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**Models and visual thinking in physical applications of differential
equation theory: three case studies during the period 1850–1950
(Bashforth, Størmer, Lemaître)**

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This paper is organized around three important works in applied mathematics that took place in the century 1850–1950: Francis Bashforth (1819–1912) on capillary action [1], Carl Størmer (1874–1957) on polar aurora [4], Georges Lemaître (1894–1966) on cosmic rays [3]. I have chosen these three figures for several reasons: they were applied mathematicians with strong theoretical training; they studied complex physical problems for which they had to create new numerical methods at the limit of the human and technical possibilities of their time; there is a natural continuity in their works, each being partially inspired by the previous one; finally, these works present the same characteristics as what we call today mathematical modeling and computer simulation.

Francis Bashforth was fellow at St. John’s College at Cambridge and later professor of mathematics at the Royal Military Academy of Woolwich. Between 1864 and 1880 he developed important experimental and theoretical research on ballistics. Before and after his professional engagement in artillery, he was also interested in capillary action. In this domain, his major aim was to compare the measured forms of drops of fluid resting on a horizontal plane, obtained by experiment, with the theoretical forms of the same drops as determined by the Laplace differential equation of capillarity.

In his research, Bashforth used, on the one hand, a new measurement process involving a micrometer of his invention and, on the other hand, a new method of numerical integration of differential equations involving finite differences of the fourth order and efficient quadrature formulas, conceived with the help of the famous astronomer John Couch Adams [5]. Bashforth, with his assistants, computed 32 integral curves, each of them with 36 points. Knowing that five auxiliary values were necessary for each point of the curve, we arrive at the total of more than 5000 numbers to be calculated. The calculation time can be estimated to at least 500 hours. The coincidence of the curves obtained by the experimental method and the numerical one was excellent and could be viewed as a mutual validation of the two approaches of the given capillary problem.

In Bashforth’s work, we may distinguish different levels of representation of the physical phenomenon concerned. Experimentation and measurement lead to what I call an “experimental model” of the forms of drops. In parallel, the mathematization of the problem gives birth to what we would call today a “mathematical

model”. This model is non-operative because we cannot integrate the differential equation analytically, so it is necessary to discretize this equation to obtain a “numerical model”. This process of discretization is not a simple translation. It would be an error to consider the continuous mathematical model and the discrete numerical model as being obviously equivalent. In fact, a discretization process often introduces significant changes in the informational content of the original model, because a numerical algorithm may be divergent, may suffer from numerical instability, and may be unadapted to the available instruments of calculation.

Carl Størmer, the second character in my story, was a Norwegian mathematician trained in Kristiania, Paris and Göttingen. For many years until his retirement, he was professor of mathematics at Kristiania University. Up to his death, the major part of his research was devoted to the study of the curious phenomenon of polar aurora, called also “aurora borealis” or “northern lights”, on which he published almost 150 papers.

Understanding that polar auroras are caused by electrically charged particles coming from outer space, Størmer decided to determine the trajectories of these particles under the action of terrestrial magnetism. In order to track these trajectories step by step from the Sun to the Earth, he had to develop new techniques of numerical integration of differential equations, inspired by those of Adams-Bashforth and British astronomers, but best suited to his specific problem. With his students, he calculated a multitude of different trajectories during three years. He himself estimated that this huge task required more than 5000 hours of work.

After that, Størmer and his assistants constructed several wire models to visualize the numerical tables issued from the calculations. These material models showed that the charged particles coming from the Sun concentrate around the polar circle, in accordance with observation. These models also explained in a convincing way why the northern lights can appear on the night side of the Earth, at the opposite of the Sun.

A few years before, a Størmer’s colleague, Kristian Birkeland, professor of physics at Kristiania University, had realized a physical simulation of the polar aurora. For that, he was sending cathode rays through an evacuated glass container against a small magnetic sphere representing the Earth, which he called “terrela”. Birkeland’s simulations showed two illuminated bands encircling the poles, in agreement with the behavior of northern lights and also with the computed trajectories obtained later by Størmer.

Finally, the physical phenomenon of polar aurora has been studied by three ways. First, by direct observations and measurements, secondly by Birkeland’s simulation, which we can consider as an “analog model”, and thirdly by Størmer’s mathematization with a continuous mathematical model consisting in a system of differential equations, a numerical model obtained by discretization and a wire material model representing concretely the trajectories. The coherence of the results obtained by these three approaches validates strongly the initial hypothesis of charged particles deviated by terrestrial magnetism.

My third and last part is devoted to the astrophysicist Georges Lemaître and his research on cosmic rays. At this time, an important problem addressed by Millikan was to explain the origin and nature of the cosmic rays detected by balloons or mountain observatories. There were two rival conceptions of these cosmic rays, one principally advocated by Millikan and the other by Arthur Compton. While Millikan held the rays to consist of high-energy photons, Compton and his collaborators argued that they were charged particles of extragalactic origin. Lemaître was interested in these cosmic rays because he saw in them the fossil traces of his “Primeval Atom hypothesis”, an ancestor of the Big Bang theory, so he wanted to prove the validity of Compton’s conception. In collaboration with the Mexican physicist Manuel Sandoval Vallarta, Lemaître engaged in complicated calculations of the energies and trajectories of charged particles in the Earth’s magnetic field.

At first, Lemaître and Vallarta tried to integrate numerically the differential equations of the trajectories with the Adams-Bashforth method, but this was not convenient. Later, they discovered the Størmer method in the literature and began to use it, but the calculations were very tedious to perform. Finally they thought of the differential analyzer constructed by Vannevar Bush at the MIT [2]. A differential analyzer is a mechanical analog machine conceived for the integration of differential equations. It is constituted by algebraic mechanisms that perform the algebraic operations and mechanical integrators that realize the integrations. Once suitably prepared, the machine is in exact correspondence with the given differential equation and when it moves from an initial given state, it traces exactly an integral curve of this equation.

For the use of the differential analyzer, Lemaître and Vallarta were helped by Samuel Hawks Caldwell, an assistant of Bush who managed the differential analyzer for the specific problem of cosmic rays. Thanks to this instrument, they could obtain hundreds of trajectories within a reasonable time. In this third situation, we find again the notions of experimental, mathematical and numerical models already analyzed in Basforth’s and Størmer’s researches, but the novelty is in the role played by the differential analyzer: this instrument being a mechanical analog model of the differential equation, it appears also, indirectly, as an analog model of the physical phenomenon of cosmic rays.

In the three situations we have studied, we encountered several representations – experimental, analog, mathematical, numerical, graphical, material – of a physical phenomenon that validate each other through the consistence and coherence of their results. Each of them brings specific information about the real phenomenon. In fact, these representations make sense when they are considered together, so I am tempted to say that this is this system of representations considered as a whole which constitutes a “model” of the phenomenon. Concretely, we can only reason and calculate in this multifaceted model, whereas the reality of the phenomenon remains definitively hidden.

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