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# INFLUENCE OF THE PARAMETRIC GEAR MESH EXCITATION ON THE DYNAMIC BEHAVIOR OF A KINEMATIC CHAIN SUBMITTED TO TORQUE FLUCTUATIONS

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**Keywords:** parametric gear mesh excitations, whining noise, spectral iterative method, coupling, upstream/downstream devices

## ABSTRACT

Gear power transmissions are responsible for upsetting vibroacoustic phenomena. The gear teeth compliance, the manufacturing errors and the tooth corrections lead to a periodic transmission error fluctuation and a parametric excitation associated to gear mesh stiffness fluctuation. Under operating conditions, these excitation sources generate dynamic mesh forces transmitted to the housing and responsible for the whining noise radiated from the gearbox.

This work is focused on the multiphysics coupling between the gear mesh internal excitation and the external excitation associated with the upstream engine and downstream receiver system. These excitations mix the high frequencies of internal meshing excitations and low frequencies of the external excitations of the rotating shafts. The coupling between excitations generates an enrichment of the vibratory frequency response.

The goal of this paper is to present the spectral iterative methodology used and the dynamic results induced by this coupling in the case of a root vacuum pump.

## INTRODUCTION / INDUSTRIAL CONTEXT

Mechanical systems presenting gear transmission are excited by the meshing process. It is usually assumed that static transmission error (STE) and gear mesh stiffness fluctuations are responsible of noise radiated by the gearbox. STE is defined as the difference between actual position of the output toothed wheel and the position it would occupy if the gear drive were perfect [1]. Its characteristics depend on the instantaneous situations of the meshing tooth pairs resulting from tooth deflections and manufacturing errors.

The STE is also responsible for variations of the meshing stiffness  $k(t)$  which is a parametric excitation. Under steady-state operating conditions, these excitations are periodic (fundamental frequency = mesh frequency  $f_c = Z \cdot f_r$ , with  $Z$  the number of teeth).

They generate dynamic mesh forces which are transmitted to the housing through wheel bodies, shafts and bearings. Housing vibratory state is directly related to the noise radiated from the gearbox (whining noise).

"Internal" excitations generated by the meshing process may be coupled with "external excitations" associated with torque variations of upstream and/or downstream devices.

The aim of this paper is to present a dynamic analysis method of coupling through the vibro-acoustic study of a roots vacuum pump submitted to a fluid excitation.

The roots vacuum pump is composed of two shafts powered by an engine associated with a 1:1 reverse spur gear that ensures the counter-rotative motion (see figure 1). The rotors have a figure-eight configuration lobes and are separated from one another and from the stator by a narrow gap. The roots vacuum pump is designed with 5 pumping stages. The fundamental frequency of the "external" sources corresponds to the rotation frequency  $f_r$ . Two different "external excitations" are considered:

- the variations of the fluid pumping torque at each stage of the rotors:  $C(t)$ ,
- the variations of the engine torque:  $C_m(t)$ .

In this context, an "iterative spectral method" has been developed [2-3] in order to solve effectively a system of parametric equations. This method is able to deal with high number of degrees of freedom system and wide frequency spectrum excitations. It has been used to analyze the whining noise generated by parallel axes mono [4-5-6-7] and multi-meshing [8] gearboxes. It has also been used in the case of an external random excitations with gyroscopic terms [9]. The objectives of this paper is to adapt this method to an additional excitation (here, the fluidic excitation of the pumping mechanism) that is coupled with the periodic parametric meshing excitation.

In the first part of this paper, the definition and characterization of the excitations due to the gear

transmission are presented. Then, dynamic equations of the finite element model chosen, in the physical and modal basis, are expressed. Furthermore, the problem is expressed in the spectral domain and the interests of the method are highlighted.

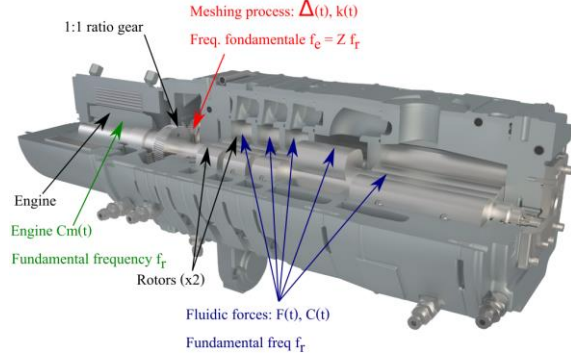


Fig.1 Example of figure

### STATIC TRANSMISSION ERROR AND MESH STIFFNESS FLUCTUATIONS

The calculation of the Static Transmission Error (STE) of a gear transmission under a load  $F$  (corresponding to the global force applied to the meshing) is well-known [10]. The principle is to solve the static contact equation between geared wheels. The excitations are described in figure 2.

For each angular position  $\theta$  of the driving wheel, a kinematic analysis of the meshing process is used to determine the theoretical contact line over the teeth (potential position of the contact on the teeth in the plane of action). These lines are discretized, in order to introduce a compliance matrix  $\mathbf{H}$  that links the teeth deflection under the load on the discrete points. The matrix  $\mathbf{H}$  is constructed with a finite element model of the wheel and teeth in a prior calculation. All the physical phenomenon that contributes to the deflection of the teeth must be taken into account. The thinner the body of the wheel, the higher the influence of the wheel elasticity and the interaction between the teeth in contact [10]. The Hertz deformation is also taken into account in the matrix  $\mathbf{H}$ . The tooth corrections the manufacturing errors are introduced through the vector  $\mathbf{e}$  that describes the initial gap between the teeth. This vector  $\mathbf{e}$  also takes into account the parallelism errors that comes from the elasto-static deformation of the whole gearbox. For each position  $\theta$ , the resolution of the contact equations allows the evaluation of the STE  $\delta(\theta)$ , and to the distributed loads  $\mathbf{P}$  along the contact lines. The corresponding system of equations is:

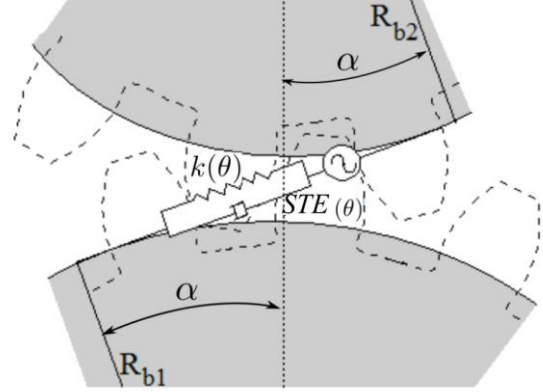
$$\begin{cases} \mathbf{H} \cdot \mathbf{P} = \delta(\theta) \cdot \mathbf{1} - \mathbf{e} \\ \mathbf{1}^T \cdot \mathbf{P} = F \end{cases} \quad (1)$$

under the constraints:

$$\begin{cases} \mathbf{H} \cdot \mathbf{P} + \delta(\theta) \cdot \mathbf{1} \geq \mathbf{e} \\ P_i \leq 0 \end{cases} \quad (2)$$

The evolution of the STE over the meshing period leads to the second parametric excitation of the gear transmission: the mesh stiffness which is a parametric excitation. It is written as the derivate of the force with respect to the STE, as follows:

$$k(\theta) = \frac{\partial F}{\partial \delta(\theta)} \quad (3)$$



$k(\theta)$ : Parametric excitation of the meshing process  
 $STE(\theta)$ : Static Error Transmission (displacement excitation)

Fig. 2 Gear Transmission

### DYNAMIC EQUATIONS AND FINITE ELEMENT MODEL

#### Linearization of the meshing force

The differential equations that define the movement can be written for the generalized coordinates  $x$  as follows:

$$\mathbf{M} \ddot{x} + \mathbf{C} \dot{x} + \mathbf{f}_{NL}(x, t) = \mathbf{F}_S \quad (4)$$

with  $\mathbf{M}$  the mass matrix,  $\mathbf{C}$  the dumping matrix. The non-linear terms  $\mathbf{f}_{NL}(x, t)$  represents the meshing force that depends on the variations of the STE and mesh stiffness.  $\mathbf{F}_S$  represents the static force transmitted by the gear. The mesh stiffness  $k(t)$  is defined as follows:

$$k(t) = \left( \frac{\partial f_{NL}}{\partial x} \right)_{(x_s, t)} \quad (5)$$

The linearized force  $F_L(x, t)$  may be written using a first-order limited development of  $f_{NL}(x, t)$  around the equilibrium position  $x_s$ :

$$\begin{aligned} F_L(x, t) &\approx f_{NL}(x_s) + \frac{\partial f_{NL}}{\partial x}(x_s, t)(x_L(t) - x_s(t)) \\ &= F_s + k(t)(x - x_s(t)) \end{aligned} \quad (6)$$

$f_{NL}(x, t)$  being periodic, so is  $F_L(x, t)$ . Introducing the linearized force (6) in the equation (4), we can then write the differential equation with parametric coefficients as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + k(t)\mathbf{x}(t) = k(t)\mathbf{x}_s(t) \quad (7)$$

### Matrix equations in the physical basis

The kinematic chain of the roots vacuum pump (2 rotors) and his gear transmission are discretized with a N-degrees of freedom finite element model. The matrix equations of the movement driving the vector  $\mathbf{x}$  (physical degrees of freedom) is written as follows:

$$\mathbf{M}_{EF}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}_{EF}\mathbf{x} + k(t)\mathbf{R}\mathbf{R}^T\mathbf{x} = k(t)\mathbf{R}\mathbf{R}^T\mathbf{x}_s(t) + \mathbf{F}(t) \quad (8)$$

with  $\mathbf{M}_{EF}$  and  $\mathbf{K}_{EF}$  the mass and stiffness matrices of the finite element model and  $\mathbf{C}$  the dumping matrix taking into account through the Basile hypothesis, a posteriori.  $\mathbf{R}$  is "geometrical structure" vector that transforms the local tooth force (1 dimension) into the force located in the closest point of the finite element model of each rotor (12 degrees of freedom in the case of beam 3D elements). It is also called "coupling vector".  $k(t)$  is the meshing periodic stiffness and  $\mathbf{F}(t)$  represents the time-dependent external.  $\mathbf{x}_s$  is a vector that corresponds to the static answer to the external static forces. Here, the STE can be written as follows:

$$\Delta^{(s)}(t) = \mathbf{R}^T\mathbf{x}_s(t) \quad (9)$$

The Dynamic Transmission Error (DTE) is defined with the following scalar function:

$$\Delta(t) = \mathbf{R}^T\mathbf{x}(t) \quad (10)$$

We set:

$$k(t) = \bar{k} + g(t) \quad (11)$$

with  $\bar{k}$  the average mesh stiffness and  $g(t)$  the centered fluctuation of the mesh stiffness. We can then introduce the  $\mathbf{K}_{AV}$  matrix that corresponds to the global stiffness matrix (averaged over time):

$$\mathbf{K}_{AV} = \mathbf{K}_{EF} + \bar{k}\mathbf{R}\mathbf{R}^T \quad (12)$$

We can then summarize the matrix equations in the following form:

$$\mathbf{M}_{EF}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}_{AV}\mathbf{x} + g(t)\mathbf{R}\mathbf{R}^T\mathbf{x} = k(t)\mathbf{R}\Delta^{(s)}(t) + \mathbf{F}\gamma(t) \quad (13)$$

with  $\mathbf{F}$  the discretized position vector of the external excitation  $\mathbf{F}(t)$  and  $\gamma(t)$  its temporal part.

### Modal equations

The idea is to extend the solution on the modal basis determined with the average characteristic of the system. This modal basis is computed with the matrices  $\mathbf{M}_{EF}$  and  $\mathbf{K}_{AV}$ . It leads to the eigen frequencies  $\omega_k(t)$  and eigen vectors  $\mathbf{V}_k$ :

$$\mathbf{B} = [\mathbf{V}_k], \omega_k \quad (14)$$

such as:

$$\mathbf{B}^{(-1)}\mathbf{M}_{EF}\mathbf{B} = \mathbf{I} \quad (15)$$

We can then define:

$$\mathbf{x} = \mathbf{B}\mathbf{q} \quad (16)$$

With the modal coordinate  $\mathbf{q}$ .

The equation (13) can then be written as follows:

$$\mathbf{B}^{-1}\mathbf{M}_{EF}\mathbf{B}\ddot{\mathbf{q}} + \mathbf{B}^{-1}\mathbf{C}\mathbf{B}\dot{\mathbf{q}} + \mathbf{B}^{-1}\mathbf{K}_{AV}\mathbf{B}\mathbf{q} + g(t)\mathbf{B}^{-1}\mathbf{R}\mathbf{R}^T\mathbf{B}\mathbf{q} = k(t)\mathbf{B}^{-1}\mathbf{R}\Delta^{(s)}(t) + \mathbf{B}^{-1}\mathbf{F}\gamma(t) \quad (17)$$

The symmetrizing properties of the eigen modes leads us to the following form:

$$\mathbf{diag}[1]\ddot{\mathbf{q}} + \mathbf{diag}[2\zeta_k\omega_k]\dot{\mathbf{q}} + \mathbf{diag}[\omega_k^2]\mathbf{q} + g(t)\mathbf{r}\mathbf{r}^T\mathbf{q} = k(t)\mathbf{r}\Delta^{(s)}(t) + \phi\gamma(t) \quad (18)$$

with  $\mathbf{r} = \mathbf{B}^{-1}\mathbf{R}$  the structural vector projected in the modal basis, and  $\mathbf{\Phi} = \mathbf{B}^{-1}\mathbf{F}$  the projection in the modal basis of the position vector of the fluctuating external forces.

The parametric-coupled equations system of the mechanism can be written in the indexed following form:

$$\ddot{q}_k + 2\zeta_k\omega_k\dot{q}_k + \omega_k^2q_k + g(t)r_k \sum_{l=1}^N r_l q_l = k(t)r_k\Delta^{(s)}(t) + \phi_k\gamma(t) \quad (19)$$

### EQUATIONS IN THE SPECTRAL DOMAIN

The first principle of the spectral method is to solve the coupled equations (19) in the spectral domain, keeping only the steady-state of the system (forced response). In fact, the free response is either a decreasing exponential (asymptotic stability) or an increasing exponential in the case of parametric instabilities. We assume that the dumping is high enough to be out of the parametric instabilities region. The Fourier transform of the coupled

equations under the indexed form gives:

$$H_k^{-1}(\omega)Q_k(\omega) + G(\omega)r_k \otimes \sum_{l=1}^N r_l Q_l(\omega) = K(\omega) \otimes r_k E^{(s)}(\omega) + \phi_k \Gamma(\omega) \quad (20)$$

or:

$$Q_k(\omega) + H_k(\omega)G(\omega)r_k \otimes \sum_{l=1}^N r_l Q_l(\omega) = H_k(\omega)K(\omega) \otimes r_k E^{(s)}(\omega) + H_k(\omega)\phi_k \Gamma(\omega) \quad (21)$$

With:

$$H_k(\omega) = \frac{1}{\omega_k^2 - \omega^2 + 2i\zeta_k \omega_k \omega} \quad (22)$$

and  $Q(\omega)$ ,  $G(\omega)$ ,  $K(\omega)$  and  $E^{(s)}(\omega)$  respectively the Fourier transform of  $q_k(t)$ ,  $g(t)$ ,  $k(t)$  and  $\Delta^{(s)}(t)$ , and finally  $\otimes$  the convolution product.

The second principle of the method is to condense the  $N$  coupled equations on the line of action in order to explicit the Dynamic Transmission Error  $E(\omega)$ . The coupled equations system (21) is then turned into one and only equation:

$$\sum_{k=1}^N r_k Q_k(\omega) + \sum_{k=1}^N r_k H_k(\omega)G(\omega)r_k \otimes \sum_{l=1}^N r_l Q_l(\omega) = \sum_{k=1}^N r_k H_k(\omega)K(\omega) \otimes r_k E^{(s)}(\omega) + \sum_{k=1}^N r_k H_k(\omega)\phi_k \Gamma(\omega) \quad (23)$$

We define the dynamic error in the spectral domain:

$$E(\omega) = \sum_{l=1}^N r_l Q_l(\omega) \quad (24)$$

We then come to:

$$E(\omega) + \sum_{k=1}^N r_k H_k(\omega)G(\omega)r_k \otimes E(\omega) = \sum_{k=1}^N r_k H_k(\omega)K(\omega) \otimes r_k E^{(s)}(\omega) + \sum_{k=1}^N r_k H_k(\omega)\phi_k \Gamma(\omega) \quad (25)$$

For the sake of simplicity, we define:

$$S(\omega) = \sum_{k=1}^N r_k^2 H_k(\omega)K(\omega) \otimes E^{(s)}(\omega) \quad (26)$$

$$W(\omega) = \sum_{k=1}^N r_k H_k(\omega)\phi_k \Gamma(\omega) \quad (27)$$

We introduce the following scalar function:

$$T(\omega) = \sum_{k=1}^N r_k^2 H_k(\omega) \quad (28)$$

We can then rewrite:

$$S(\omega) = T(\omega)K(\omega) \otimes E^{(s)}(\omega) \quad (29)$$

We finally get the scalar equation:

$$E(\omega) + T(\omega)[G \otimes E](\omega) = S(\omega) + W(\omega) \quad (30)$$

## PRINCIPLE OF RESOLUTION

The third principle of the spectral iterative method is to iterate according to the following schema:

$$E^{(n+1)}(\omega) = S(\omega) + W(\omega) - T(\omega)[G \otimes E^{(n)}](\omega) \quad (31)$$

with the following initial conditions:

$$E^{(1)}(\omega) = S(\omega) + W(\omega) \quad (32)$$

The termination criterion is based on the relative gap between two iterations:

$$\frac{E^{(n+1)}(\omega) - E^{(n)}(\omega)}{E^{(n+1)}(\omega)} < \epsilon \quad (33)$$

## DYNAMIC MESH FORCE AND PHYSICAL RESPONSE

The equation (30) and the iterative schema of the equations (31) and (32) directly give the dynamic transmission error on the meshing. The meshing overload associated with  $f_D(t)$  is written as follows:

$$f_D(t) = k(t)\mathbf{R}^T \mathbf{x}(t) - k(t)\mathbf{R}^T \mathbf{x}_s(t) \quad (34)$$

In the spectral domain:

$$F_D(\omega) = [K \otimes E](\omega) - [K \otimes E^{(s)}](\omega) \quad (35)$$

In order to compute the response of a degree of freedom  $X_i(\omega)$  of the system, it is necessary to go back to the physical basis through the matrix  $\mathbf{B}^{-1}$  composed of eigen vectors  $V_i$  of the system (see equation (16)), and use the equation (21), which gives:

$$Q_k(\omega) = H_k(\omega) \left[ r_k [K \otimes E^{(s)} - G \otimes E](\omega) + \phi_k \Gamma(\omega) \right] \quad (36)$$

We can then access to the physical basis and write the degrees of freedom  $i$  in the following way:

$$X_i(\omega) = \sum_{k=1}^N V_{ik} H_k(\omega) \left[ r_k [K \otimes E^{(s)} - G \otimes E](\omega) + \phi_k \Gamma(\omega) \right] \quad (37)$$

## CONCLUSION

This article presents a methodology for solving vibro-acoustic problems coupling internal parametric periodic excitations associated with the gear mesh process and fluctuating external forces. The specific case of a roots vacuum pump composed of a 1:1 ration gear transmission and submitted to fluidic external torque variations is presented. The spectral iterative method is characterized by:

- a use of the modal basis of the system, which is determined by characteristics of the system averaged over time
- a resolution in the spectral domain, under steady-state operating conditions
- a condensation of the N-equations system into a single equation using the "structural geometric" vector expressed in the modal basis
- a simple iterative schema

This efficient method permits a fast resolution in the spectral domain of a system excited by coupled high parametric frequencies due to a gear transmission and low

frequencies due to a fluidic pumping torque on the stage of the rotors.

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