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# Manufacturers' Coalition under a Price Elastic Market - a Quadratic Production Game Approach

Sonia Mahjoub<sup>a</sup>, Jean-Claude Hennet<sup>b,1</sup>

<sup>a</sup>LEMNA, Nantes University, Chemin de la Censive du Tertre-BP 52231,  
44322 NANTES Cedex 3, France

<sup>b</sup>LSIS, CNRS-UMR 7296, Université Paul Cézanne, Faculté de Saint Jérôme, Avenue  
Escadrille Normandie Niemen, 13397 Marseille Cedex 20, France

## Abstract

The paper addresses a supply network design problem in which a set of enterprises decides to organize itself as a multi-stage supply network through resource sharing and production coordination. A biform game theory formulation of the problem is obtained by combining the cooperative subgame in the manufacturing network with the strategic subgame of the Stackelberg type between the retailer and the manufacturers' network. As a result of the interaction between these two subgames, a new type of cooperative game, the Quadratic Production Game (QPG), is formulated to describe the supply network design problem under anticipated price elastic demands from the market. The key problem of coalitional stability is addressed through the properties of rationality and fairness of the profit sharing agreement. We prove that the game is non convex in general and thus the fair solution given by the Shapley value allocation is not always rational. The main results indicate that the coalition stability can be reinforced by minimizing the number of partner enterprises achieving the maximal expected profit and applying a profit sharing policy with guaranteed fairness restricted to the member enterprises.

*Keywords: Game Theory; Manufacturing; Enterprise networks; Cooperative Game Theory; Production Games; Biform Games; Stackelberg equilibrium.*

## 1. Introduction

In the context of international projects such as the European coordinated action CODESNET (2009), it has been observed that some networks of manufacturers have now organized themselves both internally in a cooperative manner, by sharing their products and resources, and externally as dominant strategic actors relatively to their suppliers and customers. Other empirical studies, , such as Audy et al.(2011), have stressed the fact that potential benefits and cost savings brought by resource and/or inventory pooling among enterprises must be organized and implemented and that their effectiveness critically depends on the business practices of the collaborating companies. The same authors also distinguish horizontal

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<sup>1</sup> Corresponding author. Tel.: +33491056016; fax: +33491056033  
E-mail address: jean-claude.hennet@lsis.org (J.-C. Hennet)

cooperation, vertical cooperation and lateral cooperation, which is a combination of the two, with partners at the same level (horizontal) and at different levels (vertical) of the supply chain. The manufacturing processes considered in this study are structured from the bills of materials of the final products. The structure and the flow dynamics of the supply chain can then be obtained from the MRP theory. The global efficiency of the emerging supply chain is supposed to derive from the allocation of tasks to the companies who own the most effective resources for performing them. In this view, enterprises may be complementary even if they perform tasks that are at the same level in the manufacturing process.

This study describes a supply network that consists of a price reactive market, a retailer and a manufacturing network. The term “supply network” (SN) is preferred to “virtual enterprise” (VE) simply because our analysis may apply to any types of market dynamics, rapid with agility requirements as for VE or slow with more stable business agreements. And the crucial problem of partner selection applies to supply networks as well as virtual enterprises (Crispim and Pinho de Souza, 2010). The enterprises of the manufacturing network own capital and human resources and want to make a collective use of them, to produce the quantities of products that maximize their corporate expected profit. In the recent years, many studies have been devoted to globally optimizing supply chain design problems, either in a deterministic framework (Bidhandi et al., 2009, Liu and Papageorgiou, 2013) or in stochastic or uncertain environments (Santoso et al. 2005, Baghalian et al., 2013). Here, the game theory approach allows modeling the enterprises of the network as autonomous entities with their own resources, goals and decisions. However, since it can be economically profitable for the firms to collaborate, cooperative game theory can be used to determine the maximal expected profit of the manufacturing network and the coalitions for which this profit value can be obtained. This theory also helps focusing the analysis on the coalitional stability issue, in connection with the profit allocation policy among the partners of the coalition.

In the model under study, the retailer who sells the goods on the market is considered an independent actor in the supply chain, associated with the manufacturing network through a wholesale price contract. Assuming that the manufacturing network knows the profit maximizing model of the retailer, the manufacturing network acts as a Stackelberg leader with respect to the retailer. As defined in classical game theory books (e.g. Osborne and Rubinstein, 1994), a Stackelberg game is a two-player two-stage game with perfect information in which the leader plays first. Here, the network imposes the wholesale prices of the goods produced and the retailer buys the quantities of goods that maximize his expected profit. The retailer then decides on the retail prices of the goods by anticipating the price elastic demand from

customers. It can be noted that the assumption of a single retailer is not restrictive. This model is generic and can be applied as well to a set of retailers contracting with the same manufacturing network and facing the same market.

The concept of co-opetition, coined by Brandenburger and Nalebuff (1996), can be useful to analyze such new structures of power and trade. In this paper, competition does not only emerge from the cooperative game between manufacturers. It is also the leading trend of the profit sharing mechanism between manufacturers and retailers. Biform games have been introduced by Brandenburger and Stuart (2007). They combine strategic subgames with cooperative subgames. In the context of enterprise networks, they provide a theoretical framework for describing business situations that involve strategic decisions with one or several partners and cooperative agreements between others. This hybrid model has been adopted in several SCM literatures, in particular by Anupindi et al. (2001), Plambeck and Taylor (2005), Chatain and Zemsky (2007).

Anupindi et al. (2001) analyzed a decentralized distribution system composed of independent retailers. In the first stage, before demand realization, each retailer makes its own decision on how much to order. In the second stage, after observing the demands, the retailers can cooperate by reallocating their inventories and allocating the corresponding additional profit. The authors have shown that this biform game has a non-empty core and have constructed an allocation mechanism based on dual solution and contained in the core of the game.

Plambeck and Taylor (2005) studied a model with two original equipment manufacturers (OEMs) who sell their capacity to the contract manufacturer (CM). In the first stage, the OEMs individually choose their capacity and innovation levels. In the second cooperative stage, the manufacturers pool their capacity and negotiate the allocation of the additional profit obtained from capacity pooling.

In (Chatain and Zemsky, 2007), a biform game approach is used to model a buyer-supplier relationship. First, suppliers make initial proposals and take organizational decisions. This stage is analyzed using a non-cooperative game theory approach. Then, suppliers negotiate with buyers who seek to outsource two tasks. In this stage, a cooperative game theory approach is applied to characterize the outcome of the bargaining process among the player on how to distribute the total surplus. Each supplier's share of the total surplus is the product of its added value and its relative bargaining power.

Due to their simplicity and frequent use in the industry, wholesale price contracts have been extensively studied in the literature on supply chains. In particular, Larriviere and Porteus (2001) analyzed a wholesale price only contract in a supply chain with one manufacturer and

one retailer. The manufacturer (leader) offers the terms of contract to her retailer (follower). They assumed that the retailer will accept this contract if such contract allows him to earn a non-negative profit. Under this situation, the authors studied how supply chain performance depends on the demand distribution.

Chen (2011) extended the newsvendors problem by integrating a wholesale price discount policy. In this study, the author considered a supply chain with a manufacturer acting as a Stackelberg leader and dominating the retailer. He discussed the wholesale price contract as a benchmark and proposed a contract with discount that can coordinate the supply chain. Arda and Henet (2008) also pointed at some limitations of wholesale price contracts and proposed a contract combining wholesale price and delay penalty for a better coordination of the supply chain. A more complete study on optimal contract selection is presented in Talluri and Lee (2010), but their study mainly applies to couples (manufacturer – supplier), for which a contract is generally more sophisticated than for couples (manufacturer – retailer).

Another application of the wholesale price contract with the Stackelberg model can be found in the study of Hua and Li (2008). The particularity of this study is the shifting of power from the manufacturer to the retailer who faces stochastic demand. Accordingly, the authors suppose that the retailer is in a dominant position and has the ability to influence the manufacturer's wholesale price.

In our model, the market demand for each product is supposed stochastic but its mean value curve is characterized by a constant price elasticity, and the manufacturing network is supposed to act as a Stackelberg leader with respect to the retailer. This study shows that the optimal expected profit function of the manufacturing network is a quadratic function of the expected quantities of goods sold on the market. The manufacturing network design problem then defines a quadratic production game, denoted QPG. Such a game can be viewed as an extension of the classical Linear Production Game (LPG), studied in particular by Shapley and Shubik (1972), Owen (1975) and Henet and Mahjoub (2010). However, it is shown that the properties of the QPG differ from those of the LPG, especially from the difficulty to construct for the QPG a “competitive” profit allocation policy similar to the Owen set. This new type of games is studied in terms of efficiency, rationality and fairness of payoff policies. In particular, it is shown that the core of the game, defined as the set of efficient and rational allocations policies, is not empty. However, the QPG is not a convex game in general and therefore, the fair profit allocation policy computed by the Shapley value allocation, does not always belong to the core. A modified Shapley value assignment policy is then proposed as a fair allocation policy that satisfies an additional necessary condition for rationality.

Performance evaluation appears as a critical issue for implementing the proposed collaborative scheme for the design of the multistage supply network. The gap to theoretical optimality arises from the assumed autonomy of the retailer. The paper shows how to evaluate this gap and provides an explicit expression of this gap in a particular case. A possible solution to filling this gap is to integrate the retailer as a collaborative actor of the supply chain.

The next section of the paper presents the multistage network problem. Section 3 formulates the retailer's problem of profit maximization under given wholesale prices. The manufacturers' network and the multistage manufacturing process are modelled in section 4. In section 5, the supply chain formation process is analyzed as the outcome of a biform game involving a cooperative subgame among the partners of a manufacturing network and a strategic subgame between the manufacturing network and the retailer considered as an external partner. The integrated manufacturing problem is then described as a Quadratic Production Game (QPG) and some key properties are established for this game. In section 6, the study evaluates the gap between the performance of the biform game structure under study and the global optimum of the chain. The issues of fairness and game convexity are then discussed in section 7. A new allocation policy called the restricted Shapley value solution is proposed in section 8. And finally, section 9 gives some conclusions of the study and draws some paths for future research on games and manufacturing network design problems.

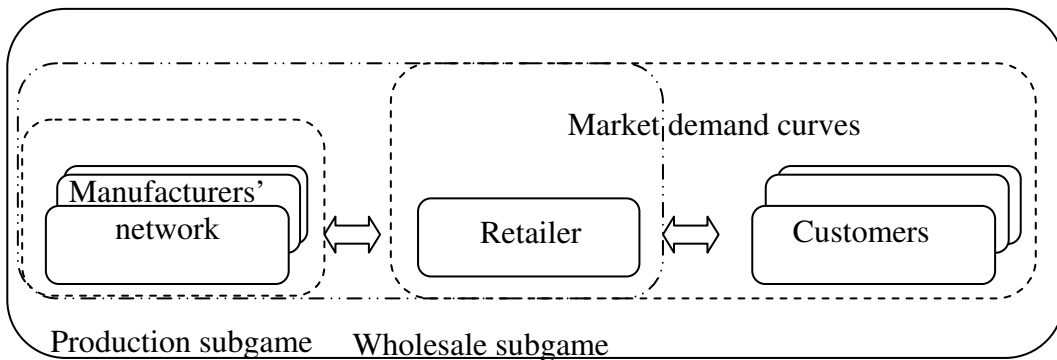
## **2. The Multistage Network Problem**

The supply chain model considered in this study focuses on a manufacturing network made of autonomous enterprises who want to collaborate to generate profit from capturing a market. The companies in the network plan to organize their activities in the form of supply chains for manufacturing goods to be sold on the market. The manufacturing network also collectively negotiates with a retailer who sells the goods on the market. The network is supposed to dominate the retailer by imposing the wholesale prices of the goods and anticipating the reaction function of the retailer, who fixes the retail prices of the goods to maximize his expected profit under prospective demand curves characterized by constant price elasticity.

The market is characterized by a price dependent expected demand curve for each end-product. These curves are supposed to be known by the retailer and by the manufacturing network in the steady state of exchanges. Figure 1 describes the model and its decomposition in terms of subgames. The wholesale subgame represents the interaction between the manufacturing network and the retailer in the form of a Stackelberg game, with the network

as the Stackelberg leader. The manufacturing subgame raises the two basic problems of cooperative games (also called TU cooperative games, where TU indicates that utilities are transferable, as it is the case for money):

- global utility maximization: determination of the maximal value function and the coalitions of players for which this value is obtained,
- allocation problem: determination of the endowments of the players by distributing the global payoff among them.



*Fig.1 A Multistage supply chain*

### 3. The retailer and the market

Consider a retailer who proposes on a market a set of products numbered  $i=1, \dots, n$  at a price vector  $p=(p_1 \dots p_n)^T$ . According to the theory of Consumer's Behavior (see e.g. Samuelson and Nordhaus, 2004), the market reacts by buying quantities of products defining vector  $y=(y_1 \dots y_n)^T$  such that a utility function  $U=F(y)$  is maximized under a budget constraint with maximal budget  $\beta: p^T y \leq \beta$ . At the equilibrium, the matrix of price elasticity of demands is:  $H = ((\frac{\partial y_i}{\partial p_j}))$ .

As it is classical in economics, we now assume that, at least locally, matrix  $H$  is constant, symmetric and negative definite. Then, its inverse,  $Q = ((\frac{\partial p_i}{\partial y_j}))$ , is also negative definite.

Under such constant price elasticities (diagonal terms of  $H$ ) and cross-elasticities (off-diagonal terms), the market game is supposed to reach a stable equilibrium for which the vector of expected demand rates  $y_i$  sold over a reference period, satisfies:

$$y = Hp + \varphi. \quad \text{with } \varphi \geq -Hp \text{ (componentwise vector inequality)}. \quad (1)$$

In stationary conditions, the retailer is supposed to know the expected market demand with respect to prices for the  $n$  products. He can then impose the prices of the goods  $p_i$  for  $i=1, \dots, n$ , so that the expected quantity sold,  $y_i$  for  $i=1, \dots, n$ , would maximize his expected profit over the reference period.

It is also assumed that the vector of wholesale prices,  $w = (w_1 \dots w_n)^T$ , is determined by the manufacturer's network, who acts as a Stackelberg leader with respect to the retailer. In addition, the retailer has to face storage costs, supposed linear in quantities, with unit cost vector  $s = (s_1 \dots s_n)^T$ . His expected profit over the reference period is:

$$\Pi^R(y) = (p - w - s)^T y = \sum_{i=1}^n (p_i - w_i - s_i) y_i \quad \text{under conditions: } p_i \geq w_i + s_i \quad \text{for } i=1, \dots, n \quad (2)$$

Then, the retailer can set the market prices of the products according to the inverse demand map (3):

$$p = Qy + \psi \quad \text{with } \psi \geq -Qy, \quad Q = H^{-1}, \quad (3)$$

The retailer's objective is to find the optimal vector  $y^*$  that maximizes  $\Pi^R$ , with:

$$\Pi^R(y) = y^T Qy + (\psi - w - s)^T y \quad (4)$$

The optimality condition takes the following form:

$$\nabla \Pi^R(y^*) = \begin{bmatrix} \frac{\partial \Pi^R}{\partial y_1}(y^*) \\ \vdots \\ \frac{\partial \Pi^R}{\partial y_n}(y^*) \end{bmatrix} = 2Qy^* + \psi - w - s = 0 \quad (5)$$

And since matrix  $Q$  is negative definite, the criterion is strictly concave and admits a single maximal solution. For each product, the optimal expected demand rate is:

$$y^* = -\frac{1}{2} H(\psi - w - s) \quad (6)$$

Accordingly, the optimal proposed retail price is derived from (3) and (6):

$$p^* = \frac{1}{2} (\psi + w + s) \quad (7)$$

From optimality condition (5), the vector of wholesale prices of products,  $w$ , to be determined by the manufacturer's network, is related to the optimal vector of expected outputs  $y^*$ , by:

$$w = \psi - s + 2Qy^* \quad (8)$$



It can be noted that that the components of vector  $w$  in (8) are decision variables, not expected values, even if they can be computed from the vector of expected output rates,  $y^*$ . From (4) and (8), the retailer's optimal expected profit can then be expressed as a quadratic function of the output vector  $y^*$ :

$$\Pi^R(y^*) = -y^{*T} Q y^*. \quad (9)$$

It can be noted that if cross elasticity of demand between the different products of the manufacturers' network is neglected, then matrix  $H = ((\frac{\partial y_i}{\partial p_j}))$  is diagonal, each diagonal term

$$H_{ii} = \frac{\partial y_i}{\partial p_i} \text{ is negative and represents the price elasticity of product } i. \text{ The inverse demand map}$$

(3) then reduces to  $n$  independent price-demand curves, and the retailer's profit maximization problem is totally decomposed by products, matrix  $Q$  also being diagonal with negative diagonal terms.

#### 4. The manufacturers' network

Consider a network of  $N$  firms represented by numbers in the set  $\mathcal{N} = \{1, \dots, N\}$ . These firms are willing to cooperate to produce commodities and set their wholesale prices. By contract, the retailer buys the products from the manufacturing network at their wholesale prices and sells them to a market.

The  $N$  manufacturers compete to be partners in a coalition  $S \subseteq \mathcal{N}$ . Each candidate enterprise is characterized by its production resources: manufacturing plants, machines, work teams, robots, pallets, storage areas, etc. Mathematically, each firm is characterized by her vector  $B^j = (B_{1j} \cdots B_{Rj})^T$ ,  $j = 1, \dots, N$  of  $R$  types of resources. These resources can be used, directly or indirectly to produce final products in quantities given by vector  $y = (y_1 \cdots y_n)^T$ .

Several authors, like Francas and Minner (2009), have explored the relations between the market properties and the manufacturing network configuration. In this study, the manufacturing process is described by the bills of materials (BOM) of the products. Each transformation of products requires resources owned by one or several companies in the network. The sequence of product transformations generates the structure of the supply chain, with the logistic organization of product flows in agreement with the manufacturing sequence

of products. The BOM technical matrix,  $T$ , is defined as follows: according to the selected manufacturing process, production of one unit of product  $i$  requires combination of components  $l \in \{1, \dots, n\}$  in quantities  $T_{li}$ . Assuming a constant lead-time at each manufacturing stage, a dynamic model of the manufacturing process can be obtained from the MRP theory, formulated in continuous time, as in (Grübbström, 1999) or in discrete time, as in (Hennet, 2003, Grubbström and Huynh, 2006). It can be noted that MRP algorithms can be easily translated into distributed ordering policies in supply networks, as shown in (Hennet, 2009). In this study, the supply chains are evaluated in stationary conditions, components being implicitly available at the right time and in the right quantity. Then, denoting  $\omega_i$  the quantity of product  $i$  produced during the reference period, the output rate vector  $y$  can be computed from the global throughput rate vector  $\omega = [\omega_1 \ \dots \ \omega_n]^T$  by the following relation:

$$y = (I - T)\omega, \quad \text{with } I \text{ the } n \times n \text{ identity matrix.} \quad (10)$$

Then, using the property that matrix  $I - T$  is an M-matrix, as defined in (Berman and Plemmons, 1979), its inverse exists and is nonnegative. Hence, the global throughput vector  $\omega$  can be computed by:

$$\omega = (I - T)^{-1}y. \quad (11)$$

Let  $m_{ri}$  be the cumulated amount of resource  $r$  necessary to produce 1 unit of product  $i$ . The resource requirements matrix is noted  $M = ((m_{ri})) \in \mathfrak{R}^{R \times n}$ . The vector of resource requirement rates,  $M\omega$ , relates to the output rate vector  $y$  in the following form:

$$M\omega = Ay \quad \text{with } A = M(I - T)^{-1}. \quad (12)$$

$A = ((A_{ri})) \in \mathfrak{R}^{R \times n}$  and  $A_{ri}$  is the total amount of resource  $r$  necessary for production of one unit of final product  $i$ .

A coalition  $S$  is defined as a subset of the set  $\mathcal{N}$  of  $N$  enterprises with characteristic vector

$$e_S \in \{0,1\}^N \quad \text{such that:} \quad \left\langle \begin{array}{l} (e_S)_j = 1 \quad \text{if } j \in S \\ (e_S)_j = 0 \quad \text{if } j \notin S \end{array} \right. \quad (13)$$

For the  $R$  types of resources considered ( $r = 1, \dots, R$ ),  $B_{rj}$  is the amount of resource  $r$  available per time unit at enterprise  $j$ ,  $B = ((B_{rj})) \in \mathfrak{R}^{R \times N}$ .

Resource capacity constraints for coalition  $S$  are thus written:

$$Ay \leq Be_S. \quad (14)$$

The coalition incurs manufacturing costs  $c = (c_1 \cdots c_n)^T$  per unit of final products and sells the products to the retailer at the wholesale price defined by vector  $w = (w_1 \cdots w_n)^T$ . The retailer acts as an intermediate party between the manufacturers' network and the final consumers.

Under the considered wholesale price contract, the manufacturers' coalition acts as the Stackelberg leader relatively to the retailer by fixing the wholesale price vector  $w$  as a take-it-or-leave-it proposal. As the follower, the retailer can only accept or reject the manufacturers' proposal. It is assumed that the retailer agrees to conclude any contract, provided that he obtains an expected profit greater than his opportunity cost which is set equal to zero by convention. After the manufacturer network has set the vector of wholesale prices,  $w$ , the retailer determines the vector of retail prices,  $p$  to maximize his expected profit. Using the expected demand curve, he is able to determine the expected rate of purchase from customers and his expected profit rate from the retail price vector. Having anticipated the retailer's reaction function (5), the coalition determines the optimal vector of wholesale prices,  $w^*$ , to maximize her expected profit. The pair of optimal vectors  $(w^*, y^*)$  can thus be determined by the manufacturers' network.

## 5. The Quadratic Production Game

In the biform game formulation of the supply chain design problem, the manufacturing network must solve two joint problems: the strategic problem of selecting the wholesale price vector  $w$ , and the cooperative problem of optimizing both the production rate vector  $y$  and the coalition characteristic vector  $e_S$ . The profit optimization problem can be formulated as follows:

$$\begin{aligned} & \text{Maximize}_{y, e_S} \Pi^M = (w - c)^T y \\ & \text{subject to } Ay \leq B e_S \\ & y \in \mathfrak{R}_+^n, e_S \in \{0, 1\}^N \end{aligned} \tag{15}$$

For given vectors  $w$  and  $c$ , problem (16) characterizes a TU cooperative game denoted  $\langle \mathcal{N}, v \rangle$  known as the Linear Production Game (LPG) and studied in (Shapley and Shubik, 1972), (Owen, 1975). In the biform game studied in this paper, variables  $w_i$  are decision variables with optimal values related to the optimal expected output rates  $y_i$  through relations (8). Acting as the Stackelberg leader in the strategic game with the retailer, the manufacturing network anticipates the optimal reaction of the retailer by substituting equation (8) into the objective function of problem (15). The obtained set of quadratic programming problems ( $P$ )

defines a quadratic production game denoted (QPG), which is different type of TU cooperative games  $\langle \mathcal{N}, v \rangle$ .

$$\begin{aligned} \text{Maximize}_{y, e_S} \Pi^M &= (\psi - s - c)^T y + 2y^T Qy \\ \text{subject to } Ay &\leq B e_S \\ y &\in \mathfrak{R}_+^n, e_S \in \{0, 1\}^N \end{aligned} \quad (P)$$

Through the assumption of exogenous prices imposed by the market, the LPG describes a competitive market situation. On the contrary, the quadratic production game (QPG) described in this paper is more appropriate to describe an oligopolistic situation in which the retailer anticipates the market reaction function to decide on the retail prices. Upstream, the manufacturing network imposes its decisions to the retailer in the form of wholesale prices.

In this context, the QPG addresses the three following issues:

- the profit maximization problem for the manufacturing network considered as a whole,
- the coalition decision problem through the choice of vector  $e_S$ ,
- the problem of profit allocation to the members of the optimal coalition.

Wholesale prices imposed to the retailer are derived from the solution of the profit maximization problem.

Consider a coalition  $S$ ,  $S \subseteq \mathcal{N}$ . The maximal profit that can be obtained by this coalition is obtained as the solution of a problem denoted  $(P_S)$ . Formally, problem  $(P_S)$  is similar to problem  $(P)$ , except for the fact that in problem  $(P)$ ,  $e_S$  is a vector of decision variables, while in problem  $(P_S)$ , vector  $e_S$  is the fixed characteristic vector of the selected coalition,  $S$ .

In terms of Cooperative Game Theory, the characteristic function of a coalition  $S \subseteq \mathcal{N}$  is the value  $v(S)$  that gives the maximal value of coalition  $S$ . Therefore,  $v(S)$  is the optimal value of criterion  $\Pi^M$  of problem  $(P_S)$ . This value is obtained for the optimal output rate vector denoted  $y_S$ .

The following results can be derived from the comparison of problems  $(P_S)$  for different coalitions  $S \subseteq \mathcal{N}$ .

**Property 1** *The grand coalition generates the optimal profit*

*Proof:* Consider the grand coalition  $\mathcal{N}$ . Its characteristic vector is  $e_{\mathcal{N}} = [1 \cdots 1]^T$ . Note that matrices  $A$  and  $B$  in  $(P_S)$  are componentwise nonnegative. For any set  $S \subseteq \mathcal{N}$ ,  $e_S \leq e_{\mathcal{N}}$  and

$Be_S \leq Be_T$ . Then, the optimal solution of  $(P_S)$  is feasible for  $(P_{\mathcal{N}})$  and the maximal expected profit, denoted  $v^*$ , can be obtained as the optimal solution of  $(P_{\mathcal{N}})$  defined by:

$$\begin{aligned} \text{Maximize}_y \Pi^M &= (\psi - s - c)^T y + 2y^T Qy \\ \text{subject to } Ay &\leq Be_{\mathcal{N}} \\ y &\in \mathfrak{R}_+^n \end{aligned} \quad . \quad (P_{\mathcal{N}})$$

It can be noted that Property 1 derives from weak monotonicity of the criterion of problem (P) with  $e_S$ . However, the QPG does not generally possess the stronger property of super-additivity. To see that the game is not super-additive, we can consider the case when the unconstrained optimal of problem (P) is reached for a coalition  $S$  strictly included in  $\mathcal{N}$ . Then,  $v(S) = v(\mathcal{N}) = v^*$ , and if  $v(\mathcal{N} - S) > 0$ , then  $v(S) + v(\mathcal{N} - S) > v(\mathcal{N})$ .

Due to Property 1, the global profit maximization problem can be solved through solving  $(P_{\mathcal{N}})$  instead of (P), with the advantage of solving a problem in which all the variables are continuous.

Additionally, the criterion of problem  $(P_{\mathcal{N}})$  is strictly concave with respect to variables  $y_i$  since it is a quadratic form with Hessian  $4Q$  definite negative. Its maximization over the convex set defined by the set of linear constraints  $Ay \leq Be_{\mathcal{N}}$  yields a unique maximum.

It can be noticed that property 1 does not mean that the grand coalition is the best strategic choice since some coalitions with a smaller cardinality than  $N$  may also yield the optimal expected profit.

**Property 2** *The QPG defined by program (P) is balanced: it satisfies the following condition:*

$$v(\mathcal{N}) \geq \sum_{S \subseteq \mathcal{N}} \lambda_S v(S) \text{ for any set of weights } \lambda_S \in [0 \ 1] \text{ for each } S \subseteq \mathcal{N} \text{ such that } \sum_{S \subseteq \mathcal{N}} \lambda_S = 1 .$$

*Proof:* The quadratic problem  $(P_{\mathcal{N}})$  is composed by a concave quadratic objective function and linear constraints taking the following form:

$$v(\mathcal{N}) = \Pi^M(y^{\mathcal{N}}) = \max_y \Pi^M(y)$$

Subject to  $Ay \leq Be_{\mathcal{N}}$

where  $y^S$  represents the vector of optimal quantities for coalition  $S \subseteq \mathcal{N}$ .

Let  $\lambda_S \in [0, 1]$  be a set of weights for each  $S \subseteq \mathcal{N}$  such that  $\sum_{S \subseteq \mathcal{N}} \lambda_S = 1$ . These weights form a

balancing set of weights and can be interpreted as the fractions of time that each player devotes to each coalition he is a member of, with a given coalition representing the same fraction of time for each player. And let  $\hat{y}_i = \sum_{S \subseteq \mathcal{N}} \lambda_S y_i^S$ ,  $\hat{y} = (\hat{y}_1 \cdots \hat{y}_n)^T$ .

First, it can be shown that  $\hat{y}_i$  verifies the resources constraints. For any resource ( $r = 1, \dots, R$ ), it holds that:

$$\begin{aligned} \sum_{i=1}^n A_{ri} \hat{y}_i &= \sum_{i=1}^n A_{ri} \sum_{S \subseteq \mathcal{N}} \lambda_S y_i^S \\ &= \sum_{S \subseteq \mathcal{N}} \lambda_S \sum_{i=1}^n A_{ri} y_i^S \leq \sum_{S \subseteq \mathcal{N}} \lambda_S \sum_{j=1}^S B_{rj} (e_S)_j \leq \sum_{j=1}^N B_{rj} \end{aligned}$$

As a consequence,  $v(\mathcal{N}) = \Pi^M(y^{\mathcal{N}}) \geq \Pi^M(\hat{y})$ .

Next, due to concavity of the objective function in problem (P),  $\Pi^M(\sum_{S \subseteq \mathcal{N}} \lambda_S y^S) \geq \sum_{S \subseteq \mathcal{N}} \lambda_S v(S)$  and thus  $v(\mathcal{N}) \geq \sum_{S \subseteq \mathcal{N}} \lambda_S v(S)$ .

Consequently, the Quadratic Production Game is balanced.

**Corollary 3** *The QPG game defined by the programming (P) has a non-empty core.*

It is recalled that, by definition, the core of a cooperative game is the set of allocation profiles  $(u_j)_{j \in \mathcal{N}}$  that are both efficient (Pareto optimal) and collectively rational (see e.g. Osborne and Rubinstein, 1994).

The allocation profile  $(u_i)_{i \in \mathcal{N}}$  is said to be efficient (or Pareto optimal) if and only if

$$u(\mathcal{N}) = \sum_{j=1}^N u_j = v^*. \quad (16)$$

The allocation profile  $(u_j)_{j \in \mathcal{N}}$  is said to be rational (or collectively rational) if and only if :

$$u(S) \geq v(S) \quad \forall S \subseteq \mathcal{N}. \quad (17)$$

It can be noted that if an allocation profile is collectively rational, it is necessarily individually rational:

$$u_j \geq v(\{j\}) \text{ for all } j \in \mathcal{N}. \quad (18)$$

Bondareva (1963) and Shapley (1967) described a necessary and sufficient condition for the non-emptiness of the core of a cooperative game. Specifically, the core is non-empty if and only if the game is balanced.

Consequently, since the quadratic production game is balanced (Property 2), it has a nonempty core.

## 6. Global performance of the biform game

### 6.1 The gap to global supply chain optimality

From objective functions (2) and (15) and relation (3), the problem of globally optimizing the expected profit of the supply chain can be written as follows:

$$\begin{aligned} & \text{Maximize } \Pi^G(y) = (p - c - s)^T y \\ & \text{subject to } Ay \leq Be_S \\ & \quad p = Qy + \psi \\ & y \in \mathfrak{R}_+^n, p \in \mathfrak{R}_+^n, e_S \in \{0,1\}^N. \end{aligned}$$

By replacing  $p$  by its expression as a function of  $y$ , the problem takes the form:

$$\begin{aligned} & \text{Maximize}_{y, e_S} \Pi^G(y) = (\psi - s - c)^T y + y^T Qy \\ & \text{subject to } Ay \leq Be_S \quad . \\ & y \in \mathfrak{R}_+^n, e_S \in \{0,1\}^N \end{aligned} \quad (P')$$

Note that the structure of problem  $(P')$  is very similar to that of problem  $(P)$ . All the properties of the QPG, in particular Property 1, also apply to the game associated with problem  $(P')$ . The global optimal profit of the supply network is the optimum of problem  $(P'_N)$ , obtained by replacing  $e_S$  in  $(P')$  by  $e_N = \mathbf{1}$ .

Clearly, the solution of problem  $(P)$  is feasible for problem  $(P')$  because it satisfies the same set of constraints. However, the objective functions of the two problems differ. Therefore, the global supply chain profit is smaller than the optimal one because the optimal output  $y^*$  for problem  $(P)$  differs from the optimal one  $y^{**}$  that solves problem  $(P')$ . Furthermore, even if the manufacturing network acts as a Stackelberg leader with respect to the retailer, the manufacturing network is unable to capture the expected profit of the retailer, which is given by (9):  $\Pi^R(y^*) = -y^{*T} Qy^*$ .

Consider the particular case when the unconstrained optimum of the manufacturing network profit function  $\Pi^M(y)$ , satisfies constraints  $Ay \leq B\mathbf{1}$ . Then, the optimality conditions of problem  $(P)$  take the form:  $4Qy^* + \psi - s - c = 0$ . The optimal output vector is then given by :

$$y^* = -\frac{1}{4} H(\psi - s - c) . \quad (19)$$

The global profit of the supply chain is then distributed among the manufacturing network and the retailer in the proportions  $2/3, 1/3$ , since

$$\Pi^M(y) = -2y^{*T} Qy^* \quad \text{and} \quad \Pi^R(y^*) = -y^{*T} Qy^* .$$

In this case, the global optimal of problem ( $P'$ ), if it also satisfies constraints  $Ay \leq B1$ , is obtained by solving the unconstrained optimality conditions for ( $P'$ ):  $2Qy^{**} + \psi - s - c = 0$  .

The global optimal output vector is then twice the output vector of the decentralized case:

$$y^{**} = -\frac{1}{2}H(\psi - s - c) = 2y^* \quad . \quad (20)$$

In this case, the global profit of the supply chain is:

$$\Pi^G(y^{**}) = -y^{**T} Qy^{**} = -4y^{*T} Qy^*$$

The suboptimality gap (loss of income) of the supply chain profit function is equal to the profit value of the retailer:  $\Pi^G(y^{**}) - \Pi^M(y^*) - \Pi^R(y^*) = -y^{*T} Qy^*$  .

Several techniques may be investigated to reduce the optimality gap. Cachon (2003), Arda and Henet (2008) and other authors have shown that the limit to the system global performance is related to the price-only contract between the retailer and the manufacturing network. Such a gap to optimality could thus be overcome by using other contracts, such as revenue sharing or penalty contracts.

## 6.2 Optimality of a direct interaction manufacturers-customers

Another option to reach optimality could be investigated. It can be noted that the global optimization problem ( $P'$ ) is exactly formulated as the manufacturing production game in which the manufacturing network also plays the role of the retailer by directly selling the products on the market, with the associated unit storage costs of end-products  $s_i$  for  $i \in \{1, \dots, n\}$ :

$$\begin{aligned} \text{Maximize}_{y, e_S} \quad & \Pi^G(y) = (\psi - s - c)^T y + y^T Qy \\ \text{subject to} \quad & Ay \leq Be_S \\ & y \in \mathfrak{R}_+^n, \quad e_S \in \{0, 1\}^N \end{aligned}$$

The following property can thus be derived.

### ***Property 4***

*If the manufacturing network also plays the role of the retailer with respect to the market, her production achieves global optimality and she captures the maximal global profit.*



This result provides strategic insights for formation and management of virtual enterprises. However, the maximal achievable degree of integration of retail with manufacturing is generally limited by business segmentation in different sectors, often imposed by national and international regulations.

## 7. The issues of Fairness and Game Convexity

From the definition of the core, any profit allocation policy in the core of the QPG is efficient and rational. Other properties can differentiate allocations. In particular, it is desirable to relate the profit share of the players to their marginal contribution to the value function.

Some definitions and properties related to fairness are now recalled from Definition 5 to Property 10. Then, a specific result for the QPG is established in Property 11.

### *Definition 5: Marginal contribution*

Classically (see e.g. Osborne and Rubinstein, 1994), the marginal contribution of player  $j$  to coalition  $S \subseteq \mathcal{N}$  with  $j \notin S$  is defined by:

$$\Delta_j(S) = v(S \cup \{j\}) - v(S) \quad (21)$$

### *Definition 6: Fairness*

A profit allocation policy, denoted  $x_j(\mathcal{N}, v)$ , is fair in  $\mathcal{N}$  if and only if it satisfies the balanced contributions property:

$$x_j(\mathcal{N}, v) - x_j(\mathcal{N} - \{k\}, v^{\mathcal{N}-\{k\}}) = x_k(\mathcal{N}, v) - x_k(\mathcal{N} - \{j\}, v^{\mathcal{N}-\{j\}}) \quad \forall j \in \mathcal{N}, \forall k \in \mathcal{N}$$

where by definition, for all  $S \subseteq \mathcal{N}$ ,  $(S, v(S))$  is the subgame of  $(\mathcal{N}, v)$  defined by  $v(T) = v(S) \quad \forall T \subseteq S$ .

### *Definition 7: Shapley value*

A particular allocation policy, introduced by Shapley (1953), has been shown to possess the properties of balance and fairness. It is called the Shapley value, and is defined by:

$$\varphi_j(\mathcal{N}, v) = \frac{1}{N!} \sum_{q \in Q} \Delta_j(S_j(q)) \quad (22)$$

for each  $j$  in  $\mathcal{N}$  where  $Q$  is the set of all  $N!$  orderings of  $\mathcal{N}$ , and  $S_j(q)$  is the set of players strictly preceding  $j$  in the ordering  $q$  ( $j$  not being included in  $S_j(q)$ ).

### *Property 8*

*The Shapley value is the only fair allocation policy in  $\mathcal{N}$ .*

This property is general. It derives from Definition 7 and from the fact that the unique value that satisfies the balanced contribution property is the Shapley value (see e.g. [23]).

The game convexity property will now be introduced to relate fairness with the core of a cooperative game.

**Definition 9: Convexity**

*A cooperative game is convex if  $\forall j \in \mathcal{N}$  and  $\forall (S, T)$  such that  $S \subseteq T \subseteq \mathcal{N} - \{j\}$ ,*

$$v(S \cup \{j\}) - v(S) \leq v(T \cup \{j\}) - v(T) \quad (23)$$

This relation means that in a convex games, each player contribution to the worth of a coalition increases as the coalition increases in size. In other words, it is more effective for a player to join a larger coalition.

As in [27, 28], the convexity property (23) can be rewritten in the following form, also known as the supermodularity property:

$$v(S \cup T) \geq v(S) + v(T) - v(S \cap T) \quad \forall S \subseteq \mathcal{N}, T \subseteq \mathcal{N} \quad (24)$$

Convex games exhibit the following important property.

**Property 10**

*In a convex game, the Shapley value is a member of the core.*

This property indicates the existence for a convex game of an allocation policy that is efficient, rational and fair. It is the Shapley value. Such properties of the allocation policy are very useful to guarantee the robust stability of the grand coalition.

Unfortunately, when a game is not convex, it is not always possible to construct a fair allocation that belongs to the core of the game. This difficulty will be illustrated by an example for the QPG.

**Property 11 Non convexity of the QPG**

*The QPG game defined problem (P) is not convex in general*

*Proof:* To show this property, it suffices to exhibit an example of a non convex QPG.

Consider the following numerical example:

$$A = \begin{bmatrix} 1 & 0.5 \\ 0.7 & 0.8 \end{bmatrix}, B = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 20 \end{bmatrix}, \beta = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \varphi = \begin{bmatrix} 20 \\ 40 \end{bmatrix}, c = \begin{bmatrix} 5 \\ 5 \end{bmatrix}. \text{ The unconstrained optimum of}$$

the QPG is:  $v^* \cong 32.29$  for  $y^* = [2.5 \ 6.25]$ . The optimal value  $v^*$  is the value of the

characteristic function for coalitions  $\{1,2,3\}$ ,  $\{1,2\}$ ,  $\{1,3\}$ , and  $v(S) = 0$  for  $S = \{1\}, \{2\}, \{3\}, \{2,3\}$ .

Thus, this QPG is not convex since property (22) does not apply, for instance, if

$$S = \{1,2\}, T = \{1,3\}.$$

In addition to non convexity, this numerical example is a case for which the Shapley value does not belong to the core of the QPG. Indeed, for this problem, the core allocation is unique:

$$(v^*, 0, 0) \text{ and the Shapley allocation is: } \left(\frac{2}{3}v^*, \frac{1}{6}v^*, \frac{1}{6}v^*\right).$$

To complete the results for this example, the vector of wholesale prices is:  $w \equiv \begin{bmatrix} 7.5 \\ 9.17 \end{bmatrix}$ , and the

$$\text{optimal vector of retail prices is: } p \equiv \begin{bmatrix} 8.75 \\ 11.25 \end{bmatrix}.$$

It is then possible to differentiate coalitional rationality (relation (17), not verified in general) from individual rationality (18). Finally, the manufacturers' game can be solved in a fair, efficient and individually rational manner through the following steps:

- Solve problem  $(P_{\mathcal{N}})$  to obtain the maximal profit and the optimal output vector  $y^*$ ,
- Set the wholesale price vector  $w$  computed by (8),
- Set the market price vector  $p$  computed by (7),
- Compute the Shapley value allocation (22) to allocate the expected profit among the partners.

Computation of the Shapley value allocation requires computing the solution of all the problems  $(P_S)$  for  $S \subseteq \mathcal{N}$ , and this, of course, can be very time consuming for large sets of manufacturing partners.

## 8. A restricted Shapley value solution

### 8.1 How can we deal with the non-convexity issue?

The results of the preceding sections indicate that when the QPG is convex, the grand coalition is optimal and can be stabilized by the Shapley value profit allocation policy, that is efficient, rational and fair.

The non-convexity issue raises 2 questions:

- 1 In what cases the QPG is non-convex?
- 2 What compromise can be found between rationality and fairness?

To answer the first question, it is interesting to mention the results of Aumann (1964) related to purely competitive markets. Such markets can be represented as cooperative games having the property that for a large number of firms, the core of the game coincides with the set of competitive equilibria. To show this result, Aumann introduced the idealized concept of replicated agents, who own the same bundle of resources. In any core allocation, identical agents should receive the same share of the profit. More generally, if two agents own the same resource and if this resource is not scarce, the two agents do not receive any reward for this resource in any core allocation policy. In this statement, the definition of non-scarcity may vary with the problem formulation. In Linear Production Games, a resource is not scarce if its marginal price is 0, that is if it is in excess in the optimal production plan (Hennet and Mahjoub, 2010).

For Quadratic Production Games, consider the set of all the coalitions that achieve the maximal profit value :  $O = \{S \subseteq \mathcal{N}; v(S) = v^*\}$ . The following property can be exhibited.

**Property 12**

*A necessary condition for an allocation policy  $(u_j)_{j \in \mathcal{N}}$  to belong to the core of the QPG is:*

$$u_j = \mathbf{0} \quad \forall j \in \mathcal{N}; \exists S \in O, j \notin S . \quad (25)$$

*Proof:* Suppose  $u_j > \mathbf{0}$  and  $\exists S \in O, j \notin S$ . Then  $\sum_{j \in S} u_j \leq v^* - u_j < v^*$ . Then, the allocation policy is not efficient in  $S$ , which contradicts the assumption that it belongs to the core of the game.

In the set  $O$ , the coalition with minimal cardinality may be unique or not. But it is always possible to select one subset,  $S^* \in O$  with minimal cardinality, denoted  $s^*$ , so that:

$$\begin{cases} v(S^*) = v^* \\ \text{Card}(S^*) = s^* \end{cases} \quad (26)$$

It can be noted that for a Shapley value allocation, denoted  $(x_j)_{j \in \mathcal{N}}$ , any player in  $\mathcal{N}$  receives a positive allocation, provided that he is not a dummy player:  $x_j > \mathbf{0}$  if  $\exists S \in \mathcal{N}, \Delta_j(S) > \mathbf{0}$ .

For the QPG, this property is one of the main reasons for the Shapley allocation not to belong to the core of the game.

Then, as a possible answer to question 2, a compromise between fairness and rationality can be found by restricting the game to a set  $S^*$  with minimal cardinality as defined above (26).

The cooperative game under study becomes  $\langle S^*, v \rangle$  in place of  $\langle \mathcal{N}, v \rangle$ . For this new game, the

grand coalition,  $S^*$ , is the only coalition achieving the maximal profit value. A possible allocation policy can then be the Shapley value allocation for the restricted game  $\langle S^*, v \rangle$ :

$$\begin{cases} \psi_j(S^*, v) = \frac{1}{S^{*!}} \sum_{q \in Q} \Delta_j(S_j(q)) & \text{if } j \in S^* \\ \psi_j(S^*, v) = 0 & \text{if } j \notin S^* \end{cases} \quad (27)$$

for each  $j$  in  $S^*$  where  $Q$  is the set of all  $S^{*!}$  orderings of  $S^*$ , and  $S_j(q)$  is the set of players (strictly) preceding  $j$  in the ordering  $q$ .

By construction, the proposed allocation policy (27) is efficient. It is also fair in  $S^*$ . But in general, it is not fair in  $\mathcal{N}$  if  $S^* \neq \mathcal{N}$ . Policy (27) is individually rational in  $S^*$  but collective rationality in  $S^*$  has to be tested, Property 12 being only a necessary condition for collective rationality in  $S^*$ . In a similar manner, convexity of the QPG restricted to  $\langle S^*, v \rangle$  is more likely to be verified than for  $\langle \mathcal{N}, v \rangle$ , but it is not guaranteed and has to be tested.

## 8.2 A numerical example

In the case of the example above,  $S^* = \{1, 2\}$  or  $S^* = \{1, 3\}$  can be indifferently selected as optimal coalitions with minimal cardinality. Suppose, for example that coalition  $S^* = \{1, 2\}$  is selected. Then, the studied TU cooperative game is  $\langle S^*, v \rangle$  in place of  $\langle \mathcal{N}, v \rangle$ . The restricted QPG  $\langle S^*, v \rangle$  has the same optimal solution than  $\langle \mathcal{N}, v \rangle$ :  $v^* \cong 32.29$  for  $y^* = [2.5 \ 6.25]$ . But it can be easily verified that it is convex and that the restricted Shapley value allocation:

$$\psi_1(S^*, v^*) = \psi_2(S^*, v^*) = \frac{v^*}{2}$$

is fair, efficient and rational for the TU game  $\langle S^*, v \rangle$ .

Similar results are obtained when selecting  $S^* = \{1, 3\}$ , by simply replacing index 2 by index 3.

## 8.3 Discussion

The restricted Shapley value policy has been proposed to eliminate one of the major causes of non-rationality of the global Shapley policy. If a firm owns resources useful in some but not all the possible coalitions to which she belongs, then its Shapley value is strictly positive but its profit allocation for any core allocation policy should be null. To escape this contradiction between fairness and stability, it is proposed to exclude such a firm from the partnership, since she could not be fairly rewarded anyway. Having selected an optimal coalition with minimal cardinality, some causes of non-convexity may remain

Some managerial insights can be drawn from this study, to improve the design process of supply networks. In particular, the coalition stability requirement stated in Property 12 implies

that owners of non-scarce resource should not receive a share of the common profit. As a consequence, it may be profitable for a firm to join an enterprise network only if she owns resources useful to produce the products that are globally the most profitable ones and if these resources are sufficiently scarce among the other partners. Another observation, derived from the assumption of a price elastic market, is that the retailer is always able to generate a positive profit for himself for any set of wholesale prices imposed by the manufacturing network.

## **9. Conclusions**

In the study reported in this paper, a supply network design problem has been studied as a biform game involving a network of manufacturing companies and a retailer facing a market with price dependent demands for the manufactured goods.

The manufacturing network organizes the production resources in the most profitable manner and imposes the wholesale prices of the goods to the retailer. This situation has generated a cooperative game called QPG (Quadratic Production Game), not yet studied in the literature. The paper evaluates the performance gap between the global optimum of the system and the game equilibrium between the network and the retailer. This gap can be interpreted as the cost of the retailer's autonomy with respect to the manufacturing network.

It has been shown that in general, the QPG is not convex and therefore coalitional rationality and fairness of the profit allocation policy are not always compatible. To reach a compromise between fairness and rationality, it has been proposed to limit the coalition to the minimal number of partners able to produce the optimal output, and to distribute the expected profit among them. The particular profit allocation policy proposed has been called the "restricted Shapley value solution". A practical requirement for implementation of this policy in an open enterprise network would be the introduction of an additional mechanism, such as a tariff that would deter participants to enter or leave the coalition. Other extensions of the presented work are also under investigation. One possibility is to propose a contract for a better coordination between the manufacturing network and the retailer. Another interesting problem to be studied is the reverse case in terms of leadership, when it is not the manufacturing network but the retailer who is the Stackelberg leader. Such an assumption would give rise to a different biform game in which the reaction function of the manufacturers' network cannot be easily expressed analytically.

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