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# A Succinct Language for Dynamic Epistemic Logic (long version)

Tristan Charrier<sup>1</sup> and François Schwarzentruber<sup>2</sup>

<sup>1</sup>IRISA, 263 Avenue du General Leclerc - CS 74205, 35042 Rennes Cedex, France

<sup>2</sup>ENS Rennes, Avenue Robert Schuman, 35170 Bruz, France

## Abstract

Dynamic epistemic logic (DEL) is an extension of modal multi-agent epistemic logic with dynamic operators. We propose a succinct version of DEL where Kripke models and event models are described succinctly. Our proposal relies on Dynamic logic of propositional assignments (DLPA): epistemic relations are described with so-called accessibility programs written in DLPA. We give examples of models that are exponentially more succinct in our framework. Interestingly, the model checking of DEL is PSPACE-complete and we show that it remains in PSPACE for the succinct version.

## 1 Introduction

Agents take decisions according to their knowledge about the world and their knowledge about other agents' knowledge (higher-order knowledge). Dynamic epistemic logic ([4], [18]) is a framework designed for expressing higher-order knowledge properties and dynamics.

Actions in dynamic epistemic logic are described by means of graphs called *event models*. Specific kinds of actions have been considered in the literature. For instance, public announcements are represented by single-node event models.

Attention-based announcements [8] where some agents may listen to the source or not can be represented too by event models, but their sizes are exponential in the number of agents in the system.

However, we claim that this exponential blow-up in the representation is artificial and that is why we address succinctness in dynamic epistemic logic.

Usually, if the description language is (exponentially) more succinct, algorithmic problems become (exponentially) harder. For instance, deciding the existence of an Hamiltonian cycle is NP-complete but it becomes NEXPTIME-complete [14] when the input graph is described succinctly. A succinct representation of a graph with  $2^b$  nodes is a Boolean circuit  $C$  such that there is an edge  $(i, j) \in \{0, \dots, 2^b - 1\}^2$  iff  $C$  accepts the binary representations of the  $b$ -bit integers  $i, j$  as inputs.

In dynamic epistemic logic, the results are surprising. Whereas the model checking of DEL is PSPACE-complete [1], we would expect that the succinct version of the model checking is EXPSPACE-complete. Actually, we provide a framework where the representation of actions such as attention-based announcements is exponentially more succinct whereas the model checking remains in PSPACE.

In our framework, we do not use Boolean circuits traditionally used for representing instances for succinct decision problems (see [14], Chapter 20.1) but *accessibility programs* based on Dynamic Logic of Propositional Assignments (DLPA) ([3], [2]) as developed in [12], where they were called mental programs. The reason of that choice is that our model checking algorithm directly relies on DLPA. It extends the language of Van Benthem *et al.* [15] by including DLPA programs and postconditions for event models.

After having recalled some background on Dynamic epistemic logic in Section 2, the main contribution is to provide a succinct version of DEL in Section 3: we provide succinct models for DEL, prove that DEL can be embedded in succinct DEL (and *vice versa*), and prove that succinct DEL is exponentially more succinct than DEL. The succinctness of succinct Kripke models is rather easy to prove but the proof of the

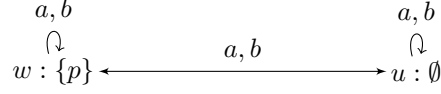


Figure 1: Example of an epistemic model

succinctness of succinct event models relies on the definition of *action emulations* [20]. In Section 4, we prove that the model checking problem in succinct DEL is (still) in PSPACE. More details may be found in [13].

## 2 Background on DEL

We first start by some notations about valuations, then present Dynamic epistemic logic (DEL). Let  $AP$  be a finite set of atomic propositions and  $Ag$  be a finite set of agents.

### 2.1 Valuations

The set of valuations over  $AP$  is denoted by  $\mathcal{V}(AP)$ . Valuations are denoted  $\mathfrak{w}, \mathfrak{u}, \dots$  and they are represented by the set of true atomic propositions. The restriction of  $\mathfrak{w}$  to a subset of propositions  $AP'$  is  $\mathfrak{w} \cap AP'$  and is noted  $\mathfrak{w}|_{AP'}$ . For any valuation  $\mathfrak{w}$  over  $AP$ , we note  $desc(\mathfrak{w})$  the formula  $\bigwedge_{p \in \mathfrak{w}} p \wedge \bigwedge_{p \in AP \setminus \mathfrak{w}} \neg p$  that describes the valuation  $\mathfrak{w}$  ( $AP$  is made implicit for simplifying the notation). For a set  $AP$ , we note  $\exists!(AP)$  the formula  $\bigvee_{p \in AP} (p \wedge \bigwedge_{q \in AP | q \neq p} \neg q)$ . The size of a valuation defined over  $AP$  is the cardinal of  $AP$ , noted  $|AP|$  (we implement such a valuation by a bit array of size  $AP$ ).

### 2.2 Epistemic models

Kripke models represent static epistemic situations and are defined as follows.

**Definition 1.** A Kripke model  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  is defined by a non-empty set  $W$  of epistemic states/worlds, epistemic relations  $(\rightarrow_a)_{a \in Ag} \subseteq W \times W$  and a valuation function  $V : W \rightarrow 2^{AP}$ .

Typically,  $W$  represent all possible configurations.  $V$  is a labeling for the states for describing the configuration. The intuitive meaning of  $w \rightarrow_a u$  is that agent  $a$  considers state  $u$  as possible when the actual state is  $w$ . We do not require  $\rightarrow_a$  to be an equivalence relation. The size of a Kripke model  $M$  is implemented by a labeled graph and each relation  $\rightarrow_a$  is represented by an adjacency list. The size of  $M$  is thus  $O(|Ag| \times |W|^2 + |W| \times |AP|)$ .

**Example 1.** Figure 1 depicts the model  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  given by  $W = \{w, u\}$ ,  $\rightarrow_a = \rightarrow_b = W \times W$ ,  $V(w) = \{p\}$  and  $V(u) = \emptyset$ . Agents  $a$  and  $b$  do not distinguish  $w$  from  $u$ , therefore they do not know the truth value of  $p$  (in both  $w$  and  $u$ ).

**Example 2.** We consider the extension of the classical muddy children puzzle [17] where children may pay attention to the father or not (as in [8]). We introduce atomic proposition  $m_a$  meaning that  $a$  is muddy and atomic proposition  $h_a$  meaning that agent  $a$  hears (i.e. pays attention to) the announcements of the father. We consider the model  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  where  $W = \mathcal{V}(\{m_a, h_a \mid a \in Ag\})$ ,  $\mathfrak{w} \rightarrow_a \mathfrak{u}$  if  $(\mathfrak{w} \models h_a \text{ iff } \mathfrak{u} \models h_a)$  and for all  $b \neq a$  ( $\mathfrak{w} \models m_b \text{ iff } \mathfrak{u} \models m_b$ ), and  $V(\mathfrak{w}) = \mathfrak{w}$  for all  $w \in W$ .

In  $M$ , an agent  $a$  will not distinguish two worlds  $\mathfrak{w}$  from  $\mathfrak{u}$  as long he sees the same forehead states for the other agents and his pay attention status is the same in both worlds.

A Kripke model represents the state of mind of agents. A pointed Kripke model is a pair  $(M, w)$  with  $w \in W$ , modeling the fact that the current epistemic state is  $w$ .

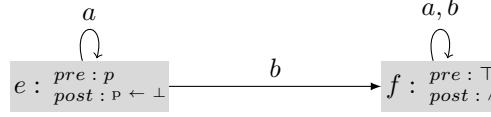


Figure 2: Example of an event model

### 2.3 Syntax of epistemic language

The language  $\mathcal{L}_{EL}$  extends the propositional language  $\mathcal{L}_{Prop}$  with modal operators  $K_a$  and is defined by the following BNF:  $\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid K_a\varphi$ , with  $p \in AP$ ,  $a \in Ag$ . The formula  $K_a\varphi$  is read “agent  $a$  knows that  $\varphi$  holds”. We define the usual abbreviations  $(\varphi_1 \wedge \varphi_2)$  for  $\neg(\neg\varphi_1 \vee \neg\varphi_2)$  and  $\bar{K}_a\varphi$  for  $\neg K_a\neg\varphi$ .

### 2.4 Event models

The dynamics of the system is modeled by event models.

**Definition 2.** An event model  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  is defined by a non-empty set of events  $E$ , epistemic relations  $(\rightarrow_a^\mathcal{E})_{a \in Ag} \subseteq E \times E$ , a precondition function  $pre : E \rightarrow \mathcal{L}_{EL}$  and a postcondition function  $post : E \times AP \rightarrow \mathcal{L}_{Prop}$ .

The precondition function defines whether a given event is executable or not. The postcondition updates the values of atomic propositions after executing an event  $e$ : the truth of  $p$  is assigned to the value  $post(e, p)$ .

**Remark 1.** Some authors ([19]) consider epistemic postcondition functions  $post : E \times AP \rightarrow \mathcal{L}_{EL}$ . Actually, it is always possible to compute an equivalent event model with a propositional postcondition function from an event model with an epistemic postcondition function. Such a transformation is given in [19] but is exponential. Fortunately, there exists a polynomial alternative transformation, stated in Proposition 5 in the Appendix.

A event model is *without postconditions* (i.e. it does not change physically the current state) when  $post(e, p) = p$  for all  $e \in E$  and all atomic propositions  $p$ , that is when  $post$  is *trivial*. In that case, we usually omit the  $post$  function and we write  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre)$ .

We introduce the notation  $e \in \mathcal{E}$  for  $e \in E$ . A pointed event model is a pair  $(\mathcal{E}, e)$  with  $e \in E$ . A multi-pointed event model is a pair  $(\mathcal{E}, E')$  with  $E' \subseteq E$ . The size of  $\mathcal{E}$  is similarly defined than the size of a Kripke model.

**Example 3.** Figure 2 shows the event model  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  where  $E = \{e, f\}$ ,  $\rightarrow_a^\mathcal{E} = \{(e, e), (f, f)\}$ ,  $\rightarrow_b^\mathcal{E} = \{(f, f), (e, f)\}$ ,  $pre(e) = p$ ,  $pre(f) = \top$ ,  $post(e, p) = \perp$  and  $post(f, p) = p$ .

**Example 4.** We focus on the notion of attention-based announcement of  $p$  as shown in [8]. In addition to classic atomic propositions, we add propositions  $h_a$  for “agent  $a$  is listening to the announcement”. The attention-based announcement of  $p$  can be then represented by the event model  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre)$  where:

- $E = \mathcal{V}(\{p\} \cup \{h_a \mid a \in Ag\}) \cup \{idle\}$ ;
- for all  $a$ ,  $\mathbf{e} \rightarrow_a^\mathcal{E} \mathbf{f}$  if  $\mathbf{e} \neq idle$ ,  $\mathbf{f} \neq idle$ ,  $\mathbf{e} \models h_a$ ,  $\mathbf{f} \models p$ ;  $\mathbf{e} \rightarrow_a^\mathcal{E} idle$  if  $\mathbf{e} \neq idle$  and  $\mathbf{e} \not\models h_a$ ;  $idle \rightarrow_a^\mathcal{E} idle$ ;
- $pre(e) = desc(e)$  if  $e \neq idle$ ,  $\top$  otherwise.

Figure 5 shows the event model of the attention-based announcement of  $p$  for two agents  $a_1$  and  $a_2$ .

Event  $idle$  is the event where nothing happens. The relation  $\rightarrow_a^\mathcal{E}$  is defined as follows: if  $a$  is listening ( $\mathbf{e} \models h_a$ ) then  $a$  believes that  $p$  has been announced ( $\mathbf{f} \models p$ ). If  $a$  is not listening ( $\mathbf{e} \not\models h_a$ ) then  $a$  believes nothing happens. The precondition is defined to match the fact that attentive agents listen to the

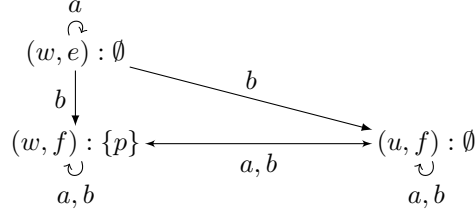


Figure 3: Example of a product

announcement of  $p$  (thus leading to events where  $p$  holds) and that other agents believe that nothing happened (thus the  $\top$  precondition on idle). The announcement is purely epistemic so the postcondition function is trivial.

## 2.5 Product

The update of a Kripke model  $M$  with an event model  $\mathcal{E}$  is defined by the synchronous product of both models, noted  $M \otimes \mathcal{E}$ , and defined as follows.

**Definition 3.** Let  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  be a Kripke model. Let  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  be an event model. The product of  $M$  and  $\mathcal{E}$  is  $M \otimes \mathcal{E} = (W', (\rightarrow_a)', V')$  where:

- $W' = \{(w, e) \in W \times E \mid M, w \models pre(e)\};$
- $(w, e) \rightarrow'_a (w', e')$  iff  $w \rightarrow_a w'$  and  $e \rightarrow_a^\mathcal{E} e'$ ;
- $V'((w, e)) = \{p \in AP \mid M, w \models post(e, p)\}.$

**Example 5.** Figure 3 shows the product of the Kripke model of Figure 1 and the event model of Figure 2.

## 2.6 Syntax

The language  $\mathcal{L}_{DEL}$  extends  $\mathcal{L}_{EL}$  and is defined by the following BNF.

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \vee \psi) \mid K_a\varphi \mid \langle \mathcal{E}, E' \rangle \varphi$$

with  $p \in AP$ ,  $a \in Ag$ . Formula  $\langle \mathcal{E}, E' \rangle \varphi$  is read “There exists an execution of  $(\mathcal{E}, E')$  such that  $\varphi$  holds”.

## 2.7 Semantics

We now define the semantics of a formula  $\varphi$  of  $\mathcal{L}_{DEL}$ .

**Definition 4.** We define  $M, w \models \varphi$  ( $\varphi$  is true in the pointed Kripke model  $M, w$ ) by induction on  $\varphi$ :

- $M, w \models p$  iff  $p \in V(w)$ ;  $M, w \models \neg\varphi$  iff  $M, w \not\models \varphi$ ;
- $M, w \models (\varphi \vee \psi)$  iff  $M, w \models \varphi$  or  $M, w \models \psi$ ;
- $M, w \models K_a\varphi$  iff for all  $u \in W$ ,  $w \rightarrow_a u$  implies  $M, u \models \varphi$ ;
- $M, w \models \langle \mathcal{E}, E' \rangle \varphi$  iff there exists  $e \in E'$  s.th.  $M, w \models pre(e)$  and  $M \otimes \mathcal{E}, (w, e) \models \varphi$ .

**Example 6.** We consider the Kripke model  $M$  of Figure 1 and the event model of Figure 2. As  $p \notin V(u)$  and  $w \rightarrow_a u$ , we have  $M, w \models \neg K_a p$ . As  $\neg p$  holds in all  $\rightarrow_a$ -successors of  $(w, e)$  (Figure 3),  $M \otimes \mathcal{E}, (w, e) \models K_a \neg p$ , we have  $M, w \models \langle \mathcal{E}, \{e\} \rangle K_a \neg p$ .

## 2.8 Bisimulation and equivalence

We define notions of equivalence for models.

### 2.8.1 Bisimulations for Kripke models

The usual notion of equivalence for Kripke models is bisimulation.

**Definition 5.** Let  $AP$  be a finite set of atomic propositions,  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  and  $M' = (W', (\rightarrow'_a)_{a \in Ag}, V')$  be two Kripke models. A relation  $B \subseteq W \times W'$  is a  $AP$ -bisimulation between  $M$  and  $M'$  iff for all  $w \in W, w' \in W'$  such that  $wBw'$ :

- *Invariance:*  $V(w)|_{AP} = V'(w')|_{AP}$ ;
- *Zig:* for all  $u \in W$  such that  $w \rightarrow_a u$  there exists  $u' \in W'$  such that  $w' \rightarrow'_a u'$  and  $uBu'$ ;
- *Zag:* for all  $u' \in W'$  such that  $w' \rightarrow'_a u'$  there exists  $u \in W$  such that  $w \rightarrow_a u$  and  $uBu'$ .

Two pointed Kripke models  $(M, w)$  and  $(M', w')$  are  $AP$ -bisimilar if there is a  $AP$ -bisimulation  $B$  between  $M$  and  $M'$  with  $wBw'$ .  $(M, w)$  and  $(M', w')$  are  $AP$ -bisimilar iff  $((M, w) \models \varphi$  iff  $(M', w') \models \varphi)$  for all formulas  $\varphi$  of  $\mathcal{L}_{DEL}$  [6]<sup>1</sup>, whose propositions are in  $AP$ . When  $AP$  is clear from the context, we say bisimilar instead of  $AP$ -bisimilar. Two Kripke models  $M$  and  $M'$  are bisimilar if there exists  $w \in W$  and  $w' \in W'$  such that  $(M, w)$  and  $(M', w')$  are bisimilar.

### 2.8.2 Equivalence of event models

For event models, the equivalence is defined as follows.

**Definition 6.** Let  $\mathcal{E}$  and  $\mathcal{E}'$  be two pointed event models. They are equivalent if for all pointed Kripke models  $(M, w)$ , for all  $e \in \mathcal{E}$ , there exists  $e' \in \mathcal{E}'$  such that  $(M \otimes \mathcal{E}, (w, e))$  and  $(M \otimes \mathcal{E}', (w, e'))$  are bisimilar (and vice versa).

Equivalence of event models without postconditions is characterized by *action emulations* ([20]) defined as follows.

**Definition 7.** Let  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre)$  and  $\mathcal{E}' = (E', (\rightarrow_a^{\mathcal{E}'})_{a \in Ag}, pre')$  be two event models without postconditions. Let  $\Sigma$  be the set of preconditions appearing in  $\mathcal{E}$  and  $\mathcal{E}'$ . Let  $\hat{\Sigma}$  be the set of formulas containing  $\Sigma$  and closed under sub-formulas and negation (see [20] for more details). Let  $CS(\hat{\Sigma})$  be the set of maximal consistent subsets of  $\hat{\Sigma}$ . An action emulation  $AE$  is a set of relations  $\{AE_\Gamma\}_{\Gamma \in CS(\hat{\Sigma})} \subseteq E \times E'$  such that whenever  $eAE_\Gamma e'$ :

- *Invariance:*  $pre(e) \in \Gamma$  and  $pre(e') \in \Gamma$ ;
- *Zig:* For all  $f \in E$  and  $\Gamma' \in CS(\hat{\Sigma})$ , if  $e \rightarrow_a^\mathcal{E} f$ ,  $pre(f) \in \Gamma'$  and the formula  $(\bigwedge_{\psi \in \Gamma} \psi \wedge \hat{K}_a \bigwedge_{\psi' \in \Gamma'} \psi')$  is consistent then there exists  $f' \in E'$  such that  $e' \rightarrow_a^{\mathcal{E}'} f'$  and  $fAE_{\Gamma'} f'$ ;
- *Zag:* symmetric of Zig for  $E'$ .

Action emulation is similar to bisimulation in the sense that the types of rules are the same, except that we ask of preconditions to be in a same maximal consistent subset of  $\hat{\Sigma}$ . Action emulation characterizes equivalence for event models without postconditions:

**Proposition 1.**  $\mathcal{E}$  and  $\mathcal{E}'$  are equivalent iff there is an action emulation  $AE$  between  $\mathcal{E}$  and  $\mathcal{E}'$  such that for all  $\Gamma \in CS(\hat{\Sigma})$  such that for all  $e \in \mathcal{E}$  with  $pre(e) \in \Gamma$ , there exists  $e' \in \mathcal{E}'$  such that  $eAE_\Gamma e'$  (and vice versa).

<sup>1</sup>Actually this result is shown for any modal logic, so the result for  $\mathcal{L}_{DEL}$  is a corollary.

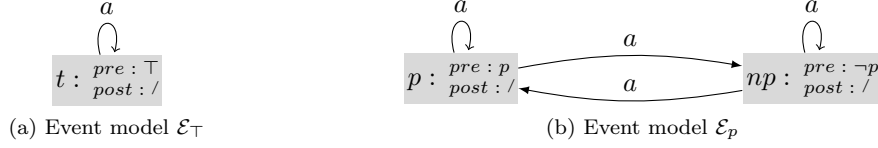


Figure 4: Two equivalent event models  $\mathcal{E}_\top$  and  $\mathcal{E}_p$

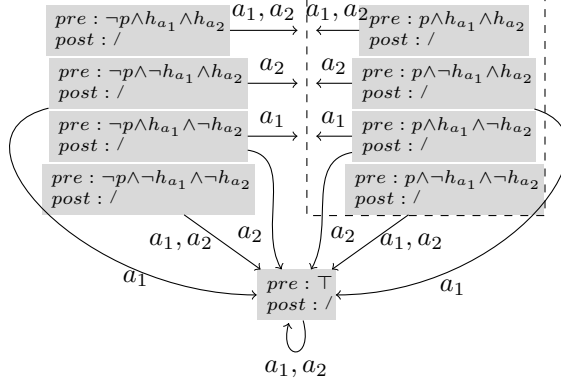


Figure 5: The event model  $\mathcal{A}_p$  corresponding to an attention-based announcement  $p$  for two agents  $a_1$  and  $a_2$ . The six edges pointing to the dashed box point to all four events in the box.

**Example 7.** Consider the event models  $\mathcal{E}_\top$  and  $\mathcal{E}_p$  in Figure 4. Here  $CS(\hat{\sigma}) = \{\Gamma_p, \Gamma_{\neg p}\}$  with  $\Gamma_p = \{\top, p\}$  and  $\Gamma_{\neg p} = \{\top, \neg p\}$ . The action emulation  $AE$  between  $\mathcal{E}_\top$  and  $\mathcal{E}_p$  is defined as  $tAE_{\Gamma_p}p$  and  $tAE_{\Gamma_{\neg p}}np$ . Let us verify that  $AE$  is an action emulation:

- *Invariance:*  $pre(t) = \top \in \Gamma_p, pre(t) = \top \in \Gamma_{\neg p}, pre(p) = p \in \Gamma_p, pre(np) = \neg p \in \Gamma_{\neg p}$ .
- *Zig:* For  $tAE_{\Gamma_p}p$ , the only possibility is  $t \xrightarrow{\mathcal{E}_\top} t$ .
  - $pre(t) \in \Gamma_p$  and the formula  $\top \wedge p \wedge \hat{K}_a(\top \wedge \neg p)$  is consistent. The event  $f'$  such that  $tAE_{\Gamma_p}f'$  and  $p \xrightarrow{\mathcal{E}_p} f'$  is  $f' = p$
  - Same reasoning for  $\Gamma_{\neg p}$ , except that  $f' = np$  in this case.

The reasoning is similar for  $tAE_{\Gamma_{\neg p}}np$ .

- *Zag:* in all cases,  $e' = t$  is the chosen state.

Therefore,  $\mathcal{E}_\top$  and  $\mathcal{E}_p$  are equivalent.

## 2.9 Model checking

The model checking problem takes as input a pointed Kripke model  $M, w$ , a formula  $\varphi$  of  $\mathcal{L}_{DEL}$  and returns yes if  $M, w \models \varphi$ ; no otherwise. Sets  $AP$  and  $Ag$  are implicitly part of the input: the number of atomic propositions and the number of agents is unbounded and specified by  $M, w, \varphi$ .

**Theorem 1.** ([1]) The model checking problem is PSPACE-complete<sup>2</sup>.

<sup>2</sup>Actually, hardness was proven in [1] for a language with non-deterministic choice  $\cup$  in the language. For a proof without such a constraint, see [7].

### 3 Succinct DEL

In classical DEL Kripke models and event models are represented explicitly. It faces some practical limits when sizes of models are exponential in the number of atomic propositions or the number of agents (see Examples 2 and 4). In this section, we provide a symbolic succinct representation for both Kripke and event models. It relies on *accessibility programs*, presented in Subsection 3.1, written in a PDL-dialect called Dynamic logic of propositional assignments (DLPA).

In Subsections 3.2 and 3.3, we define respectively succinct Kripke models and succinct event models. For both cases, we give the *semantics* (how Kripke/event models are computed from their succinct representations), the *expressiveness* (all standard DEL models can be represented in our succinct version) and the *succinctness* (some succinct models are strictly more succinct than standard DEL). In next Subsections, we present a succinct representation of the product update and the language of succinct DEL.

#### 3.1 Language of accessibility programs

An *accessibility program*, simply called *program*, succinctly describes a relation between valuations. We write  $u \xrightarrow{\pi} v$  for “ $v$  is a successor of  $u$  by  $\pi$ ”. The syntax for accessibility programs is defined by the following BNF.

$$\pi ::= p \leftarrow \beta \mid \beta? \mid (\pi; \pi) \mid (\pi \cup \pi) \mid (\pi \cap \pi) \mid \pi^{-1}$$

where  $p \in AP$ ,  $\beta$  is a Boolean formula. Program  $p \leftarrow \beta$  is read “assign atomic proposition  $p$  to the truth value of  $\beta$ ”. Program  $\beta?$  is read “test  $\beta$ ”. Program  $\pi_1; \pi_2$  is read “execute  $\pi_1$  then  $\pi_2$ ”. Program  $(\pi_1 \cup \pi_2)$  is read “either execute  $\pi_1$  or  $\pi_2$ ”. Program  $(\pi_1 \cap \pi_2)$  is the intersection of  $\pi_1$  and  $\pi_2$ . Program  $\pi^{-1}$  is the inverse of  $\pi$ . We write  $set(p_1, \dots, p_n) = (p_1 \leftarrow \perp \cup p_1 \leftarrow \top); \dots; (p_n \leftarrow \perp \cup p_n \leftarrow \top)$  for the program setting arbitrary values to  $p_1, \dots, p_n$ .

The semantics of accessibility programs is defined by induction as follows:

- $w \xrightarrow{p \leftarrow \beta} u$  iff  $(u = w \setminus \{p\} \text{ and } w \not\models \beta) \text{ or } (u = w \cup \{p\} \text{ and } w \models \beta)$ ;
- $w \xrightarrow{\beta?} u$  iff  $w = u$  and  $w \models \beta?$ ;
- $w \xrightarrow{\pi_1; \pi_2} u$  iff there exists  $v$  s. th.  $w \xrightarrow{\pi_1} v$  and  $v \xrightarrow{\pi_2} u$ ;
- $w \xrightarrow{\pi_1 \cup \pi_2} u$  iff  $w \xrightarrow{\pi_1} u$  or  $w \xrightarrow{\pi_2} u$ ;
- $w \xrightarrow{\pi_1 \cap \pi_2} u$  iff  $w \xrightarrow{\pi_1} u$  and  $w \xrightarrow{\pi_2} u$ ;
- $w \xrightarrow{\pi^{-1}} u$  iff  $u \xrightarrow{\pi} w$ .

The size of an accessibility program corresponds to the number of operators needed to write the program. For instance, the accessibility program  $(p \leftarrow \top) \cup (q?; p \leftarrow \perp)$  has size 10. The models are succinctly described by means of accessibility programs. We are interested here in describing Kripke models, event models and the product update.

#### 3.2 Succinct Kripke models

We first define the succinct representation of Kripke models, then show how to extract a Kripke model from a succinct one and *vice versa* and finally we give an example where the succinct representation is indeed strictly more succinct.

### 3.2.1 Definition

**Definition 8.** A succinct Kripke model is a tuple  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  where  $AP_M$  is a finite set of atomic propositions,  $\beta_M$  is a Boolean formula over  $AP_M$ , and  $\pi_a$  is a program over  $AP_M$  for each agent  $a$ .

The Boolean formula  $\beta_M$  succinctly describes the set of epistemic states. Intuitively, each  $\pi_a$  succinctly describes the accessibility relation  $\rightarrow_a$  for an agent  $a$ . A pointed succinct Kripke model is a pair  $\mathfrak{M}, \mathbf{w}$  with  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  is a succinct Kripke model and  $\mathbf{w}$  is a valuation satisfying  $\beta_M$ .

### 3.2.2 From succinct Kripke models to Kripke models

We define the explicit Kripke model  $\hat{M}(\mathfrak{M})$  associated to the succinct Kripke model  $\mathfrak{M}$ : the set of worlds is the set of valuations satisfying  $\beta_M$  and the epistemic relation  $\rightarrow_a$  is the relation described by  $\pi_a$ .

**Definition 9.** Given a succinct Kripke model  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$ , the Kripke model represented by  $\mathfrak{M}$ , noted  $\hat{M}(\mathfrak{M})$  is the model  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  where:

- $W = \{\mathbf{w} \in \mathcal{V}(AP_M) \mid \mathbf{w} \models \beta_M\}$ ;
- $\rightarrow_a = \{(\mathbf{w}, \mathbf{u}) \in W^2 \mid \mathbf{w} \xrightarrow{\pi_a} \mathbf{u}\}$ ;
- $V(\mathbf{w}) = \mathbf{w}$ .

### 3.2.3 From Kripke models to succinct models

We define a succinct Kripke model  $\mathfrak{M}_M$  representing the Kripke model  $M$  with respect to a set of propositions  $AP$ .

**Definition 10.** Let  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  be a Kripke model. We define the succinct Kripke model  $\mathfrak{M}_M = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  where:

- $AP_M = AP \cup \{p_w \mid w \in W\}$ ;
- $\beta_M = \exists!(\{p_w \mid w \in W\}) \wedge \bigwedge_{w \in W} p_w \rightarrow \text{desc}(V(w))$ ;
- $\pi_a = \bigcup_{w \rightarrow_a u} p_w?; \text{set}(AP_M); p_u?$ .

The intended meaning of the fresh atomic propositions  $p_w$  is to designate the world  $w$  (as *nominals* in hybrid logic [5]). Formula  $\beta_M$  describes the set  $W$  and the valuation  $V$ . Program  $\pi_a$  performs a non-deterministic choice over edges  $w \rightarrow_a u$  and then simulate the transition  $w \rightarrow_a u$ . The following proposition states that  $\mathfrak{M}_M$  indeed represents  $M$ .

**Proposition 2.**  $(\hat{M}(\mathfrak{M}_M), \{p_w\} \cup V(w))$  and  $(M, w)$  are  $AP$ -bisimilar.

*Proof.* We note  $M = (W, (\rightarrow_a)_{a \in Ag}, V)$  and  $\hat{M}(\mathfrak{M}_M) = (W', (\rightarrow'_a)_{a \in Ag}, V')$ . We define the relation  $B := \{(u, p_u \cup V(u)) \mid u \in W\}$ . Let us prove that  $B$  is a  $AP$ -bisimulation. Invariance of  $B$  is routine. For Zig, consider  $w \in W$ . For all  $u \in W$ , if  $w \rightarrow_a u$  then we can verify that  $\hat{M}(\mathfrak{M}_M), u \cup V(u) \models \beta_M$  and that  $p_w \cup V(w) \xrightarrow{\pi_a} p_u \cup V(u)$  so we have  $w \rightarrow'_a u$ . The Zag property is symmetrical.  $\square$

**Example 8.** The Kripke model  $M$  from Figure 1 is modeled by the succinct Kripke model  $\mathfrak{M}_M = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  with  $AP_M = \{p, p_w, p_u\}$ ,  $\beta_M = \exists!(\{p_u, p_w\}) \wedge (p_w \rightarrow p) \wedge (p_u \rightarrow \neg p)$  and  $\pi_a = \bigcup_{v_1, v_2 \in W} p_{v_1}?; \text{set}(AP_M); p_{v_2}?$ .

**Example 9.** We consider the Russian cards puzzle [16]. Three agents, Alice, Bob and Crow have respectively in their hands three cards, three cards and one card<sup>3</sup>. The cards range from 1 to 7. The goal of Alice and Bob is to share their hands by performing public announcements such that Crow does not know any of their cards. It is forbidden for Alice and Bob to announce something they do not know.

The Kripke model associated to this problem is defined on the set of atomic propositions  $AP = \{p_{a,i}, a \in Ag, i \in \{1, \dots, 7\}\}$  with  $Ag = \{Alice, Bob, Crow\}$ . The proposition  $p_{a,i}$  stands for “Agent  $a$  own card  $i$ ”.

One succinct Kripke model for this puzzle is  $\mathfrak{M} = \langle AP, \beta, (\pi_a)_{a \in Ag} \rangle$ . To define  $\beta$  we define the notation  $\exists^3(S)$  for “Exactly 3 atomic propositions are true in  $S$ ”. We have

$$\beta = \exists^3(\{p_{Alice,i}, i \in \{1, \dots, 7\}\}) \wedge \exists^3(\{p_{Bob,i}, i \in \{1, \dots, 7\}\}) \wedge \exists!(\{p_{Crow,i}, i \in \{1, \dots, 7\}\}) \\ \wedge \bigwedge_{i=1}^7 \exists!(\{p_{Alice,i}, p_{Bob,i}, p_{Crow,i}\})$$

Formula  $\beta$  states that each player has the right number of cards and each card is in exactly one hand. The accessibility program for agent  $a$  is  $\pi_a = set(\bigcup_{i \in \{1, \dots, 7\}} \bigcup_{b \in Ag \setminus a} \{p_{a,i}\})$ . It means that agent  $a$  considers as possible any combination of hands for the other players.

### 3.2.4 Succinctness of succinct Kripke models

To show the succinctness of succinct Kripke models, we describe a family of Kripke models having exponentially more compact representations with succinct Kripke models.

Let us consider the family of models  $M$  given in Example 2 for all number of agents  $n$ . The Kripke model  $M$  is succinctly represented by the succinct Kripke model  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  defined by  $\beta_M = \top$ ,  $AP_M = \{h_a, m_a \mid a \in Ag\}$ ,  $\pi_a = set(\{m_a\} \cup \{h_b \mid b \neq a\})$ .

Formula  $\beta_M = \top$  means that the set of possible worlds is the set of *all* valuations. Program  $\pi_a$  changes propositions agent  $a$  is uncertain of ( $m_a$  and  $h_b$  for all  $b \neq a$ ) while the truth values of other propositions remain unchanged. Pointed Kripke models  $M, \mathbf{w}$  and  $\hat{M}(\mathfrak{M}), \mathbf{w}$  are bisimilar. The number of worlds in  $M$  is  $2^{2^n}$  while the size of  $\mathfrak{M}$  is  $O(n^2)$  (each program  $\pi_a$  is of size  $O(n)$ ).

Furthermore, there is no bisimilar Kripke model  $M'$  with less worlds than  $M$  since all valuations appear once in  $M$ .

## 3.3 Succinct event models

We adopt a similar method for succinct event models.

### 3.3.1 Definition

**Definition 11.** A succinct event model is a tuple  $\mathfrak{E} = \langle AP_M, AP_E, \chi_E, (\pi_{a,E})_{a \in Ag}, \text{post} \rangle$  where:

- $AP_M$  and  $AP_E$  are two finite disjoint sets of atomic propositions;
- $\chi_E$  is a formula of  $\mathcal{L}_{EL}$  over  $AP_M \cup AP_E$  where atomic propositions of  $AP_E$  are not under the scope of a modal operator  $K_a$ ;
- $\pi_{a,E}$  is a program over  $AP_M \cup AP_E$  for all  $a \in Ag$ ;
- $\text{post}$  is a program over  $AP_M \cup AP_E$ .

$AP_M$  is the set of propositions used to describe preconditions while  $AP_E$  are new fresh propositions to potentially encode distinct events with the same precondition. The formula  $\chi_E$  describes the set of events and their preconditions. Each  $\pi_{a,E}$  corresponds to the symbolic representation of an accessibility relation  $\rightarrow_a^E$  for an agent  $a$ .  $\text{post}$  encodes the postcondition function.

<sup>3</sup>The problem can be extended to  $n, m, k$  cards [16].

### 3.3.2 From succinct event models to event models

Let  $sub(\chi_E)$  be the union of  $AP_M$  and the set of sub-formulas of  $\chi_E$  of the form  $K_a\psi$  not under the scope of another  $K_{a'}$  operator.

**Example 10.** If  $AP_M = \{p, q\}$  and  $AP_E = \{p_e, p_f\}$  then  $sub((-p_f \wedge K_a \neg K_b p) \vee K_a q) = \{p, q, K_a \neg K_b p, K_a q\}$ .

**Definition 12.** Given a succinct event model

$\mathfrak{E} = \langle AP_M, AP_E, \chi_E, (\pi_{a,E})_{a \in Ag}, \mathbf{post} \rangle$ , the event model represented by  $\mathfrak{E}$ , noted  $\hat{\mathcal{E}}(\mathfrak{E})$  is the model  $(E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  where

- $E = \{(\mathbf{v}_{pre}, \mathbf{v}_{post}) \in \mathcal{V}(sub(\chi_E) \cup AP_E) \times \mathcal{V}(AP_M) \text{ s.th. } \mathbf{v}_{pre}|_{AP_M \cup AP_E} \xrightarrow{\mathbf{post}} \mathbf{v}_{post} \cup (\mathbf{v}_{pre}|_{AP_E}) \text{ and } \mathbf{v}_{pre} \models \chi_E\}$ ;
- $\rightarrow_a^\mathcal{E} = \{((\mathbf{v}_{pre}, \mathbf{v}_{post}), (\mathbf{v}_{pre}', \mathbf{v}_{post}')) \in E^2 \text{ s.th. } \mathbf{v}_{pre} \xrightarrow{\pi_{a,E}; set(sub(\chi_E))} \mathbf{v}_{pre}'\}$ ;
- $pre((\mathbf{v}_{pre}, \mathbf{v}_{post})) = desc(\mathbf{v}_{pre}|_{sub(\chi_E)})$ ;
- $post((\mathbf{v}_{pre}, \mathbf{v}_{post}), p) = \begin{cases} \top & \text{if } p \in \mathbf{v}_{post} \\ \perp & \text{otherwise.} \end{cases}$

In an event  $e = (\mathbf{v}_{pre}, \mathbf{v}_{post})$ ,  $\mathbf{v}_{pre}$  represents the valuation before the execution of  $e$  (it encodes the precondition and the truth value of propositions in  $AP_E$ ).  $\mathbf{v}_{post}$  represents the valuation after the execution of  $e$  (it takes into account the effect of the postconditions). The relation  $\rightarrow_a^\mathcal{E}$  is defined with the program  $\pi_{a,E}$  but we also change arbitrarily propositions of  $sub(\chi_E)$ . The precondition of an event  $e = (\mathbf{v}_{pre}, \mathbf{v}_{post})$  is the description of  $\mathbf{v}_{pre}$  restricted to  $sub(\chi_E)$ . For instance, if  $\mathbf{v}_{pre} = \{p, p_e, K_a r\}$  then  $pre((\mathbf{v}_{pre}, \mathbf{v}_{post})) = p \wedge K_a r$ . The postcondition is inferred from  $\mathbf{v}_{post}$ .

### 3.3.3 From event models to succinct event models

We define a succinct event model  $\mathfrak{E}_\mathcal{E}$  representing the event model  $\mathcal{E}$ .

**Definition 13.** Let  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  be an event model. We define the succinct event model  $\mathfrak{E}_\mathcal{E} = \langle AP_M, AP_E, \chi_E, (\pi_{a,E})_{a \in Ag}, \mathbf{post} \rangle$  where

- $AP_M$  is a superset of propositions appearing in  $pre$ ;
- $AP_E = \{p_e \mid e \in E\}$ ;
- $\chi_E = \exists!(AP_E) \wedge \bigwedge_{e \in E} (p_e \rightarrow pre(e))$ ;
- $\pi_{a,E} = \bigcup_{e \rightarrow_a^\mathcal{E} f} p_e?; set(AP_M \cup AP_E); p_f?$ ;
- $\mathbf{post} = \bigcup_{e \in E} p_e?; \left( \bigcap_{p \in AP_M} p \leftarrow post(e, p) \right)$ .

The fresh atomic proposition  $p_e$  designates event  $e$ . Formula  $\chi_E$  describes the set  $E$  and the precondition  $pre$ . Program  $\pi_{a,E}$  performs a non-deterministic choice in the same spirit of  $\pi_a$  in Definition 10. Program  $\mathbf{post}$  non-deterministically chooses the current event  $e$  and applies the postcondition assignments in parallel. The following proposition states that  $\mathfrak{E}_\mathcal{E}$  indeed represents  $\mathcal{E}$ .

**Proposition 3.**  $\hat{\mathcal{E}}(\mathfrak{E}_\mathcal{E})$  and  $\mathcal{E}$  are equivalent.

*Proof.* Let  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  and  $\hat{\mathcal{E}}(\mathfrak{E}_\mathcal{E}) = (E', (\rightarrow_a^{\mathcal{E}'})_{a \in Ag}, pre', post')$ . Let  $M = (W, (\rightarrow_a^M)_{a \in Ag}, V)$  be a Kripke model. Let  $B$  be the set of tuples  $((w, e), (w, e'))$  with  $w \in W$ ,  $e \in E$  and  $e' = (\mathbf{v}_{pre}^e, \mathbf{v}_{post}^e) \in E'$  such that  $\mathbf{v}_{pre}^e = \{p_e\} \cup \mathbf{v}_e$  with  $\mathbf{v}_e \subseteq sub(\chi_E)$ ,  $V(w) \subseteq \mathbf{v}_e$ ,  $\mathbf{v}_e \models pre(e)$  and  $\mathbf{v}_{post}^e = V(w, e)$ .  $W$ . Such an  $e'$  is well defined by Definitions 12 and 13. We prove that  $B$  is a bisimulation.

We have  $V'((w, e')) = V((w, e))$  by Definition 12 so invariance holds. For Zig, if  $(w, e) \rightarrow_a^{M \otimes \mathcal{E}} (u, f)$  then  $M, u \models \text{pre}(f)$ . Therefore there exists  $\mathbf{v}_f \subseteq \text{sub}(\chi_E)$  such that  $V(u) \subseteq \mathbf{v}_f$ ,  $\mathbf{v}_f \models \text{pre}(f)$  and  $\{p_f\} \cup \mathbf{v}_f \models \chi_E$ . By definition of **post**, we have  $V(u) \cup \{p_f\} \xrightarrow{\text{post}} V((u, f)) \cup \{p_f\}$ . We deduce that  $f' = (\{p_f\} \cup \mathbf{v}_f, V((u, f))) \in E'$ . We have  $w \rightarrow_a^M u$  so  $\{p_e\} \cup \mathbf{v}_e \xrightarrow{\pi_{a,E}; \text{set}(\text{sub}(\chi_E))} \{p_f\} \cup \mathbf{v}_f$ . We conclude that  $(u, f) \rightarrow_a^{E'} (u, f')$ . The Zag property is proven similarly.  $\square$

**Example 11.** *The event model  $\mathcal{E}$  of Figure 2 is modeled by the succinct event model  $\mathfrak{E}_{\mathcal{E}} = \langle AP_M, AP_E, \chi_E, (\pi_{a,E})_{a \in Ag}, \text{post} \rangle$  with  $AP_M = \{p, p_w, p_u\}$ ,  $AP_E = \{p_e, p_f\}$ ,  $\chi_E = \exists!(AP_E) \wedge (p_e \rightarrow p) \wedge (p_f \rightarrow \top)$ ,  $\pi_{a,E} = \bigcup_{e_1 \rightarrow_a e_2} p_{e_1}?$ ;  $\text{set}(AP_M \cup AP_E)$ ;  $p_{e_2}?$  and  $\text{post} = (p_e?; p \leftarrow \perp) \cup (p_f?)$  (trivial postcondition counterpart in **post** has been omitted).*

### 3.3.4 Succinctness of succinct event models

As for succinct Kripke models, we provide a family of event models having exponentially more compact representations with succinct event models.

Let us consider the family of models  $(\mathcal{E}_n)_{n \in \mathbb{N}}$  given in Example 4 for all number of agents  $n$ . The event model  $\mathcal{E}$  is succinctly represented by the succinct event model  $\mathfrak{E} = \langle AP_M, AP_E, \chi_E, (\pi_{a,E})_{a \in Ag}, \text{post} \rangle$  defined by

- $\chi_E = \top$ ;
- $\pi_{a,E} = (\neg p_{idle}?: (h_a?: p \leftarrow \top; \text{set}(\{h_b, b \in Ag\})) \cup (\neg h_a?: \text{set}(AP_M); p_{idle} \leftarrow \top)) \cup (p_{idle}?: \text{set}(AP_M))$ ;
- $\text{post} = \top?$ .

Atomic proposition  $p_{idle}$  intuitively means that the event *idle* is occurring (at the bottom in Figure 5). Formula  $\chi_E = \top$  means that the set of possible events is unconstrained. Program  $\pi_{a,E}$  works as follows: if  $p_{idle}$  is false, if  $h_a$  is true, assign  $\top$  to  $p$  and arbitrarily change  $h_b$  for all  $b \neq a$ ; if  $h_a$  is false, change valuations of propositions in  $AP_M$  and set  $p_{idle}$  to true; otherwise if  $p_{idle}$  is true, change all truth values of propositions in  $AP_M$ . The number of worlds in  $\mathcal{E}$  is  $2^{n+1} + 1$  while the size of  $\mathfrak{E}$  is  $O(n^2)$  (each program  $\pi_{a,E}$  is of size  $O(n)$ ).

Now we prove that standard event models cannot represent  $\mathcal{E}_n$  as succinctly as succinct event models.

**Theorem 2.** *There is no propositional event model  $\mathcal{E}'_n$  equivalent to  $\mathcal{E}_n$  with less than  $2^n$  events.*

*Proof.* By contradiction, we use the characterization of Proposition 1. We suppose that there  $\mathcal{E}'_n$  has less than  $2^n$  events, and that there is an action emulation  $AE$  between  $\mathcal{E}_n$  and  $\mathcal{E}'_n$ . Let  $\Sigma$  be the set of preconditions of  $\mathcal{E}_n$  and  $\mathcal{E}'_n$ . Note that  $\hat{\Sigma}$  (defined as in Definition 7) is a set of propositional formulas.  $\mathcal{E}'_n$  has less than  $2^n$  events, so there exists  $\mathbf{e}_1, \mathbf{e}_2 \in \mathcal{V}(\{p\} \cup \{h_a \mid a \in Ag\})$  with  $\mathbf{e}_1 \models p$  and  $\mathbf{e}_2 \models p$  and  $\mathbf{e}_1 \neq \mathbf{e}_2$ , and there exists an event  $e'$  of  $\mathcal{E}'_n$ ,  $\Gamma_1, \Gamma_2$  such that  $\mathbf{e}_1 AE_{\Gamma_1} e'$  and  $\mathbf{e}_2 AE_{\Gamma_2} e'$ . As  $\mathbf{e}_1 \neq \mathbf{e}_2$ , there is an agent  $a$  such that  $\mathbf{e}_1 \models \neg h_a$  and  $\mathbf{e}_2 \models h_a$  (we swap  $\mathbf{e}_1$  and  $\mathbf{e}_2$  if  $\mathbf{e}_1 \models h_a$  and  $\mathbf{e}_2 \models \neg h_a$ ). Then  $\mathbf{e}_1 \rightarrow_a^{\mathcal{E}'_n} \text{idle}$ . We consider the maximal consistent subset  $\Gamma' = \{\varphi \in \hat{\Sigma} \mid \{h_a, a \in Ag\} \models \varphi\}$ . We have  $\text{pre}(\text{idle}) \in \Gamma'$  and the formula  $(\bigwedge_{\psi \in \Gamma_1} \psi \wedge \hat{K}_a \bigwedge_{\psi' \in \Gamma'} \psi')$  is consistent (because  $\Gamma_1$  and  $\Gamma'$  are propositional). By Zig, there exists  $f' \in E'$  such that  $e' \rightarrow_a^{\mathcal{E}'_n} f'$  with  $\text{idle} AE_{\Gamma'} f'$ . By Zag, as  $\mathbf{e}_2 AE_{\Gamma_2} e'$  and the formula  $(\bigwedge_{\psi \in \Gamma_2} \psi \wedge \hat{K}_a \bigwedge_{\psi' \in \Gamma'} \psi')$  is consistent, there exists  $\mathbf{f} \in E$  such that  $\mathbf{e} \rightarrow_a^{\mathcal{E}_n} \mathbf{f}$  and  $\mathbf{f} AE_{\Gamma'} f'$ . By invariance we obtain  $\text{pre}(\mathbf{f}) \in \Gamma'$ . However  $\text{pre}(\mathbf{f}) = \text{desc}(\mathbf{f})$  and  $p \in \mathbf{f}$  so  $\{h_a, a \in Ag\} \not\models \text{pre}(\mathbf{f})$ . We derive a contradiction, so  $\mathcal{E}'_n$  has at least  $2^n$  events.  $\square$

## 3.4 Succinct product updates

Now, we generalize Definition 9 to obtain a succinct representation for updates.

**Definition 14.** Let  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  be a succinct Kripke model. Let  $\mathfrak{E}_1, \dots, \mathfrak{E}_n$  be a sequence of succinct event models with  $\mathfrak{E}_i = \langle AP_M, AP_{E_i}, \chi_{E_i}, (\pi_{a, E_i})_{a \in Ag}, \mathbf{post}_i \rangle$  with  $AP_{E_1}, \dots, AP_{E_n}$  being disjoint sets. The succinct product update of  $\mathfrak{M}$  and  $\mathfrak{E}_1, \dots, \mathfrak{E}_n$ , noted  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n$ , is  $\mathfrak{M}_n = \langle AP_n, L_n, (\pi_a^n)_{a \in Ag} \rangle$  defined by induction on  $n$ :

- $\mathfrak{M}_0 = \langle AP_M, [\beta_M], (\pi_a)_{a \in Ag} \rangle$ ;
- For  $n \geq 1$ ,  $\mathfrak{M}_n = \langle AP_{n-1} \cup AP_{E_n}, L_{n-1} :: [\chi_{E_n}; \mathbf{post}_n], (\pi_a^n)_{a \in Ag} \rangle$   
with  $\pi_a^n = \mathbf{post}_n^{-1}; ((\pi_a^{n-1}; \mathit{set}(AP_{\mathfrak{E}_n})) \cap \pi_{a, E_n}); \mathbf{post}_n$  and  $::$  is the concatenation operator.

Contrary to Definition 9, we now represent the set of worlds by a list  $L_n$ . For  $n = 0$ , Definition 14 and Definition 9 coincides in the sense that  $[\beta_M]$  is considered the same as  $\beta_M$ . For  $n \geq 1$ , we push  $\chi_{E_n}; \mathbf{post}_n$  at the end of  $L_{n-1}$ . The intuition behind the definition of accessibility programs  $\pi_a^n$  is as follows: we undo the effect of the postconditions of  $\mathfrak{E}_n$ ; we then simulate the conjunction “ $w \rightarrow_a w'$  and  $e \rightarrow_a^{\mathcal{E}} e'$ ” of Definition 3 by executing the intersection of program  $\pi_a^{n-1}; \mathit{set}(AP_{\mathfrak{E}_n})$  and  $\pi_{a, E_n}$ ; finally we reapply the postconditions of  $\mathfrak{E}_n$ .

Next definition explains how to build the Kripke model that corresponds to the succinct product update.

**Definition 15.** Given a succinct Kripke model  $\mathfrak{M} = \langle AP_M, \beta_M, (\pi_a)_{a \in Ag} \rangle$  and succinct event models  $\mathfrak{E}_1, \dots, \mathfrak{E}_n$  where  $\mathfrak{E}_i = \langle AP_M, AP_{E_i}, \chi_{E_i}, (\pi_{a, E_i})_{a \in Ag}, \mathbf{post}_i \rangle$ , the Kripke model represented by the product update of  $\mathfrak{M}$  and  $\mathfrak{E}_1, \dots, \mathfrak{E}_n$ , noted  $\hat{M}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$  is the model  $M_n = (W_n, (\rightarrow_{a, n})_{a \in Ag}, V_n)$ , defined by induction on  $n$ :

- $M_0 = \hat{M}(\mathfrak{M})$ ;
- For  $n \geq 1$ :  $M_n = (W_n, (\rightarrow_{a, n})_{a \in Ag}, V_n)$  where
  - $W_n = \{ \mathbf{w} \in \mathcal{V}(AP_M \cup \bigcup_{i=1}^n AP_{E_i}), \text{ s.t. there exists } \mathbf{v} \in W_{n-1} \text{ and } \mathbf{e} \in \mathcal{V}(AP_{E_n}) \text{ s.t.h. } M_{n-1}, \mathbf{v} \cup \mathbf{e} \models \chi_{E_n} \text{ and } \mathbf{v} \cup \mathbf{e} \xrightarrow{\mathbf{post}_n} \mathbf{w}; \}$
  - $\rightarrow_{a, n} = \left\{ (\mathbf{w}, \mathbf{u}) \in W_n^2 \mid \mathbf{w} \xrightarrow{\pi_a^n} \mathbf{u} \right\}$ ;
  - $V_n(\mathbf{w}) = \mathbf{w}$ .

We say that  $\mathbf{w}$  is a state of  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n$  if  $\mathbf{w} \in W_n$  where  $W_n$  is given in Definition 15.

**Proposition 4.** Let  $\mathfrak{M}$  be a succinct Kripke model and  $\mathfrak{E}_1, \dots, \mathfrak{E}_n$  a sequence of succinct event models. The models  $\hat{M}(\mathfrak{M}) \otimes \hat{\mathcal{E}}(\mathfrak{E}_1) \otimes \dots \otimes \hat{\mathcal{E}}(\mathfrak{E}_n)$  and  $\hat{M}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$  are  $AP_M \cup \bigcup_{i=1}^n AP_{E_i}$ -bisimilar.

*Proof.* The result is proven by recurrence on  $n$ . Case  $n = 0$  is direct. For the case  $n > 1$ , with the induction hypothesis it suffices to prove that  $\hat{M}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$  and  $\hat{M}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_{n-1}) \otimes \hat{\mathcal{E}}(\mathfrak{E}_n)$  are  $AP_M \cup \bigcup_{i=1}^n AP_{E_i}$ -bisimilar. The proof technique is similar to the proof of Proposition 3, the details are left to the reader.  $\square$

When the context is clear, we write  $\mathfrak{M} \otimes \vec{\mathfrak{E}}$  for  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n$ .

### 3.5 Language of succinct DEL

We define the language  $\mathcal{L}_{succDEL}$ :

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid K_a\varphi \mid \langle \mathfrak{E}, \beta_0 \rangle \varphi$$

where  $\mathfrak{E}$  is a succinct event model over  $(AP_M, AP_E)$  and  $\beta_0$  is a Boolean formula over  $AP_M \cup AP_E$ .

The syntax of succinct DEL is similar to the syntax of DEL itself except that operators  $\langle \mathcal{E}, E' \rangle$  (where  $\mathcal{E}, E'$  is a multi-pointed event model) are replaced by  $\langle \mathfrak{E}, \beta_0 \rangle$  where  $\beta_0$  is a Boolean formula that succinctly represents a set of events  $E'$ . The semantics of  $\langle \mathfrak{E}, \beta_0 \rangle \varphi$  is:  $M, w \models \langle \mathfrak{E}, \beta_0 \rangle \varphi$  iff there exists  $e \in \hat{\mathcal{E}}(\mathfrak{E})$  such that  $e \models \beta_0$ ,  $M, w \models \mathit{pre}(e)$  and  $M \otimes \hat{\mathcal{E}}(\mathfrak{E}), (w, e) \models \varphi$  where  $\mathit{pre}(e)$  is the precondition of  $e$  in  $\hat{\mathcal{E}}(\mathfrak{E})$ .

We write  $\mathfrak{M} \otimes \vec{\mathfrak{E}}, w \models \varphi$  for  $\hat{M}(\mathfrak{M} \otimes \vec{\mathfrak{E}}), w \models \varphi$ .

## 4 Model checking

The succinct model checking problem takes as input a pointed succinct Kripke model  $\mathfrak{M}, \mathfrak{w}$  and a formula  $\varphi$  of  $\mathcal{L}_{succDEL}$ , and returns yes if  $\mathfrak{M}, \mathfrak{w} \models \varphi$ , no otherwise.

As the model checking of DLPA is PSPACE-hard [2][3], the succinct model checking problem is PSPACE-hard. Now we provide the upper bound:

**Theorem 3.** *The succinct model checking problem is in PSPACE.*

To prove Theorem 3, as  $APTIME = PSPACE$  [9] (where  $APTIME$  is to the class of problems decided by an alternating Turing machine in polynomial time), we provide an alternating algorithm deciding the succinct model checking problem in polynomial time.

### 4.1 Background on alternating algorithms

The internal nodes of the computation tree of our algorithm on an input  $\mathfrak{M}, \mathfrak{w}, \varphi$  are either existential configurations (as for non-deterministic algorithms) or universal configurations. Player  $(\exists)$  (respectively  $(\forall)$ ) chooses the next configuration in existential (universal) configurations. Leafs are either accepting or rejecting configurations. Player  $(\exists)$  wins if the game reaches an accepting configuration.

The algorithm accepts its input  $\mathfrak{M}, \mathfrak{w}, \varphi$  if player  $(\exists)$  has a winning strategy. The running time of an alternating algorithm is the height of the computation tree.

### 4.2 Description of our algorithm

The full pseudo-code of our algorithm is given in Figure 6 and extends algorithms for variants of DLPA given in [10] and [12] in order to handle succinct event models. The main procedure *main* for model checking succinct DEL first calls  $mc_{yes}(\mathfrak{M}, w, \varphi)$ . If this call is not rejecting the input (that is if  $\mathfrak{M}, w \models \varphi$ ) then it accepts its input  $\mathfrak{M}, w, \varphi$ .

We implicitly define the dual versions  $mc_{no}$ ,  $stateof_{no}$ ,  $issucc_{no}$  obtained from procedures  $mc_{yes}$ ,  $stateof_{yes}$ ,  $issucc_{yes}$  by switching  $(\exists)$  with  $(\forall)$ , and with **or**, and indices *yes* with indices *no*. The specifications of all procedures are given in the following Theorem:

**Theorem 4.** *For any succinct product update  $\mathfrak{M} \otimes \vec{\mathcal{E}}$ , any valuations  $\mathfrak{w}, \mathfrak{u}$  any formula  $\varphi$  of  $\mathcal{L}_{succDEL}$  and any accessibility program  $\pi$ :*

- $mc_{yes}$  rejects  $\mathfrak{M} \otimes \vec{\mathcal{E}}, w, \varphi$  iff  $\mathfrak{M} \otimes \vec{\mathcal{E}}, \mathfrak{w} \not\models \varphi$ ;
- $mc_{no}$  rejects  $\mathfrak{M} \otimes \vec{\mathcal{E}}, \mathfrak{w}, \varphi$  iff  $\mathfrak{M} \otimes \vec{\mathcal{E}}, \mathfrak{w} \models \varphi$ ;
- $stateof_{yes}$  rejects  $\mathfrak{w}, \mathfrak{M} \otimes \vec{\mathcal{E}}$  iff  $\mathfrak{w}$  is not a state of  $\mathfrak{M} \otimes \vec{\mathcal{E}}$ ;
- $stateof_{no}$  rejects  $\mathfrak{w}, \mathfrak{M} \otimes \vec{\mathcal{E}}$  iff  $\mathfrak{w}$  is a state of  $\mathfrak{M} \otimes \vec{\mathcal{E}}$ ;
- $issucc_{yes}$  rejects  $\mathfrak{w}, \mathfrak{u}, \pi$  iff  $\mathfrak{w} \not\stackrel{\pi}{\rightarrow} \mathfrak{u}$ ;
- $issucc_{no}$  rejects  $\mathfrak{w}, \mathfrak{u}, \pi$  iff  $\mathfrak{w} \stackrel{\pi}{\rightarrow} \mathfrak{u}$ .

**Procedures  $mc_{yes}$  and  $mc_{no}$ .** The cases  $\varphi = p$  and  $\varphi = (\varphi_1 \vee \varphi_2)$  in  $mc_{yes}$  are straightforward. The case  $\varphi = \neg\psi$  consists in calling the dual procedure  $mc_{no}$  with  $\psi$ . In the case  $\varphi = K_a\psi$ , we compute the program  $\pi_a^n$  from the Kripke model  $(\mathfrak{M} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_n)$ , given in Definition 14, then universally choose a valuation  $\mathfrak{u}$  and finally check that one of the three conditions holds:

- $\mathfrak{u}$  is not in the set of states of  $(\mathfrak{M} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_n)$  (the corresponding call to  $stateof_{no}$  does not reject its input), meaning that the chosen valuation  $\mathfrak{u}$  is irrelevant.
- $\mathfrak{w} \stackrel{\pi_a}{\rightarrow} \mathfrak{u}$  (the call  $issucc_{no}$  does not reject  $\mathfrak{w}, \mathfrak{u}, \pi_a$ ), meaning that the chosen valuation  $\mathfrak{u}$  is irrelevant.

- or that  $(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n), \mathbf{u} \models \psi$  (the call  $mc_{yes}$  does not reject  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n, \mathbf{u}, \psi$ ). This condition is particularly relevant when  $\mathbf{u}$  is actually in the set of states of  $(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$  and  $\mathbf{w} \xrightarrow{\pi_a} \mathbf{u}$ .

In the case  $\varphi = \langle \mathfrak{E}, \beta_0 \rangle \psi$ , player  $(\exists)$  chooses a valuation  $\mathbf{e}$  with  $\mathbf{e} \models \beta_0$ . Then the algorithm checks that  $\mathbf{w} \cup \mathbf{e}$  is a state of  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n$  and that  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n, \mathfrak{E}, (\mathbf{w} \cup \mathbf{e}) \models \psi$ .

**Procedures  $stateof_{yes}$  and  $stateof_{no}$ .** The call  $stateof_{yes}(w, \mathfrak{M}, \mathfrak{E}_1, \dots, \mathfrak{E}_n)$  rejects if and only if the valuation  $u$  does not correspond to a state of  $\mathfrak{M}, \mathfrak{E}_1, \dots, \mathfrak{E}_n$ . In order to check that the valuation  $\mathbf{u}$  is not a state of  $\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n$ , the procedure mimics Definition 14 and distinguishes two cases. In the case  $n = 0$ , we check that  $\beta_M$  holds in  $\mathbf{w}$ . In the case  $n \geq 1$ , we first check  $\chi_{E_n}$ . Then we simulate the program  $\text{post}_n$  by universally choosing a valuation  $\mathbf{u}$  such that  $\mathbf{u} \xrightarrow{\text{post}_n} \mathbf{w}$ . Finally, we check that  $\chi_{E_n}$  holds in  $\mathbf{u}$  (we check that the precondition of  $\mathfrak{E}_n$  holds).

**Procedures  $issucc_{yes}$  and  $issucc_{no}$ .** The cases over  $\pi$  follow the semantics for accessibility programs (Subsection 3.1).

### 4.3 Sketch of proofs

**Lemma 1.** *The procedure main runs in polynomial time in the size of  $(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n, \mathbf{w}, \varphi)$ .*

*Proof.* The size of the argument of the procedures is strictly decreasing at each step, so the execution is in linear time in respect to the size of the input. Thus,  $Mc$  runs in polynomial time in respect to the size of  $(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n, \mathbf{w}, \varphi)$ . Therefore, the height of the computation tree is polynomial in the size of the input.  $\square$

Theorem 4 is proved by a mutual induction for  $mc_{no}$ ,  $stateof_{no}$ ,  $issucc_{no}$  obtained from  $mc_{yes}$ ,  $stateof_{yes}$ ,  $issucc_{yes}$  on the size of their inputs.

### 4.4 Impact

The translation, with straightforward base cases,

$$tr(\langle \mathcal{E}, E' \rangle \varphi) = \langle \mathfrak{E}_{\mathcal{E}}, \bigvee_{e \in E'} \left( p_e \wedge \bigwedge_{e \notin E'} \neg p_e \right) \rangle tr(\varphi) \quad (*)$$

is such that  $M, w \models \varphi$  iff  $\mathfrak{M}_M, p_w \wedge desc(V(w)) \models tr(\varphi)$ . Thus, Theorem 3 gives an alternative proof for the PSPACE upper bound of the model checking of DEL.

Interestingly, if for some  $\mathcal{E}, E'$  whose size depends on the input (as the attention-based announcement in Example 4) and for which there exists a succinct pointed event model  $\mathfrak{E}, \beta_0$  of polynomial size in the input, then, instead of (\*), we use:

$$tr(\langle \mathcal{E}, E' \rangle \varphi) = \langle \mathfrak{E}, \beta_0 \rangle tr(\varphi)$$

and the model checking remains in PSPACE. For instance, the model checking of an epistemic formula that contains an arbitrary finite number of agents and attention-based announcement modalities  $[p!]_{at}$  is in PSPACE.

## 5 Conclusion and perspectives

In this paper, we provide an exponentially more succinct version of Dynamic epistemic logic (DEL) whose model checking problem remains in PSPACE. We conjecture that a model checking procedure for our succinct language may use BDD techniques of [15].

It opens a long-term research track for studying algorithmic problems in DEL such as satisfiability problem and epistemic planning in their succinct versions.

Procedure $main(\mathfrak{M}, \mathfrak{w}, \varphi)$   $mc_{yes}(\mathfrak{M}, \mathfrak{w}, \varphi)$   <b>accept</b>
Procedure $mc_{yes}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n, \mathfrak{w}, \varphi)$   Case $\varphi = p$ : <b>if</b> $\mathfrak{w} \not\models p$ <b>then reject</b>   Case $\varphi = (\varphi_1 \vee \varphi_2)$ :     $(\exists)$ choose $i \in \{1, 2\}$     $mc_{yes}((\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n), \mathfrak{w}, \varphi_i)$   Case $\varphi = \neg\psi$ : $mc_{no}((\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n), \mathfrak{w}, \psi)$ .   Case $\varphi = K_a\psi$ :     Get $\pi_a^n$ from $(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$ .     $(\forall)$ choose $\mathfrak{u}$ over $AP_{\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n}$     $(\exists) stateof_{no}(\mathfrak{u}, (\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n))$     <b>or</b> $issucc_{no}(\mathfrak{w}, \mathfrak{u}, \pi_a)$ <b>or</b> $mc_{yes}((\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n), \mathfrak{u}, \psi)$   Case $\varphi = \langle \mathfrak{E}, \beta_0 \rangle \psi$ :     $(\exists)$ choose $e$ such that $\mathfrak{e} \models \beta_0$     $(\forall) stateof_{yes}(\mathfrak{w} \cup \mathfrak{e}, (\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n \otimes \mathfrak{E}_n))$     <b>and</b> $mc_{yes}(\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n \otimes \mathfrak{E}, (\mathfrak{w} \cup \mathfrak{e}), \psi)$ .
Procedure $stateof_{yes}(\mathfrak{w}, \mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n)$   Case $n = 0$ : $mc_{yes}(\mathfrak{M}, \mathfrak{w}, \beta_M)$   Case $n > 0$ :     $(\exists)$ choose $\mathfrak{u} \in \mathcal{V}(AP_{\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_n})$     $(\forall) issucc_{yes}(\mathfrak{u}, \mathfrak{w}, \text{post}_n)$     <b>and</b> $stateof_{yes}(\mathfrak{u} _{AP_{\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_{n-1}}}, \mathfrak{M} \otimes \mathfrak{E}_1, \dots, \otimes \mathfrak{E}_{n-1})$     <b>and</b> $mc_{yes}(\mathfrak{M}, \mathfrak{E}_1, \dots, \mathfrak{E}_{n-1}, \mathfrak{u} _{AP_{\mathfrak{M} \otimes \mathfrak{E}_1 \otimes \dots \otimes \mathfrak{E}_{n-1}}}, \chi_{E_n})$
Procedure $issucc_{yes}(\mathfrak{w}, \mathfrak{u}, \pi)$   Case $\pi = p \leftarrow \beta$ : <b>if</b> $(\mathfrak{w} \models \beta \text{ and } \mathfrak{u} \neq \mathfrak{w} \cup \{p\})$ <b>or</b> $(\mathfrak{w} \not\models \beta \text{ and } \mathfrak{u} \neq \mathfrak{w} \setminus \{p\})$ <b>then reject</b>   Case $\pi = \beta?$ : <b>if</b> $\mathfrak{w} \neq \mathfrak{u}$ <b>or</b> $\mathfrak{w} \not\models \beta$ <b>then reject</b>   Case $\pi = (\pi_1; \pi_2)$ :     $(\exists)$ choose a valuation $\mathfrak{v}$ $(\forall) issucc_{yes}(\mathfrak{w}, \mathfrak{v}, \pi_1)$ <b>and</b> $issucc_{yes}(\mathfrak{v}, \mathfrak{u}, \pi_2)$   Case $\pi = (\pi_1 \cup \pi_2)$ :     $(\exists)$ choose $i \in \{1, 2\}$     $issucc_{yes}(\mathfrak{w}, \mathfrak{u}, \pi_i)$   Case $\pi = (\pi_1 \cap \pi_2)$ :     $(\forall)$ choose $i \in \{1, 2\}$     $issucc_{yes}(\mathfrak{w}, \mathfrak{u}, \pi_i)$   Case $\pi = \pi'^{-1}$ : $issucc_{yes}(\mathfrak{u}, \mathfrak{w}, \pi')$ .

Figure 6: Pseudo-code

The succinct satisfiability problem is the following decision problem: determine whether a formula  $\varphi \in \mathcal{L}_{succDEL}$  is satisfiable. There exists a tableau method for determining whether a formula  $\varphi \in \mathcal{L}_{DEL}$  is satisfiable [1] that yields a non-deterministic algorithm in exponential time for the satisfiability problem. We conjecture that the tableau method can be adapted for a formula  $\varphi \in \mathcal{L}_{succDEL}$  in order to prove that the succinct satisfiability problem is in NEXPTIME.

Charrier et al. [11] provides a succinct language for iterations of event models: they provide modalities  $\langle (\mathcal{E}, e)^i \rangle$  where  $i$  is an integer written in binary instead of the explicit sequence  $(\mathcal{E}, e); \dots; (\mathcal{E}, e)$ . The corresponding model checking of DEL is PSPACE-complete even with such succinct iterations, when preconditions are propositional and with trivial postconditions. It would be interesting to study the extension

of our succinct DEL with succinct iterations.

Also, interestingly, the satisfiability problem of attention-based announcement logic is PSPACE-complete [8] even if corresponding event models are exponential in the number of agents (see Figure 5). So the gap between PSPACE and NEXPTIME is not due to the size of models but in the very structure of event models. We claim that our succinct version DEL may help in characterizing fragments of DEL with lower complexity for model checking/satisfiability problem.

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# A Appendix

## A.1 Get rid of epistemic postconditions (Remark 1)

**Proposition 5.** *For any pointed event model  $\langle \mathcal{E}, E' \rangle$  over  $AP$ , we can effectively compute in polynomial time in the size of  $\langle \mathcal{E}, E' \rangle$   $|AP| + 2$  pointed event models  $\langle \mathcal{E}_{tag}, E'_{tag} \rangle, \langle \mathcal{E}_1, E_1 \rangle, \dots, \langle \mathcal{E}_{|AP|}, E_{|AP|} \rangle, \langle \mathcal{E}_{remove}, E_{remove} \rangle$  with propositional postconditions over  $AP' \supseteq AP$  such that for any model  $M$  (where atomic propositions outside  $AP$  are false in all worlds), any world  $w \in M$  and any formula  $\varphi$  over  $AP$ :*

$$(M, w) \models \langle \mathcal{E}, E' \rangle \varphi \text{ iff } (M, w) \models \langle \mathcal{E}_{tag}, E'_{tag} \rangle \langle \mathcal{E}_1, E_1 \rangle \dots \langle \mathcal{E}_{|AP|}, E_{|AP|} \rangle \langle \mathcal{E}_{remove}, E_{remove} \rangle \varphi.$$

*Proof. Construction.* Let  $AP = \{p_1, \dots, p_{|AP|}\}$ . Let  $\mathcal{E} = (E, (\rightarrow_a^\mathcal{E})_{a \in Ag}, pre, post)$  be an event model with epistemic postconditions defined over  $AP$  and  $E' \subseteq E$ . We define  $|AP| + 2$  new event models  $\mathcal{E}_{tag}, \mathcal{E}_1, \dots, \mathcal{E}_{|AP|}, \mathcal{E}_f$  with propositional postconditions that simulate the behaviour of  $\mathcal{E}$ . They are defined over a new set of atomic propositions  $AP' = AP \cup \{\alpha_p, p \in AP\} \cup \{p_e, e \in E\}$ . Intuitively,  $\alpha_p$  will be a temporary atomic proposition to store the new value of  $p$ ,  $p_e$  stands for ‘‘event  $e$  will take place’’. Let us define all these event models:

- $\mathcal{E}_{tag} = (E_{tag}, (\rightarrow_{a,tag})_{a \in Ag}, pre_{tag}, post_{tag})$  with  $E_{tag} = \{e_{tag}, e \in E\}$ ,  $\rightarrow_{a,tag}$  is the universal relation  $E_{tag} \times E_{tag}$ ,  $pre_{tag}(e_{tag}) = \top$ ,  $post_{tag}(e_{tag}, p_e) = \top$ ,  $post_{tag}(e_{tag}, p) = p$  for  $p \neq p_e$ . This event model duplicates and tags all states with an event of  $\mathcal{E}$ . Note that this event model has trivial preconditions. Thus, in event  $e_{tag}$  (that comes from event  $e$ ) the postcondition is  $post(e_{tag}, p_e) = \top$  and is trivial for other atomic propositions. The corresponding pointed event set is  $E'_{tag} = \{e_{tag} \in E_{tag}, e \in E'\}$ .
- For all  $j \in \{1, \dots, |AP|\}$ ,  $\mathcal{E}_j = (E_j, (\rightarrow_{a,j})_{a \in Ag}, pre_j, post_j)$  with  $E_j = \{e_j^\top, e \in E\} \cup \{e_j^\perp, e \in E\}$ ,  $pre_j(e_j^b) = p_e \wedge (post(e, p_j) \leftrightarrow b)$  for  $b \in \{\top, \perp\}$ ,  $\rightarrow_{a,j}$  is the universal relation  $E_j \times E_j$ ,  $post_j(e_j^b, \alpha_{p_j}) = b$ ,  $post_j(e_j^b, p) = p$  if  $p \neq \alpha_{p_j}$ . Intuitively,  $e_j^b$  tests that  $e$  has occurred and that the truth value of the postcondition of  $e$  for  $p_j$  is  $b$ , and then stores the value  $b$  in  $\alpha_{p_j}$ . The corresponding pointed event set is  $E_j$ .
- $\mathcal{E}_{remove} = (E_{remove}, (\rightarrow_{a,remove})_{a \in Ag}, pre_{remove}, post_{remove})$  with  $E_{remove} = \{e_{remove}, e \in E\}$ ,  $e_{remove} \rightarrow_{a,remove} f_{remove}$  iff  $e \rightarrow_a^\mathcal{E} f$ ,  $pre_{remove}(e_{remove}) = (p_e \wedge pre(e))$ ,  $post_f(e_{remove}, p_j) = \alpha_{p_j}$  for all  $j \in \{1, \dots, |AP|\}$  and  $post_{remove}(e_{remove}, p) = \perp$  for  $p \in AP' \setminus AP$ .  $\mathcal{E}_{remove}$  is a copy of  $\mathcal{E}$  such that every event  $e$  is replaced by an event  $e_{remove}$ , where the precondition is the conjunction of  $p_e$  and  $pre(e)$ . The corresponding pointed event set is  $E_{remove}$ .

**Correctness.** To show Proposition 5, we give a  $AP'$ -bisimulation between  $M \otimes \mathcal{E}$  and  $M \otimes \mathcal{E}_{tag} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_{|AP|} \otimes \mathcal{E}_f$  such that the pointed states in  $M \otimes \mathcal{E}_{tag} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_{|AP|} \otimes \mathcal{E}_f$  are bisimilar to the pointed states in  $M \otimes \mathcal{E}$ . The candidate bisimulation is:

$$B = \{((w, e), (w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove})) \text{ s.t. } b_j = \top \text{ iff } M, w \models post(e, p_j)\}.$$

- Invariance: let  $p_j \in AP$ .  $p_j \in V((w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove}))$  iff  $\alpha_{p_j} \in V((w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}))$  (definition of  $\mathcal{E}_{remove}$ ).  $\alpha_{p_j}$ 's value is only changed in  $\mathcal{E}_j$  and its value is the truth value of  $post(e, p_j)$  in  $(M \otimes \mathcal{E}_{tag} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_{j-1}, (w, e_{tag}, e_1, \dots, e_{j-1}))$ .  $(M \otimes \mathcal{E}_{tag} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_{j-1}, (w, e_{tag}, e_1, \dots, e_{j-1}))$  and  $M, w$  are  $AP$ -bisimilar. Therefore,  $\alpha_{p_j}$ 's value is the truth value of  $post(e, p_j)$  in  $M, w$ .

Therefore,  $p_j \in V((w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove}))$  iff  $M, w \models post(e, p_j)$ .

To summarize,  $p_j \in V((w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove}))$  iff  $p_j \in V((w, e))$ . For atomic propositions in  $AP' \setminus AP$ , they were false in  $M$  are reset to false in  $\mathcal{E}_{remove}$ . To conclude,

$$V((w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove})) = V((w, e)).$$

- **Zig:** in the rest of the proof, we call  $\rightarrow_a^{Zig}$  the relations in  $M \otimes \mathcal{E}$  and  $\rightarrow_a^{Zag}$  the relations in  $M \otimes \mathcal{E}_{tag} \otimes \mathcal{E}_1 \otimes \dots \otimes \mathcal{E}_{|AP|} \otimes \mathcal{E}_{remove}$ . Suppose that  $(w, e) \rightarrow_a^{Zig} (w, f)$  and  $(w, e)B(w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove})$ . We have that  $(w, f)B(w, f_{tag}, f_1^{b'_1}, \dots, f_{|AP|}^{b'_{|AP|}}, f_{remove})$ , where  $b'_j = \top$  iff  $M, w \models post(e, p_j)$ .

By definition of the event models, we have

$$(w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove}) \rightarrow_a^{Zag} (w, f_{tag}, f_1^{b'_1}, \dots, f_{|AP|}^{b'_{|AP|}}, f_{remove}).$$

- **Zag:** if  $(w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove}) \rightarrow_a^{Zag} (w, f_{tag}, f_1^{b'_1}, \dots, f_{|AP|}^{b'_{|AP|}}, f_{remove})$  and  $(w, e)B(w, e_{tag}, e_1^{b_1}, \dots, e_{|AP|}^{b_{|AP|}}, e_{remove})$  then in particular,  $M, w \models pre(f)$  and  $e \rightarrow_a^{\mathcal{E}} f$ . We conclude directly that  $(w, e) \rightarrow_a^{Zig} (w, f)$ .

□