolve toward a more abstract and conventional representation of mathematical objects.

According to Fyfe and al. (2014), there is no opposition between the use of concrete materials and abstract materials in such learning experience, but a continuity that helps students build the mathematical concepts in that it fades away the characteristics of concrete material. Following Bruner's theory, they design a concreteness fading instructional process in three steps aiming at

organizing a progressive evolution of students' mathematical understanding. They stress the necessity of three distinct steps pointing out obvious links between them. The process begins with a situation of the manipulation of concrete objects, providing "embodied perceptual and physical experiences" (Fyfe et al., 2014, p. 12). It goes on describing a situation where students interact with analogical representations of these objects (graphics and pictures) that help them disregard properties irrelevant to the mathematics in play. It ends with a situation involving symbolic and abstract representations, useful for the generalization and transfer to other situations. Such an interesting proposal is consistent with Vergnaud's "conceptual field" (2009). He defines a conceptual field that invests with meaning a given mathematical concept by the set of situations in which it is carried out:

"It is at the same time a set of situations and a set of concepts tied together. By this, I mean that a concept's meaning does not come from one situation only but from a variety of situations and that, reciprocally, a situation cannot be analysed with one concept alone, but rather with several concepts, forming systems." (Vergnaud, 2009, p. 86).

He concludes that students come to face contrasting situations about a given concept. The concreteness fading process is a method providing a set of distinct situations. But it raises the question of how to characterize and design situations and manipulatives at each step of the process for a given mathematical concept. In this regard, Fyfe's proposal isn't fully complete. According to Fyfe and al., the first step may include virtual manipulatives. But any virtual environment, either on a tablet or a computer screen deals with computer representations of objects, engaging distinct operational invariants and situations. It is an evolution in comparison with the manipulation of concrete objects. In that regard, it may also pertain to the second step. Therefore, the characteristic of objects and models for each step may be further investigated, tested and defined.

Some recent studies explore the combination of concrete and virtual manipulatives for conceptual learning in mathematics (Ladel & Kortenkamp, 2015), (Soury-Lavergne & Maschietto, 2015). But currently there does not exist a methodology to design such duos. The purpose of this ICME contribution is to explicit some principles that might be efficient for the design of duo of artefacts and to discuss their possible application to the concreteness fading process. Mainly, we use the theory of didactical situations (Brousseau, 1997) and especially the concept of feedback in the interaction subject-milieu to design didactical situations including concrete and virtual artefacts (Mackrell, Maschietto, & Soury-Lavergne, 2013).

## **THE PASCALINE AND THE E-PASCALINE, FOR THE LEARNING OF NUMBERS, PLACE VALUE AND COMPUTATION**

We have designed a first duo of artefacts composed of a concrete manipulative, the pascaline, and a virtual counterpart, the e-pascaline, with the principle that each artefact complements the other one (Figure 1). The pascaline is an arithmetic machine developed after the historical machine of the French mathematician Blaise Pascal. Produced by Italian research on mathematical machines, it is already used in Italy (Maschietto, 2015). The e-pascaline has been developed with the Cabri Elem technology and is a fixture of a collection of e-books (Soury-Lavergne & Maschietto, 2015).

The duo pascaline and e-pascaline aims at teaching place value notation and computation ; moreover it offers a rich mathematical experience on numbers to the students.

### From the pascaline to the e-pascaline

The pascaline is a simple mechanical machine made up of gears providing a symbolic representation of three digit numbers, thus adequate for arithmetic operations. Each of the five wheels has ten cogs. The digits from 0 to 9 are stamped on the lower yellow wheels that display units, tens and hundreds from the right to the left. When the units wheel initialized to 0 rotates fully clockwise, the right upper wheel makes the tens wheel rotate in the same direction one tooth forward. This automatic mechanical motion of each lower wheel illustrates the idea of packing ten units into one ten or ten tens into one hundred. Likewise, the jerky motion of the wheel supports the recursive approach to numbers as it rotates one tooth at a time, adding or subtracting 1 according to the rotation clockwise or anticlockwise. It links addition and subtraction as inverse operations.

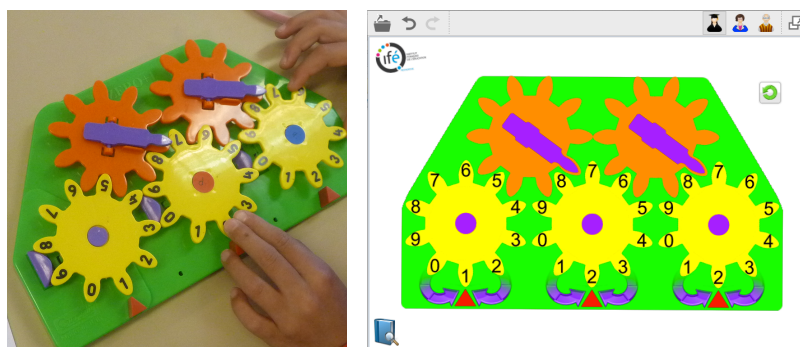


Figure 1. The pascaline (right) and the e-pascaline in a Cabri Elem e-book (left), both displaying number 122 (the 3 digits above the red triangles).

We have designed a virtual machine with some chosen elements of continuities and discontinuities in relation with the physical pascaline (Maschietto & Soury-Lavergne, 2013). The aim was to support the transfer of students' ideas concerning relevant mathematical meanings and to impede those irrelevant to mathematical interpretation at primary school level. Therefore, we have analyzed both students' schemes of uses as well as the way they use the pascaline in order to know what physical components and actions of the machine they have clearly identified. We have selected some of these elements to design the e-pascaline. As a result, the e-pascaline looks like the pascaline with just a few meaningful differences. For instance, with the e-pascaline, it is no longer possible to directly move the upper wheels. Blocking the upper wheels is a means to reinforce the association between a direction of rotation and an operation (the upper wheels turn in the opposite direction due to the principle of gear rotation). Likewise it is no longer possible to subtract 1 from 0 or to add 1 to 999. The e-pascaline only allows operations between 0 and 999, which is the full range of numbers displayed by the e-pascaline and the pascaline.

Moreover, the e-pascaline comes with additional components such as action arrows. Indeed, the rotation of the physical pascaline wheels produces sound and haptic feedback each time a tooth revolves. Students use these clicks to control their action on the machine and to perform operations. The e-pascaline makes it explicit by displaying two arrows on each side of the wheels. These arrows are buttons on which the user clicks to actuate the e-pascaline wheels. They express a possible action on the virtual machine and the direction of the wheel rotation. Other examples of additional components are the reset button (to reset the three wheels to zero) or the "counter of clicks" to display the number of clicks performed by the user since the last reset of the counter. All

these e-pascaline components created added value and can be used to design problem-solving situations.

The last additional element of the virtual part to the duo of artefacts is the collection of e-books offering several didactical situations possible with the e-pascaline.

### **Three different levels of feedback in the duo of artefacts**

The didactical situations with the e-pascaline are developed in e-books created with the Cabri Elem technology. From one page to the other, the designers must set the didactical variable values and implement appropriate feedback to provoke the evolution of the students' solving strategies and thus learning (Mackrell et al., 2013). One of the most important design principles at the basis of Cabri Elem authoring environment is direct manipulation (Laborde & Laborde, 2011) which involves both action and feedback on action. In the process of situation design with the authoring environment, we have identified three kinds of feedback, one of them being "direct manipulation feedback":

*Direct manipulation feedback* is the response of the environment to any student's action and may be combined to produce the other two types of feedback. An example of direct manipulation feedback is the fact that the rotation of the e-pascaline wheels is displayed continuously when a student clicks on the action arrow. Direct manipulation feedback resides mainly in choosing what elements are displayed or hidden, the successive positions of these elements and their dynamic update (the counter of click is automatically updated when clicking on a wheel).

*Strategy feedback* aims at supporting a student in his solving strategy. It is a response to a student's strategy with a mathematical value. To implement strategy feedback, the designers need to identify (i) configurations that are typical of a strategy and induce a diagnosis of this strategy and (ii) new objects or actions that can provide help to the student without changing the nature of the task or giving the answer. Such feedback may consist of help alerts or signs pointing out some contradictory elements in the student's strategy that call his attention to his current strategy limitations. It may also consist of canceling direct manipulation feedback. Below, we present examples of strategy feedback in the e-pascaline e-books.

*Evaluation feedback* is related to the completion of the task. Such feedback is necessary for the students to know how successful their strategies are to solve the problem. In the e-pascaline collection of e-books, it is mainly a smiling smiley displayed on the page that indicates success and a sad smiley that indicates error. Moreover, the successive smileys obtained after each problem remain on the page. They provide information about the global achievement of the students. If the evaluation feedback is automatically displayed, it may happen independently of the students' request. Then, the students may develop a trial and error strategy seeking the unplanned pop-up of a smiley without looking for a solution. So, in the e-pascaline e-books, the evaluation is given only after an explicit request from the students.

These three levels of feedback appear relevant to design and to analyse didactical situations including each kind of artefact, either physical or virtual.

### Adding with the pascaline and the e-pascaline

There are two main procedures to add two numbers with the pascaline, both starting from the first term displayed on this device. Once the first term is displayed, the *iteration procedure* consists in repeating the operation of pushing the units wheel, one tooth at a time clockwise until the number of clicks corresponds to the second term of the sum. For instance, when adding 26 by iteration, the student clicks 26 times on the units wheel. The *decomposition procedure* consists in pushing each of the three wheels by a number of clicks equal to the corresponding digit of the second term. For instance, when adding 26 by decomposition, the user clicks 6 times on the units wheel and twice on the tens wheel (the order between the wheels does not matter). The iteration procedure is based on the quantity represented by the number while the decomposition procedure is based on the meaning of the digits in place value notation. Hence, the evolution of students' procedures from iteration to decomposition corresponds to the transition from a procedure based on the quantity represented by the number (adding by counting one by one) to a procedure based on place value notation. It indicates an evolution of the mathematical meanings associated to place value notation and their possible use for performing operation.

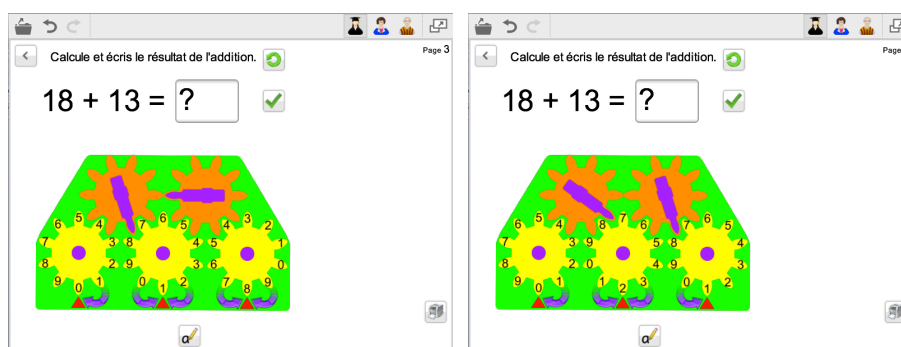


Figure 2. On the left, the first term 18 is written, the e-pascaline is waiting for the second term. On the right, the second term is added, by using the units wheel. After 3 clicks, the adding unit arrow disappears.

In term of feedback of the pascaline when adding two numbers, there is an asymmetric direct manipulation feedback corresponding to the two terms of the addition. When the students add two numbers, the pascaline continuously displays the first term and never the second one. For the second term, the feedback is reduced to the clicks produced by the moving wheels and the boosted haptic feedback when two or three of the lower wheels turn simultaneously. About the strategy feedback, using the physical pascaline enables the students to realize that the two procedures are not equivalent. For instance, adding a large number like 100 requires 100 clicks on the units wheel and only one click on the hundreds wheel. Students may be conscious of this difference but the physical machine does not compel the transfer from one to the other procedure. Finally, there is no evaluation feedback with the pascaline. This last level of feedback relies on the intervention of a human agent, mainly a teacher.

The addition e-book deals with the crucial and tricky passage from the iteration procedure to the decomposition procedure. Most six-year-old pupils apply the iteration procedure even with large numbers (Soury-Lavergne & Maschietto, 2015). The e-book consists of three pages with the same structure and components (Figure 2). The differences from one page to another concern the size of

the proposed numbers for addition (up to 30 in pages 1 and 2, up to 69 in page 3) and the type of feedback given by the e-pascaline in response to the students' procedures. We have implemented feedback to compel students' procedures to evolve from iteration to decomposition. We used the possibility of hiding the action arrows on the units wheel to compel the students to consider and use another wheel, the tens one. It is possible and efficient because the iteration procedure requires only addition on the unit wheels, although decomposition procedure requires the use of the units wheel and the tens wheel as soon as the second term is a two-digit number. In the first page of the e-book, all procedures are feasible. It supports appropriation and devolution of the situation. In the next two pages, the unit wheel can only be used a number of times equal to the sum of the unit digits of the two terms. For example, to add  $18+13$  (Figure 2), the user can only click  $8+3$  times on the units wheel before the addition arrow disappears. The iteration procedure, which needs 13 clicks on the units wheel is no longer possible. In such a way, students have to look for another strategy to perform the addition. The fact that the action arrow is concealed corresponds to a strategy feedback. It occurs in response to the iteration procedure and makes explicit to the students that iteration on the units wheel is no longer possible, yet does not give the appropriate procedure. Such a feedback is not possible with the pascaline. The possibility to design different kinds of feedback contributes to the added value of the e-pascaline to the duo of artefacts.

The differences between the situations with the pascaline and the e-pascaline are sufficiently clear-cut to support students' evolution of procedures and therefore conceptualization of place value notation. Situations including the pascaline and the e-pascaline have characteristics of each of the three steps of the concreteness fading process. They involve concrete manipulatives (step 1), iconic and pictorial model (step 2) and symbolic representations (step 3) with strong links between the three aspects. The duo of artefacts permits to create situations and problems that link physical manipulations to different kinds of representation and particularly symbolic representation. As Fyfe and her colleagues claim: "*it links the concrete and the abstract instantiations as mutual references*" (2014, p. 12).

But in the case of the duo of artefacts, it is up to the students to make the connection between the pascaline and e-pascaline, just as they do between each step of the concreteness fading process. There is no direct interaction between the two types of artefacts independently from the users. An action of the pupils on one of the duo of artefacts does not produce feedback from the other. It is now possible to connect the world of physical manipulatives to the one of digital manipulatives.

## **A ROBOT TO CONNECT PHYSICAL AND DIGITAL MANIPULATIVES**

The OCINAE project<sup>1</sup> – Connected Objects and Digital Interfaces for Learning at Primary School – aims to explore and design mathematics learning situations with a system of connected objects. The OCINAE system is a set of interacting devices either concrete like cards or dice, or digital, like tablets and smartphones. Connection between the two classes of objects is actuated by a mobile robot that can read physical elements such as cards or any printed material. The robot itself has properties of the two classes. Like any concrete manipulative, it has mechanical and physical properties. However, it is also a digital artefact because its behavior results from instructions given

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<sup>1</sup> Funded by the French Bank for Public Investments, it is a partnership between two companies, digiSchool and Awabot, and two public institutions, Erasme and the French Institute of Education.

by a digital environment (automatically generated or piloted by a user). We are currently trying to define the different kinds of feedback that such a complex environment can provide in response to students' actions in a learning situation.



Figure 3. Some of the connected objects of OCINAEE project in scenario “the target number”: moving robot, smartphone displaying the target number 12, game board and sets of cards.

For instance, in a scenario named “the target number”, students have to choose 3 numbers out of 6 whose sum is the target number. In the current OCINAEE version, the target number is displayed on a smartphone and the numbers to be added are printed on cards with their symbolic writing. Students have to present cards to the robot. Then the robot moves on a line toward the target and stops before or after the target according to the sum. It also announces if the number of presented cards is correct or not. To implement such scenario with OCINAEE devices, we had to create direct manipulation feedback but also choose whether it is to be produced by a concrete or virtual object. For instance, the movement of the robot on the board is a strategy feedback in the concrete space because it tells students something about their strategies (too small or too big). Moreover, even though it is a rather simple kind of feedback, this simple movement of the robot mediates the notion of number line. It is also an evaluation feedback, since reaching the target point indicates success. Another example of feedback is the fact that the robot's eyes flash each time a card is presented but then, no indication of the numbers of cards already presented is revealed. This choice results from a didactical analysis: students need to know that the system takes a card into account but they have to deal with the fact that the result is a sum of three terms. They have to control it and they can succeed if they manage the cards and separate the ones they have already presented from the others. We are also creating a tablet version of the scenario to compare students' strategies with concrete cards or virtual representations of numbers.

## CONCLUSION

With the idea of duo of artefacts, we propose a hands-on tool to design learning situations that take advantage of both physical and digital manipulatives. With the connected objects, we have a new opportunity to design situations and enlarge the possibilities of action and feedback. Moreover, it may be now possible to create learning situations with concrete manipulatives for a range of mathematical concepts by selecting the behavior of the robot and implementing appropriate feedback that may be either physical, in the world of concrete objects or virtual, at the interface of the digital world.



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