# Families of deformations of the thirteen peregrine breather solutions to the NLS equation depending on twenty four parameters 

Pierre Gaillard, Mickaël Gastineau

## - To cite this version:

Pierre Gaillard, Mickaël Gastineau. Families of deformations of the thirteen peregrine breather solutions to the NLS equation depending on twenty four parameters. Journal of Basic and Applied Research International, 2017, 21 (3), pp.130-139. hal-01492325

HAL Id: hal-01492325

## https://hal.science/hal-01492325

Submitted on 21 Mar 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The thirteenth Peregrine breather and its deformations depending on twenty four parameters solutions to the NLS equation. 

+Pierre Gaillard, ${ }^{\times}$Mickaël Gastineau<br>+ Université de Bourgogne,<br>9 Av. Alain Savary, Dijon, France: Dijon, France :<br>e-mail: Pierre.Gaillard@u-bourgogne.fr,<br>× IMCCE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ Paris 06, Univ. Lille, 77 Av. Denfert-Rochereau, 75014 Paris, France :<br>e-mail: gastineau@imcce.fr

March 21, 2017


#### Abstract

We go on with the study of the solutions to the focusing one dimensional nonlinear Schrödinger equation (NLS). We construct here the thirteen's Peregrine breather ( $P_{13}$ breather) with its twenty four real parameters, creating deformation solutions to the NLS equation. New families of quasi-rational solutions to the NLS equation in terms of explicit ratios of polynomials of degree 182 in $x$ and $t$ multiplied by an exponential depending on $t$ are obtained. We present characteristic patterns of the modulus of these solutions in the $(x ; t)$ plane, in function of the different parameters.


PACS: 35Q55, 37K10.
Keywords : NLS equation, Peregrine breather, rogue waves.

## 1 Introduction

From the first quasi-rational solution to NLS equation constructed in 1983 by Peregrine [3], a lot of works have carried out. Akhmediev, Eleonskii and Kulagin obtained the first higher order analogue of the Peregrine breather [4, 5] in 1986 and construted also other families of higher order 3 and 4 were constructed in a series of articles $[6,7]$, using Darboux transformations.
Since the years 2010, many works were published using various methods, in
particular in 2013, it was found in [8] solution in terms of wronskians and in [9], solutions expressed in terms of determinants of order $2 N$ depending on $2 N-2$ real parameters. A new representation has been found as a ratio of a determinant of order $N+1$ by another one of order $N$ by Ling and Zhao in [10]. Very recently in 2014, another approach have been given in [11] using a dressing method where the solutions are expressed as the quotient of a determinant of order $N+1$ by another one of order $N$.

We construct here new solutions to the focusing one dimensional nonlinear Schrödinger equation which appear as deformations of the (analogue) Peregrine breather of order 13 with 24 real parameters. These solutions are completely expressed as a ratio of two polynomials of degree 182 in $x$ and $t$ by an exponential depending on $t$. We cannot give the solutions in terms of polynomials of $x$ and $t$ in this text because of their lengths; only plots in the $(x ; t)$ plane of the modulus of the solutions to analyze the evolution of the solutions in function of the different parameters are presented in details here.

## 2 Solutions to NLS equation as a ratio of two determinants

We recall the following result which have been proved in $[12,13]$ :
Theorem 2.1 The function $v$ defined by

$$
v(x, t)=\frac{\operatorname{det}\left(\left(n_{\left.j k)_{j, k \in[1,2 N]}\right)}\right)\right.}{\operatorname{det}\left(\left(d_{j k)_{j, k \in[1,2 N]}}\right)\right.} e^{(2 i t-i \varphi)}
$$

is a quasi-rational solution to the NLS equation

$$
i v_{t}+v_{x x}+2|v|^{2} v=0
$$

where

$$
\begin{aligned}
& n_{j 1}=f_{j, 1}(x, t, 0), \quad n_{j k}=\frac{\partial^{2 k-2} f_{j, 1}}{\partial \epsilon^{2 k-2}}(x, t, 0), \\
& n_{j N+1}=f_{j, N+1}(x, t, 0), \quad n_{j N+k}=\frac{\partial^{2 k-2} f_{j, N+1}}{\partial \epsilon^{2 k-2}}(x, t, 0), \\
& d_{j 1}=g_{j, 1}(x, t, 0), \quad d_{j k}=\frac{\partial^{2 k-2} g_{j, 1}}{\partial \epsilon^{2 k-2}}(x, t, 0), \\
& d_{j N+1}=g_{j, N+1}(x, t, 0), \quad d_{j N+k}=\frac{\partial^{2 k-2} g_{j, N+1}}{\partial \epsilon^{2 k-2}}(x, t, 0), \\
& 2 \leq k \leq N, 1 \leq j \leq 2 N
\end{aligned}
$$

The functions $f$ and $g$ are defined for $1 \leq k \leq N$ by :

$$
\begin{align*}
& f_{4 j+1, k}=\gamma_{k}^{4 j-1} \sin A_{k}, \quad f_{4 j+2, k}=\gamma_{k}^{4 j} \cos A_{k}, \\
& f_{4 j+3, k}=-\gamma_{k}^{4 j+1} \sin A_{k}, \quad f_{4 j+4, k}=-\gamma_{k}^{4 j+2} \cos A_{k}, \\
& f_{4 j+1, N+k}=\gamma_{k}^{2 N-4 j-2} \cos A_{N+k}, \quad f_{4 j+2, N+k}=-\gamma_{k}^{2 N-4 j-3} \sin A_{N+k}, \\
& f_{4 j+3, N+k}=-\gamma_{k}^{2 N-4 j-4} \cos A_{N+k}, \quad f_{4 j+4, k}=\gamma_{k}^{2 N-4 j-5} \sin A_{N+k},  \tag{1}\\
& g_{4 j+1, k}=\gamma_{k}^{4 j-1} \sin B_{k}, \quad g_{4 j+2, k}=\gamma_{k}^{4 j} \cos B_{k}, \\
& g_{4 j+3, k}=-\gamma_{k}^{4 j+1} \sin B_{k}, \quad g_{4 j+4, k}=-\gamma_{k}^{4 j+2} \cos B_{k}, \\
& g_{4 j+1, N+k}=\gamma_{k}^{2 N-4 j-2} \cos B_{N+k}, \quad g_{4 j+2, N+k}=-\gamma_{k}^{2 N-4 j-3} \sin B_{N+k}, \\
& g_{4 j+3, N+k}=-\gamma_{k}^{2 N-4 j-4} \cos B_{N+k}, \quad g_{4 j+4, N+k}=\gamma_{k}^{2 N-4 j-5} \sin B_{N+k},
\end{align*}
$$

The arguments $A_{\nu}$ and $B_{\nu}$ of these functions are given for $1 \leq \nu \leq 2 N$ by

$$
A_{\nu}=\kappa_{\nu} x / 2+i \delta_{\nu} t-i x_{3, \nu} / 2-i e_{\nu} / 2, \quad B_{\nu}=\kappa_{\nu} x / 2+i \delta_{\nu} t-i x_{1, \nu} / 2-i e_{\nu} / 2
$$

The terms $\kappa_{\nu}, \delta_{\nu}, \gamma_{\nu}$ are defined by $1 \leq \nu \leq 2 N$

$$
\begin{align*}
& \kappa_{j}=2 \sqrt{1-\lambda_{j}^{2}}, \quad \delta_{j}=\kappa_{j} \lambda_{j}, \quad \gamma_{j}=\sqrt{\frac{1-\lambda_{j}}{1+\lambda_{j}}}  \tag{2}\\
& \kappa_{N+j}=\kappa_{j}, \quad \delta_{N+j}=-\delta_{j}, \quad \gamma_{N+j}=1 / \gamma_{j}, \quad 1 \leq j \leq N
\end{align*}
$$

where $\lambda_{j}$ are given for $1 \leq j \leq N$ by :

$$
\begin{equation*}
\lambda_{j}=1-2 j^{2} \epsilon^{2}, \lambda_{N+j}=-\lambda_{j} . \tag{3}
\end{equation*}
$$

The terms $x_{r, \nu}(r=3,1)$ are defined for $1 \leq \nu \leq 2 N$ by :

$$
\begin{equation*}
x_{r, \nu}=(r-1) \ln \frac{\gamma_{\nu}-i}{\gamma_{\nu}+i} . \tag{4}
\end{equation*}
$$

The parameters $e_{\nu}$ are given by

$$
\begin{align*}
& e_{j}=i \sum_{k=1}^{N-1} a_{j} \epsilon^{2 k+1} j^{2 k+1}-\sum_{k=1}^{N-1} b_{j} \epsilon^{2 k+1} j^{2 k+1} \\
& e_{N+j}=i \sum_{k=1}^{N-1} a_{j} \epsilon^{2 k+1} j^{2 k+1}+\sum_{k=1}^{N-1} b_{j} \epsilon^{2 k+1} j^{2 k+1}, \quad 1 \leq j \leq N \tag{5}
\end{align*}
$$

## 3 Solutions of order 13 with twenty four parameters

We construct here deformations of the Peregrine breather $P_{13}$ of order 13 depending on 24 parameters. We cannot give the analytic expression of the solution to NLS equation of order 13 with twenty four parameters because of their lengths. For simplicity, we denote

$$
d_{3}:=\operatorname{det}\left(\left(n_{j k)_{j, k \in[1,2 N]}}\right), \quad d_{1}:=\operatorname{det}\left(\left(d_{j k)_{j, k \in[1,2 N]}}\right)\right.\right.
$$

In this study, one reaches the current limits of capacity of calculation; all the explicit expressions were calculated for the parameters $a_{i}$ and $b_{i}$ for indices $i$
from 2 to 12. The only expression which was not calculated is that corresponding to $i$ equal to 1 : the calculation of the expression needing 32 core stopped at the end of 6 months.
On the other hand, one could generate the corresponding images in this case $i$ equal to 1 without calculating the determinants explicitly.
Calculations are the more long as indices $i$ are close to 1 . Precisely for $a_{12}$, calculations last approximately 1 hour on 20 cores; for $a_{2}$, it is approximately about 3 months on 32 cores.

The number of terms of the polynomials of the numerator $d_{3}$ and denominator $d_{1}$ of the solutions are shown in the table below (Table 1) when other $a_{i}$ and $b_{i}$ are set to 0 .

| $a_{i}$ | $d_{3}$ | $d_{1}$ |
| :---: | :---: | :---: |
| 2 | 1051741 | 531675 |
| 3 | 559480 | 282841 |
| 4 | 348403 | 176084 |
| 5 | 245676 | 124198 |
| 6 | 183131 | 92581 |
| 7 | 143053 | 72324 |
| 8 | 115599 | 58448 |
| 9 | 94974 | 48014 |
| 10 | 76243 | 38529 |
| 11 | 55015 | 27787 |
| 12 | 30003 | 15154 |

Table 1: Number of terms for the polynomials $d_{3}$ and $d_{1}$ of the solutions to the NLS equation in the case $N=13$.

We construct figures to show deformations of the Peregrine breather of order 13. We get different types of symmetries in the plots in the $(x, t)$ plane. We give some examples of this fact in the following discussion.

### 3.1 Peregrine breather of order 13

If we choose $a_{i}=b_{i}=0$ for $1 \leq i \leq 10$, we obtain the classical thirteenth Peregrine breather (here, the peak going until 27 has been truncated)


Figure 1: Solution to NLS, $\mathrm{N}=13$, all parameters equal to 0 , Peregrine breather $P_{13}$.

### 3.2 Variation of parameters

With other choices of parameters, we obtain all types of configurations : triangles and multiple concentric rings configurations with a maximum of 91 peaks. The configuration of the solutions of the NLS equation in the $(x ; t)$ plane being the same for $a_{j}$ or $b_{j}$, we restrict ourself to present the solutions only in the case of the parameters $a_{j}$, for $1 \leq j \leq 12$. We present figures of deformations with only one parameter non equal to zero, all other parameters are fixed equal to zero. For more accuracy, one presents in the following only the views of top.


Figure 2: Solution to NLS, $\mathrm{N}=13, a_{1}=10^{3}$ : triangle with 91 peaks, sight from top.


Figure 3: Solution to NLS, $\mathrm{N}=13, a_{2}=10^{4}: 11$ rings with $5,10,10,10,10,10$, $10,5,5,10,5$ peaks and one peak in the center, sight from top.


Figure 4: Solution to NLS, N=13, $a_{3}=10^{6}: 10$ rings with $7,14,7,7,14,14$, $7,7,7,7$, peaks, sight from top.


Figure 5: Solution to NLS, N=13, $a_{4}=10^{8}: 5$ rings with $9,18,18,18,18$, peaks, in the center $P_{4}$, sight from top.


Figure 6: Solution to NLS, $\mathrm{N}=13, a_{5}=10^{9}: 7$ rings of $11,11,22,11,11,11$, 11 peaks one peak in he center $P_{2}$, sight from top.


Figure 7: Solution to NLS, N=13, $a_{6}=10^{12}: 7$ rings with 13 peaks without peak in the center, sight of top.


Figure 8: Solution to NLS, $\mathrm{N}=13, a_{7}=10^{14}: 6$ rings with 15 peaks and in the center one peak, sight from top.


Figure 9: Solution to NLS, $\mathrm{N}=13, a_{8}=10^{15}: 5$ rings with 17 peaks and in the center the Peregrine breather of order 3 , sight from top.


Figure 10: Solution to NLS, $\mathrm{N}=13, a_{9}=10^{19}: 4$ rings with 19 peaks and in the center the Peregrine breather of order 5 , sight from top.


Figure 11: Solution to NLS, N=13, $a_{10}=10^{20}$ : three rings with 21 peaks and in the center the Peregrine breather of order 7 , sight from top.


Figure 12: Solution to NLS, $\mathrm{N}=13, a_{11}=10^{20}$ : two rings with 23 peaks and in the center the Peregrine breather of order 9 , sight from top.


Figure 13: Solution to NLS, $N=13, a_{12}=10^{20}$ : one ring with 25 peaks and in the center the Peregrine breather of order 11, sight from top.

## 4 Conclusion

We have given here an explicit construction of multi-parametric solutions to the NLS equation of order 13 with 24 real parameters. We cannot give the explicit representation in terms of polynomials of degree 182 in $x$ and $t$ because of his length. The properties about the shape of the breather in the $(x, t)$ coordinates, the maximum of amplitude equal to $2 N+1$ and the degree of polynomials in $x$ and $t$ here equal to $N(N+1)$ are checked.
We obtained different patterns in the $(x ; t)$ plane by different choices of these parameters. So we obtain a classification of the rogue waves at order 13.
We obtain two types of patterns : the triangles and the concentric rings for the same index $i$ for $a_{i}$ or $b_{i}$ non equals to 0 .
In the cases $a_{1} \neq 0$ or $b_{1} \neq 0$ we obtain triangles with a maximum of 91 peaks; for $a_{2} \neq 0$ or $b_{2} \neq 0$, we have rings with 5 or 10 peaks. For $a_{3} \neq 0$ or $b_{3} \neq 0$, we obtain rings with 7 or 14 peaks with in the center $P_{2}$. For $a_{4} \neq 0$ or $b_{4} \neq 0$, we have 5 rings with $9,18,18,19,18$, peaks with in the center the Peregrine $P_{4}$. For $a_{5} \neq 0$ or $b_{5} \neq 0$, we have 7 rings of $11,11,22,11,11,11,11$ peaks with in the center $P_{3}$. For $a_{6} \neq 0$ or $b_{6} \neq 0$, we have 7 rings with 13 peaks on each of them without peak in the center. For $a_{7} \neq 0$ or $b_{7} \neq 0$, we have 6 rings with 15 peaks on each of them and in the center the Peregrine breather of order 1 (one peak). For $a_{8} \neq 0$ or $b_{8} \neq 0$, we have 5 rings with 17 peaks on each of them and in the center the Peregrine breather of order 3 . For $a_{9} \neq 0$ or $b_{9} \neq 0$, we have 4 rings with 19 peaks and in the center the Peregrine breather of order 5. For $a_{10} \neq 0$ or $b_{10} \neq 0$, we have 3 rings with 21 peaks and in the center the Peregrine breather of order 7 . For $a_{11} \neq 0$ or $b_{11} \neq 0$, we have only two rings with 23 peaks and in the center the Peregrine breather of order 9. At least, for $a_{12} \neq 0$ or $b_{12} \neq 0$, we have only one ring with 25 peaks and in the center the Peregrine breather of order 11.
We verify in this study the following conjectures:
at order $N$, for $a_{1} \neq 0$ or $b_{1} \neq 0$, the modulus of the solution to the NLS equation presents the configuration of a triangle with $N(N+1) / 2$ peaks; this has been formulated already by different authors, in particular Akhmediev and Matveev.
at order $N$, in the case $1 \leq i \leq\left[\frac{N}{2}\right]$, for $a_{N-i} \neq 0$ or $b_{N-i} \neq 0$, the modulus of the solution to he NLS equation presents $i$ concentric rings with $2 N-2 i+1$ peaks and in the center the $P_{N-2 i}$ breather; this conjecture was given for this time by the authors of this paper in 2015.
at order $N$, in the case $\left[\frac{N}{2}\right]<i \leq N-2$, for $a_{N-i} \neq 0$ or $b_{N-i} \neq 0$, the modulus of the solution to the NLS equation presents $n_{k}$ rings of $k(2 N-2 i+1)$ peaks, for $\left.1 \leq k \leq r, r n_{r}(2 N-2 i+1)<N(N+1) / 2\right)$ with
in the center the $P_{N-2 i}$ breather, verifying

$$
\sum_{n_{k}=1}^{r} n_{k} k(2 N-2 i+1)=2 i N-2 i^{2}+i
$$

This conjecture was also given for this time by the authors of this paper in 2015.

It would be relevant to prove the preceding conjectures.

## References

[1] V. E. Zakharov, Stability of periodic waves of finite amplitude on a surface of a deep fluid, J. Appl. Tech. Phys, V. 9, 86-94, (1968)
[2] V. E. Zakharov, A.B. Shabat Exact theory of two dimensional self focusing and one dimensinal self modulation of waves in nonlinear media, Sov. Phys. JETP, V. 34, 62-69, (1972)
[3] D. Peregrine, Water waves, nonlinear Schrödinger equations and their solutions, J. Austral. Math. Soc. Ser. B, V. 25, 16-43, (1983)
[4] N. Akhmediev, V. Eleonski, N. Kulagin, Generation of periodic trains of picosecond pulses in an optical fiber : exact solutions, Sov. Phys. J.E.T.P., V. 62, 894-899, (1985)
[5] N. Akhmediev, V. Eleonski, N. Kulagin, Exact first order solutions of the nonlinear Schrödinger equation, Th. Math. Phys., V. 72, N. 2, 183-196, (1987)
[6] N. Akhmediev, A. Ankiewicz, J.M. Soto-Crespo, Rogue waves and rational solutions of nonlinear Schrödinger equation, Physical Review E, V. 80, N. 026601, (2009)
[7] N. Akhmediev, A. Ankiewicz, P.A. Clarkson, Rogue waves, rational solutions, the patterns of their zeros and integral relations, J. Phys. A : Math. Theor., V. 43, 122002-1-9, (2010)
[8] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. 44, 1-15, (2011)
[9] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, Jour. Of Math. Phys., V. 54, 013504-1-32, (2013)
[10] L. Ling, L.C. Zhao, Trajectory Characters of rogue waves, arXiv : 1305.5599v1, 24 May, (2013)
[11] A.A. Gelash, V.E. Zakharov, Superregulier solitonic solutions : a novel scenario for the nonlinear stage of modulation instability, Non linearity, V. 27, 1-39, (2014)
[12] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, Journal of Math. Sciences : Advances and Applications, V. 13, N. 2, 71-153, (2012)
[13] P. Gaillard, Multi-parametric deformations of the Peregrine breather of order N solutions to the NLS equation and multi-rogue waves, Adv. Res., V. 4, 346-364, 2015
[14] P. Gaillard, Other 2N-2 parameters solutions to the NLS equation and $2 \mathrm{~N}+1$ highest amplitude of the modulus of the N-th order AP breather, J. Phys. A: Math. Theor., V. 48, 145203-1-23, 2015
[15] M. Gastineau, J. Laskar, TRIP 1.3.8 : TRIP Reference manual, IMCCE, Paris Observatory, 2014, http://www.imcce.fr/trip/
[16] P. Gaillard, V.B. Matveev, Wronskian addition formula and its applications, Max-Planck-Institut für Mathematik, MPI 02-31, V. 161, 2002
[17] P. Gaillard, A new family of deformations of Darboux-Pöschl-Teller potentials, Lett. Math. Phys., V. 68, 77-90, 2004
[18] P. Gaillard, V.B. Matveev, New formulas for the eigenfunctions of the two-particle Calogero-Moser system, Lett. Math. Phys., V. 89, 1-12, 2009
[19] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, RIMS Kyoto, N. 1653, 1-19, 2009
[20] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, J. Phys A : Math. Theor., V. 42, 1-16, 2009
[21] P. Gaillard, Wronskian representation of solutions of NLS equation and seventh order rogue waves, J. Mod. Phys., V. 4, N. 4, 246-266, 2013
[22] P. Gaillard, V.B. Matveev, Wronskian addition formula and Darboux-Pöschl-Teller potentials, J. Math., V. 2013, ID 645752, 1-10, 2013
[23] P. Gaillard, Two parameters deformations of ninth Peregrine breather solution of the NLS equation and multi rogue waves, J. Math., V. 2013, 1-111, 2013
[24] P. Gaillard, Two-parameters determinant representation of seventh order rogue waves solutions of the NLS equation, J. Theor. Appl. Phys., V. 7, N. 45, 1-6, 2013
[25] P. Gaillard, Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation, J. Math. Phys., V. 54, 073519-1-22, 2013
[26] P. Gaillard, Deformations of third order Peregrine breather solutions of the NLS equation with four parameters, Phys. Rev. E, V. 88, 042903-1-9, 2013
[27] P. Gaillard, Ten parameters deformations of the sixth order Peregrine breather solutions of the NLS equation, Phys. Scripta, V. 89, 015004-1-7, 2014
[28] P. Gaillard, The fifth order Peregrine breather and its eight-parameters deformations solutions of the NLS equation, Commun. Theor. Phys., V. 61, 365-369, 2014
[29] P. Gaillard, Higher order Peregrine breathers, their deformations and multirogue waves, J. Of Phys. : Conf. Ser., V. 482, 012016-1-7, 2014
[30] P. Gaillard, M. Gastineau, Eighteen parameter deformations of the Peregrine breather of order ten solutions of the NLS equation, Int. J. Mod. Phys. C, V. 26, N. 2, 1550016-1-14, 2014
[31] P. Gaillard, Two parameters wronskian representation of solutions of nonlinear Schrödinger equation, eight Peregrine breather and multi-rogue waves, J. Math. Phys., V. 5, 093506-1-12, 2014
[32] P. Gaillard, Hierarchy of solutions to the NLS equation and multi-rogue waves, J. Phys. : Conf. Ser., V. 574, 012031-1-5, 2015
[33] P. Gaillard, Tenth Peregrine breather solution of the NLS, Ann. Phys., V. 355, 293-298, 2015
[34] P. Gaillard, M. Gastineau, The Peregrine breather of order nine and its deformations with sixteen parameters solutions of the NLS equation, Phys. Lett. A., V. 379, 13091313, 2015
[35] P. Gaillard, Higher order Peregrine breathers solutions to the NLS equation, Jour. Phys. : Conf. Ser., V. 633, 012106-1-6, 2016
[36] P. Gaillard, M. Gastineau Patterns of deformations of Peregrine breather of order 3 and 4, solutions to the NLS equation with multi-parameters, Journal of Theoretical and Applied Physics, V. 10,1-7, 2016
[37] P. Gaillard, M. Gastineau Twenty parameters families of solutions to the NLS equation and the eleventh Peregrine breather, Commun. Theor. Phys, V. 65, 136-144, 2016
[38] P. Gaillard, Rational solutions to the KPI equation and multi rogue waves, Annals Of Physics, V. 367, 1-5, 2016
[39] P. Gaillard, M. Gastineau Twenty two parameters deformations of the twelfth Peregrine breather solutions to the NLS equation, Adv. Res., V., 1-11, 2016
[40] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, Theor. And Math. Phys., V. 189, 1440-1449, 2016
[41] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, Jour. of Math. Phys., V. 57, 063505-1-13, doi: 10.1063/1.4953383, 2016

