Why estimating relative differences by Ln(A/B) in percentage and why naming it *geometric* difference

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Introduction

We know since Weber, Fechner and Stevens, that *relative* differences express dissimilarities better than *absolute* differences. For example, differential thresholds from a standard (Ref) are better defined by a *Weber ratio* JND/Ref rather than by several *Just Noticeable Differences* (JND).
Relative differences are meaningfully expressed in percent. I demonstrate elsewhere (1) that **Ln(A/B)** may be turned directly into a percentage to express the relative difference between A and B in %. Naming it the "***geometric difference***" emphasizes the relationship between logarithmic scale and relative differences.

(1) **Reference**

C. Graff (2014). Expressing relative differences (in percent) by the difference of natural logarithms. *Journal of Mathematical Psychology* 60, 82–85.

Arithmetic difference

* The word *difference* bears numerous meanings, including dissimilarities.
* Its use in mathematics is essentially dedicated to the result of the *arithmetic* operation called subtraction.
* Thus I will specify the result of the subtraction of A by B as the *arithmetic* difference: D = A – B.

Geometric difference & other relative differences

A dissimilarity, e.g. between **A=150g** and **B=125g**, may be expressed by the *arithmetic* difference A – B = 25g. The *geometric difference* is one estimate of relative difference, as well as (A – B) / B or (A-B) / [(A+B) /2]. It always sits between the two extreme, better-known, estimates:

(A – B)/B = 0.200 = 20.0%
Ln(A/B) = 0.182 = 18.2%
(A – B)/A = 0.167 = 16.7%

Thus (A – B)/A < Ln(A/B) < (A – B)/B. This advantage, specific to Ln, the natural logarithm (log to the base *e*), adds to the following properties: additivity, symmetry and agreement between inverted units (1).

Arithmetic & geometric means

* The *arithmetic mean* M_a= (A+B)/2 between two values A and B is such that the *arithmetic difference* between either of the two values and their *arithmetic mean* are equal (but opposite):

$$(M_a–A) = – (M_a–B).$$

* The *geometric mean* $M_g = \sqrt{A*B}$ between two values A and B is such that the *geometric difference* between either of the two values and their *geometric mean* are equal (but opposite):

$$\mathrm{Ln}(M_g/B) = – \mathrm{Ln}(M_g/A).$$

The sum of geometric differences from a geometric mean is *null* also when more than two values are averaged, as for arithmetic differences and mean.

Arithmetic & geometric progression

* An *arithmetic progression* is a sequence of values (…, A_i, A_{i+1}, ...) such that $A_{i+1} = A_i + C$, thus

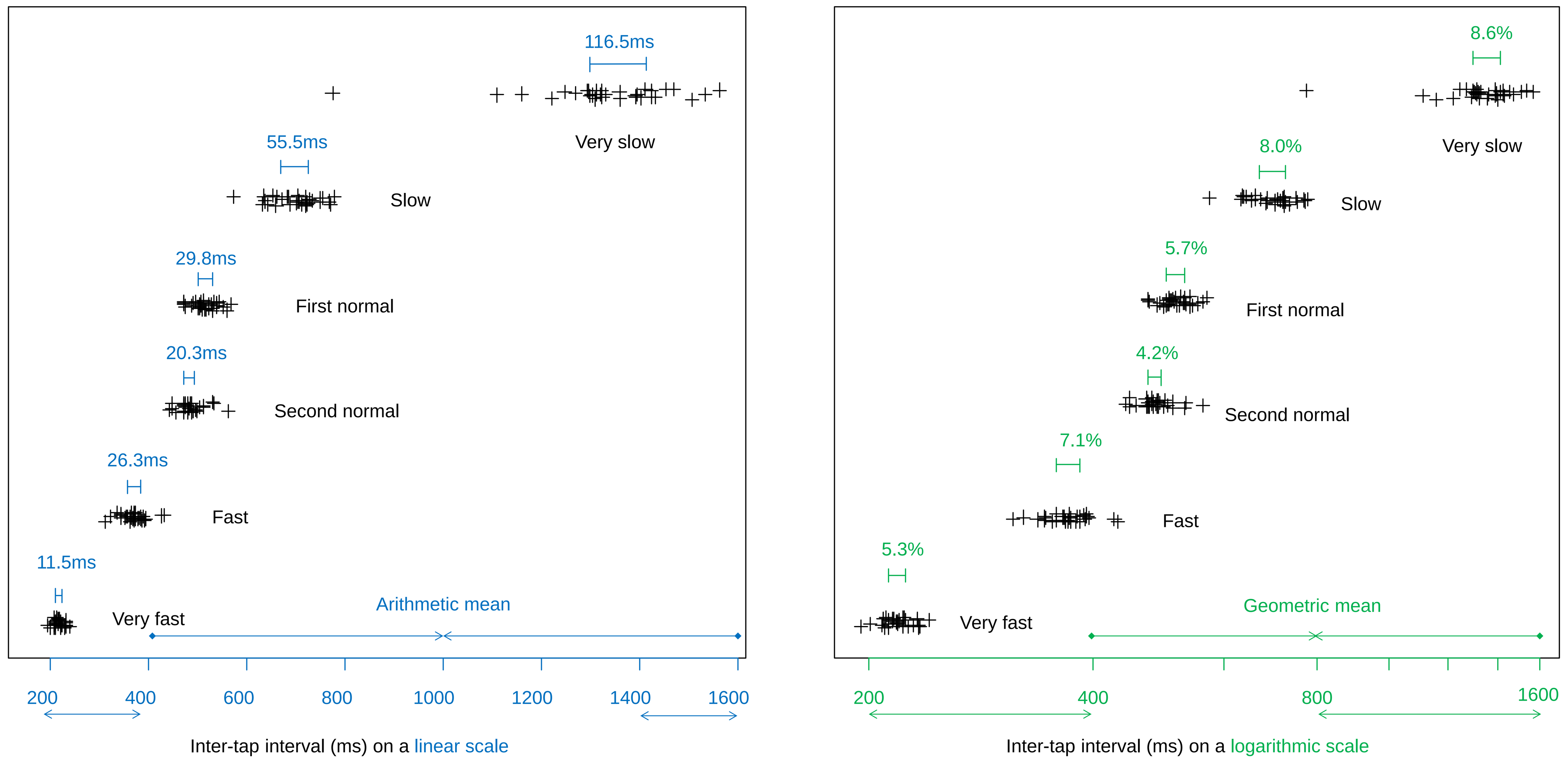
$$C = A_{i+1} – A_i.$$

Two consecutive values A_{i+1} and A_{i-1} are separated by a constant *arithmetic* difference C.

* A *geometric progression* is a sequence of values (…, G_i, G_{i+1}, …) such that $G_{i+1} = G_i*K$, K being a constant ratio.

Two consecutive values G_{i+1} and G_{i-1} are separated by a constant *geometric* difference

$$\mathrm{Ln}(K) = \mathrm{Ln}(G_{i+1}/G_i).$$



Inter-tap interval plots of isochronous finger-tapping at various paces by one participant. Each inter-quartile range is shown as *arithmetic* difference on a *linear* scale (left), and as *geometric* difference in % on a *logarithmic* scale (right).

Linear & logarithmic scales

* On a *linear* scale, an *equal* distance represents *same* dissimilarities as estimated by equal *arithmetic* differences.
* On a *logarithmic* scale, an *equal* distance represents *same* dissimilarities as estimated by equal *geometric* differences (%).

Conclusion

Relative differences are central to psychophysics. Taking the step to present the geometric difference as a percentage will facilitate comparisons between stimuli and between performances. The expression "*geometric* difference" for Ln(A/B) = Ln(A)–Ln(B) may contribute to comprehend it among many related mathematical tools.