



HAL
open science

Estimation of traffic density on motorways

Neila Bhourri, Habib Haj Salem, Markos Papageorgiou, Jean Marc Blosseville

► **To cite this version:**

Neila Bhourri, Habib Haj Salem, Markos Papageorgiou, Jean Marc Blosseville. Estimation of traffic density on motorways. IFAC/IFIP/IFORS International Symposium (AIPAC'89), Jul 1989, Nancy, France. pp.579-583. hal-01485770

HAL Id: hal-01485770

<https://hal.science/hal-01485770>

Submitted on 27 Mar 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Estimation of traffic density on motorways

by Neila Bhouri, Habib Hadj-Salem, Markos Papageorgiou and Jean-Marc Blosseville

Département Analyse et Régulation du Trafic (DART)

Institut National de Recherche sur les Transports et leur Sécurité (INRETS)

A scalar Kalman filter is applied to traffic density estimation in a 500-metre section of the Boulevard Périphérique in Paris using presence detector data. After derivation of the model and filter equations, estimates are compared to real traffic density measurements extracted from a film. Results indicate accuracy of estimates; excellent tracking capabilities; and low sensitivity with respect to changing application conditions.

Introduction. Traffic density records are needed in real time for motorway surveillance and control. However direct density measurement on motorways is only possible on the basis of aerial photographs obtained by recently-developed video camera detectors¹. On the other hand, traffic occupancy measurements obtained by standard induction loops are known to be related to the traffic density around the measurement location. In fact, several algorithms have been proposed in the past for the estimation of traffic density from presence detector data. Most of these algorithms²⁻⁵ assume detectors to be placed at roughly 500-metre intervals along the road. Further algorithms⁶⁻⁷ have also been developed on the basis of more sophisticated traffic flow models, requiring detectors at 2-3 km spacing.

A particularly simple and efficient method for density estimation in the case of relatively short (typically 500-m) distances between detectors was proposed and tested on the basis of microscopic simulation data^{4,5}. The method applies a Kalman filter in order to reduce density observation errors included in the occupancy measurements. This paper presents the application of the basic estimation technique^{4,5} to a section of the Boulevard Périphérique in Paris, along with testing against real traffic data. The impact of on-ramp and off-ramp traffic, of section length and of sample time interval length on estimate quality is also examined.

Definitions and problem statement

Consider a section of motorway of length Δ with m lanes and detector loops located at each lane at both ends of the section. No on-ramps and off-ramps are considered at this point, although they will be introduced for the application later on. Let $n_i(k)$ denote the number of cars passing a given detector station at lanes $i = 1, \dots, m$. Integer argument k corresponds to car count during the time interval $[(k-1)T, kT]$ where T is the sample time interval. Corresponding traffic volumes are given by $q_i(k) = n_i(k)/T$.

The occupancy time of a detector i due to the passage of a vehicle j is $t_{ij} = (l_{ij} + d)/y_{ij}$ where l_{ij} , y_{ij} are the vehicle's length and speed respectively, and d is the length of the detector loop which is equal for all detectors. Occupancy $o_i(k)$ of detector i is then given by

$$o_i(k) = 100/T \sum_{j=1}^{n_i(k)} t_{ij} \quad \dots (1)$$

Introducing the effective vehicle length $L_{ij} = l_{ij} + d$, and substituting t_{ij} in Equation (1), we obtain:

$$o_i(k) = 100/T \sum_{j=1}^{n_i(k)} (L_{ij}/y_{ij}) \quad \dots (2)$$

Measurement errors are mainly caused by lane changes of vehicles activating either both or neither of two adjacent detectors. Furthermore, occupancy measurement errors may be the result of variable vehicle height. Hence, the volume Q_i and occupancy O_i measurements provided by the detectors will be disturbed, i.e.

$$q_i(k) = Q_i(k) + \eta_i(k) \quad \dots (3)$$

$$o_i(k) = O_i(k) + \xi_i(k) \quad \dots (4)$$

where η_i , ξ_i are the corresponding measurement errors.

Station volume q resp. Q is defined as the sum of lane volumes q_i resp. Q_i whilst station occupancy o resp. O is the average of lane occupancies o_i resp. O_i . Traffic density $\rho(k)$ is defined as the number of cars in the segment at time kT divided by the segment length Δ . The problem statement is as follows:

Given measurements (3), (4) for the upstream and downstream detector stations, provide estimates of traffic density $\rho_i(k)$ in the segment.

Kalman filter equations. The problem statement and equations of a scalar, discrete-time Kalman filter (one-step prediction) is provided here in a simplified form which is sufficient for the present

application, see Jazwinsky⁸ for details. Consider a process state equation:

$$\rho(k+1) = \rho(k) + u(k) + \gamma(k) \quad \dots (5)$$

and an observation equation:

$$z(k) = \rho(k) + \zeta(k) \quad \dots (6)$$

where ρ, u are the system state resp. input. $\gamma(k)$, $\zeta(k)$, the system resp. measurement noise, are white gaussian sequences with zero means and known variances Γ resp. Z , with $\Gamma > 0$, $Z > 0$. For simplicity, γ and ζ are assumed to be uncorrelated.

On the basis of known measurements $z(k)$, $u(k)$ at time k , we wish to predict the state at time $k+1$ so as to minimise the mean squares error. Ignoring the initial estimation phase, the steady-state filter equation is the following

$$\hat{\rho}(k+1) = (1-H)\hat{\rho}(k) + H.z(k) + u(k) \quad \dots (7)$$

H is the constant filter gain, given by

$$H = (\beta + \sqrt{\beta^2 + 4\beta}) / (2 + \beta + \sqrt{\beta^2 + 4\beta}) \quad \dots (8)$$

where $\beta = \Gamma/Z$. From Equation (8), it is easily seen that $0 \leq H \leq 1$, depending upon the value of the noise variance ratio β . In fact, if β is small, which means that the state equation (5) is more accurate than the observation (6), then H becomes small, and the estimation equation (7) approximates (5). On the other hand, if β is high, then H approximates 1, and (7) is based essentially on the observations.

In summary, application of filter (7) to traffic density estimation requires derivation of a state and an observation equation of the form (5) resp. (6) with known noise variances.

State equation. A state equation is readily derived from conservation of cars. In fact, we have for traffic density:

$$\rho(k+1) = \rho(k) + T/\Delta \cdot [q^u(k) - q^d(k)] \quad \dots (9)$$

where upper indices u , d stand for upstream resp. downstream measurement station. Substituting the measurements (3) in (9), we obtain (5) with $u = T/\Delta (Q^u - Q^d)$ and $\gamma = T/\Delta (\eta^u - \eta^d)$. Clearly, γ will not be zero means white noise if there is any reason for systematically more (or less) lane changes in the upstream segment and as compared to the downstream end.

Furthermore, the noise variance Γ may only be approximately known. However, it will be seen that the estimation results are fairly insensitive with respect to β - variations.

If volume measurements were accurate, i.e. $\gamma = 0$, and the initial density $\rho(0)$ were known, Equation (9) would furnish accurate estimates of density. However, both detector errors and initial value errors are unavoidable, and hence an observation equation is necessary for reliable density estimation.

Observation equation. The local mean speed around a detector i is defined as the harmonic mean of vehicle speeds y_{ij} :

$$w_i = n_i / \sum_{j=1}^{n_i} (1/y_{ij}) \quad \dots (10)$$

Combining Equations (2) and (10), we get

$$w_i = 100 \cdot L_i \cdot q_i / o_i \quad \dots (11)$$

where L_i is the mean effective vehicle length for lane i , defined as

$$L_i = \sum_{j=1}^{n_i} t_{ij} / \sum_{j=1}^{n_i} (1/y_{ij}) \quad \dots (12)$$

Recalling $t_{ij} = L_{ij}/y_{ij}$, it is easily deduced from Equation (12) that, if car speeds y_{ij} are equal, then L_i equals the arithmetic mean of the individual car lengths. On the other hand, if all effective car lengths are equal to, say, λ then $L_i = \lambda$ even if car speeds are not equal.

The station local mean speed w is given by:

$$w = n / \sum_{i=1}^m \sum_{j=1}^{n_i} (1/y_{ij}) = 100 \cdot q / \sum_{i=1}^m (o_i/L_i) \quad \dots (13)$$

If we assume space-time homogeneous conditions, then $q = \bar{\rho} \cdot w$ holds with $\bar{\rho}$ the density around the detector station. From this and Equation (13) we get:

$$\bar{\rho} = 0.01 \sum_{i=1}^m (o_i/L_i) \quad \dots (14)$$

Our observation of the segment density will be more accurate if we take the average of the two detector stations, in order to approximate the density of the motorway segment, i.e.:

$$\rho(k) = 0.01 \sum_{i=1}^m \{ [o_i^u(k) + o_i^d(k)] / 2L_i \} \quad \dots (15)$$

Equation (15) presents a linear density-occupancy relationship which may serve as observation equation. The following remarks are in order:

(1) Equation (15) requires the availability of mean effective vehicle lengths L_i for each lane. Good estimates of L_i are required, otherwise the observation equation and eventually the filter estimates may be biased. However, L_i estimates can be easily derived in real-

time from Equation (12) if there is one single speed-detector station along the whole motorway.

(2) If the effective lengths of the individual lanes are equal, i.e. $L_i = L$, we deduce from Equation (14)

$$\bar{\rho} = 0.01 / L \cdot m \cdot o \quad \dots (16)$$

whilst for equal occupancy values for each lane, i.e. $o_i = o$, we get:

$$\rho = 0.01 / \bar{L} \cdot m \cdot o \quad \dots (17)$$

where $\bar{L} = m \cdot (\sum_{i=1}^m 1/L_i)^{-1}$ is the harmonic mean of the mean effective lengths of the lanes.

(3) Errors (noise) in the observation equation are due to two reasons. The first source are measurement errors arising if one substitutes Equation (4) into Equation (15). The second source are modelling errors due to the space-time homogeneity assumption behind the derivation of Equation (14) resp. Equation (15). Again, the resulting total observation error $\zeta(k)$ may not correspond exactly to the theoretical requirements of the Kalman filter. Furthermore, the noise variance can hardly be estimated. Finally, observation noise may be correlated to state noise with an unknown, but presumably small, covariance due to the common source of measurement errors. However, the remark made for the system noise applies here as well, as will be clear after presentation of the results.

(3) The observation equation may be directly used for density estimates. Clearly, the estimation error in this case will be equal to Z . Hence, application of Kalman filtering should lead to an estimation error variance which is less than Z , the observation error variance.

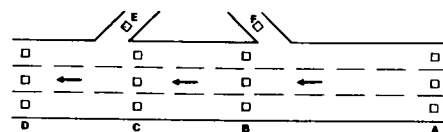


Fig 1. Test sections of the Brancion on-ramp and Boulevard Périphérique in Paris.

Application segment. The traffic density estimation, Equation (7), was applied¹⁰ to a three-lane section of the Boulevard Périphérique in Paris (on-ramp Brancion, Fig 1). More particularly, the algorithm was applied to five sections, namely AB, BC, CD, AC, AD with lengths 221 m, 125 m, 130 m, 346 m and 476 m respectively. Detector stations A, B, C, D, E, F provide traffic volume and occupancy measurements on the basis of $T = 5$ sec.

Real density values, needed for comparison purposes, were extracted for each section from a film for a duration of 12 minutes. The density in the segment during

this time-period is around the critical density which is 120 veh/km. Particularly in the stretch DC, density obtains values up to 180 veh/km. Unfortunately, no stop-and-go traffic conditions are present in the available data material.

For each section, the conservation equation was appropriately modified to consider on-ramp and/or off-ramp volumes, if any. Furthermore, the observation equation was appropriately modified in order to take into account the systematically inhomogeneous space distribution of density due to the on-ramp and the off-ramp. Besides, since mean vehicle lengths for each lane were not available, Equation (16) rather than Equation (14) provided a basis for the observation. More specifically, the observations for sections AB, BC, CD, AC read:

$$z_{AB}(k) = \alpha \cdot O_A(k) \quad \dots (18)$$

$$z_{BC}(k) = \alpha/2 \cdot [O_B(k) + O_C(k)] \quad \dots (19)$$

$$z_{CD}(k) = \alpha \cdot O_D(k) \quad \dots (20)$$

$$z_{AC}(k) = \alpha [O_A O_A(k) \cdot \bar{A}\bar{B} + (O_B(k) + O_C(k)) \cdot \bar{B}\bar{C}2] / \bar{A}\bar{C} \quad \dots (21)$$

with $\alpha = 0.01 \cdot 3/L$. For section AD, two alternative observations were studied:

$$z_{AD}(k) = \alpha [O_A(k) \cdot \bar{A}\bar{C} + O_D(k) \cdot \bar{C}\bar{D}] / \bar{A}\bar{D} \quad \dots (22)$$

$$z_{AD}(k) = \alpha [O_A(k) \cdot \bar{A}\bar{B} + (O_B(k) + O_C(k)) \cdot \bar{B}\bar{C}2 + O_D(k) \cdot \bar{D}\bar{C}] / \bar{A}\bar{D} \quad \dots (23)$$

Since mean effective length L was not available, it was derived such that the observation error subject to the known real density values by minimum. A value of $L = 6.25$ m resulted regardless of the particular section considered.

As a first approach, the state error variance Γ was calculated using the real density values available. The values obtained ranged from 75 (veh/km)² for section AD to 358 (veh/km)² for section CD. In a similar way, calculation of observation error variance Z was performed leading to values 170 (veh/km)² for section AD with observation (23) up to 603 (veh/km)² for section CD.

There are several reasons why measurement error variances differ for different sections. First, there is a systematic space inhomogeneity in some sections which is not visible in the observations. For example, vehicles entering from the on-ramp form a density hump which appears much smoother when observed at station D. Second, lane-change behaviour is different in each section depending upon the existence (or not) of an on-ramp resp. off-ramp. Finally, the section length may have an impact on the accuracy of the observation effected at the section's ends.

Of course, for a normal application, real density values are not available and the variances Γ , Z should be known *a priori*. However, we will see that the filter estimates are fairly insensitive for a broad range of error variance ratio values β . The values of β calculated as above, were found to range from 0.4 for section AB to 2 for section BC.

Fig 2. Estimation error variance versus filter gain H .

Results

Above values of Γ resulted in filter gains H ranging from 0.47 (section AB) to 0.62 (section BC). However, in order to study the sensitivity of the estimates (Equation (7)) with respect to the gain H , the filter was applied for all possible H -values and the variance of the estimation error was observed for each section. The corresponding diagrams are depicted in Fig 2, which also depicts the variances of observation for each section, which are, of course, independent of H and appear as horizontal lines in the diagram. The diagram gives rise to the following remarks applying to all sections:

- (1) The gain H calculated by using Equation (8) is not necessarily the optimal one. This is probably due to the simplifications introduced and the uncertainty of calculation of error variances.
- (2) Estimates are clearly better than observations for a broad range of H -values ($0.05 \leq H \leq 0.8$). Hence, ameliorations achieved by application of filter (7) are substantial as compared to direct utilisation of observations, e.g. Equation (18).
- (3) Estimates are fairly insensitive with respect to moderate H -changes. For H in the range $0.1 \leq H \leq 0.4$, amelioration of filter error variance compared to observation error variance is fairly constant, in the order of 50 per cent. In view of these results, a constant gain $H = 0.2$ may be chosen for all sections.

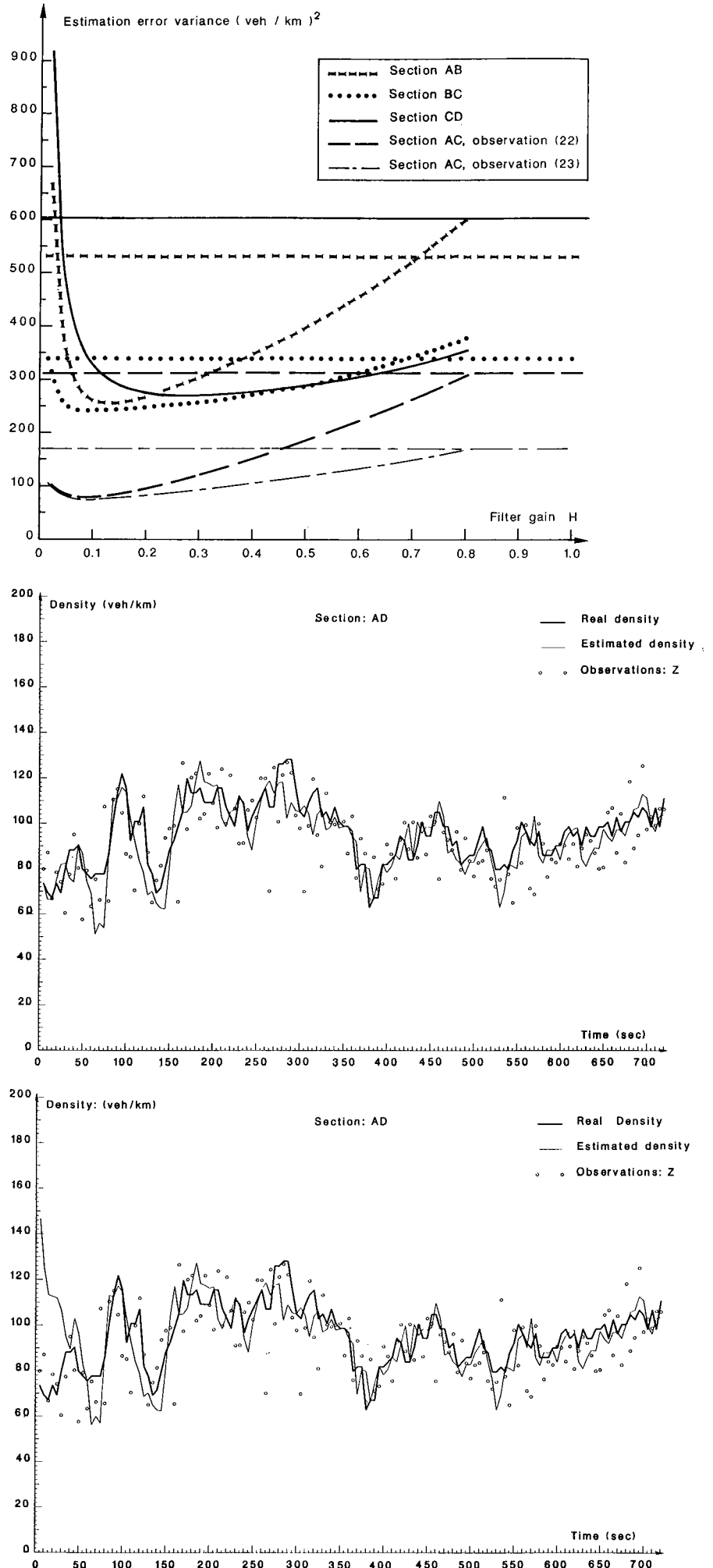
Fig 3. Filter results.

Figure 3 depicts the real and estimated density trajectories for $H = 0.2$, section AD with observation (23), using an initial condition $\hat{\rho}(0)$ which equals the real one. Estimates are seen to follow fairly well even sudden changes for real density. Detailed results may be found in Bhouri¹⁰.

In order to investigate the tracking capability of filter (7), initial estimate $\hat{\rho}(0)$ was set equal to twice the real density value resp. equal to zero. In both cases and for all sections, estimates reached the real density in approximately one minute. Figure 4 depicts results for $\rho(0) = 2 \cdot \hat{\rho}(0)$, section AD, with observation (23).

The influence of the length of the sample time interval T was also investigated. Results obtained with $T = 10$ sec. were comparable to the ones obtained so far whilst $T = 15$ sec. led to a substantial deterioration, see Bhouri¹⁰ for details.

Fig 4. Investigation of tracking capabilities.



Conclusions

A particularly simple Kalman filter for traffic density estimation was tested against real density measurements extracted from a film. The estimation error variance to the half. Sensitivity of estimates with respect to filter gain and sample time interval is low. Tracking behaviour in the case of a wrong initial estimate was found to be rapid, in the order of one minute.

ACKNOWLEDGMENT

The authors would like to thank the engineering staff of Ville de Paris for the data material they made available for purposes of this study.

REFERENCES

- ¹BLOSSEVILLE, J. M., F. LENOIR and S. BEUCHER. Traffic spatial measurement using video image-processing. *Advances in Intelligent Robotic Systems (SPIE's Cambridge Symposium on Optical and Optoelectronic Engineering, November 1987)*, Cambridge, Mass. (to appear).
- ²NAHI N. E. and A. N. TRIVEDI. Recursive estimation of traffic variables: section density and average speed. *Transpn Sci.*, 7 (1973), 269-286.
- ³GHOSH D. and C. H. KNAPP. Estimation of traffic variables using a linear model of traffic flow. *Transpn Res.*, 12 (1978), 395-402.
- ⁴KURKJIAN A., S. B. GERSHWIN, P. K. HOUP, A. S. WILLSKY and E. Y. CHOW. Estimation of roadway traffic density on freeways using presence detector data. *Transpn Sci.*, 14 (1980), 232-261.
- ⁵PAYNE H. J., D. BROWN and J. TODD. Demand-responsive strategies for interconnected freeway ramp control systems: Vol. I. Metering strategies. Report No. FHWA/RD-85/109, U.S. Department of Transportation, Washington, DC, March 1985.
- ⁶CREMER M., M. PAPAGEORGIOU and G. SCHMIDT. Einsatz regelungstechnischer Mittel zur Verbesserung des Verkehrsablaufs auf Schnellstrassen. *Forschung Strassenbau und Strassenverkehrstechnik*, 307 (1980), 1-44.
- ⁷PAPAGEORGIOU, M. *Applications of Automatic Control Concepts to Traffic Flow Modelling and Control*. Springer-Verlag, Berlin/Heidelberg/New York/Tokyo, 1983.
- ⁸JAWWINSKY, A. H. *Stochastic Processes and Filtering Theory*. Academic Press, 1970.
- ⁹WARDROP, J. G. Some theoretical aspects of road traffic research. *Proc., Inst. Civil Engrs.*, Part II, Vol. 1, 1952, 325-362.
- ¹⁰BHOURI, N. Estimation de la densité dans une section d'autoroute. Institut National de Recherche sur les Transports et leur Sécurité, DART, Arcueil, France, 1987.

The authors' address: Département Analyse et Régulation du Trafic (DART), Institut National de Recherche sur les Transports et leur Sécurité (INRETS), 2 avenue du Général Malleret-Joinville, B.P. 34, F-94114 Arcueil Cedex, France.

'No contraflow' widening on M5

Secretary of State for Transport Paul Channon last month unveiled innovative plans for widening the final two-lane section of the M5 motorway in Hereford and Worcester, the 13 miles between Warndon (Junction 6) and Strensham (Junction 8). The new approach, designed by the DTP in conjunction with consultants Howard Humphreys and Partners, will do away with the need for contraflow and provide benefits in terms of speed construction, accident prevention (saving, perhaps, 190 personal injuries and 10 deaths) and reduced traffic delays.

Called 'parallel widening', it involves the construction of three new lanes and a hard shoulder alongside one of the carriageways. While work is going on, traffic on the

existing motorway will be generally unaffected. When the new lanes and bridges are built, traffic will be transferred from one carriageway of the existing motorway onto the new length. The existing motorway will then be adapted to make three new lanes and a hard shoulder.

The result is no contraflow, a minimum two-lane flow with hard shoulder in each direction throughout the works, and a central reserve between opposing flows of traffic at all times.

Mr Channon said that this section of the M5 is particularly suited to parallel widening: 'in other areas, proximity of property or other physical or engineering constraints may still point to the more usual approach as the most economic and viable solution'.

AUSTRALIAN ROAD



A selection of publications from the Australian Road Research Board can now be obtained direct from *Traffic Engineering + Control* 29 Newman Street London W1P 3PE

RESEARCH BOARD Remittances made payable to PRINTERHALL LTD must accompany your order

Please supply ___ copy/copies of the undermentioned titles:

Special Reports:

- SR-34: *Storm drainage design in small urban catchments: a handbook for Australian practice* (John Argue) ___ @ £15.50 Post Paid £ ___
- SR-36: *Modelling shopping destination choices: a theoretical and empirical investigation* (P. O. Barnard) ___ @ £15.50 Post Paid £ ___
- SR-38: *Australian personal travel characteristics* (M. R. Wigan) ___ @ £16.20 Post Paid £ ___

Technical Manuals:

- ATM-21: *The ARRB video vehicle detector* (J. S. Dods) ___ @ £12.00 Post Paid £ ___

Research Reports:

- ARR-141: *Truck size and weight limits: overseas practices* (P. F. Sweatman) ___ @ £ 7.50 Post Paid £ ___
- ARR-142: *Strain measurements in asphalt layers* (K. G. Sharp) ___ @ £ 7.50 Post Paid £ ___
- ARR-143: *The influence of cues from the road and the roadside environment on estimates of operating speeds and speed limits* (P. Cairney) ___ @ £ 7.50 Post Paid £ ___
- ARR-144: *Overtaking lane practice in Canada and Australia* (C. J. Hoban and J. F. Morrall) ___ @ £ 7.50 Post Paid £ ___
- ARR-145: *Image-processing applied to roads: surface texture, mensuration, vehicle shape and number detection* (M. R. Wigan and M. C. Cullinan) ___ @ £ 7.50 Post Paid £ ___
- ARR-146: *Planning and design for rural roads: observations on a year in America* (C. J. Hoban) ___ @ £ 7.50 Post Paid £ ___
- ARR-147: *Towards more fuel-efficient control practices at isolated urban intersections* (D. P. Bowyer and D. C. Biggs) ___ @ £ 7.50 Post Paid £ ___
- ARR-148: *Introduction to SIDRA-2 for Signal Design: 13th ARRB Conference Workshop papers and discussions* (edited by R. Akçelik) ___ @ £ 7.50 Post Paid £ ___

Total remittance herewith £ ___

NAME _____

ADDRESS _____

Telephone: _____

Please note that *Traffic Engineering + Control* can only service orders from the U.K. and Eire — Readers in other countries should contact the Australian Road Research Board at P.O. Box 156, Nunawading, Victoria 3131 — Telephone: (03) 235 1555; Telex: AA 33113; Fax: (03) 233 8878 for details of other national stockists