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Discretization of space and time in wave mechanics: the validity limit

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Abstract

Assuming that space and time can only have discrete values, it is shown that wave mechanics must necessarily have a specific applicability limit: in a discrete context, unlike in a continuous one, frequencies can not have arbitrarily high values.

1. Introduction

Let's assume, as work hypothesis, the existence of both discrete space and discrete time, namely spatial and temporal intervals not further divisible; this assumption leads to some interesting consequences. Here, as a first result, we find the limit of applicability of wave mechanics (and consequently also of quantum mechanics).

So, if we suppose that neither space nor time are continuous, but that instead both are discrete, it will not be possible to establish arbitrary values for a spatial coordinate or the time: any length must be an integer multiple of the fundamental length and any time interval must be an integer multiple of the fundamental time. Let's name l_0 the fundamental length and t_0 the fundamental time.

2. The applicability limit of wave mechanics

The values of l_0 and t_0 are not essential, at least for the moment: what matters is to establish the principle that $l_0 > 0$ and $t_0 > 0$.

Of course, if l_0 is the minimum length, neither a wavelength can be less than it; so l_0 is also the minimum wavelength. For an electromagnetic wave, as shown by the relation $\lambda\nu = c$, at the minimum wavelength $\lambda = l_0$ corresponds the maximum frequency ν_{max} , if we want to keep c constant. Then, if ν is the frequency of an electromagnetic wave, it must be $\nu \leq \nu_{max}$; if we consider $\lambda = l_0$ (and consequently $\nu = \nu_{max}$) we have $l_0 \nu_{max} = c$, from which we obtain:

$$\nu_{max} = \frac{c}{l_0} \quad (1)$$

It is evident that in a continuous context ($l_0 \rightarrow 0$) there is no limit for the frequency.

The frequency can be expressed as $\nu = 1/T$ where T is the period; the maximum value for the frequency is reached when T assumes the minimum value t_0 .

So from Eq. (1) we obtain:

$$\frac{l_0}{t_0} = c \quad (2)$$

that indicates the relationship between l_0 and t_0 .

The energy associated to a wave is $E = h\nu$, so for Eq. (1) there is also an upper limit for the energy a wave can have. In particular:

$$E_{max} = h \nu_{max} = \frac{hc}{l_0} \quad (3)$$

Also for energy, in a continuous context ($l_0 \rightarrow 0$) there is no upper limit.

The relation energy-mass $E = mc^2$ allows us to write

$$E_{max} = h \nu_{max} = m_{max} c^2 = \frac{hc}{l_0} \quad (4)$$

obtaining

$$m_{max} = \frac{h}{cl_0} \quad (5)$$

that represents the maximum value for the mass that in a discrete context can be treated from the point of view of wave mechanics. Again, also for mass, in a continuous context ($l_0 \rightarrow 0$) there is no upper limit.

We can obtain the same result starting from the expression^{[1][2]} of the Compton wavelength $\lambda_c = h/mc$ and imposing $\lambda_c = \lambda_{min} = l_0$ and $m = m_{max}$.

3. Conclusion

The assumption that both space and time are discrete has led to find the applicability limit of wave and quantum mechanics: in a discrete context (contrary to what happens in a continuous context) no object, having mass greater than m_{max} , as expressed by Eq. (5), can be treated from the point of view of wave or quantum mechanics.

References

[1] Paul S. Wesson and James M. Overduin “A scalar field and the Einstein vacuum in modern Kaluza-Klein theory” [2012]. Available at <https://arxiv.org/pdf/1209.5703.pdf>

[2] Subir Ghosh “Planck Scale Effect in the Entropic Force Law” [2011]. Available at <https://arxiv.org/pdf/1003.0285.pdf>