

3D X-RAY COMPUTED TOMOGRAPHY RECONSTRUCTION USING SPARSITY ENFORCING HIERARCHICAL MODEL BASED ON HAAR TRANSFORMATION

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Abstract

In this paper, we consider the 3D X-ray CT reconstruction problem by using the Bayesian approach with a hierarchical prior model. A generalized Student-t distributed prior model is used to enforce the sparse structure of the multilevel Haar Transformation of the image. Comparisons with some state of the art methods are presented, showing that the proposed method gives more accurate reconstruction results and a faster convergence. Simulation results are also provided to show the effectiveness of the proposed hierarchical model for a reconstruction with more limited projections.

Computed Tomography

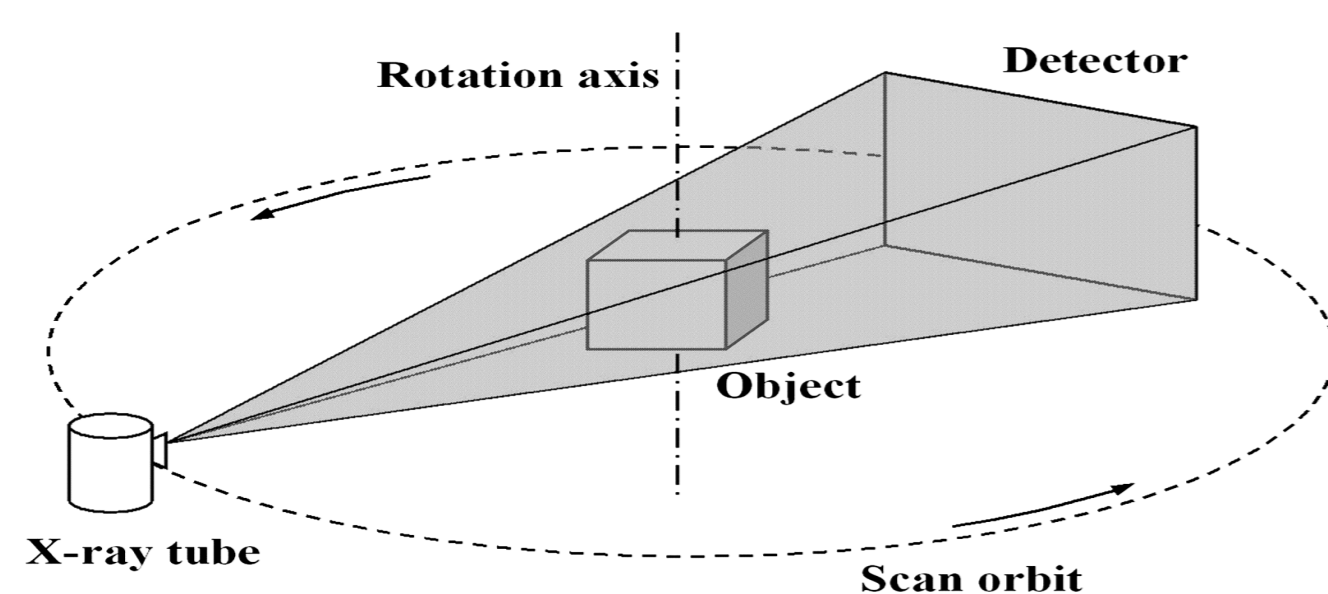


Figure: X-ray Computed Tomography.

Discrete forward model:

$$g = Hf + \epsilon \quad (1)$$

where ϵ represents the noise vector, and matrix H corresponds to the projection operator.

Mathematical Model:

(a) Projection: $g = Hf$

(b) Back-projection: $f = H^t g$

(c) Filtered Back-projection: $f = H^t(H^t H)^{-1} g$

Regularization methods:

$$J(f) = \|g - Hf\|^2 + \lambda R(f)$$

$$R(f) = \|Df\|_2^2 \Rightarrow \text{QR}$$

$$R(f) = \|f\|_1 \Rightarrow \text{LASSO}$$

$$R(f) = \|Df\|_1 \Rightarrow \text{TV}$$

JMAP and VBA Algorithm

Joint Posterior

$$\begin{aligned} \rho(f, z, v_z, v_\epsilon, v_\xi | g) \\ \propto \rho(g|f, v_\epsilon) \rho(f|z, v_\xi) \rho(z|v_z) \rho(v_z|\alpha_{z_0}, \beta_{z_0}) \cdot \\ \rho(v_\epsilon|\alpha_{\epsilon_0}, \beta_{\epsilon_0}) \rho(v_\xi|\alpha_{\xi_0}, \beta_{\xi_0}) \end{aligned}$$

JMAP Method

$$(\hat{f}, \hat{z}, \hat{v}_z, \hat{v}_\epsilon, \hat{v}_\xi) = \arg \max_{f, z, v_z, v_\epsilon, v_\xi} \rho(f, z, v_z, v_\epsilon, v_\xi | g) \quad (5)$$

Optimization

$$\text{iter} : \hat{f}^{(k+1)} = \hat{f}^{(k)} - \hat{\gamma}_f^{(k)} \nabla \mathcal{J}(\hat{f}^{(k)});$$

$$\text{iter} : \hat{z}^{(k+1)} = \hat{z}^{(k)} - \hat{\gamma}_z^{(k)} \nabla \mathcal{J}(\hat{z}^{(k)});$$

$$\hat{v}_{z_j} = \left(\beta_{z_0} + \frac{1}{2} \hat{z}_j^2 \right) / (\alpha_{z_0} + 3/2);$$

$$\hat{v}_{\epsilon_i} = \left(\beta_{\epsilon_0} + \frac{1}{2} (g_i - [H\hat{f}]_i)^2 \right) / (\alpha_{\epsilon_0} + 3/2);$$

$$\hat{v}_{\xi_j} = \left(\beta_{\xi_0} + \frac{1}{2} (\hat{f}_j - [D\hat{z}]_j)^2 \right) / (\alpha_{\xi_0} + 3/2), \quad (6)$$

$\forall j \in [1, M]$ and $\forall i \in [1, N]$, where

$$\mathcal{J}(f) = \frac{1}{2} \|V_\epsilon^{-1/2} (g - Hf)\|^2 + \frac{1}{2} \|V_\xi^{-1/2} (f - Dz)\|^2;$$

$$\mathcal{J}(z) = \frac{1}{2} \|V_\xi^{-1/2} (f - Dz)\|^2 + \frac{1}{2} \|V_z^{-1/2} z\|^2, \quad (7)$$

Numerical Results

Relative Means Square Error (RMSE):

$$\delta_f = \frac{\|\hat{f} - f\|^2}{\|f\|^2}$$

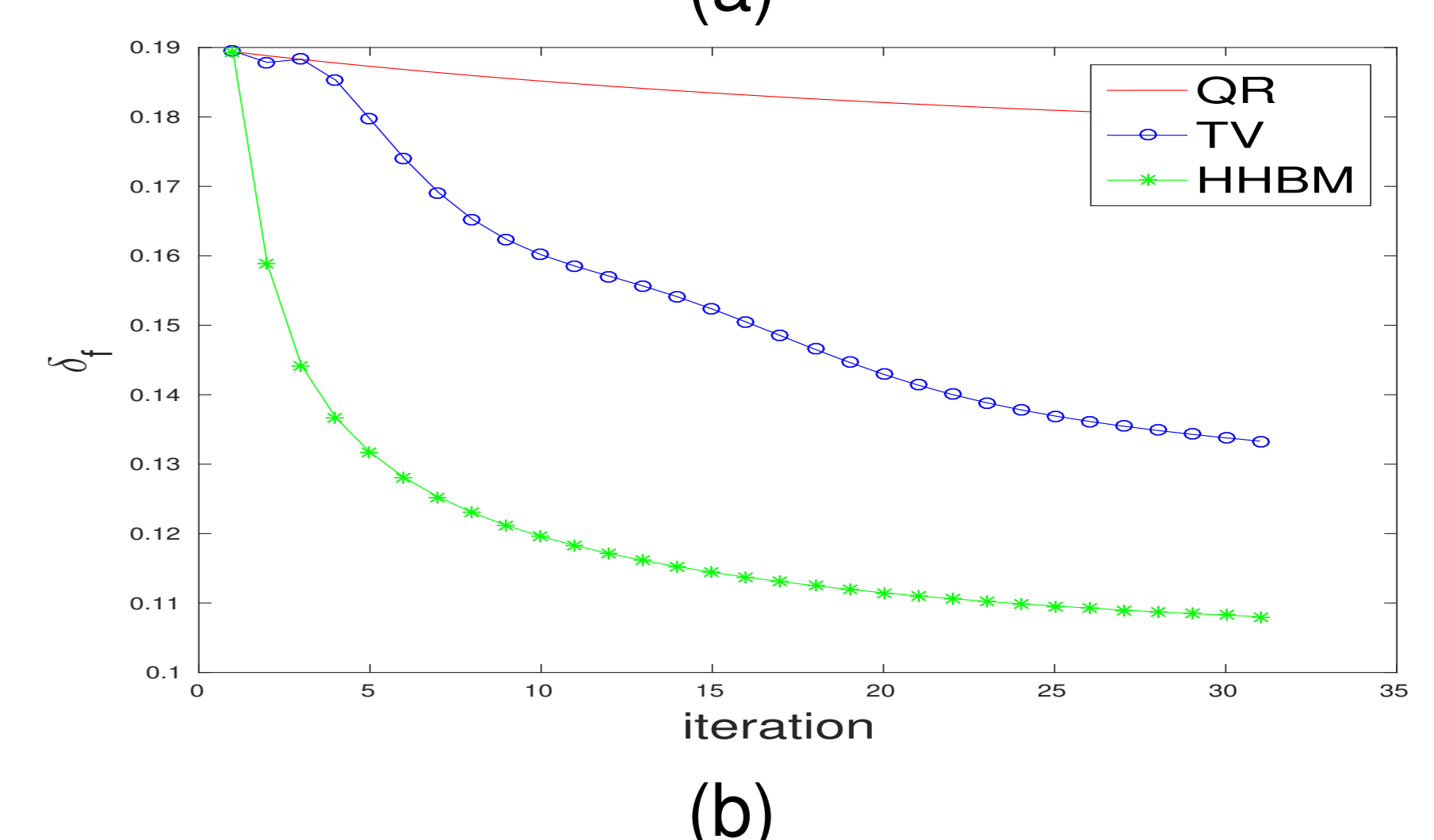
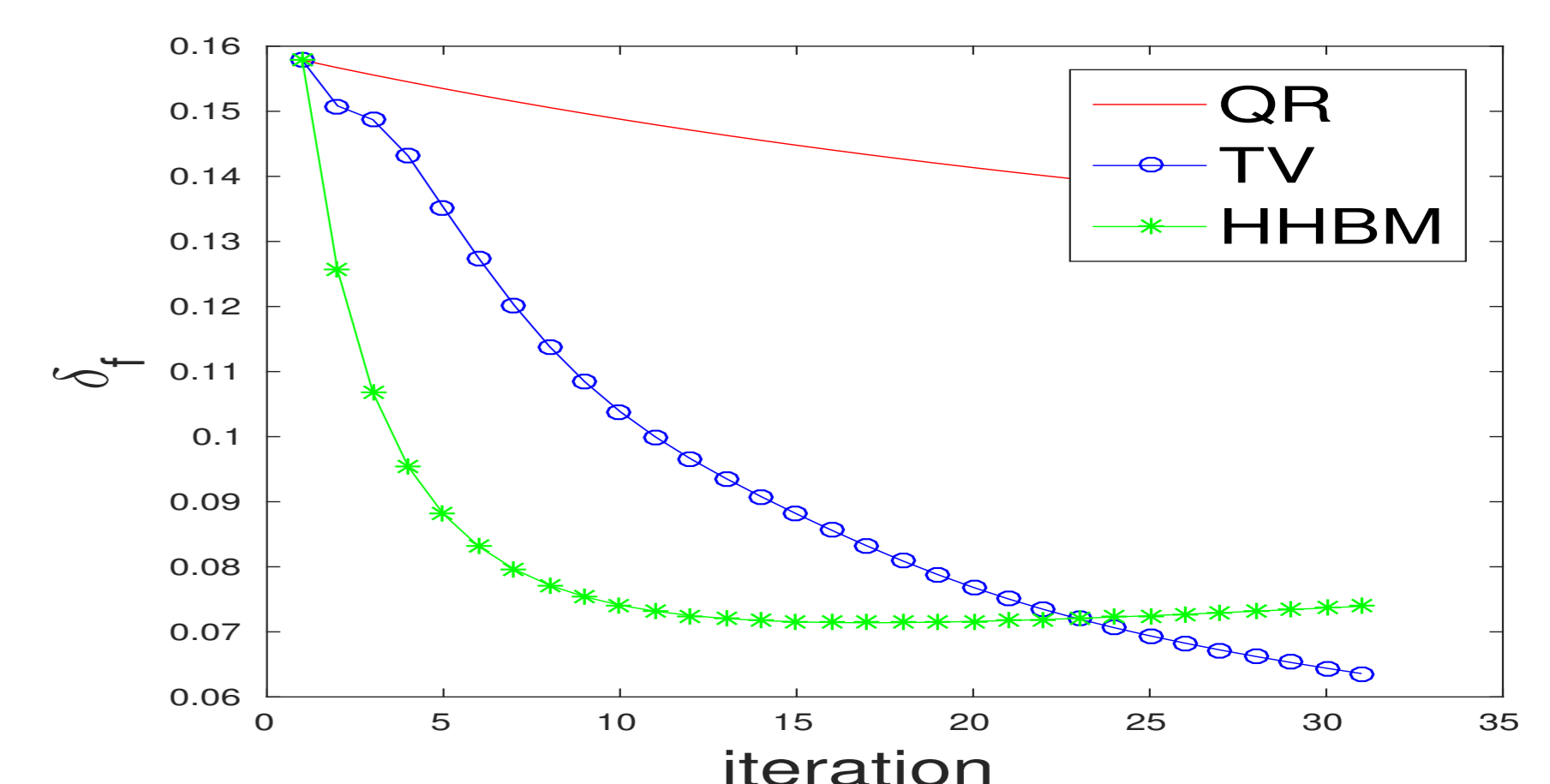


Figure: Comparison of NMSE of reconstructed Shepp Logan phantom of size 64^3 by using different methods with respectively 64 and 32 projections and a low SNR=20dB.

Bayesian Approach

Bayes rule

$$\rho(f|g) = \frac{\rho(g|f)\rho(f)}{\rho(g)} \propto \rho(g|f)\rho(f) \quad (2)$$

Hierarchical model

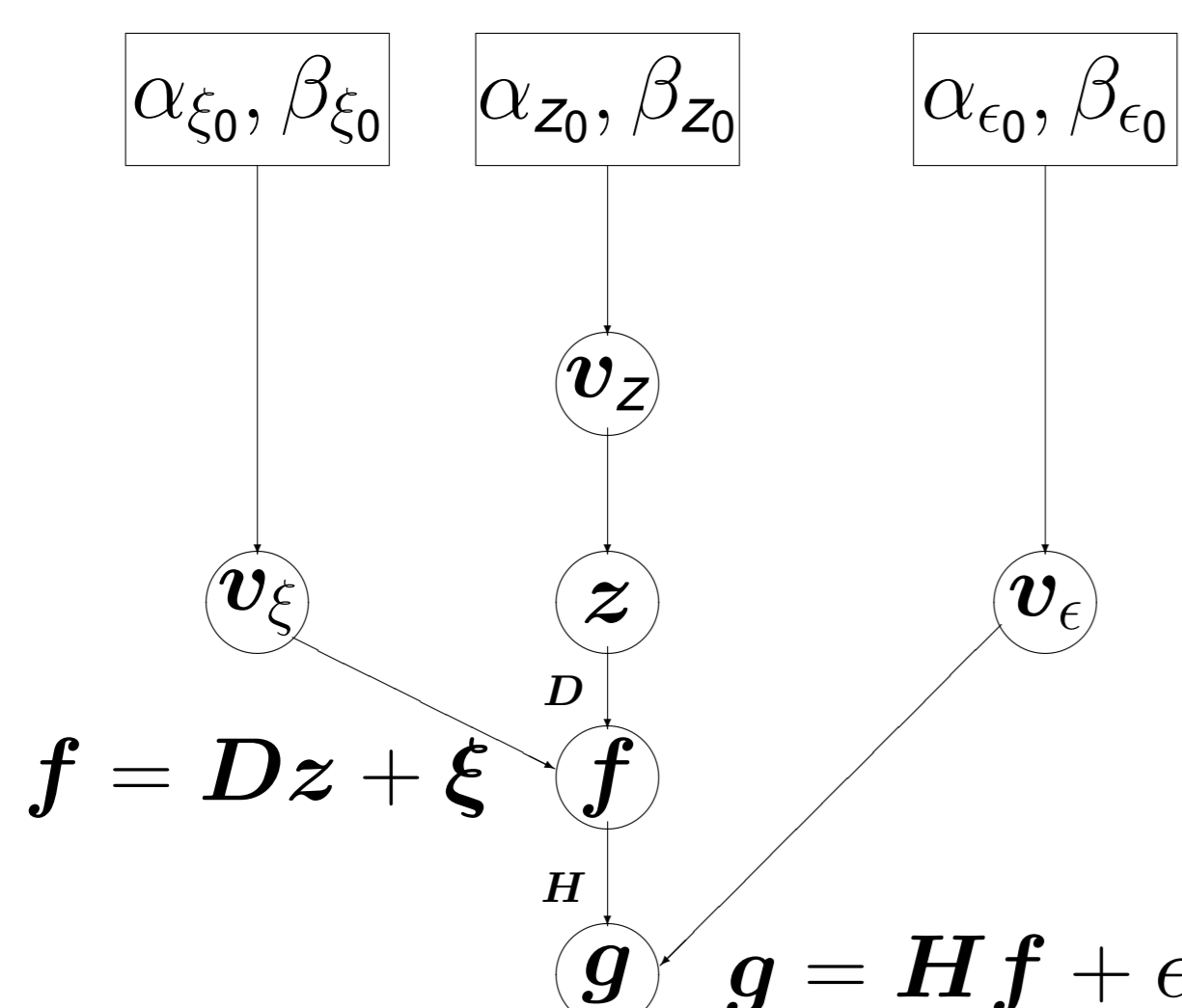


Figure: The Hierarchical Model.

z : multilevel Haar transformation coefficient.

Hierarchical Structure of Student-t Distribution

$$\begin{aligned} \rho(z|\alpha_{z_0}, \beta_{z_0}) &= St_g(z|\alpha_{z_0}, \beta_{z_0}) \\ &= \int_0^\infty \mathcal{N}(z|0, v_z) \mathcal{IG}(v_z|\alpha_{z_0}, \beta_{z_0}) dv_z \end{aligned} \quad (3)$$

Hierarchical Model

$$\begin{cases} \rho(g|f, v_\epsilon) = \mathcal{N}(g|Hf, V_\epsilon) \\ \rho(f|z, v_\xi) = \mathcal{N}(f|Dz, V_\xi) \\ \rho(z|v_z) = \mathcal{N}(z|0, V_z) \\ \rho(v_z|\alpha_{z_0}, \beta_{z_0}) = \mathcal{IG}(v_z|\alpha_{z_0}, \beta_{z_0}) \\ \rho(v_\epsilon|\alpha_{\epsilon_0}, \beta_{\epsilon_0}) = \mathcal{IG}(v_\epsilon|\alpha_{\epsilon_0}, \beta_{\epsilon_0}) \\ \rho(v_\xi|\alpha_{\xi_0}, \beta_{\xi_0}) = \mathcal{IG}(v_\xi|\alpha_{\xi_0}, \beta_{\xi_0}) \end{cases} \quad (4)$$

$V_i = \text{diag}[v_i]$ and $v_i = [v_{i_1}, \dots, v_{i_N}]$

Numerical Results

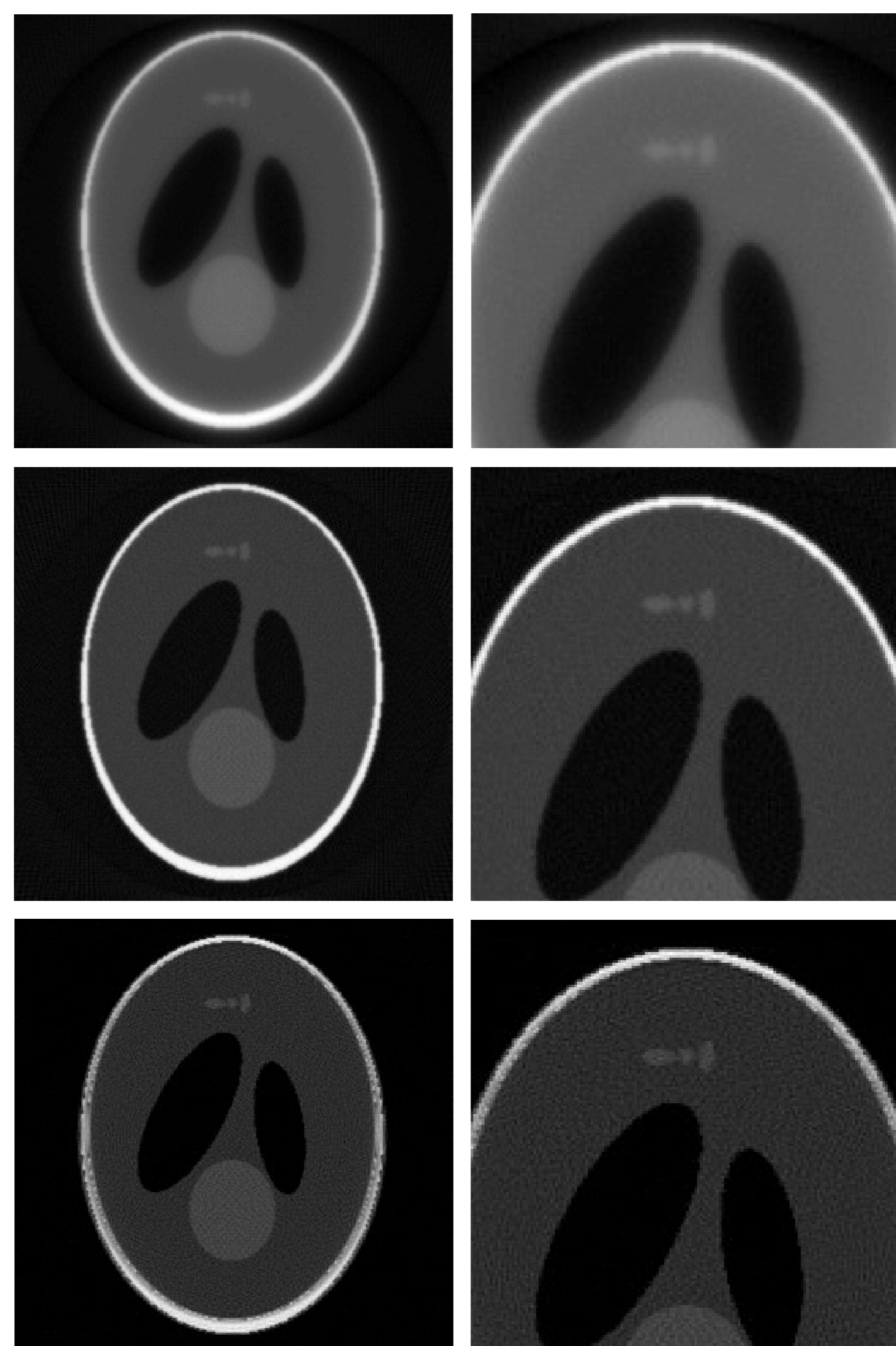


Figure: The middle slices and zones of the reconstructed 3D Shepp-Logan phantom by using different methods with 1/4 of total projections with noise of SNR=40dB. top: QR, middle: TV, bottom: HHBM.

Conclusions and Perspectives

- ◇ A three-level hierarchical model is proposed.
- ◇ The heavy-tailed Student-t distribution is used to define the prior model.
- ◇ Semi-supervised model. Parameters are estimated and hyper-parameters are easy to fix.
- ◇ Preserve edges.

[1] Ali Mohammad-Djafari, "Bayesian inference with hierarchical prior models for inverse problems in imaging systems," in Systems, Signal Processing and their Applications (WoSSPA), 2013 8th International Workshop on, IEEE, 2013, pp. 7-18.
[2] Dimitris G Tzikas, CL Likas, and Nikolaos P Galatsanos, "The variational approximation for bayesian inference," Signal Processing Magazine, IEEE, vol. 25, no. 6, pp. 131-146, 2008.