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Why estimating relative differences by $\ln(A/B)$ in percentage and why naming it *geometric difference*

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(1) Reference

C. Graff (2014). Expressing relative differences (in percent) by the difference of natural logarithms. *Journal of Mathematical Psychology* 60, 82–85.

Introduction

We know since Weber, Fechner and Stevens, that *relative differences* express dissimilarities better than *absolute differences*. For example, differential thresholds from a standard (Ref) are better defined by a *Weber ratio JND/Ref* rather than by several *Just Noticeable Differences (JND)*. Relative differences are meaningfully expressed in percent. I demonstrate elsewhere (1) that $\ln(A/B)$ may be turned directly into a percentage to express the relative difference between A and B in %. Naming it the “*geometric difference*“ emphasizes the relationship between logarithmic scale and relative differences.

Arithmetic difference

- * The word *difference* bears numerous meanings, including dissimilarities.
- * Its use in mathematics is essentially dedicated to the result of the *arithmetic* operation called **subtraction**.
- * Thus I will specify the result of the subtraction of A by B as the *arithmetic difference*: $D = A - B$.

Geometric difference & other relative differences

A dissimilarity, e.g. between $A=150g$ and $B=125g$, may be expressed by the *arithmetic difference* $A - B = 25g$. The *geometric difference* is one estimate of relative difference, as well as $(A - B) / B$ or $(A-B) / [(A+B)/2]$. It always sits between the two extreme, better-known, estimates:

$$(A - B)/B = 0.200 = 20.0\%$$

$$\ln(A/B) = 0.182 = 18.2\%$$

$$(A - B)/A = 0.167 = 16.7\%$$

Thus $(A - B)/A < \ln(A/B) < (A - B)/B$. This advantage, specific to \ln , the natural logarithm (log to the base e), adds to the following properties: additivity, symmetry and agreement between inverted units (1).

Arithmetic & geometric means

- * The *arithmetic mean* $M_a = (A+B)/2$ between two values A and B is such that the *arithmetic difference* between either of the two values and their *arithmetic mean* are equal (but opposite):

$$(M_a - A) = - (M_a - B)$$
.
- * The *geometric mean* $M_g = \sqrt{A \cdot B}$ between two values A and B is such that the *geometric difference* between either of the two values and their *geometric mean* are equal (but opposite):

$$\ln(M_g/B) = - \ln(M_g/A)$$
.

The sum of geometric differences from a geometric mean is *null* also when more than two values are averaged, as for arithmetic differences and mean.

Arithmetic & geometric progression

- * An *arithmetic progression* is a sequence of values $(..., A_i, A_{i+1}, ...)$ such that $A_{i+1} = A_i + C$, thus

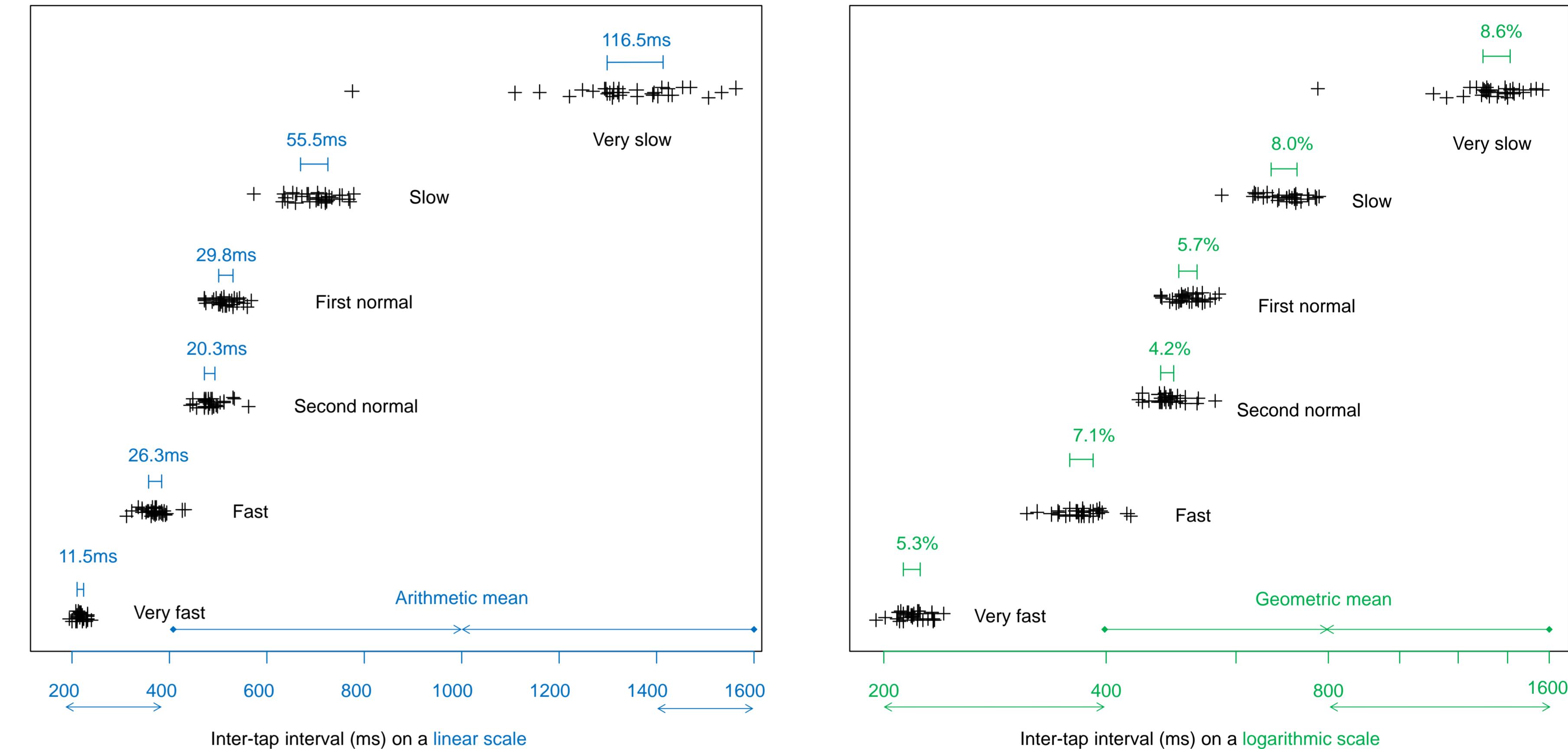
$$C = A_{i+1} - A_i$$

Two consecutive values A_{i+1} and A_{i-1} are separated by a *constant arithmetic difference* C.

- * A *geometric progression* is a sequence of values $(..., G_i, G_{i+1}, ...)$ such that $G_{i+1} = G_i \cdot K$, K being a constant ratio.

Two consecutive values G_{i+1} and G_{i-1} are separated by a *constant geometric difference*

$$\ln(K) = \ln(G_{i+1}/G_i)$$



Inter-tap interval plots of isochronous finger-tapping at various paces by one participant. Each inter-quartile range is shown as *arithmetic difference* on a *linear scale* (left), and as *geometric difference in %* on a *logarithmic scale* (right).

Linear & logarithmic scales

* On a *linear scale*, an *equal distance* represents *same dissimilarities* as estimated by equal *arithmetic differences*.

* On a *logarithmic scale*, an *equal distance* represents *same dissimilarities* as estimated by equal *geometric differences (%)*.

Conclusion

Relative differences are central to psychophysics. Taking the step to present the geometric difference as a percentage will facilitate comparisons between stimuli and between performances. The expression “*geometric difference*” for $\ln(A/B) = \ln(A) - \ln(B)$ may contribute to comprehend it among many related mathematical tools.