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# On the Efficiency of Nash Equilibria in the Interference Channel with Noisy Feedback

Victor Quintero, Samir M. Perlaza, and Jean-Marie Gorce

**Abstract**—In this paper, the price of anarchy (PoA) and the price of stability (PoS) of the  $\eta$ -Nash equilibria ( $\eta$ -NEs), of the two-user linear deterministic interference channel (IC) with noisy channel-output feedback are characterized for all  $\eta > 0$ . The price of anarchy is the ratio between the sum-rate capacity and the smallest sum-rate at an  $\eta$ -NE. Alternatively, the price of stability is the ratio between the sum-rate capacity and the biggest sum-rate at an  $\eta$ -NE. Some of the main conclusions of this work are the following: (a) When both transmitter-receiver pairs are in the low-interference regime, the PoA can be made arbitrarily close to one as  $\eta$  approaches zero, subject to a particular condition. More specifically, there are scenarios in which even the worst  $\eta$ -NE (in terms of sum-rate) is arbitrarily close to the Pareto boundary of the capacity region. (b) The use of feedback plays a fundamental role on increasing the PoA in some interference regimes. This is basically because in these regimes, the use of feedback increases the sum-capacity, whereas the smallest sum-rate at an  $\eta$ -NE remains the same as in the case without feedback. (c) The PoS is equal to one in all the interference regimes. This implies that there always exists an  $\eta$ -NE in the Pareto boundary of the capacity region. The conclusions of this work reveal the relevance of jointly using equilibrium selection methods and channel-output feedback for reducing the effect of anarchical behavior of the network components in the  $\eta$ -NE sum-rate of the interference channel.

**Index Terms**—Nash equilibrium, Linear Deterministic Interference Channel, Price of Anarchy, Price of Stability.

## I. LINEAR DETERMINISTIC INTERFERENCE CHANNEL WITH NOISY CHANNEL-OUTPUT FEEDBACK

Consider the two-user decentralized linear deterministic interference channel with noisy channel-output feedback (D-LD-IC-NOF) depicted in Figure 1. For all  $i \in \{1, 2\}$ , with  $j \in \{1, 2\} \setminus \{i\}$ , the number of bit-pipes between transmitter  $i$  and its corresponding intended receiver is denoted by  $\vec{n}_{ii}$ ; the number of bit-pipes between transmitter  $i$  and its corresponding non-intended receiver is denoted by  $n_{ji}$ ; and the number of bit-pipes between receiver  $i$  and its corresponding transmitter is denoted by  $\overleftarrow{n}_{ii}$ . These six non-negative integer parameters describe the LD-IC-NOF in Figure 1.

At transmitter  $i$ , the channel-input  $\mathbf{X}_{i,n}$  at channel use  $n$ , with  $n \in \{1, 2, \dots, N_i\}$ , is a  $q$ -dimensional binary vector

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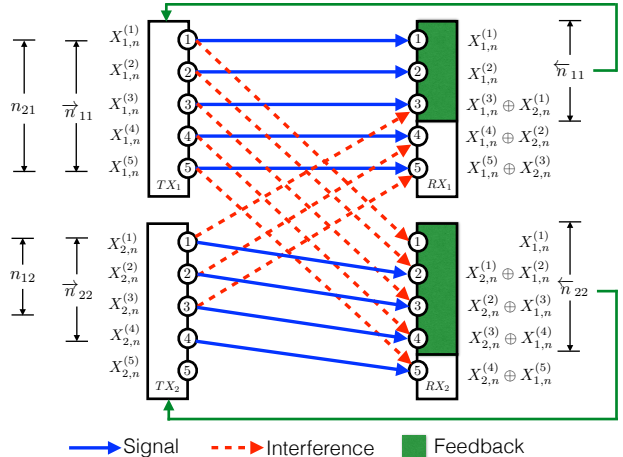


Fig. 1. Two-user linear deterministic interference channel with noisy channel-output feedback at channel use  $n$ .

$$\mathbf{X}_{i,n} = \left( X_{i,n}^{(1)}, X_{i,n}^{(2)}, \dots, X_{i,n}^{(q)} \right)^T \in \mathcal{X}_i, \text{ with } \mathcal{X}_i = \{0, 1\}^q, \quad (1)$$

$$q = \max(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}),$$

and  $N_i \in \mathbb{N}$  is the block-length of transmitter-receiver pair  $i$ . At receiver  $i$ , the channel-output  $\vec{\mathbf{Y}}_{i,n}$  at channel use  $n$ , with  $n \in \{1, 2, \dots, \max(N_1, N_2)\}$ , is also a  $q$ -dimensional binary vector  $\vec{\mathbf{Y}}_{i,n} = \left( \vec{Y}_{i,n}^{(1)}, \vec{Y}_{i,n}^{(2)}, \dots, \vec{Y}_{i,n}^{(q)} \right)^T$ . Let  $\mathbf{S}$  be a  $q \times q$  binary lower shift matrix of the form:

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}. \quad (2)$$

The input-output relation during channel use  $n$  is given by

$$\vec{\mathbf{Y}}_{i,n} = \mathbf{S}^{q-\vec{n}_{ii}} \mathbf{X}_{i,n} + \mathbf{S}^{q-n_{ij}} \mathbf{X}_{j,n}, \quad (3)$$

where  $\mathbf{X}_{i,n} = (0, 0, \dots, 0)^T$  for all  $n > N_i$ . The feedback signal  $\overleftarrow{\mathbf{Y}}_{i,n}$  available at transmitter  $i$  at the end of channel use  $n$  is

$$\overleftarrow{\mathbf{Y}}_{i,n} = \mathbf{S}^{(\max(\vec{n}_{ii}, n_{ij}) - \overleftarrow{n}_{ii})^+} \vec{\mathbf{Y}}_{i,n-d}, \quad (4)$$

where  $d \in \mathbb{N}$  is a finite delay and additions and multiplications are defined over the binary field.

Without any loss of generality, the feedback delay is assumed to be equal to one channel use, i.e.,  $d = 1$ . Let  $\mathcal{W}_i$  be the set of message indices of transmitter  $i$ . Transmitter

$i$  sends the message index  $W_i \in \mathcal{W}_i$  by transmitting the codeword  $\mathbf{X}_i = (\mathbf{X}_{i,1}, \mathbf{X}_{i,2}, \dots, \mathbf{X}_{i,N_i}) \in \mathcal{X}_i^{N_i}$ , which is a binary  $q \times N_i$  matrix. The encoder of transmitter  $i$  can be modeled as a set of deterministic mappings  $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$ , with  $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \{0, 1\}^q$  and for all  $n \in \{2, 3, \dots, N_i\}$ ,  $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \{0, 1\}^{q \times (n-1)} \rightarrow \{0, 1\}^q$ , such that

$$\mathbf{X}_{i,1} = f_{i,1}^{(N)}(W_i, \Omega_i) \text{ and} \quad (5a)$$

$$\mathbf{X}_{i,n} = f_{i,n}^{(N)}(W_i, \Omega_i, \overleftarrow{\mathbf{Y}}_{i,1}, \overleftarrow{\mathbf{Y}}_{i,2}, \dots, \overleftarrow{\mathbf{Y}}_{i,n-1}), \quad (5b)$$

where  $\Omega_i$  is a randomly generated index known by both transmitter  $i$  and receiver  $i$ , while unknown by transmitter  $j$  and receiver  $j$ .

The decoder of receiver  $i$  is defined by a deterministic function  $\psi_i^{(N)} : \{0, 1\}^{q \times N} \times \mathbb{N} \rightarrow \mathcal{W}_i$ . At the end of the communication, receiver  $i$  uses the  $q \times N$  binary matrix  $(\overleftarrow{\mathbf{Y}}_{i,1}, \overleftarrow{\mathbf{Y}}_{i,2}, \dots, \overleftarrow{\mathbf{Y}}_{i,N})$  and  $\Omega_i$  to obtain an estimate  $\widehat{W}_i \in \mathcal{W}_i$  of the message index  $W_i$ , i.e.,  $\widehat{W}_i = \psi_i^{(N)}(\overleftarrow{\mathbf{Y}}_{i,1}, \overleftarrow{\mathbf{Y}}_{i,2}, \dots, \overleftarrow{\mathbf{Y}}_{i,N}, \Omega_i)$ . Let  $W_i$  be written as  $c_{i,1} c_{i,2} \dots c_{i,M_i}$  in binary form, with  $M_i = \lceil \log_2 |\mathcal{W}_i| \rceil$ . Let also  $\widehat{W}_i$  be written as  $\widehat{c}_{i,1} \widehat{c}_{i,2} \dots \widehat{c}_{i,M_i}$  in binary form.

A transmit-receive configuration for transmitter-receiver pair  $i$ , denoted by  $s_i$ , can be described in terms of the block-length  $N_i$ , the number of bits per block  $M_i$ , the channel-input alphabet  $\mathcal{X}_i$ , the codebook, the encoding functions  $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$ , the decoding function  $\psi_i^{(N)}$ , etc.

The average bit error probability at decoder  $i$  given the configurations  $s_1$  and  $s_2$ , denoted by  $p_i(s_1, s_2)$ , is given by

$$p_i(s_1, s_2) = \frac{1}{M_i} \sum_{\ell=1}^{M_i} \mathbb{1}_{\{\widehat{c}_{i,\ell} \neq c_{i,\ell}\}}. \quad (6)$$

Within this context, a rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is said to be achievable if it complies with the following definition.

*Definition 1 (Achievable Rate Pairs):* A rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is achievable if there exists at least one pair of configurations  $(s_1, s_2)$  such that the decoding bit error probabilities  $p_1(s_1, s_2)$  and  $p_2(s_1, s_2)$  can be made arbitrarily small by letting the block-lengths  $N_1$  and  $N_2$  grow to infinity.

The aim of transmitter  $i$  is to autonomously choose its transmit-receive configuration  $s_i$ , in order to maximize its achievable rate  $R_i(s_1, s_2)$ . Note that the rate achieved by transmitter-receiver  $i$  depends on both configurations  $s_1$  and  $s_2$  due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel. The following section models this interaction using tools from game theory.

## II. THE TWO-USER INTERFERENCE CHANNEL AS A GAME

The competitive interaction of the two transmitter-receiver pairs in the decentralized interference channel can be modeled by the following game in normal-form:

$$\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}). \quad (7)$$

The set  $\mathcal{K} = \{1, 2\}$  is the set of players, that is, the set of transmitter-receiver pairs. The sets  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are the sets of

actions of players 1 and 2, respectively. An action of a player  $i \in \mathcal{K}$ , which is denoted by  $s_i \in \mathcal{A}_i$ , is basically its transmit-receive configuration as described above. The utility function of player  $i$  is  $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$  and it is defined as the information rate of transmitter  $i$ ,

$$u_i(s_1, s_2) = \begin{cases} R_i = \frac{M_i}{N_i}, & \text{if } p_i(s_1, s_2) < \epsilon \\ 0, & \text{otherwise,} \end{cases} \quad (8)$$

where  $\epsilon > 0$  is an arbitrarily small number. This game formulation was first proposed in [1] and [2].

A class of transmit-receive configurations  $s^* = (s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$  that are particularly important in the analysis of this game is referred to as the set of  $\eta$ -Nash equilibria ( $\eta$ -NE), with  $\eta > 0$ . This type of configuration satisfy the following definition.

*Definition 2 ( $\eta$ -Nash equilibrium):* In the game  $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ , an action profile  $(s_1^*, s_2^*)$  is an  $\eta$ -Nash equilibrium if for all  $i \in \mathcal{K}$  and for all  $s_i \in \mathcal{A}_i$ , there exists an  $\eta > 0$  such that

$$u_i(s_i, s_j^*) \leq u_i(s_i^*, s_j^*) + \eta. \quad (9)$$

Let  $(s_1^*, s_2^*)$  be an  $\eta$ -Nash equilibrium action profile of the game in (7). Then, none of the transmitters can increase its own information transmission rate more than  $\eta$  bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for  $\eta$  sufficiently large, from Definition 2, any pair of configurations can be an  $\eta$ -NE. Alternatively, for  $\eta = 0$ , the classical definition of Nash equilibrium is obtained [3]. In this case, if a pair of configurations is a Nash equilibrium ( $\eta = 0$ ), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible  $\eta$ -NE rate pairs  $(R_1, R_2)$  of the game in (7) with the smallest  $\eta$  for which there exists at least one equilibrium configuration pair. The set of rate pairs that can be achieved at an  $\eta$ -NE is known as the  $\eta$ -Nash equilibrium region.

*Definition 3 ( $\eta$ -NE Region):* Let  $\eta > 0$  be fixed. An achievable rate pair  $(R_1, R_2)$  is said to be in the  $\eta$ -NE region of the game  $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$  if there exists a pair  $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$  that is an  $\eta$ -NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \text{and} \quad u_2(s_1^*, s_2^*) = R_2. \quad (10)$$

The  $\eta$ -NE region of the game in (7) is characterized in [4] in terms of two regions: the capacity region, denoted by  $\mathcal{C}(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$ , and a convex region, denoted by  $\mathcal{B}_\eta(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$ . This region was first characterized in [5] for the case without feedback and in [6] for the case of perfect channel-output feedback.

In the following, the tuple  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  is used only when needed. The capacity region  $\mathcal{C}$  of the two-user LD-IC-NOF is described in Lemma 1 (at the top of the next page) and the region  $\mathcal{B}_\eta$  for all  $\eta > 0$ , is described as follows:

$$\mathcal{B}_\eta = \left\{ (R_1, R_2) : L_i \leq R_i \leq U_i, \text{ for all } i \in \{1, 2\} \right\}, \quad (12)$$

*Lemma 1 (Theorem 1 in [7]): The capacity region  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  of the two-user LD-IC-NOF is the set of non-negative rate pairs  $(R_1, R_2)$  that satisfy for all  $i \in \{1, 2\}$ , with  $j \in \{1, 2\} \setminus \{i\}$ :*

$$R_i \leq \min(\max(\vec{n}_{ii}, n_{ji}), \max(\overleftarrow{n}_{ii}, n_{ij})), \quad (11a)$$

$$R_i \leq \min\left(\max(\vec{n}_{ii}, n_{ji}), \max(\overleftarrow{n}_{ii}, \overleftarrow{n}_{jj} - (\vec{n}_{jj} - n_{ji})^+)\right), \quad (11b)$$

$$R_1 + R_2 \leq \min\left(\max(\vec{n}_{22}, n_{12}) + (\vec{n}_{11} - n_{12})^+, \max(\vec{n}_{11}, n_{21}) + (\vec{n}_{22} - n_{21})^+\right), \quad (11c)$$

$$R_1 + R_2 \leq \max\left(\left(\vec{n}_{11} - n_{12}\right)^+, n_{21}, \vec{n}_{11} - (\max(\vec{n}_{11}, n_{12}) - \overleftarrow{n}_{11})^+\right) \\ + \max\left(\left(\vec{n}_{22} - n_{21}\right)^+, n_{12}, \vec{n}_{22} - (\max(\vec{n}_{22}, n_{21}) - \overleftarrow{n}_{22})^+\right), \quad (11d)$$

$$2R_i + R_j \leq \max(\vec{n}_{ii}, n_{ji}) + (\vec{n}_{ii} - n_{ij})^+ + \max\left(\left(\vec{n}_{jj} - n_{ji}\right)^+, n_{ij}, \vec{n}_{jj} - (\max(\vec{n}_{jj}, n_{ji}) - \overleftarrow{n}_{jj})^+\right). \quad (11e)$$

where,

$$L_i = \left((\vec{n}_{ii} - n_{ij})^+ - \eta\right)^+ \quad \text{and} \quad (13a)$$

$$U_i = \max(\vec{n}_{ii}, n_{ij}) - \left(\min\left(\left(\vec{n}_{jj} - n_{ji}\right)^+, n_{ij}\right) \quad (13b)$$

$$- \left(\min\left(\left(\vec{n}_{jj} - n_{ji}\right)^+, n_{ji}\right) - (\max(\vec{n}_{jj}, n_{ji}) - \overleftarrow{n}_{jj})^+\right)^+ + \eta,$$

with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ .

The following lemma characterizes the  $\eta$ -NE region of the two-user LD-IC-NOF.

*Lemma 2 (Theorem 1 in [4]): Let  $\eta > 0$  be fixed. The  $\eta$ -NE region of the two-user LD-IC-NOF with parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ , is*

$$\mathcal{N}_\eta = \mathcal{C} \cap \mathcal{B}_\eta. \quad (14)$$

### III. PRELIMINARIES

#### A. Definitions

Let  $\alpha_i \in \mathbb{Q}$  be the interference parameter of transmitter-receiver pair  $i$ , with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , such that:

$$\alpha_i = \frac{n_{ij}}{\vec{n}_{ii}}. \quad (15)$$

The scenario in which the desired signal is stronger than the interference ( $\alpha_i < 1$ ) is referred to as the low-interference regime (LIR). Alternatively, the scenario in which the desired signal is weaker than or equal to the interference ( $\alpha_i \geq 1$ ) is referred to as the high-interference regime (HIR). The main results of this paper are presented using a list of events (Boolean variables) that are fully determined by the parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}$ , and  $n_{21}$ . The event in which the number of interference-free bit-pipes at receiver  $i$  is bigger than or equal to the number of bit pipes in the cross-interference link in receiver  $j$  is denoted by  $A_{1,i}$ , i.e.,

$$A_{1,i}: \vec{n}_{ii} - n_{ij} \geq n_{ji}. \quad (16)$$

The event in which the number of bit-pipes from transmitter  $i$  to receiver  $i$  is bigger than or equal to the number of bit pipes in the cross-interference link in receiver  $j$  is denoted by  $A_{2,i}$ , i.e.,

$$A_{2,i}: \vec{n}_{ii} \geq n_{ji}. \quad (17)$$

In the following, given an event, e.g.  $A_{2,i} : \vec{n}_{ii} \geq n_{ji}$ , the notation  $\overline{A}_{2,i}$  implies  $\vec{n}_{ii} < n_{ji}$  (logical complement). Combining the events (16) and (17), ten different conditions are identified:

$$B_1 : A_{1,1} \wedge A_{1,2}, \quad (18a)$$

$$B_{2,i} : A_{1,i} \wedge \overline{A}_{1,j} \wedge A_{2,j}, \quad (18b)$$

$$B_{3,i} : A_{1,i} \wedge \overline{A}_{1,j} \wedge \overline{A}_{2,j}, \quad (18c)$$

$$B_4 : \overline{A}_{1,1} \wedge \overline{A}_{1,2} \wedge A_{2,1} \wedge A_{2,2}, \quad (18d)$$

$$B_{5,i} : \overline{A}_{1,1} \wedge \overline{A}_{1,2} \wedge \overline{A}_{2,i} \wedge A_{2,j}, \quad (18e)$$

$$B_6 : \overline{A}_{1,1} \wedge \overline{A}_{1,2} \wedge \overline{A}_{2,2} \wedge \overline{A}_{2,2}, \quad (18f)$$

$$B_7 : A_{1,1}, \quad (18g)$$

$$B_8 : \overline{A}_{1,1} \wedge A_{2,1} \wedge A_{2,2}, \quad (18h)$$

$$B_9 : \overline{A}_{1,1} \wedge \overline{A}_{2,1} \wedge A_{2,2}, \quad (18i)$$

$$B_{10} : \overline{A}_{1,1} \wedge \overline{A}_{2,2}. \quad (18j)$$

For all  $i \in \{1, 2\}$ ,  $j \in \{1, 2\} \setminus \{i\}$  when both transmitter-receiver pairs are in the LIR, i.e.,  $\vec{n}_{11} > n_{12}$  and  $\vec{n}_{22} > n_{21}$ , the events  $B_1, B_{2,i}, B_{3,i}, B_4, B_{5,i}$ , and  $B_6$  exhibit the property stated by the following lemma.

*Lemma 3 (Both transmitter-receiver pairs are in the LIR): For all  $i \in \{1, 2\}$ ,  $j \in \{1, 2\} \setminus \{i\}$  and for all  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$  with  $\vec{n}_{11} > n_{12}$  and  $\vec{n}_{22} > n_{21}$ , only one of the events  $B_1, B_{2,i}, B_{3,i}, B_4, B_{5,i}$ , and  $B_6$  holds true.*

*Proof:* The proof follows from verifying that for all  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , when both transmitter-receiver pairs are in the LIR, i.e.,  $\vec{n}_{11} > n_{12}$  and  $\vec{n}_{22} > n_{21}$ , the events (18a)-(18f) are mutually exclusive. ■

For all  $i \in \{1, 2\}$ ,  $j \in \{1, 2\} \setminus \{i\}$  when transmitter-receiver pair 1 is in the LIR and transmitter-receiver pair 2 is in the HIR, i.e.,  $\vec{n}_{11} > n_{12}$  and  $\vec{n}_{22} \leq n_{21}$ , the events  $B_7, B_8, B_9$ , and  $B_{10}$  exhibit the property stated by the following lemma.

*Lemma 4 (Transmitter-receiver pair 1 in the LIR and transmitter-receiver pair 2 in the HIR): For all  $i \in \{1, 2\}$ ,  $j \in \{1, 2\} \setminus \{i\}$  and for all  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$  with  $\vec{n}_{11} > n_{12}$  and  $\vec{n}_{22} \leq n_{21}$ , only one of the events  $B_7, B_8, B_9$ , and  $B_{10}$  holds true.*

*Proof:* The proof of Lemma 4 follows along the same lines of the proof of Lemma 3. ■

#### IV. MAIN RESULTS: EFFICIENCY OF AN $\eta$ -NE

This section characterizes the efficiency of the set of  $\eta$ -NEs of the game in (7) using two metrics: price of anarchy (PoA) and price of stability (PoS). The PoA measures the loss of performance due to decentralization by comparing the maximum sum-rate achieved by a centralized LD-IC-NOF with the minimum sum-rate achieved by a decentralized LD-IC-NOF at an  $\eta$ -NE. Alternatively, the PoS measures the loss of performance due to decentralization by comparing the maximum sum-rate achieved by a centralized LD-IC-NOF with the maximum sum-rate achieved by a decentralized LD-IC-NOF at an  $\eta$ -NE [8].

##### A. Price of Anarchy

Let  $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$  be the set of all possible action profiles and  $\mathcal{A}_{\eta\text{-NE}} \subset \mathcal{A}$  be the set of  $\eta$ -NE strategies of the game in (7) (Definition 2).

*Definition 4 (Price of Anarchy [9]):* Let  $\eta > 0$ . The PoA of the game  $\mathcal{G}$ , denoted by  $\text{PoA}(\eta, \mathcal{G})$ , is given by:

$$\text{PoA}(\eta, \mathcal{G}) = \frac{\max_{(s_1, s_2) \in \mathcal{A}} \sum_{i=1}^2 R_i(s_1, s_2)}{\min_{(s_1^*, s_2^*) \in \mathcal{A}_{\eta\text{-NE}}} \sum_{i=1}^2 R_i(s_1^*, s_2^*)}. \quad (19)$$

Let  $\bar{\Sigma}_C$  denote the solution to the optimization problem in the numerator of (19), which correspond to the maximum sum-rate in the centralized case. Let also  $\underline{\Sigma}_N$  denote the solution to the optimization problem in the denominator of (19). Closed-form expressions of  $\bar{\Sigma}_C$  and  $\underline{\Sigma}_N$  are presented in [4].

The following theorems describe the  $\text{PoA}(\eta, \mathcal{G})$  in particular interference regimes of the LD-IC-NOF. In all the cases, it is assumed that  $\bar{n}_{ii} \leq \max(\bar{n}_{ii}, n_{ij})$  for all  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ . If  $\bar{n}_{11} > \max(\bar{n}_{11}, n_{12})$  or  $\bar{n}_{22} > \max(\bar{n}_{22}, n_{21})$ , the analysis is the same as in the case of perfect channel-output feedback, i.e.,  $\bar{n}_{11} = \max(\bar{n}_{11}, n_{12})$  or  $\bar{n}_{22} = \max(\bar{n}_{22}, n_{21})$ .

*Theorem 1 (Both transmitter-receiver pairs in the LIR):*

For all  $i \in \{1, 2\}$ ,  $j \in \{1, 2\} \setminus \{i\}$  and for all  $(\bar{n}_{11}, \bar{n}_{22}, n_{12}, n_{21}, \bar{n}_{11}, \bar{n}_{22}) \in \mathbb{N}^6$  with  $\bar{n}_{11} > n_{12}$  and  $\bar{n}_{22} > n_{21}$ , the  $\text{PoA}(\eta, \mathcal{G})$  satisfies:

$$\text{PoA}(\eta, \mathcal{G}) = \begin{cases} \frac{\bar{\Sigma}_{C1}}{\underline{\Sigma}_{N1}} & \text{if } B_1 \\ \frac{\bar{\Sigma}_{C2,i}}{\underline{\Sigma}_{N1}} & \text{if } B_{2,i} \\ \frac{\bar{n}_{ii}}{\underline{\Sigma}_{N1}} & \text{if } B_{3,i} \\ \frac{\bar{\Sigma}_{C3}}{\underline{\Sigma}_{N1}} & \text{if } B_4 \\ \frac{\bar{n}_{ii}}{\underline{\Sigma}_{N1}} & \text{if } B_{5,i} \\ \frac{\min(\bar{n}_{11}, \bar{n}_{22})}{\underline{\Sigma}_{N1}} & \text{if } B_6, \end{cases} \quad (20)$$

where,

$$\begin{aligned} \bar{\Sigma}_{C1} = & \min \left( \bar{n}_{22} + \bar{n}_{11} - n_{12}, \bar{n}_{11} + \bar{n}_{22} - n_{21}, \right. \\ & \max \left( \bar{n}_{11} - n_{12}, \bar{n}_{11} \right) + \max \left( \bar{n}_{22} - n_{21}, \bar{n}_{22} \right), \\ & 2\bar{n}_{11} - n_{12} + \max \left( \bar{n}_{22} - n_{21}, \bar{n}_{22} \right), \\ & \left. 2\bar{n}_{22} - n_{21} + \max \left( \bar{n}_{11} - n_{12}, \bar{n}_{11} \right) \right); \quad (21a) \end{aligned}$$

$$\begin{aligned} \bar{\Sigma}_{C2,i} = & \min \left( \bar{n}_{22} + \bar{n}_{11} - n_{12}, \bar{n}_{11} + \bar{n}_{22} - n_{21}, \right. \\ & \max \left( \bar{n}_{11} - n_{12}, \bar{n}_{11} \right) + \max \left( \bar{n}_{22} - n_{21}, \bar{n}_{22} \right), \\ & 2\bar{n}_{ii} - n_{ij} + \max \left( n_{ij}, \bar{n}_{jj} \right), \\ & \left. 2\bar{n}_{jj} - n_{ji} + \max \left( \bar{n}_{ii} - n_{ij}, \bar{n}_{ii} \right) \right); \quad (21b) \end{aligned}$$

$$\begin{aligned} \bar{\Sigma}_{C3} = & \min \left( \bar{n}_{22} + \bar{n}_{11} - n_{12}, \bar{n}_{11} + \bar{n}_{22} - n_{21}, \right. \\ & \max \left( n_{21}, \bar{n}_{11} \right) + \max \left( n_{12}, \bar{n}_{22} \right), \\ & 2\bar{n}_{11} - n_{12} + \max \left( n_{12}, \bar{n}_{22} \right), \\ & \left. 2\bar{n}_{22} - n_{21} + \max \left( n_{21}, \bar{n}_{11} \right) \right); \text{ and} \quad (21c) \end{aligned}$$

$$\underline{\Sigma}_{N1} = \bar{n}_{11} - n_{12} + \bar{n}_{22} - n_{21} - 2\eta. \quad (21d)$$

*Proof:* The proof is presented in [4].  $\blacksquare$

From Theorem 1, the following conclusions can be drawn.

When both transmitter-receiver pairs are in the LIR, and at least one of the conditions  $B_{3,i}$ ,  $B_{5,i}$ , or  $B_6$  holds true, with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , then the  $\text{PoA}(\eta, \mathcal{G})$  does not depend on the feedback parameters  $\bar{n}_{11}$  and  $\bar{n}_{22}$ . However, this is not always the case as shown in the following corollaries.

*Corollary 1:* For any  $(\bar{n}_{11}, \bar{n}_{22}, n_{12}, n_{21}, \bar{n}_{11}, \bar{n}_{22}) \in \mathbb{N}^6$  with  $\bar{n}_{11} > n_{12}$  and  $\bar{n}_{22} > n_{21}$ , that satisfies  $B_1$ , it follows that:

$$1 < \frac{\bar{\Sigma}_{C4}}{\underline{\Sigma}_{N1}} \leq \text{PoA}(\eta, \mathcal{G}) \leq \frac{\bar{\Sigma}_{C5}}{\underline{\Sigma}_{N1}}, \quad (22)$$

where,

$$\bar{\Sigma}_{C4} = \bar{n}_{11} + \bar{n}_{22} - n_{12} - n_{21}, \text{ and} \quad (23)$$

$$\bar{\Sigma}_{C5} = \bar{n}_{11} + \bar{n}_{22} - \max(n_{12}, n_{21}). \quad (24)$$

The lower bound in (22) is obtained assuming that  $\bar{n}_{11} = 0$  and  $\bar{n}_{22} = 0$  in (20). That is, when feedback is not available. The upper bound in (22) is obtained assuming that  $\bar{n}_{11} = \max(\bar{n}_{11}, n_{12}) = \bar{n}_{11}$  and  $\bar{n}_{22} = \max(\bar{n}_{22}, n_{21}) = \bar{n}_{22}$  in (20). That is, when perfect channel-output feedback is available at both transmitter-receiver pairs.

Note also that for any  $\eta$  arbitrarily small, when both transmitter-receiver pairs are in the LIR; condition  $B_1$  holds

true;  $\overleftarrow{n}_{11} \leq \overrightarrow{n}_{11} - n_{12}$ ; and  $\overleftarrow{n}_{22} \leq \overrightarrow{n}_{22} - n_{21}$ , the sum-rate capacity approaches to the minimum sum-rate at an  $\eta$ -NE ( $\text{PoA}(\eta, \mathcal{G}) \approx 1$ ). Alternatively, when both transmitter-receiver pairs are in the LIR; condition  $B_1$  holds true; and at least one the following conditions:  $\overleftarrow{n}_{11} > \overrightarrow{n}_{11} - n_{12}$  or  $\overleftarrow{n}_{22} > \overrightarrow{n}_{22} - n_{21}$  holds true, the use of feedback in transmitter-receiver pair 1 or transmitter-receiver pair 2, respectively, enlarges both the capacity region and the  $\eta$ -NE region. Nonetheless, the PoA increases as the smallest sum-rate at an  $\eta$ -NE remains unchanged with respect to the case without feedback.

*Corollary 2:* For any  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$  with  $\overrightarrow{n}_{11} > n_{12}$  and  $\overrightarrow{n}_{22} > n_{21}$ , that satisfies  $B_{2,i}$  for all  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , it follows that:

$$1 < \frac{\overrightarrow{n}_{ii}}{\sum_{N1}} \leq \text{PoA}(\eta, \mathcal{G}) \leq \frac{\overline{\Sigma}_{C5}}{\sum_{N1}}. \quad (25)$$

Note that when both transmitter-receiver pairs are in the LIR and for a given  $i \in \{1, 2\}$  condition  $B_{2,i}$  holds true;  $\overleftarrow{n}_{ii} \leq \overrightarrow{n}_{ii} - n_{ij}$ ; and  $\overleftarrow{n}_{jj} \leq n_{ij}$ , the use of feedback in either transmitter-receiver pair does not enlarge the capacity region or the  $\eta$ -NE region. Then, the  $\text{PoA}(\eta, \mathcal{G})$  is equal to the lower bound in (25), i.e.,  $\text{PoA}(\eta, \mathcal{G}) = \frac{\overrightarrow{n}_{ii}}{\sum_{N1}}$ . Conversely, when both transmitter-receiver pairs are in the LIR and for a given  $i \in \{1, 2\}$  condition  $B_{2,i}$  holds true; and at least one of the following conditions:  $\overleftarrow{n}_{ii} > \overrightarrow{n}_{ii} - n_{ij}$  or  $\overleftarrow{n}_{jj} > n_{ij}$  holds true, the use of feedback enlarges both the capacity region and the  $\eta$ -NE region.

The lower and upper bounds in (25) are obtained as in the case of (22).

*Corollary 3:* For any  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$  with  $\overrightarrow{n}_{11} > n_{12}$  and  $\overrightarrow{n}_{22} > n_{21}$ , that satisfies  $B_4$ , it follows that:

$$1 < \frac{\min(\overline{\Sigma}_{C5}, n_{12} + n_{21})}{\sum_{N1}} \leq \text{PoA}(\eta, \mathcal{G}) \leq \frac{\overline{\Sigma}_{C5}}{\sum_{N1}}. \quad (26)$$

Note that when both transmitter-receiver pairs are in the LIR; condition  $B_4$  holds true; and  $\overline{\Sigma}_{C5} \leq n_{12} + n_{21}$ , then the  $\text{PoA}(\eta, \mathcal{G})$  does not depend on the feedback parameters  $\overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ . When both transmitter-receiver pairs are in the LIR; condition  $B_4$  holds true;  $\overline{\Sigma}_{C5} > n_{12} + n_{21}$ ;  $\overleftarrow{n}_{11} \leq n_{21}$ ; and  $\overleftarrow{n}_{22} \leq n_{12}$ , then the  $\text{PoA}(\eta, \mathcal{G})$  is equal to the lower bound in (26), i.e.,  $\text{PoA}(\eta, \mathcal{G}) = \frac{n_{12} + n_{21}}{\sum_{N1}}$ . Conversely, when both transmitter-receiver pairs are in the LIR; condition  $B_4$  holds true;  $\overline{\Sigma}_{C5} > n_{12} + n_{21}$ ; and at least one of the following conditions:  $\overleftarrow{n}_{11} > n_{21}$  or  $\overleftarrow{n}_{22} > n_{12}$  holds true, the use of feedback in transmitter-receiver pair 1 or transmitter-receiver pair 2, respectively, enlarges the capacity region and the  $\eta$ -NE region.

*Theorem 2 (Transmitter-receiver pair 1 in the LIR and transmitter-receiver pair 2 in the HIR):* For all  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$  with  $\overrightarrow{n}_{11} > n_{12}$  and

$\overrightarrow{n}_{22} \leq n_{21}$ , the  $\text{PoA}(\eta, \mathcal{G})$  satisfies:

$$\text{PoA}(\eta, \mathcal{G}) = \begin{cases} \frac{\overrightarrow{n}_{11}}{\overrightarrow{n}_{11} - n_{12} - \eta} & \text{if } B_7 \\ \frac{\overrightarrow{n}_{11}}{\overrightarrow{n}_{11} - n_{12} - \eta} & \text{if } B_8 \\ \frac{\min(\overrightarrow{n}_{22} + \overrightarrow{n}_{11} - n_{12}, n_{21})}{\overrightarrow{n}_{11} - n_{12} - \eta} & \text{if } B_9 \\ \frac{\overrightarrow{n}_{11}}{\overrightarrow{n}_{11} - n_{12} - \eta} & \text{if } B_{10}. \end{cases} \quad (27)$$

Note that in the cases in which transmitter-receiver pair 1 is in the LIR and transmitter-receiver pair 2 is in the HIR, the  $\text{PoA}(\eta, \mathcal{G})$  does not depend on the feedback parameters. This is basically because the use of feedback in this scenario can enlarge the capacity region but it does not increase the sum-rate capacity (Theorem 4 in [10]).

In the case in which transmitter-receiver pair 1 is in the HIR and transmitter-receiver pair 2 is in the LIR, i.e.,  $\overrightarrow{n}_{11} \leq n_{12}$  and  $\overrightarrow{n}_{22} > n_{21}$ , the  $\text{PoA}(\eta, \mathcal{G})$  for the D-LD-IC-NOF is characterized as in Theorem 2 interchanging the indices of the parameters.

*Theorem 3 (Both transmitter-receiver pairs in the HIR):* For all  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$  with  $\overrightarrow{n}_{11} \leq n_{12}$  and  $\overrightarrow{n}_{22} \leq n_{21}$ , the  $\text{PoA}(\eta, \mathcal{G})$  satisfies:

$$\text{PoA}(\eta, \mathcal{G}) = \infty. \quad (28)$$

The result on Theorem 3 is due to the fact that  $\left( (\overrightarrow{n}_{11} - n_{12})^+ - \eta \right)^+ + \left( (\overrightarrow{n}_{22} - n_{21})^+ - \eta \right)^+ = 0$ . That is, when  $\overrightarrow{n}_{11} \leq n_{12}$  and  $\overrightarrow{n}_{22} \leq n_{21}$ , none of the transmitter-receiver pairs is able to transmit at a strictly positive rate at the worst  $\eta$ -NE, i.e.,  $\sum_N = 0$ .

In general, in any interference regime in which the  $\text{PoA}(\eta, \mathcal{G})$  depends on the feedback parameters  $\overleftarrow{n}_{11}$  or  $\overleftarrow{n}_{22}$ , there exist a value in the feedback parameter  $\overleftarrow{n}_{11}$  or the feedback parameter  $\overleftarrow{n}_{22}$  beyond which the  $\text{PoA}(\eta, \mathcal{G})$  increases. These values correspond to those values beyond which the sum capacity can be enlarged (Theorem 4 in [10]).

## B. Price of Stability

In this section, the efficiency of the  $\eta$ -NEs of the game  $\mathcal{G}$  in (7) is analyzed by using the PoS.

*Definition 5 (Price of stability [11]):* Let  $\eta > 0$ . The PoS of the game  $\mathcal{G}$ , denoted by  $\text{PoS}(\eta, \mathcal{G})$ , is given by:

$$\text{PoS}(\eta, \mathcal{G}) = \frac{\max_{(s_1, s_2) \in \mathcal{A}} \sum_{i=1}^2 R_i(s_1, s_2)}{\max_{(s_1^*, s_2^*) \in \mathcal{A}_{\eta\text{-NE}}} \sum_{i=1}^2 R_i(s_1^*, s_2^*)}. \quad (29)$$

Let  $\overline{\Sigma}_N$  denote the solution to the optimization problem in the denominator of (29). A closed-form expression of  $\overline{\Sigma}_N$  is presented in [4].

The following proposition characterizes the PoS of the game  $\mathcal{G}$  in (7) for the LD-IC-NOF.

*Proposition 1 (PoS):* For all  $(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$  and for all  $\eta > 0$  arbitrary small, the PoS in the game  $\mathcal{G}$  of the LD-IC-NOF is:

$$\text{PoS}(\eta, \mathcal{G}) = 1. \quad (30)$$

Note that the fact that the price of stability is equal to one, independently of the parameters  $\vec{n}_{11}$ ,  $\vec{n}_{22}$ ,  $n_{12}$ ,  $n_{21}$ ,  $\overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ , implies that despite the anarchical behavior of both transmitter-receiver pairs, the biggest  $\eta$ -NE sum-rate is equal to the sum-rate capacity, i.e.,  $\overline{\Sigma}_C = \overline{\Sigma}_{\mathcal{N}_\eta}$ . This implies that in all interference regimes, there always exist at least one Pareto optimal  $\eta$ -NE.

## V. SYMMETRIC GAME

Denote by  $\text{PoA}_{\text{NFS}}(\mathcal{G})$  the PoA of the game (7) with an arbitrarily small value of  $\eta$ , i.e.,

$$\text{PoA}_{\text{NFS}}(\mathcal{G}) = \lim_{\eta \rightarrow 0} \text{PoA}(\eta, \mathcal{G}), \quad (31)$$

and symmetric conditions, i.e.,  $\vec{n}_{11} = \vec{n}_{22} = \vec{n}$ ,  $n_{12} = n_{21} = m$ , and  $\overleftarrow{n}_{11} = \overleftarrow{n}_{22} = \overleftarrow{n}$ . Let  $\alpha$  and  $\beta$  be the normalized parameters of the symmetric LD-IC-NOF, i.e.,  $\alpha = \frac{m}{\vec{n}}$  and  $\beta = \frac{\overleftarrow{n}}{\vec{n}}$ . The  $\text{PoA}_{\text{NFS}}(\mathcal{G})$  can be obtained as a special case of Theorem 1 and Theorem 3 as shown by the following corollary.

*Corollary 4 (Corollaries 1, 2, 3 and 4 in [8]):* The  $\text{PoA}_{\text{NFS}}(\mathcal{G})$  satisfies for all  $\alpha \in [0, \frac{1}{2}]$ :

$$\text{PoA}_{\text{NFS}}(\mathcal{G}) = \begin{cases} 1 & \text{if } \beta \leq 1 - \alpha \\ \min\left(\frac{\beta}{1-\alpha}, \frac{2-\alpha}{2(1-\alpha)}\right) & \text{if } 1 - \alpha < \beta \leq 1 \\ \frac{2-\alpha}{2(1-\alpha)} & \text{if } \beta > 1; \end{cases} \quad (32a)$$

for all  $\alpha \in (\frac{1}{2}, \frac{2}{3}]$ :

$$\text{PoA}_{\text{NFS}}(\mathcal{G}) = \begin{cases} \frac{\alpha}{1-\alpha} & \text{if } \beta \leq \alpha \\ \min\left(\frac{\beta}{1-\alpha}, \frac{2-\alpha}{2(1-\alpha)}\right) & \text{if } \alpha < \beta \leq 1 \\ \frac{2-\alpha}{2(1-\alpha)} & \text{if } \beta > 1; \end{cases} \quad (32b)$$

for all  $\alpha \in (\frac{2}{3}, 1)$ :

$$\text{PoA}_{\text{NFS}}(\mathcal{G}) = \frac{2-\alpha}{2(1-\alpha)}; \quad (32c)$$

and for all  $\alpha \in [1, \infty)$ :

$$\text{PoA}_{\text{NFS}}(\mathcal{G}) = \infty. \quad (32d)$$

Denote by  $\text{PoA}_{\text{WFS}}(\mathcal{G})$  the PoA of the game in (7) with an arbitrarily small value of  $\eta$ , i.e.,

$$\text{PoA}_{\text{WFS}}(\mathcal{G}) = \lim_{\substack{\eta \rightarrow 0 \\ \beta \rightarrow 0}} \text{PoA}(\eta, \mathcal{G}), \quad (33)$$

without feedback, i.e.,  $\overleftarrow{n}_{11} = \overleftarrow{n}_{22} = 0$ , and symmetric conditions, i.e.,  $\vec{n}_{11} = \vec{n}_{22} = \vec{n}$  and  $n_{12} = n_{21} = m$ . The  $\text{PoA}_{\text{WFS}}(\mathcal{G})$  can be obtained as a special case of Theorem 1 and Theorem 3 as shown by the following corollary.

*Corollary 5 (PoA without feedback under symmetric conditions in [12]):* The  $\text{PoA}_{\text{WFS}}(\mathcal{G})$  satisfies:

$$\text{PoA}_{\text{WFS}} = \begin{cases} 1 & \text{if } 0 \leq \alpha \leq \frac{1}{2} \\ \frac{\alpha}{1-\alpha} & \text{if } \frac{1}{2} < \alpha \leq \frac{2}{3} \\ \frac{2-\alpha}{2(1-\alpha)} & \text{if } \frac{2}{3} < \alpha < 1, \\ \infty & \text{if } \alpha \geq 1. \end{cases} \quad (34)$$

## VI. CONCLUSIONS

The PoA and the PoS of the  $\eta$ -NE of the two-user LD-IC-NOF have been characterized, with  $\eta > 0$  arbitrarily small. It has been shown that when both transmitter-receiver pairs are in the LIR, the PoA can be made arbitrarily close to one as  $\eta$  approaches zero, subject to a particular condition. This immediately implies that in this regime even the worst  $\eta$ -NE (in terms of sum-rate) is arbitrarily close to the Pareto boundary of the capacity region. More importantly, it has been shown that the use of feedback increases the PoA in some interference regimes. This is basically because in these regimes, the use of feedback increases the sum-capacity, where as the smallest sum-rate at an  $\eta$ -NE is not changed with respect to the case without feedback. In some cases, the PoA can be infinity due to the fact that both transmitter-receiver pairs are in the HIR, the smallest sum-rate at an  $\eta$ -NE is zero bits per channel use. In other regimes, the use of feedback does not have any impact in the PoA as it does not increase the sum-capacity. Finally, the PoS is shown to be equal to one in all interference regimes. This implies that there always exists an  $\eta$ -NE in the Pareto boundary of the capacity region. The main results of this work highlight the relevance of designing equilibrium selection methods such that decentralized networks can operate at efficient  $\eta$ -NE points. The need of these methods becomes more relevant when channel-output feedback is available as it might increase the PoA.

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