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Space vehicle design taking into account multidisciplinary couplings and mixed epistemic / aleatory uncertainties

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Abstract

Space vehicle design is a complex process involving numerous disciplines such as aerodynamics, structure, propulsion and trajectory. These disciplines are tightly coupled and may involve antagonistic objectives that require the use of specific methodologies in order to assess trade-offs between the disciplines and to obtain the global optimal configuration. Generally, there are two ways to handle the system design. On the one hand, the design may be considered from a disciplinary point of view (a.k.a. Disciplinary Design Optimization): the designer of each discipline has to design its subsystem (*e.g.* engine) taking the interactions between its discipline and the others (interdisciplinary couplings) into account. On the other hand, the design may also be considered as a whole: the design team addresses the global architecture of the space vehicle, taking all the disciplinary design variables and constraints into account at the same time. This methodology is known as Multidisciplinary Design Optimization (MDO) and requires specific mathematical tools to handle the interdisciplinary coupling consistency.

In the first part of this chapter, we present the main classical techniques to efficiently tackle the interdisciplinary coupling satisfaction problem. In particular, an MDO decomposition strategy based on the "Stage-Wise decomposition for Optimal Rocket Design" formulation is described. This method allows the design process to be decentralized according to the different subsystems (*e.g.* launch vehicle stages) and reduces the computational cost compared to classical MDO methods.

Furthermore, when designing an innovative space vehicle including breakthrough technologies (*e.g.* launch vehicle with new kind of propulsion, new aerodynamics configuration), one has to cope with nu-

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merous uncertainties relative to the involved technology models (epistemic uncertainties) and the effects of these on the global design and on the interdisciplinary coupling satisfaction. Moreover, aleatory uncertainties inherent to the physical phenomena occurring during the space vehicle mission (*e.g.* solar fluxes, wind gusts) must also be considered in order to accurately estimate the performance and reliability of the vehicle. The combination of both epistemic and aleatory uncertainties requires dedicated techniques to manage the computational cost induced by uncertainty handling.

The second part of this chapter is devoted to the handling of design process in the presence of uncertainties. Firstly, we describe a design methodology that enables to define the design rules (*e.g.* safety factors) taking both aleatory and epistemic uncertainties into account. Secondly, we present new MDO methods that allow to decompose the design process while maintaining the interdisciplinary functional coupling relationships between the disciplines in the presence of uncertainties.

Keywords: Multidisciplinary Design Optimization, Launch vehicle design, Aleatory / Epistemic uncertainties

Nomenclature

$\mathbf{z}$	Design variable vector
$\mathbf{Y}$	Input coupling variable vector
$\mathbf{U}$	Uncertain variable vector
f	Objective function
$\mathbf{g}$	Inequality constraint vector
$\mathbf{h}$	Equality constraint vector
$\mathbf{c}$	Coupling function vector
Ξ	Objective function uncertainty measure
$\mathbf{K}$	Inequality function uncertainty measure
$\mathbb{E}$	Expected value
σ	Standard deviation
ϕ	Joint Probability Density Function (PDF)
$\boldsymbol{\theta}_{\boldsymbol{Y}}$	Parameter vector of the uncertain coupling variables $\mathbf{Y}$
$\hat{y}$	Polynomial Chaos Expansion based surrogate model of the coupling y
$\boldsymbol{\alpha}$	Polynomial Chaos Expansion coefficient vector
J	Interdisciplinary coupling constraint
s	Safety margin
$\widehat{m}$	Low fidelity model

1 Introduction

Aerospace vehicle designs are long term projects (often around 10 years) involving important budgets and requiring a dedicated design organization. NASA and ESA [62] stress the need to reduce the cost and to increase the effectiveness of space missions and satellite launches. Improving the design process for aerospace vehicles is essential to obtain low cost, high reliability, and effective launch capabilities [12]. This design is a complex multidisciplinary optimization process: the objective is to find the vehicle architecture and characteristics that provide the optimal performance [34] while satisfying design requirements and ensuring a certain level of reliability. The slightest mistake in the design process may induce economical, material and human disastrous consequences (*e.g.* explosion of the Brazilian VLS launch vehicle in 2002).

As a representative example of aerospace vehicles, the design of launch vehicles involves several disciplines (*e.g.* propulsion, aerodynamics, trajectory, mass and structure) and is customarily decomposed into interacting submodels (Fig. 1). Each discipline may rely on computing-intensive simulations such as Finite Element analyses for the structure discipline or Computational Fluid Dynamics analyses for the aerodynamics discipline. The aerospace vehicle performance estimation which results from flight performance, safety, reliability and cost, requires coupled disciplinary analyses. The different disciplines are a primary source of trade-offs due to the antagonist disciplinary effects on launcher performance.

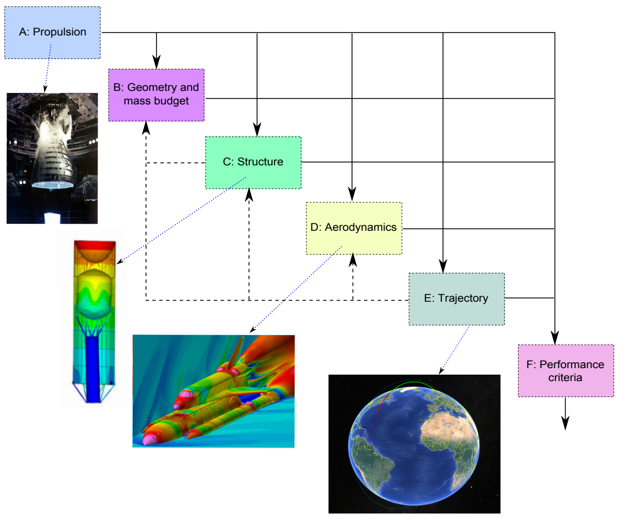


Figure 1: Example of launch vehicle analysis process of interacting submodels

Two approaches to handle system design may be distinguished:

- **Disciplinary Design Optimization** (DDO). The designer of each discipline has to design its subsystem (*e.g.* propulsion system) taking the interactions between its discipline and the others (interdisciplinary couplings) into account through specifications that can take the form of simulation parameters or optimization constraints that will be updated at each iteration. The process generally consists of loops between different disciplinary optimizations (Fig. 2). At each iteration of this loop, each discipline is re-processed based on the updated data from the previous discipline optimizations.

This approach is particularly suited to the design process of industrial companies which is often broken down according to the different engineering team expertise. However, the difficulty of this approach lies in the handling of other discipline interactions with the designed discipline in the global optimization process.

- **Multidisciplinary Design Optimization** (MDO). MDO deals with the global design problem as a whole by taking advantage of the inherent synergies and couplings between the disciplines involved in the design process (Fig. 3) to decrease the computational cost and/or to improve the quality of the optimal design [54]. Unlike the sequential disciplinary optimizations, the interactions between the disciplines are directly incorporated in the MDO methods [7]. However, the complexity of the problem is significantly increased by the simultaneous handling of all the disciplines.

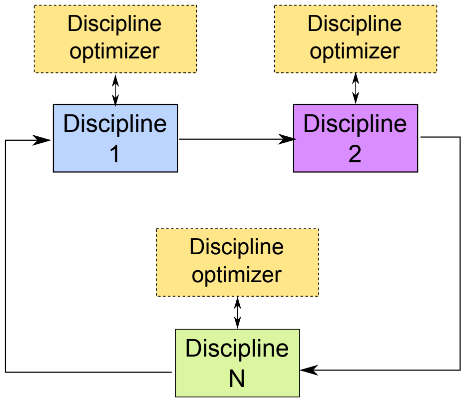


Figure 2: DDO design process

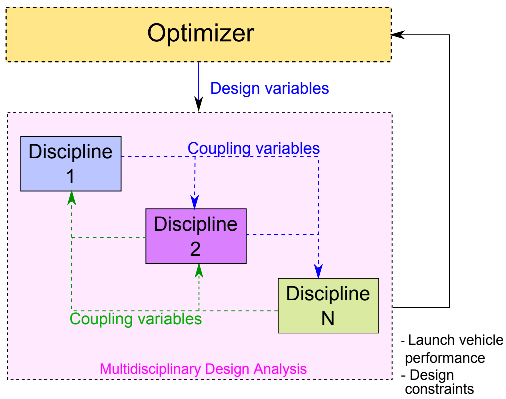


Figure 3: Example of MDO design process

In the next sections, these two approaches are discussed within the context of Launch Vehicle Design (LVD). In Section 2, the main classical techniques to efficiently tackle the interdisciplinary coupling satisfaction in MDO problems are introduced. In particular, an MDO decomposition strategy based on the "Stage-Wise decomposition for Optimal Rocket Design" (SWORD) is described. This method allows the design process to be decentralized according to launch vehicle stages and reduces the computational cost compared to classical MDO methods.

Then, in Section 3, the handling of uncertainty in the design process is discussed. First, a methodology to define design rules (*e.g.* safety margins) taking epistemic and aleatory uncertainties into account is described. Then an approach allowing to ensure multidisciplinary feasibility in the presence of uncertainty for space vehicle design while reducing the computational cost is presented and compared to classical uncertainty-based MDO methods.

2 MDO decomposition strategy for launch vehicle design

In the aerospace industry, a new system follows a typical development process involving several specific phases (Conceptual design, Preliminary design, Detailed design, Manufacturing) [12] (Fig. 4). For an aerospace vehicle, the conceptual design phase is decisive for the success of the whole design process.

It has been estimated that at least 80% of the life-cycle cost of a vehicle is locked in by the chosen concept during the conceptual phase [12]. The design space at this early design phase is large since few characteristics of the system are fixed, and traditional design approaches lead to freeze some system characteristics to focus only on alternatives selected by experts [61]. MDO techniques are useful for the conceptual design phase since they are able to handle large design spaces in a multidisciplinary environment. In [41], the authors mention that the global system performance can be enhanced by using MDO at early design phases, and design cycle duration and cost can be decreased.

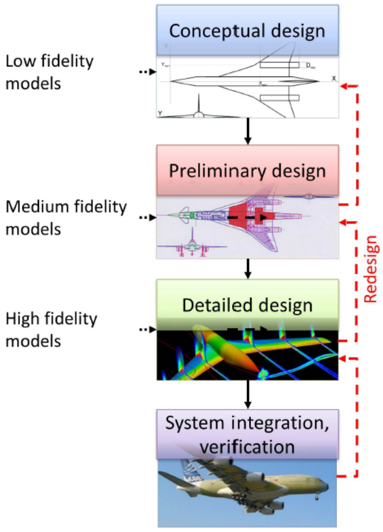


Figure 4: Phases of design process

To overcome the complexity induced by handling all the disciplines at the same time in the system design process, various MDO formulations have been developed. In the 90's, several surveys classed MDO formulations into two general types of architectures: single-level methods [10, 21], and multi-level methods [3, 38]. Multi-level approaches introduce disciplinary level optimizers in addition to the system-level optimizer involved in single-level methods, in order to facilitate the MDO process convergence.

2.1 General MDO formulation and review of main MDO approaches

A general single-level MDO problem can be formulated as follows [9]:

$$\min \quad f(\mathbf{z},\mathbf{y},\mathbf{x}) \qquad [1]$$

$$\text{w.r.t.} \quad \mathbf{z},\mathbf{y},\mathbf{x}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{z},\mathbf{y},\mathbf{x}) \leq 0 \qquad [2]$$

$$\mathbf{h}(\mathbf{z},\mathbf{y},\mathbf{x}) = 0 \qquad [3]$$

$$\forall (i,j) \in \{1,...,N\}^2 \; i \neq j, \; \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i,\mathbf{y}_{.i},\mathbf{x}_i) \qquad [4]$$

$$\forall i \in \{1,...,N\}, \; \mathbf{r}_i(\mathbf{z}_i,\mathbf{y}_{.i},\mathbf{x}_i) = 0 \qquad [5]$$

$$\mathbf{z_{min}} \leq \mathbf{z} \leq \mathbf{z_{max}} \qquad [6]$$

All the variables and functions are described in the following. Three types of variables are involved in a deterministic MDO problem:

- $\mathbf{z}$ is the design variable vector. The design variables evolve all along the optimization process in order to find their optimal values with respect to the optimization problem. Design variables may be shared between several disciplines ($\mathbf{z}_{sh}$) or specific to the discipline i ($\bar{\mathbf{z}}_i$). We note $\mathbf{z}_i = \{\mathbf{z}_{sh}, \bar{\mathbf{z}}_i\}$ the input design variable vector of the discipline $i \in \{1, ..., N\}$ with N the number of disciplines and $\mathbf{z} = \bigcup_{i=1}^{N} \mathbf{z}_i$ without duplication. Typical design variables involved in aerospace vehicle design are stage diameters, pressures in the combustion chambers, propellant masses, *etc.*

- In a multidisciplinary environment, the disciplines exchange coupling variables, $\mathbf{y}$ (Fig. 5). The latter link the different disciplines to model the interactions between them. $\mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}, \mathbf{x}_i)$ is a coupling function used to compute the *output coupling* variable vector which is calculated by discipline i and input to discipline j. $\mathbf{y}_{.i}$ refers to the vector of all the *input coupling* variables of discipline i and $\mathbf{y}_{ij}$ is the *input coupling* variable vector which is input to discipline j and output from discipline i. We note $\mathbf{y} = \bigcup_{i=1}^{N} \mathbf{y}_{.i} = \bigcup_{i=1}^{N} \mathbf{y}_{i.}$ without duplication. From the design variables and the input coupling variables to the discipline i, the output coupling variables are computed with the coupling function: $\mathbf{c}_{i.}(\mathbf{z}_i, \mathbf{y}_{.i}, \mathbf{x}_i)$ and $\mathbf{y}_{i.} = (\mathbf{y}_{i1}, \ldots, \mathbf{y}_{iN})$ is the vector of the outputs of discipline i and the input coupling variable vector to all the other disciplines.

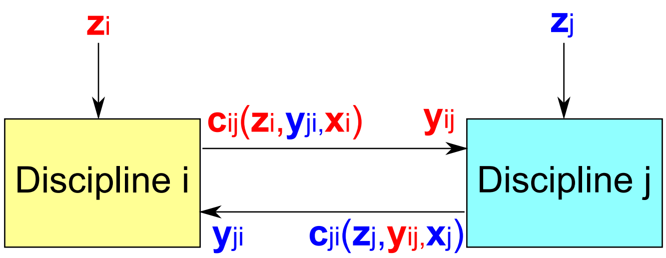


Figure 5: Couplings between the discipline i and the discipline j

For example, the sizing discipline computes the launch vehicle dry mass which is transferred to the trajectory discipline for a simulation of the launch vehicle flight. Another example is the classical aero-structure analysis (Fig. 6) [20, 24, 35]. For a launch vehicle, aero-structure analysis involves coupled analyses between aerodynamics discipline (which requires the launch vehicle geometry and the deformations) and the structure discipline (which requires the aerodynamics loads on the launch vehicle structure). For coupled systems, it is important to keep in mind that their design involves goals which are often conflicting with each other, for instance reducing weight may lead to higher stresses and the global optimum is a compromise between all the different disciplinary objectives.

- $\mathbf{x}$ is the state variable vector. Unlike $\mathbf{z}$, the state variables are not independent degrees of freedom but depend on the design variables, the coupling variables $\mathbf{y}$ and the state equations characterized by the residuals $\mathbf{r}(\cdot)$. These variables are often defined by implicit relations that require specific

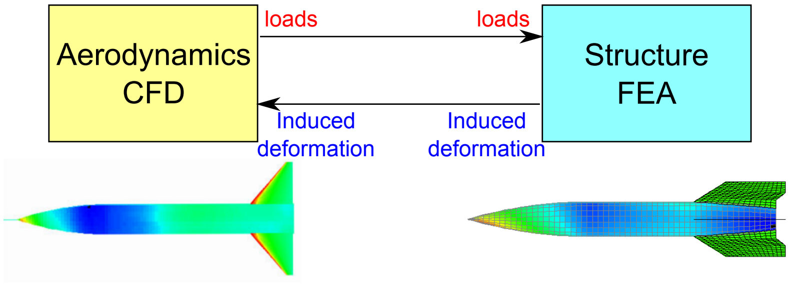


Figure 6: Couplings between aerodynamics and structure disciplines

numerical methods for solving complex industrial problems. For instance, the guidance law (*e.g.* modeled by pitch angle interpolation with respect to a set of crossing points) in a launch vehicle trajectory discipline has to be determined in order to ensure payload injection into orbit. The guidance law is often the result of an optimization problem minimizing the discrepancy between the target orbit injection and the real orbit injection. In such a modeling, the pitch angle crossing points are state variables $\mathbf{x}$ and the orbit discrepancy is the residual $\mathbf{r}(\cdot)$ to be canceled. Sometimes, the coupling variables $\mathbf{y}$ can be a subset of state variables $\mathbf{x}$.

In order to solve the MDO problem Eqs.(1-6), we are looking for:

- **Inequality and equality constraint feasibility**: the MDO solution has to satisfy the inequality constraints imposed by $\mathbf{g}(\cdot)$ and the equality constraints imposed by $\mathbf{h}(\cdot)$. These constraints are used to represent the requirements for the system in terms of targeted performance, safety, flexibility, *etc.* For example, a target orbit altitude for a launch vehicle payload is an equality constraint to be satisfied.

- **Individual disciplinary feasibility**: the MDO solution has to ensure the disciplinary state equation satisfaction expressed by the residuals $\mathbf{r}_i(\cdot)$. The latter $\mathbf{r}_i(\cdot)$ quantify the satisfaction of the state equations in discipline i. The state variables $\mathbf{x}_i$ are the roots of the state equations of discipline i. For instance, state equations may be used to represent thermodynamics equilibrium between the chemical components in rocket engine combustion. In the rest of the chapter, it is assumed that the satisfaction of the disciplinary feasibility is directly ensured by the disciplines (disciplinary analysis [9]), therefore, no more references to state variables and residuals will be done, without loss of generality.

- **Multidisciplinary feasibility**: the MDO solution has to satisfy the interdisciplinary equality constraints between the input coupling variable vector $\mathbf{y}$ and the output coupling variable vector $\mathbf{c}(\cdot)$ resulting from the discipline simulations. The couplings between the disciplines i and j are said to be *satisfied* (also called *feasible* or *consistent*) when the following interdisciplinary system of equations is verified:

$$\begin{cases} \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}) \\ \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{z}_j, \mathbf{y}_{.j}) \end{cases} \quad [7]$$

When all the couplings are satisfied, *i.e.* when Eqs.(7) are satisfied $\forall (i,j) \in \{1,...,N\}^2 \ i \neq j$, the system is said to be *multidisciplinary feasible*. The satisfaction of the interdisciplinary couplings is essential as it is a necessary condition for the modeled system to be physically realistic. Indeed, in the aero-structure example, if the aerodynamics discipline computes a load of 10MPa, it is necessary that the structure discipline uses as input 10MPa and not another value otherwise the aero-structure analysis is not consistent.

- **Optimal MDO solution**: $f(\cdot)$ is the objective function (also called performance) to be optimized. The objective function characterizes the system performance and is a measure of its quality expressed with some metrics (*e.g.* launch vehicle life cycle cost, Gross Lift-Off Weight (GLOW)). Several performance measures may be considered together by using multi-objective optimization. Multi-objective optimization is not considered in this chapter, so that the interest reader may consult [40].

In MDO, coupled and decoupled approaches may be distinguished to satisfy the interdisciplinary couplings [10].

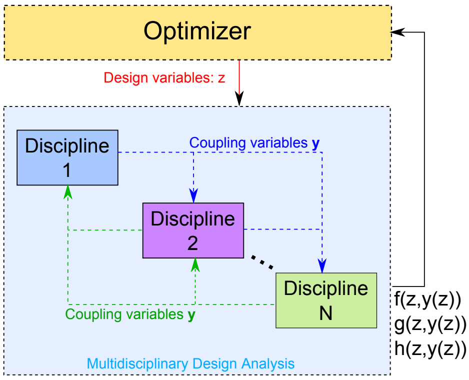


Figure 7: Multidisciplinary Design Optimization, **coupled** approach

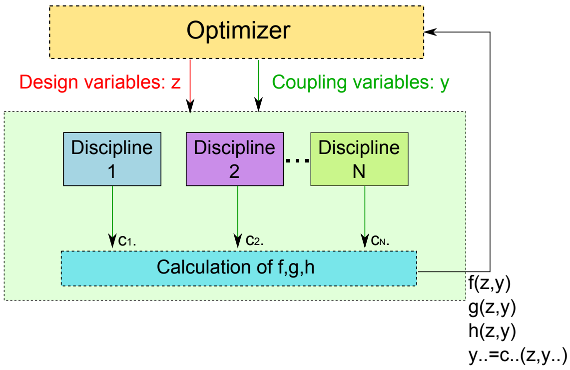


Figure 8: Multidisciplinary Design Optimization, **decoupled** approach

◇ *Coupled approaches* (Fig. 7) use a specific process, called MultiDisciplinary Analysis (MDA), in order to satisfy the interdisciplinary couplings at each iteration of the system-level optimization. MDA is an auxiliary process used to find a numerical equilibrium between the disciplines by solving the system of interdisciplinary equations Eqs.(7) [20]. MDA enables to find the numerical value of the input coupling variables $\mathbf{y}$ in order to solve the system of equations Eqs.(7). MDA can be performed by using classical techniques such as Fixed Point Iteration [9], or by an auxiliary optimization problem allowing to reduce the discrepancy between the input coupling vector and the output coupling vector.

◇ *Decoupled approaches* (Fig. 8) aim at removing MDA and involve equality constraints on the coupling variables in the MDO formulation at the system-level Eq.(4) to ensure the interdisciplinary coupling satisfaction only for the optimal design, and not at each MDO process iteration in $\mathbf{z}$ such as coupled approaches do. These additional equality constraints are imposed between the input and the output coupling variables in the MDO formulation at the same level as the system constraints $\mathbf{g}(\cdot)$ and $\mathbf{h}(\cdot)$: $\forall (i,j) \in \{1,...,N\}^2\ i \neq j,\ \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i})$. The basic idea is to define the coupling variables $\mathbf{y}$ as optimization variables. Consequently, the system-level optimizer has to control both the design variables and the input coupling variables. Hence, the additional degrees of freedom introduced by expanding the optimization variable set handled by the system-level optimizer are controlled by the coupling equality constraints. The equality constraints on coupling variables may not be satisfied at each iteration but allow to guide the search of optimal design.

Several MDO formulations have been proposed in literature to efficiently solve general and specific engineering problems. Some articles [3, 9, 10, 15, 41] provide a review of the different methods and compare them qualitatively and numerically on MDO problem benchmarks [57, 60]. Classical MDO formulations may be classified in four categories (Fig. 9) according to the *coupled* or *decoupled* and to the *single-level* or *multi-level* approaches. The single level *vs.* multi-level formulations are differentiated by the number of optimizers. Single level formulations have only one system optimizer to solve the MDO problem whereas in multi-level formulations, in addition to the system optimizer, discipline optimizers are introduced in order to distribute the problem complexity over different dedicated discipline optimizations. The four categories are:

- Single-level approaches with MDA: *e.g. Multi Discipline Feasible* (MDF) [10],
- Multi-level approaches with MDA: *e.g. Concurrent SubSpace Optimization* (CSSO) [52], *Bi-Level Integrated System Synthesis* (BLISS)[53],
- Single-level approaches with equality constraints on the coupling variables: *e.g. Individual Discipline Feasible* (IDF) [10], *All At Once* (AAO) [10],
- Multi-level approaches with equality constraints on the coupling variables: *e.g. Collaborative Optimization* (CO) [13], *Analytical Target Cascading* (ATC) [4], *QuasiSeparable Decomposition* (QSD) [29].

MDF is the most used method in literature [9]. MDF is a single-level optimization formulation involving one system-level optimizer and a MDA to solve the interdisciplinary coupling equations. CSSO and BLISS use MDA to ensure coupling satisfaction but enable parallel discipline optimizations. AAO, ATC, CO, IDF and QSD are fully decoupled formulations with satisfaction of the couplings by incorporating additional variables and corresponding equality constraints. The decoupled MDO formulations offer several advantages compared to MDF [9, 41]:

- Parallel analyses of the disciplines,
- Reduction of the number of calls to the computationally expensive discipline codes (because MDA is removed),

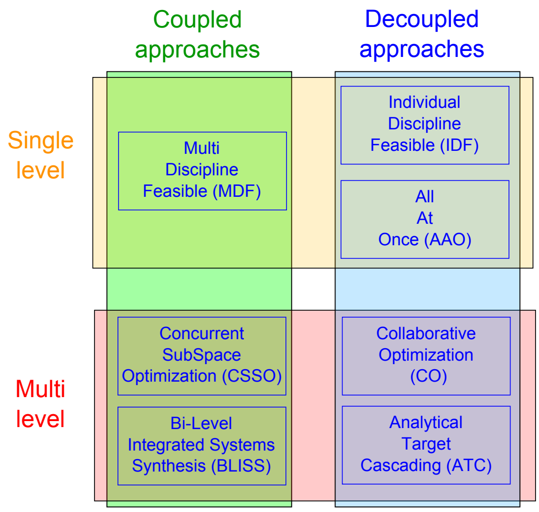


Figure 9: Classification of the main MDO formulations

- Improvement of the system optimization process convergence, however, most of the time there is no proof that the convergence is to the same optimum,
- Distribution of the optimization problem complexity: discipline optimizers only control local design variables and system-level optimizer only handles the shared design variables between several disciplines and the coupling variables.

However, in order to be competitive with respect to MDF, decoupled MDO formulations require an appropriate interdisciplinary coupling handling. Moreover, these formulations involve more variables and more constraints. In [7], the authors performed a detailed review of classical MDO formulations applied to LVD. This study points out that LVD present particularities, notably the importance of the trajectory discipline compared to the other disciplines. Exploiting these specificities in an MDO formulation might improve the LVD process. Dedicated formulations for LVD have been proposed such as the Stage-Wise decomposition for Optimal Rocket Design (SWORD) [7]. The next section focuses on these dedicated formulations.

2.2 Stage-Wise decomposition for Optimal Rocket Design

2.2.1 Theoretical formulations

In literature, the classical way to design a launch vehicle is to decompose the design process according to the involved disciplines (propulsion, aerodynamics, sizing, trajectory, *etc.*). The decomposition according to the disciplines has been coupled with single level methods (Individual Discipline Feasible, All At Once [18]) or multi-level methods (Collaborative Optimization [14], Concurrent SubSpace Optimization [51], Bi-Level Integration Systems Synthesis [50], *etc.*). In these methods, the trajectory is also optimized as a whole and is often considered as a "black box" for the optimization. The SWORD formulations [7] allow to decompose the LVD according to the different stages in order to improve the efficiency of the MDO process. In these formulations, the subsystems are not the disciplines but the different stage optimizations incorporating all the required disciplines involved in the stage design. SWORD are multi-level decoupled MDO formulations [8]. Four different formulations have been proposed depending on the decomposition process and the interdisciplinary coupling constraint handling (Fig. 10). This type of decomposition is proposed in the context of LVD but is generalizable to systems for which the system-level objective function can be decomposed into a sum of subsystem contributions, as involved in the QSD formulation [29]. According to the comparison of the methods on launch vehicle application cases implemented in [7], the third formulation is the most efficient to solve MDO problems (with respect to the number of discipline evaluations) due to its hierarchical decomposition of the design process and only this formulation is detailed in the following for the sake of conciseness (Fig. 11). For more details about the other SWORD formulations, one may consult [8]. In SWORD, the objective function $f(\cdot)$ is assumed to be decomposed such as $f(\cdot) = \sum_{j=1}^{n} f_j(\cdot)$ with n the number of stages. In practice, the Gross Lift-Off Weight (GLOW) is often minimized in LVD process [9, 19] and it can be decomposed as the sum of the stage masses and upper composite. The MDO formulation of the LVD problem using SWORD is given by:

At the system-level:

$$\begin{aligned}
\min \quad & f(\mathbf{z}, \mathbf{y}) && [8]\\
\text{w.r.t.} \quad & \mathbf{z}_{sh}, \mathbf{y} \\
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{z}, \mathbf{y}) \leq 0 && [9]\\
& \forall i \in \{1, \ldots, n\}, \mathbf{g}_i(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i^*, \mathbf{y}) \leq 0 && [10]\\
& \forall i \in \{1, \ldots, n\}, \mathbf{h}_i(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i^*, \mathbf{y}) = 0 && [11]\\
& \forall i, j \in \{1, \ldots, n\}^2 i \neq j, \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i^*, \mathbf{y}_{.i}) && [12]\\
& \mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} && [13]
\end{aligned}$$

At the subsystem-level:

$i = n$

While $i > 0$

For the i^{th} stage:

Given $\mathbf{z}_{sh}, \mathbf{y}_{i+1}, ..., \mathbf{y}_n$ (for launch vehicle: the optimal masses of the stages $i+1$ to n):

$$\begin{aligned}
\min \quad & f_i(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i, \mathbf{y}) && [14]\\
\text{w.r.t.} \quad & \overline{\mathbf{z}}_i \\
\text{s.t.} \quad & \mathbf{g}_i(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i, \mathbf{y}) \leq 0 && [15]\\
& \mathbf{h}_i(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i, \mathbf{y}) = 0 && [16]\\
& \mathbf{y}_{i.} = \mathbf{c}_{i.}(\mathbf{z}_{sh}, \overline{\mathbf{z}}_i, \mathbf{y}_{.i}) && [17]\\
& \overline{\mathbf{z}}_{i_{\min}} \leq \overline{\mathbf{z}}_i \leq \overline{\mathbf{z}}_{i_{\max}} && [18]
\end{aligned}$$

$i \leftarrow i-1$

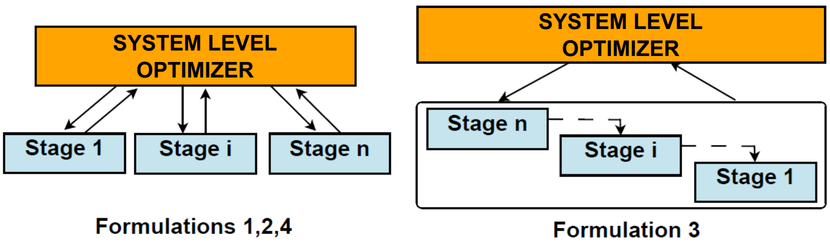


Figure 10: SWORD formulations (formulations 1,2,4 are parallel and formulation 3 is hierarchical)

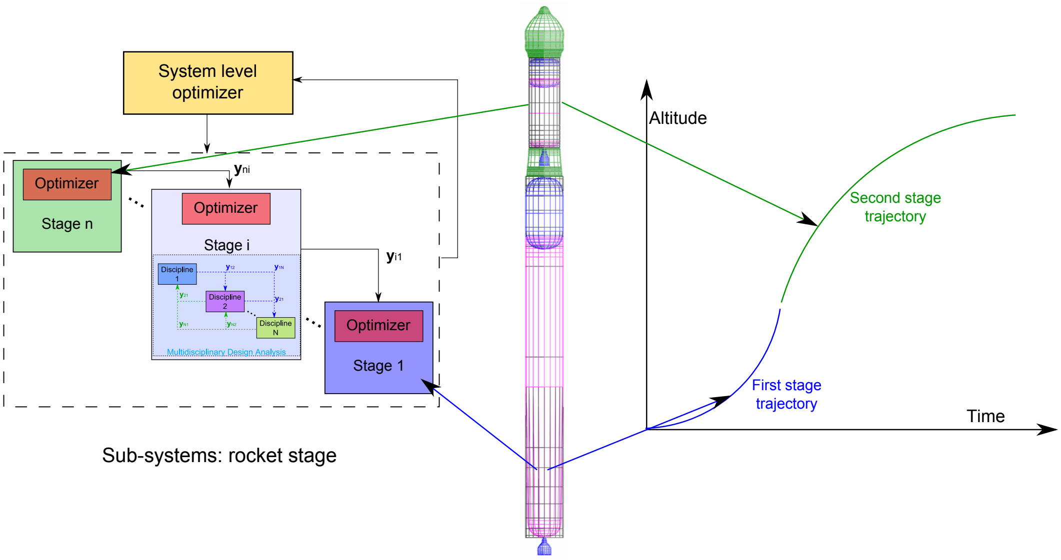


Figure 11: 3rd SWORD formulation

where $\bar{\mathbf{z}}_i^*$ is the optimal variable vector found by the i^{th} subsystem optimizer. This formulation allows to separately optimize each stage in a hierarchical process. The last stage is optimized first and the first stage is optimized last. The result of the previous optimization is passed to the next launch vehicle stage optimization (Fig. 10). In order to decouple the different stage optimizations, the added coupling variables $\mathbf{y}$ are the state vectors (position and velocity) at stage separations (to ensure the consistency of the trajectory) and the estimation of the stage masses. Furthermore, in order to ensure the trajectory consistency, additional constraints concerning the reach of each stage separation point (and final orbit for the upper stage) are involved at the subsystem-level. The different stage optimizations cannot be performed in parallel which may be a drawback in terms of computational cost when parallelization is possible. For more details on the SWORD formulations, see [7]. In the following Section, this formulation is applied to the design of a three-stage-to-orbit launch vehicle [7] and is compared to MDF.

2.2.2 Application of SWORD to launch vehicle design

Description of the test case

The proposed design problem consists in optimizing a three-stage-to-orbit expendable launch vehicle.

The selected criterion is the GLOW minimization. The payload mass is fixed at 4 metric tons. The target orbit is a 250×35786 km Geostationary Transfer Orbit (GTO). The considered disciplines are propulsion, aerodynamics, mass budget, and trajectory (Fig. 12), using low fidelity models [7, 19, 33, 56]. The considered design variables are summarized in Table 1. The constraints taken into account are relative to the reach of the target orbit, the maximal angle of attack during the trajectory, the geometry of the nozzle and the nozzle exit pressure. For more details about the problem description, one can consult [6].

The considered design variables are the chamber pressures Pc, the nozzle exit pressures Pe, the thrust to weight ratios TW, the propellant masses M_p, the mixture ratios Rm, the stage diameters D and the control law $\mathbf{u}$. All the stages are cryogenic propulsion stages (LOX/LH2). The propulsion module consists in computing the specific impulse (Isp) from Pc, Pe, Rm and TW. The aerodynamics module computes the drag coefficient Cd from the geometry characteristics of the launch vehicle. We use a zero-lift hypothesis in this test-case. The weight and sizing module is responsible for determining the dry mass of the different stages by computing the masses of the launch vehicle elements (tanks, combustion chamber, nozzle, pumps, pressurization system, *etc.*). Finally, the trajectory module consists in defining some crossing points of the pitch angle ($\mathbf{u}$) and optimizing them in order to reach the orbit requirements and improving the objective function.

Table 1: Design variables for the three stage LVD problem

Design Variables	Symbols
Stage diameters	D_1, D_2, D_3
Stage propellant masses	M_{p1}, M_{p2}, M_{p3}
Stage mixture ratio	Rm_1, Rm_2, Rm_3
Stage chamber pressure	Pc_1, Pc_2, Pc_3
Stage nozzle exit pressure	Pe_1, Pe_2, Pe_3
Stage thrust to weight ratio	TW_1, TW_2, TW_3
Stage control law parameter vector	$\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$

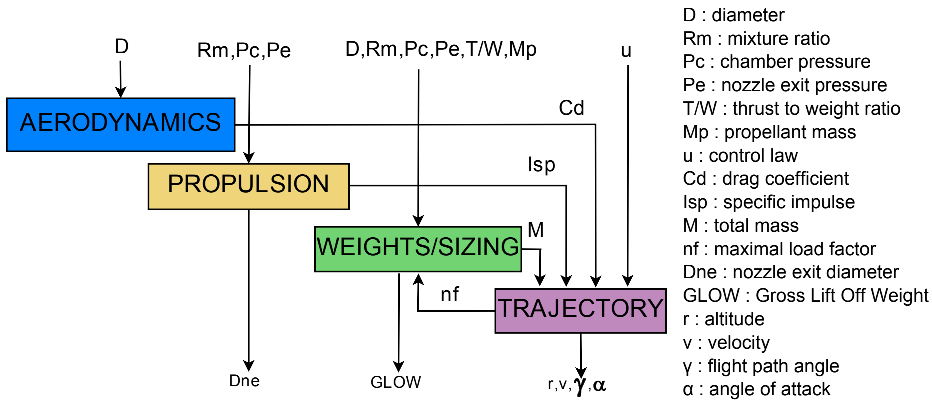
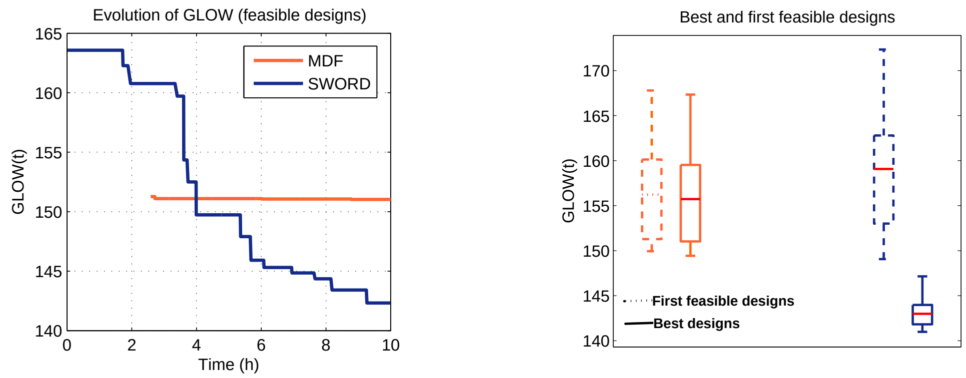


Figure 12: N2 chart for one stage

Results

At the system-level (including the MDF optimizer), a genetic algorithm with 100 individuals per generation is used (a penalization technique is used to take the constraints into account). At the subsystem-level (only for SWORD), a SQP algorithm is used. The optimization problem at the system-level is stopped after 10 hours of run. Due to the stochastic nature of GA, this study has been performed with 10 random initializations and very large variation domains concerning the design variables (global search). Statistics of the obtained results are detailed in the following.

Figure 13 shows the evolution of the objective function (GLOW) with respect to the computation time for only the feasible designs, for one representative initialization. At the stopping time of the optimization process, SWORD allows to obtain a better design than MDF, although it finds a worse first feasible design. Moreover, MDF presents some difficulties in improving the objective function (Fig. 14) while SWORD allows a decrease of the launch vehicle mass of 10% in mean. The relatively bad results obtained for the MDF can be explained by the important number of optimization variables at the system-level that makes global search very difficult and lead to relatively inconsistent results.



(a) Evolution of GLOW for one representative initialization

(b) Boxplot of best and first feasible designs

Figure 13: Comparison of SWORD and MDF

Figure 14: Improvement of objective function during the optimization

The best advantage of using a dedicated MDO formulation such as SWORD is to benefit from the specificities of the design problem to solve, that allows in this case, to reduce the search domain dimension at the system-level and to move the design complexity from the system-level to the subsystem-level. Indeed, the analysis of the problem dimension shows that the number of variables at the system-level can be reduced threefold with using SWORD. Since the search domain dimension and number of constraints are reduced, SWORD is more adapted to the exploratory search than MDF and the dispersion of the obtained results is lower than MDF (Fig. 13). This test-case illustrates the advantage to particularize classical MDO methods to specific design problem in order to improve the design process, both in terms of found results and robustness to initialization.

3 Introduction of uncertainty in the design process

At the conceptual design stage, a designer often needs to discriminate among innovative technologies that offer high performance but at a high risk, and established technologies that offer lower performance but with less uncertainty. The early design phases are characterized by the use of low fidelity analyses as well as by the lack of knowledge about the future system design and performance. This lack of knowledge in the models is reducible by increasing the model fidelity and is classified as epistemic uncertainty; uncertainty due to variability is irreducible and classified as aleatory uncertainty. The low fidelity analyses are employed due to the necessity to evaluate a high number of system architectures to explore the design space. This global exploration results in repeated discipline evaluations which is impossible to perform at an affordable computational cost with high fidelity models. Moreover, to increase the performance of the aerospace vehicles and to decrease their costs, space agencies and industries introduce new technologies (new propellant mixture, reusable rocket engines) and new architectures (reusable first stage for launch vehicles) which present a high level of uncertainty in the early design phases. If uncertainties are not taken into account at these phases, the detailed design phase might reveal that the optimal design previously found violates specific requirements and constraints. In this case, either the designers go back to the previous design phase to find a set of design alternatives, or they perform design modifications at the detailed design phase that could result in loss of performance. Both options would lead to a loss of time and money due to the re-run of complex simulations. Incorporating uncertainties in design methodologies for aerospace vehicle design has thus become a necessity to offer improvements in terms of [61]:

- reduction of design cycle time, cost and risk,
- robustness of LVD to uncertainty along the development phase,
- increasing system performance while meeting the reliability requirements,
- robustness of the launch vehicle to aleatory events during a flight (*e.g.* wind gust).

In classical design processes, both epistemic and aleatory uncertainties are usually controlled by using safety margins and the design problem is deterministically solved [34]. This may lead to over conservative or unreliable solutions depending on the choice of the margins. When breakthrough technologies are used in the design process (and no historic data are available), historically chosen margins may not be appropriate and a specific process to determine them is required to improve performance or restore safety. In the first part of this section, we detail a method to select optimal design rules and margins. This method accounts for the uncertainty reduction that occurs when refining the design models in later design phases.

Another way to take the uncertainty into account is to perform a probabilistic design (*i.e.* reliability-based design optimization). In the MDO context, Uncertainty-based Multidisciplinary Design Optimization (UMDO) aims at solving MDO problems under uncertainty. UMDO methods are recent and still under development and they have not reached sufficient maturity to efficiently estimate the final system performance and reliability [59, 61]. Incorporating uncertainty in MDO methodologies raises a number of challenges which need to be addressed. Being able, in the early design phases, to design a multidisciplinary system taking the interactions between the disciplines into account and to handle the inherent uncertainties is often computationally prohibitive. For example, a straightforward implementation of UMDO would consist in repeated sampling of the uncertain parameters (Monte Carlo simulations) and a multidisciplinary analysis (solving Equations 7) for each sample, therefore multiplying the already important cost of MDO by the number of Monte Carlo repetitions. In order to satisfy the designer requirements, it is necessary to find the system architecture which is optimal in terms of system performance while ensuring the robustness and reliability of the optimal system with respect to uncertainty. In the second part of this section, we describe a method to handle interdisciplinary coupling satisfaction in the presence of uncertainty and to decouple the design process.

3.1 Optimization of design rules and safety margins taking into account mixed epistemic / aleatory uncertainties

At the initial design stage engineers must often rely on low fidelity models that have high epistemic model uncertainty. It is important to make a distinction between *epistemic model uncertainty* and *aleatory parameter uncertainty.* Model uncertainty is defined as the discrepancy between the model and reality when the true model inputs are known [36, 46]. The model uncertainty is classified as epistemic because [22, 26]: (1) There is only a single true model, but it is unknown (2) The model uncertainty is reducible by gaining more knowledge. Parameter uncertainty is defined as uncertainty regarding the model inputs [36, 46]. In general, parameter uncertainty may be either aleatory or epistemic. Here we classify the parameter uncertainty as aleatory because [22, 26]: (1) It arises due to inherent or natural variability (2) It is irreducible. For example, wind gusts and variations of material properties are aleatory. While probability theory is generally accepted as the appropriate method for modeling aleatory uncertainty, several alternatives methods have been proposed for modeling epistemic uncertainty [27]. In the proposed method, both aleatory and epistemic uncertainty are modeled using probability theory because this theory is well suited for representing model uncertainty.

When considering both aleatory parameter and epistemic model uncertainties, the objective of the

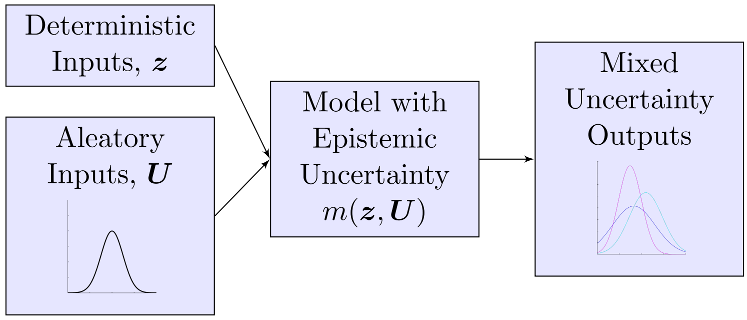


Figure 15: The propagation of aleatory parameter uncertainty through a model with epistemic model uncertainty results in a different distribution for each realization of epistemic model uncertainty (represented here by colored output curves).

design process is to find a design that is reliable with respect to the natural variability that will be experienced in service (*i.e.* aleatory parameter uncertainty). The fidelity of the model has no impact on the true reliability of the final design because the true reliability only depends on the true model and the aleatory uncertainty. For example, if the same design is selected using two different models, the designs still have the same true reliability regardless of the fidelity of the models used in the design process. However, the model fidelity determines the accuracy of the design reliability assessment which, in turns, affects the ability to selecting good designs. Therefore, the designer must also compensate for lack of knowledge regarding how well the low fidelity model agrees with reality (*i.e.* epistemic model uncertainty). The epistemic model uncertainty and aleatory parameter uncertainty are treated separately (see [31, 32, 47]) to distinguish between the quantity of interest, the true probability of failure with respect to aleatory parameter uncertainty, and the lack of knowledge regarding this quantity. The separate treatment of aleatory and epistemic uncertainties results in a distribution of probability of failure that is epistemic in nature (see Fig. 15). That is, the final design will have a single true probability of failure with respect to aleatory parameter uncertainty, but it is unknown due to the epistemic model uncertainty introduced by low fidelity modeling. When the epistemic model uncertainty is very high it may force the designer to be overly conservative if, for example, the designer is compensating for worst-case scenario epistemic model uncertainty. High epistemic model uncertainty can also prevent the designer from making any decision if the distribution of possible probability of failure spans the entire zero to one range. The proposed method is an innovative approach to the challenges of design under high epistemic model uncertainty when improved modeling will be available in the future. For the sake of simplicity, this method is described in the context of single discipline design but it can be generalized for MDO.

The proposed method [48] addresses the issue of high epistemic model uncertainty by considering the anticipated uncertainty reduction from future high fidelity modeling. This approach involves first a classical deterministic optimization with uncertainty handled through safety factors, and secondly a probability analysis to assess the reliability of the solution found. Based on these steps, the design rules and safety factors are optimized in order to comply with the reliability specifications. The use of safety margin based optimization is a necessary simplification to reduce computational cost and it agrees well with current safety margin or safety-factor based design regulations [1]. Because of the presence of epistemic uncertainty, this method emulates the possible high fidelity model outcomes, considered as future tests, in order to simulate the occurrence of redesign process. To determine the necessity of redesigning, it is also convenient to formulate a test passing criterion in terms of the safety margin calculated from the possible outcomes of simulated high fidelity model. The proposed approach is a bi-level optimization method (Fig. 17): at the upper-level, the safety margins are optimized to provide the optimal performance at the specified reliability requirements; at the lower-level, a complete design and redesign process is involved and can be decomposed into the following steps (Fig. 18):

1. Given safety margins, perform a deterministic design optimization: considering safety-factor vector $\mathbf{s}$, low fidelity model $\widehat{m}(\cdot)$, and conservative value of aleatory uncertainty $\mathbf{u}_{det}$, the classical deterministic formulation of design problem is:

$$\min \quad f(\mathbf{z}) \qquad [19]$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{z}, \mathbf{u}_{det}, \widehat{m}(\mathbf{z}, \mathbf{u}_{det})) - \mathbf{s} \leq 0 \qquad [20]$$

$$\mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} \qquad [21]$$

2. Simulate multiple possible outcomes of high fidelity model taking epistemic error into account at the optimal solution given by the previous step, and perform the test of redesign necessity (Fig. 16),

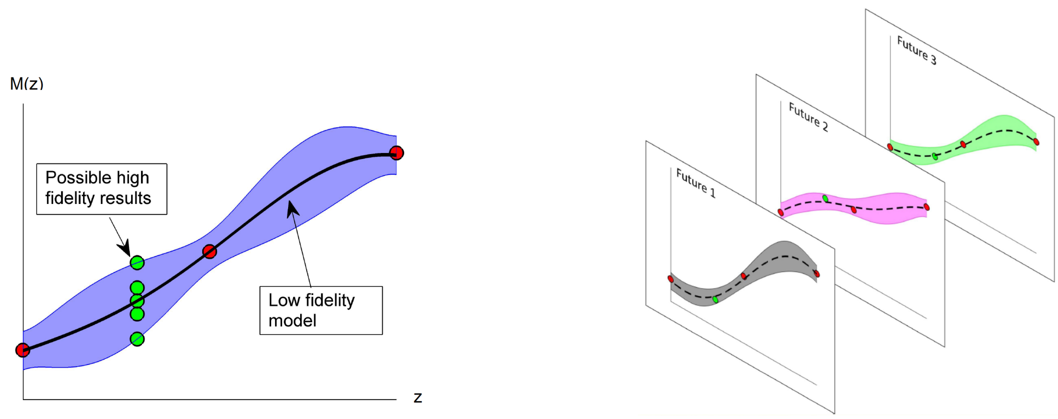


Figure 16: Possible responses of high-fidelity model (left) and simulation of high-fidelity outcomes (right)

3. If the test calls for redesign,

   (a) Calibrate the low fidelity models taking the possible high fidelity response into account,

   (b) Perform a deterministic redesign optimization with the calibrated low-fidelity model,

4. Perform a probabilistic reliability assessment.

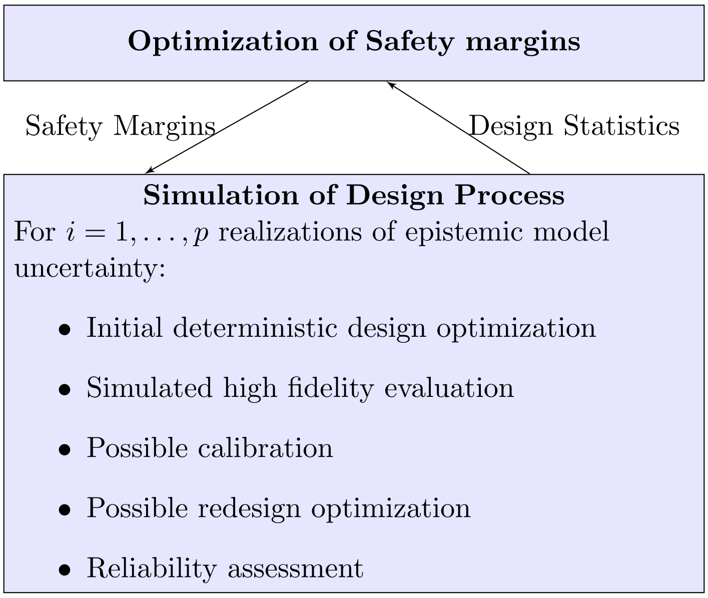


Figure 17: The safety margins that govern the deterministic design process are optimized by maximizing the expected performance while satisfying probabilistic constraints on expected reliability and probability of redesign (called the design statistics in the Figure).

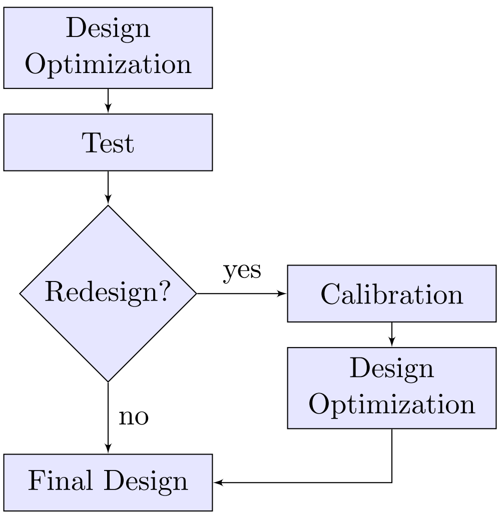


Figure 18: The deterministic design process consists of a design optimization, a test (*e.g.* high fidelity evaluation), and possible calibration and redesign

The safety margins that control the initial design, test passing criteria, and possible redesign are optimized to maximize the expected design performance while satisfying constraints on probability of redesign and expected reliability after the test (see Fig. 17). The design process is carried out deterministically for each realization of epistemic model uncertainty. The process of calculating the design statistics is basically a two-stage Monte-Carlo Simulation (MCS). The epistemic model uncertainty is sampled in the outer-loop and the reliability with respect to aleatory parameter uncertainty is calculated in the inner-loop. For each set of safety margins, the two-stage MCS is performed to calculate the probability of redesign, the expected design performance, and the expected probability of failure. The proposed method can help designers find reasonable designs while working under the burden of high epistemic model uncertainty. Furthermore, the method can be used to explore interesting questions such as whether it is better to start with a more conservative initial design and use redesign to improve performance if the initial design is revealed to be too conservative, or to start with a less conservative initial design and use redesign to restore safety if the initial design is revealed to be unsafe.

The method does not require any evaluations of the high fidelity model, only that the high fidelity evaluation and possible redesign may be performed in the future. That is, if the test is passed in the future, then the reliability of the initial design is verified to be acceptable. Similarly, alternative designs (*i.e.* redesigns) can be found that are reliable conditional on the specific epistemic realizations (*i.e.* specific test results) that will result in failing the future test (see Fig. 19). The test process can be used to not only restore safety if the initial design is revealed to be unsafe, but also to improve performance if the initial design is revealed to be overly conservative. By using the future high fidelity evaluations, the design method can be considered as the selection of multiple candidate designs instead of a single design solution. The decision to keep the initial design or redesign will be made in the future. The method considers the alternative design as a continuous epistemic random variable and relies on only specifying the optimum safety margins for locating alternative designs, rather than the explicit specification of discrete alternative designs. The preference for passing the test and keeping the initial design is controlled by a constraint on the probability of redesign in the upper-level optimization problem.

The core of the proposed method relies on the simulation of possible future test results (*i.e.* future high fidelity evaluations of initial design). Not only is it necessary to simulate the possible future high fidelity evaluations, but it is also necessary to update the distribution of epistemic model uncertainty conditional on specific realizations. By repeating the updating process for many test results it is possible to find alternative designs whose reliability is conditional on each test outcome. The method used to update the distribution of epistemic model uncertainty must account for the spatial correlations with respect to design variables. For example, it is intuitively clear that the reduction in epistemic model uncertainty from a future high fidelity evaluation is most dramatic in the immediate vicinity of the test location but decreases as the design moves away from this location. In other words, if the alternative

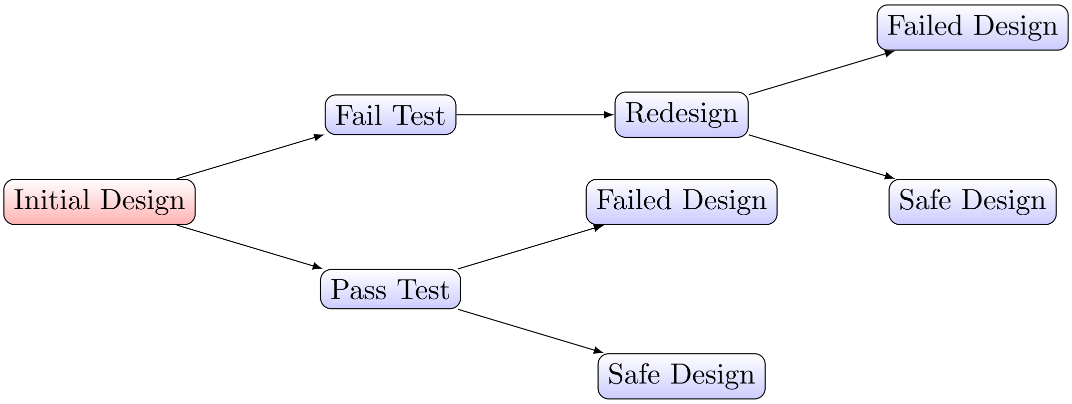


Figure 19: The final reliability (*i.e.* probability of a safe design) is conditional on passing or failing a deterministic safety margin based test. Failing the test triggers a redesign process to restore safety or improve design performance.

design is dramatically different from the initial design that was tested, then the test result might not be very useful in reducing model uncertainty regarding the new design. Early work on simulating a future test and redesign relied on the strong assumption that the model bias was a fixed but unknown constant across the design space [42, 58, 49]. More recently, the method has been extended to consider fluctuations in model uncertainty and spatial correlations through the use of Gaussian Process (GP) models to represent the model uncertainty [48]. The purpose of the GP model representation of model error is twofold: (1) The GP model provides a mathematical formulation of the intuitive idea that the reduction in the variance of the epistemic model uncertainty is greatest at the test location and decreases with distance. (2) The GP model provides a probabilistic representation of epistemic model uncertainty that allows for the propagation of mixed aleatory and epistemic uncertainties. The second point is particularly important in the proposed method because the variation of epistemic model uncertainty across the design space can alter the functional relationship with respect to aleatory parameter inputs.

The proposed method can help designers find reasonable designs while working under the burden of high epistemic model uncertainty. This method has been applied in a structural design problem [49] and an aerospace vehicle design problem [48] which is not described in this chapter for the sake of conciseness.

3.2 Uncertainty Multidisciplinary Design Optimization

Taking uncertainties in MDO into account leads to a Uncertainty-based MDO (UMDO) research field [59]. As for deterministic MDO, several UMDO formulations have been proposed in literature [23, 28, 39, 43] and the generic UMDO problem can be formulated as follows:

$$
\begin{aligned}
\min \quad & \Xi\left[f(\mathbf{z},\boldsymbol{\theta}_{\boldsymbol{Y}},\mathbf{U})\right] & [22]\\
\text{w.r.t.} \quad & \mathbf{z},\boldsymbol{\theta}_{\boldsymbol{Y}} & \\
\text{s.t.} \quad & \mathbb{K}\left[\mathbf{g}(\mathbf{z},\boldsymbol{\theta}_{\boldsymbol{Y}},\mathbf{U})\right] \leq 0 & [23]\\
& \forall (i,j) \in \{1,...,N\}^2 \; i \neq j, \; \boldsymbol{\theta}_{\mathbf{Y}_{ij}} = \mathbb{M}\left[\mathbf{c}_{ij}(\mathbf{z}_i,\boldsymbol{\theta}_{\boldsymbol{Y}.i},\mathbf{U}_i)\right] \quad \text{for statistical-based approaches} & [24]\\
& \forall (i,j) \in \{1,...,N\}^2 \; i \neq j \; \boldsymbol{\theta}_{\mathbf{Y}_{ij}} = \mathbf{c}_{ij}(\mathbf{z}_i,\boldsymbol{\theta}_{\boldsymbol{Y}.i},\mathbf{u}_i^*) \quad \text{for MPP-based approaches} & [25]\\
& \mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} & [26]
\end{aligned}
$$

Several differences exist between the UMDO and the MDO formulations and are summarized in the following:

- $\mathbf{U}$ is the uncertain variable vector. $\mathbf{U}_i$ denotes the input uncertain variable vector of the discipline i and $\mathbf{U} = \bigcup_{i=1}^{N} \mathbf{U}_i$ without duplication. In this chapter, the uncertain variables are modeled with the probability theory, with known input distributions. Aleatory and epistemic uncertainties may be considered in the UMDO problem and the proposed formulation as long as they may be modeled with the probability formalism. For instance, wind gust during a rocket launch or parameter uncertainties in the modeling of the nozzle fluid flow may be sources of uncertainty. The design variables are assumed to be deterministic, and all the uncertainties are represented by $\mathbf{U}$. $(\boldsymbol{\Omega}, \sigma_{\boldsymbol{\Omega}}, P_{\boldsymbol{\Omega}})$ is the probability space with $\boldsymbol{\Omega}$ the sample space for $\mathbf{U}$, $\sigma_{\boldsymbol{\Omega}}$ the sigma-algebra, and $P_{\boldsymbol{\Omega}}$ the probability measure. $\phi(\cdot)$ is the joint Probability Density Function (PDF) of the uncertain variable vector $\mathbf{U}$ and the realizations of $\mathbf{U}$ are noted $\mathbf{u}$.

- Due to the presence of uncertainty, the coupling variable vector $\mathbf{Y}$ is also an uncertain variable vector and therefore a function of $\mathbf{U}$. Coupled formulations derived from MDF have been proposed to handle interdisciplinary coupling variables [34, 37, 45]. For each realization of the uncertain variables, a MDA is solved in order to compute the coupling variables ensuring multidisciplinary feasibility. However, the computational cost introduced by repeated MDA solving is too prohibitive for complex system design. In order to remove the MDA, as in deterministic approaches, decoupled strategies have been developed [23, 28, 39, 43]. Because the input coupling variables are function of the uncertainty, the decoupled optimization problem to solve has an infinite dimension. Several methods focus on these types of problems such as calculus of variations [44], optimal control [63] and shape optimization [55]. To avoid to solve an infinite dimension problem, the classical approaches involve a parameterization of the uncertain coupling variable modeling and the system-level optimizer controls only a finite number of parameters (*e.g.* the statistical moments, the parameters of the probability density function defining $\mathbf{Y}$, *etc*). Two types of decoupled UMDO formulations exist in literature: the statistical-based approaches or the Most Probable Point (MPP)-based approaches. The statistical-based UMDO formulations [28, 39, 43] ensure the multidisciplinary feasibility for the statistical moments $\mathbb{M}$ of the coupling variables (*e.g.* for the expected value $\mathbb{E}$ of the coupling variables). The MPP-based UMDO formulations [23] ensure the multidisciplinary feasibility only at the Most Probable failure Point $\mathbf{u}^*$ of the uncertain variables.

The existing UMDO formulations either rely on computationally expensive MDA to rigorously ensure coupling satisfaction, or deal with incomplete coupling conditions (coupling in terms of statistical moments, at the MPP, *etc.*). The moment matching formulations are interesting since they preserve some disciplinary autonomy via parallel subsystem-level uncertainty propagation and optimizations. However, the interdisciplinary couplings are satisfied only in terms of statistical moments (expected value, standard deviation or covariance matrix) of the coupling variables.

- $\boldsymbol{\Xi}$ is the uncertain objective function measure (*e.g.* the expected value , a weighted sum of expected value and the standard deviation [11]).

  Regarding the constraint functions, two main measures exist and can be formulated as follows:

  - the robust formulation: $\mathbb{K}\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U})\right] = \mathbb{E}\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U})\right] + \eta\sigma\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U})\right]$ with $\mathbb{E}[\mathbf{g}(\cdot)]$ and $\sigma[\mathbf{g}(\cdot)]$ the vectors of expected values and standard deviation values of the constraint functions $\mathbf{g}$ and $\eta \in \mathbb{R}^+$.
  - the reliability-based formulation: $\mathbb{K}\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U})\right] = \boldsymbol{\Lambda}\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U}) > 0\right] - \boldsymbol{\Lambda}_{\mathbf{t}}$ with $\boldsymbol{\Lambda}[\mathbf{g}(\cdot)]$ the vector of the measures of uncertainty for the inequality constraint function vector. The vector of the uncertainty measures of the constraints have to be at most equal to $\boldsymbol{\Lambda}_{\mathbf{t}}$ [2]. As the uncertain variables are modeled within the probability theory, we have for the component i of the vector of failure probabilities:

$$\mathbb{K}_i\left[g_i(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U})\right] = \mathbb{P}_{[g_i(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U}) > 0]} - \mathbb{P}_{t_i} = \int_{\mathcal{I}_i} \phi(\mathbf{u}) \mathrm{d}\mathbf{u} - \mathbb{P}_{t_i} \qquad [27]$$

    with $g_i(\cdot)$ the i^{th} component of the inequality constraint vector and $\mathcal{I}_i = \{\mathbf{U} \in \boldsymbol{\Omega} | g_i(\mathbf{z}, \boldsymbol{\theta}_{\boldsymbol{Y}}, \mathbf{U}) > 0\}$.

3.2.1 Theoretical approach for interdisciplinary coupling satisfaction in the presence of uncertainty

In order to avoid the repeated MDA used in MDF under uncertainty, decoupled approaches aim at propagating uncertainty on decoupled disciplines allowing one to evaluate them in parallel and to ensure coupling satisfaction by introducing equality constraints in the UMDO formulation. However, two main challenges are faced to decouple the design process:

- The handling of uncertain input coupling variable vector $\mathbf{Y}$ by the system-level optimizer. Moreover, the uncertain variables are function and infinite-dimensional problem are complex to solve.

- The handling of coupling equality constraints between the input coupling variables $\mathbf{Y}$ and the output coupling variables computed by $\mathbf{c}(\cdot)$. Equality between two uncertain variables corresponds to an equality between two functions which is difficult to implement.

In order to understand these two challenges and the approaches described afterwards, a focus on decoupled deterministic MDO formulation is necessary. Consider two disciplines i and j and one scalar

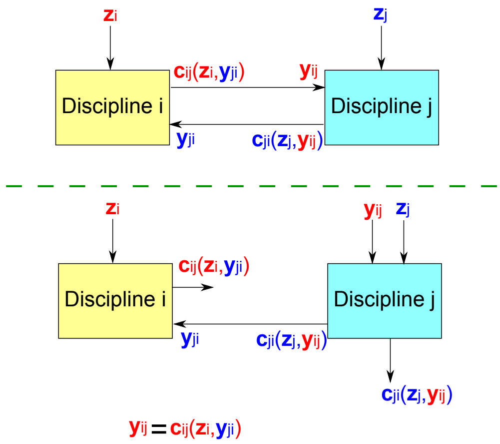


Figure 20: Two discipline coupling handling approaches

feedforward coupling y_{ij} and one scalar feedback coupling y_{ji} as illustrated in Figure 20. In deterministic decoupled MDO approach, to remove the feedforward coupling, there is only one equality constraint that has to be imposed at the system-level in the optimization formulation, Eq.(28), between the input coupling variable y_{ij} and the output coupling variable $c_{ij}(\mathbf{z}_i, y_{ji})$:

$$y_{ij} = c_{ij}(\mathbf{z}_i, y_{ji}) \qquad [28]$$

When introducing uncertainty, coupling satisfaction involves equality constraints between uncertain variables. An uncertain variable is a function. Two uncertain variables are equal, if and only if the two corresponding functions have the same initial and final sets and the same mappings. To ensure coupling satisfaction *in realizations*, an infinite number of equality constraints, Eq.(29), have to be imposed, one for each realization of the uncertain variables:

$$\forall \mathbf{u} \in \boldsymbol{\Omega}, \quad y_{ij} = c_{ij}(\mathbf{z}_i, y_{ji}, \mathbf{u}_i) \qquad [29]$$

However, it is important to point out that even if the coupling variables are random variables, for one realization $\mathbf{u}_0$ there is in general only one converged coupling variable realization that satisfies $y_{ij_0} = c_{ij}(\mathbf{z}_i, y_{ji_0}, \mathbf{u}_0)$ ensuring multidisciplinary feasibility. Indeed, the disciplines are modeled with deterministic functions, all the uncertainties arise in the discipline inputs.

Solving an optimization problem with an infinite number of constraints is a challenging task. To overcome this issue, considering an UMDO problem of N disciplines, we propose to introduce a new integral form for the interdisciplinary coupling constraint:

$$\forall (i,j) \in \{1, ..., N\}^2 \; i \neq j, \; \mathbf{J}_{ij} = \int_{\mathbf{\Omega}} \left[\mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}, \mathbf{u}_i) - \mathbf{y}_{ij}\right]^2 \phi(\mathbf{u}) \mathrm{d}\mathbf{u} = 0 \qquad [30]$$

The integrals in Eq.(30) equal to zero if the input coupling variables are equal to the output coupling variables for each realization of the uncertain variables almost surely. The interdisciplinary coupling constraints $\mathbf{J}_{ij}$ may be seen as the integration of a loss function (the difference between the input and the output coupling variables) over the entire sample space. If the new interdisciplinary coupling constraints Eq.(30) are satisfied, therefore a mathematical equivalence holds with the coupled approach because, as by using MDA, the couplings verify the following system of equations:

$$\forall \; \mathbf{u} \in \mathbf{\Omega}, \; \forall (i,j) \in \{1, ..., N\}^2 \; i \neq j, \; \begin{cases} \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}, \mathbf{u}_i) \\ \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{z}_j, \mathbf{y}_{.j}, \mathbf{u}_i) \end{cases} \qquad [31]$$

In order to be able to decouple the disciplines, the system-level optimizer has to control the input coupling variables $\mathbf{Y}$. In the proposed formulations, the considered scalar coupling variable y_{ij} is replaced by a surrogate model representing the coupling functional relations:

$$y_{ij} \rightarrow \hat{y}_{ij} \left(\mathbf{u}, \boldsymbol{\alpha}^{(ij)}\right) \qquad [32]$$

The surrogate model $\hat{y}_{ij} \left(\mathbf{u}, \boldsymbol{\alpha}^{(ij)}\right)$, allows to model a functional representation of the dependency between the uncertain variables $\mathbf{U}$ and the input coupling variables. $\boldsymbol{\alpha}^{(ij)}$ are the surrogate model parameters. In the proposed formulations, each coupling variable that is removed is replaced by a surrogate model. The metamodels are also functions, represented by parameters that may be used to decouple the UMDO problem by letting the system-level optimizer have the control on the surrogate model coefficients. Therefore, the infinite-dimensional optimization problem is transformed into a q-dimensional optimization problem with q the number of coefficients required to model all the removed coupling variables.

We propose to model the coupling functional relations with Polynomial Chaos Expansion (PCE)[25]. Indeed, this surrogate model has been successfully used to analyze and propagate uncertainty [25]. PCE are particularly adapted to represent the input coupling variables as they are dedicated to model functions that take as input uncertain variables. The scalar coupling y_{ij} is modeled by:

$$\hat{y}_{ij} \left(\mathbf{u}, \boldsymbol{\alpha}^{(ij)}\right) = \sum_{k=1}^{d_{\mathrm{PCE}}} \alpha_{(k)}^{(ij)} \Psi_k(\mathbf{u}) \qquad [33]$$

where $q = d_{\mathrm{PCE}}$ is the degree of PCE decomposition and Ψ_k is the basis of orthogonal polynomials chosen in accordance to the input uncertainty distributions.

Note that the dependency between $\hat{y}_{ij}(\cdot)$ and $\mathbf{z}$ is not taken into account in the surrogate model: $\hat{y}_{ij}(\cdot)$ is not a function of $\mathbf{z}$, it is learned only for the specific value of $\mathbf{z}$ which is the optimum of the problem. This interdisciplinary coupling satisfaction for all the realizations of the uncertain variables enables to ensure that the system is *multidisciplinary feasible*. The complex original infinite-dimensional problems are transformed into a finite-dimensional problem and the mathematical equivalence between coupled and decoupled formulations in terms of coupling satisfaction is numerically ensured.

The PCE models of the coupling functional relations is built iteratively during the system-level UMDO optimization. At the optimum, PCE models the coupling functional relations as would MDA under uncertainty do (Fig. 21). A single-level (Individual Discipline Feasible - Polynomial Chaos Expansion) and a multi-level (Multi-level Hierarchical Optimization under Uncertainty) formulations have been developed and are detailed in the following. The proposed approaches do not require any computationally expensive MDA.

Individual Discipline Feasible - Polynomial Chaos Expansion (IDF-PCE)

IDF-PCE is a single-level decoupled UMDO formulation [16] which can be formulated as follows:

$$\begin{aligned}
\min \quad & \Xi\left[f(\mathbf{z}, \boldsymbol{\alpha}, \mathbf{U})\right] && [34]\\
\text{w.r.t.} \quad & \mathbf{z}, \boldsymbol{\alpha} \\
\text{s.t.} \quad & \mathbb{K}\left[\mathbf{g}(\mathbf{z}, \boldsymbol{\alpha}, \mathbf{U})\right] \leq 0 && [35]\\
& \forall (i,j) \in \{1, ..., N\}^2 \; i \neq j, \\
& \quad \mathbf{J}_{ij} = \int_{\boldsymbol{\Omega}} \left[\mathbf{c}_{ij}\left(\mathbf{z}_i, \hat{\mathbf{y}}_{.i}\left(\mathbf{u}, \boldsymbol{\alpha}^{(.i)}\right), \mathbf{u}_i\right) - \hat{\mathbf{y}}_{ij}\left(\mathbf{u}, \boldsymbol{\alpha}^{(ij)}\right)\right]^2 \phi(\mathbf{u}) \mathrm{d}\mathbf{u} = \mathbf{0} && [36]\\
& \mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} && [37]
\end{aligned}$$

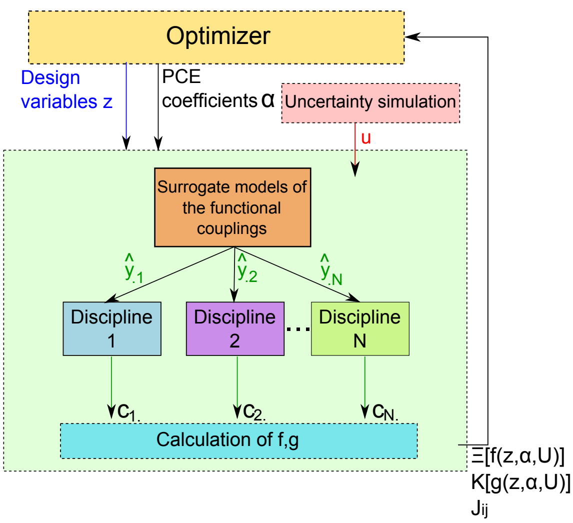


Figure 21: IDF-PCE [16]

with $\mathbf{J}_{ij}$ the interdisciplinary constraint vector of discipline i and $\hat{\mathbf{y}}_{.i}\left(\mathbf{u}, \boldsymbol{\alpha}^{(.i)}\right)$ the PCEs of all the input coupling variables. The system-level optimizer controls the design variables $\mathbf{z}$ and the PCE coefficients of the coupling variables $\boldsymbol{\alpha}$. The dimension of the design space is therefore increased with respect to the coupled approaches, by the number of parameters $\boldsymbol{\alpha}$. To ensure the multidisciplinary feasibility at the optimum, equality constraints involving the generalization error are imposed Eq.(36). The constraints have an integral form to ensure the coupling satisfaction for all the possible realizations of the uncertain variables. If we have: $\forall (i,j) \in \{1,...,N\}^2 \ \forall i \neq j, \ \mathbf{J}_{ij} = 0$, then the couplings are satisfied for all the realizations $\mathbf{u} \in \boldsymbol{\Omega}$ almost surely.

In practice, the multidimensional integrals associated to the statistical moments (expectations, standard deviations), to the coupling constraints $\mathbf{J}$ or to the probability of failure are difficult to compute. We use three techniques to estimate the statistical moments and the coupling constraints (Crude Monte Carlo, quadrature rules and decomposition of the output coupling variables over a PCE) and one to estimate the probability of failure by Subset Sampling using Support Vector Machines. Depending on the technique used to propagate uncertainty, this leads to three variants of IDF-PCE. For more details concerning IDF-PCE, one can consult [16].

Multi-level Hierarchical Optimization under Uncertainty (MHOU)

The aim of MHOU [17] is to ease the system-level optimization process by introducing a susbsystem-level optimization (Fig. 22). The formulation is inspired from SWORD. MHOU is a semi-decoupled hierarchical method that removes all the feedback interdisciplinary couplings in order to avoid the expensive disciplinary loops through MDA. The proposed approach relies on two levels of optimization and on surrogate models in order to ensure, at the convergence of the system optimization problem, the coupling functional relations between the disciplines. It allows a hierarchical design process without any loops between the subsystems. As for SWORD, this type of decomposition is proposed in the context of LVD, but it may be generalized to other design problems.

The MHOU formulation is given by:

- At the system-level:

$$\min \quad \sum_{k=1}^{N} \Xi\left[f_k(\mathbf{z}_{sh}, \mathbf{z}_k^*, \boldsymbol{\alpha}, \mathbf{U})\right] \quad [38]$$

$$\text{w.r.t.} \quad \mathbf{z}_{sh}, \boldsymbol{\alpha}$$

$$\text{s.t.} \quad \mathbb{K}\left[\mathbf{g}(\mathbf{z}_{sh}, \mathbf{z}_k^*, \boldsymbol{\alpha}, \mathbf{U})\right] \leq 0 \quad [39]$$

$$\forall (k,j) \in \{1,...,N\}^2 \ j \neq k, \ \mathbf{J}_{kj}(\mathbf{z}_{sh}, \mathbf{z}_k^*, \boldsymbol{\alpha}) = 0 \quad [40]$$

$$\forall k \in \{1,...,N\}, \ \mathbb{K}\left[\mathbf{g}_k(\mathbf{z}_{sh}, \mathbf{z}_k^*, \boldsymbol{\alpha}, \mathbf{U})\right] \leq 0 \quad [41]$$

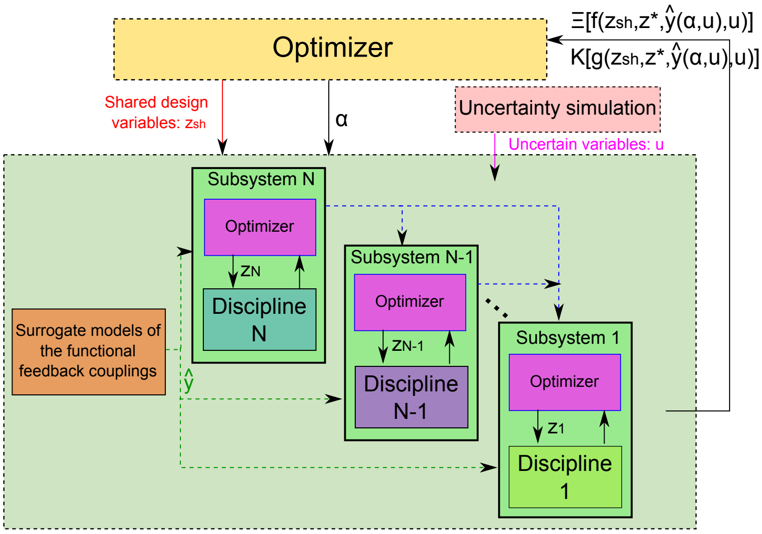


Figure 22: Multi-level Hierarchical Optimization under Uncertainty (MHOU)

- At the subsystem-level:

$k = N$

$$\text{While} \quad k > 0$$

$$\text{Given} \quad \mathbf{y}_{Nk}, \ldots, \mathbf{y}_{(k+1)k}$$

For the k^{th} subsystem

$$\min \quad \Xi\left[f_k(\mathbf{z}_{sh}, \mathbf{z}_k, \boldsymbol{\alpha}, \mathbf{U})\right] \qquad [42]$$

$$\text{w.r.t.} \quad \mathbf{z}_k$$

$$\text{s.t.} \quad \mathbb{K}\left[\mathbf{g}_k(\mathbf{z}_{sh}, \mathbf{z}_k, \boldsymbol{\alpha}, \mathbf{U})\right] \leq 0 \qquad [43]$$

$$\forall j \in \{1, \ldots, N\}\ j \neq k,\ \mathbf{J}_{kj} =$$

$$\int_{\boldsymbol{\Omega}} \left[\mathbf{c}_{kj}\left(\mathbf{z}_{sh}, \mathbf{z}_k, \hat{\mathbf{y}}_{.k}\left(\mathbf{u}, \boldsymbol{\alpha}^{(.k)}\right), \mathbf{u}_k\right) - \hat{\mathbf{y}}_{kj}\left(\mathbf{u}, \boldsymbol{\alpha}^{(kj)}\right)\right]^2 \phi(\mathbf{u})\mathrm{d}\mathbf{u} = \mathbf{0} \qquad [44]$$

$k \leftarrow k - 1$

$\mathbf{z}_k$ is the local design variable vector of discipline k and it belongs to the set $\mathcal{Z}_k$ and $\mathbf{z}_{sh}$ is the shared design variable vector between several disciplines. $\mathbf{z}_k^*$ is the optimal design variables found by the subsystem-level optimizer. This formulation allows one to optimize each subsystem separately in a hierarchical process. The system-level optimizer handles $\mathbf{z}_{sh}$ and the PCE coefficients $\boldsymbol{\alpha}$ of the feedback coupling variables. The control of PCE coefficients at the system-level allows one to remove the feedback couplings and to optimize the subsystems in sequence. The surrogate models of the functional feedback couplings provide the required input couplings to the different subsystems. The k^{th} subsystem-level optimizer handles $\mathbf{z}_k$ and the corresponding problem aims at minimizing the subsystem contribution to the system objective while satisfying the subsystem-level constraints $\mathbb{K}[\mathbf{g}_k(\cdot)]$. The interdisciplinary coupling constraint Eq.(44) ensures the couplings whatever the realization of the uncertain variables. In MHOU formulation, Eq.(44) is only considered for $k \neq N$. This formulation is particularly suited for launch vehicle in order to decompose the design process into the different stage optimizations. The decreasing order of the discipline optimization, from N to 1 is more convenient for a launch vehicle (the last stage is optimized first, then the intermediate stages and the first one is optimized last), however, in general case any order may be adopted. In practice, the disciplines are organized to have the minimal number of feedback coupling variables in order to decrease the number of coupling variables controlled at the system-level and therefore the complexity of the optimization problem.

3.2.2 Application for launch vehicle design

Two test cases have been implemented to illustrate IDF-PCE and MHOU formulations.

First test case: comparison of IDF-PCE and MDF on a two-stage-to-orbit launch vehicle design problem

The first test case [16] consists in designing a two-stage-to-orbit launch vehicle to inject a payload of 4000kg into a Geostationary Transfer Orbit from Kourou (French Guyana). MDF and IDF-PCE are compared. The LVD process consists of four disciplines: propulsion, mass budget and geometry design, aerodynamics and trajectory, using low-fidelity models [19, 33, 56] (Fig. 23). The expected value of the

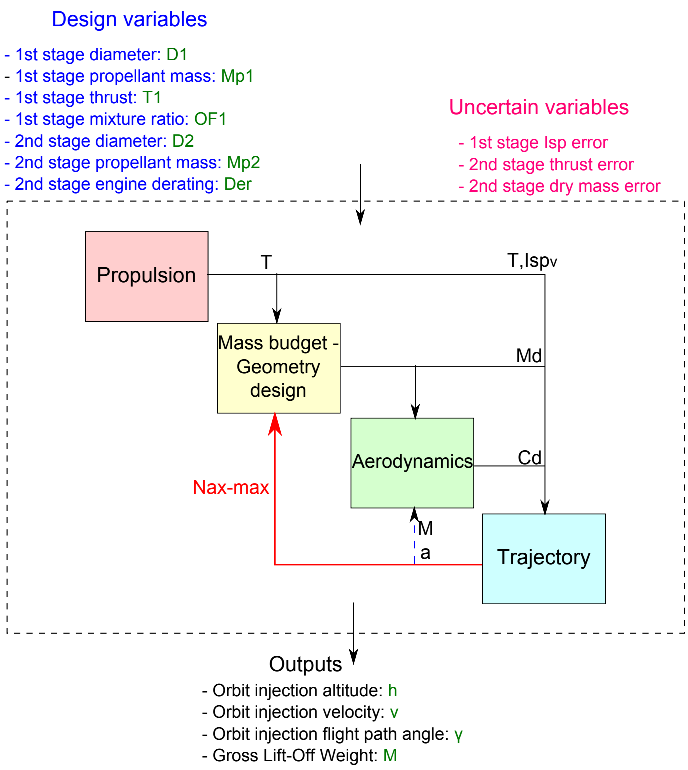


Figure 23: Design Structure Matrix for the two stage launch vehicle [16]

Gross Lift-Off Weight of the launch vehicle has to be minimized. The problem involves design variables and is initialized at a given baseline (Table 2). Three aleatory uncertain variables are present:

- 2^{nd} stage dry mass error (mass and sizing discipline),
- 1^{st} stage specific impulse error (propulsion discipline),
- 2^{nd} stage thrust error (propulsion discipline).

These errors are additional terms to the nominal value of specific impulse (Isp_{v10}), of dry mass (Me_0) and thrust (T_{20}).

Table 2: Design variables for the two stage launch vehicle

Design Variables	Symbols
Stage diameters	D_1, D_2
Stage propellant masses	M_{p1}, M_{p2}
1^{st} stage thrust	T_1
1^{st} stage mixture ratio	OF_1
2^{nd} stage engine derating coefficient	Der

One inequality constraint is considered. It is an output of the trajectory discipline and corresponds to the probability of failure of the mission (taking into account the altitude h, velocity v and flight path angle γ of the injection point). This probability of failure has to be lower than 5×10^{-2}. A failure occurs when the payload is injected outside a closed ball around the target injection point defined in the rotating frame by: $h_t = 250$km, $v_t = 9.713$km/s and $\gamma_t = 0°$. The radius of the ball corresponds to the injection tolerances and is set to be at 1% of the target altitude, at 0.5% of the target velocity and at 0.4° for the target flight path angle. The uncertainty propagation is performed with Crude Monte-Carlo (CMC). A pattern search optimization algorithm [5] is used to solve both MDF and IDF-PCE problems.

Results

Both MDF and IDF-PCE converge to the same optimum (163.7t), and the constraints are satisfied. The mean of the error between the input and the output load factor (coupling variable) is of 0.1% in IDF-PCE. IDF-PCE converges 11 times faster than MDF to the optimum as it does not require any MDA (Figs. 24). For the optimal launch vehicle, the results of uncertainty propagation for trajectory altitude are represented in Figures 25 and 26.

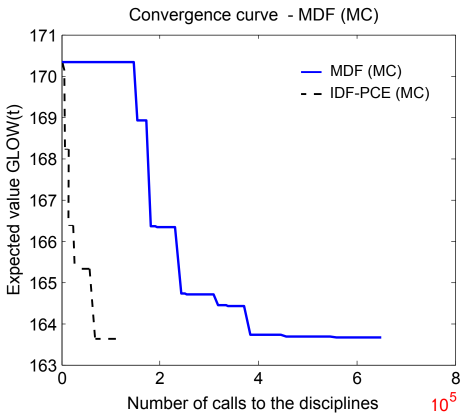


Figure 24: Convergence curves with the points satisfying the constraints

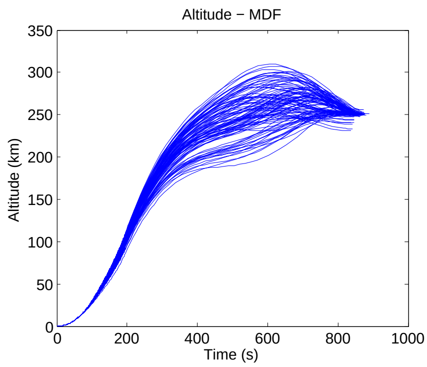


Figure 25: Optimal trajectory altitude under uncertainty - MDF

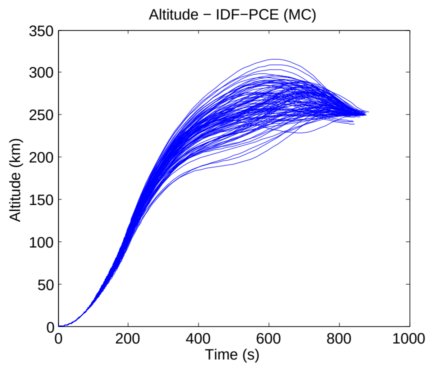


Figure 26: Optimal trajectory altitude under uncertainty - IDF-PCE

Comparison between MDO and UMDO solutions

In order to stress the need of taking the uncertainties into account in the early design phase, the deterministic MDO problem has been solved considering the uncertainties fixed to their mean values [16]. The found optimum is 158.21t (5.5t lower than the solution taking the uncertainty into account). The optimal nominal (*i.e.* without uncertainty) trajectory altitude profile is represented in Fig. 27. For the deterministic optimal launch vehicle, a propagation of uncertainty is performed by CMC and MDA with the same uncertainties as considered in the UMDO problem. In Fig. 28, the trajectory altitude is represented for CMC realizations of the uncertain variables. The deterministic optimal launch vehicle is not robust to the presence of uncertainty as the injection altitude is scattered between 200km and 250km due the lack of propellant to reach the injection point. Fig. 25 highlights the robustness of the UMDO found solution compared to the deterministic one. The deterministic MDF and the MDF under uncertainty optimal launch vehicle dimensions are represented in Figure 29.

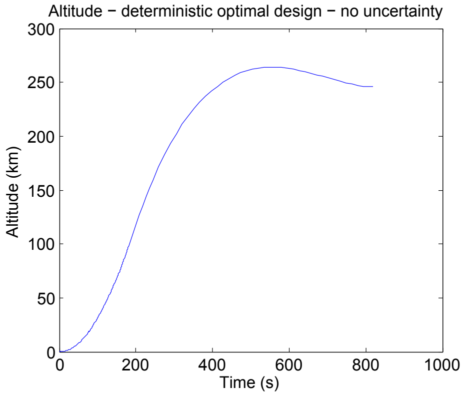


Figure 27: Trajectory altitude for the deterministic optimal launch vehicle, no uncertainty

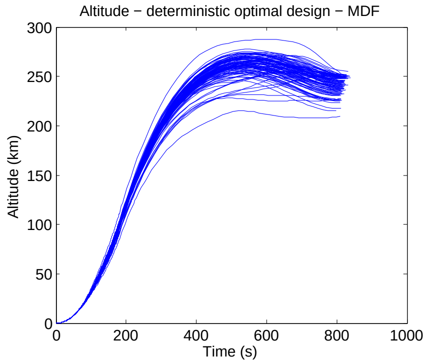


Figure 28: Uncertainty propagation - trajectory altitude - deterministic optimal launch vehicle

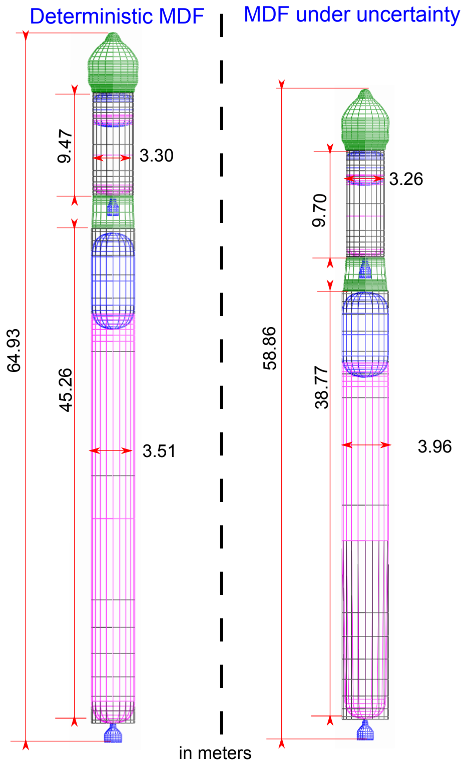


Figure 29: Comparison of optimal deterministic MDF and MDF under uncertainty launch vehicles

Second test-case: comparison of IDF-PCE, MDF and MHOU on a multi-stage sounding rocket design problem

This LVD test case consists in designing a sounding rocket with two solid stages to launch a payload of 800kg from Kourou that has to reach at least an altitude of 300km. Sounding rockets carry scientific experiments into space along a parabolic trajectory. Their overall time in space is brief and the cost factor makes sounding rockets an interesting alternative to heavier launch vehicles as they are sometimes more appropriate to successfully carry out a scientific mission and are less complex to design. Four disciplines are involved in the considered test case, the propulsion, the mass budget and geometry design, the aerodynamics and the trajectory (Fig. 30) [19, 33, 56]. The sounding rocket design is decomposed into two subsystems, one for each stage. MHOU enables a hierarchical design process decomposed into two teams, one for each sounding rocket stage. On this test case, MDF, IDF-PCE and MHOU are compared.

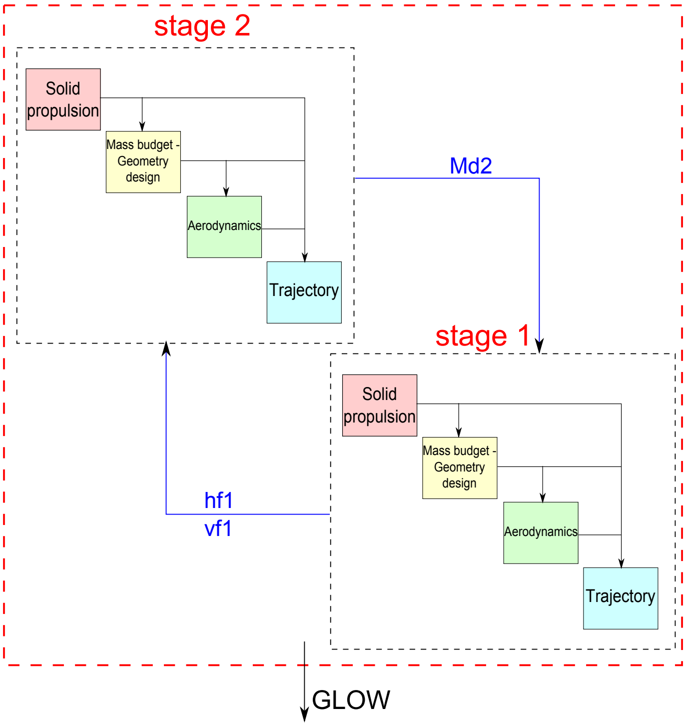


Figure 30: Design Structure Matrix for the two stage sounding rocket

Table 3: Design variables for the two stage sounding rocket.

Design Variables	Symbols
Stage diameters	D_1, D_2
Stage propellant masses	M_{p1}, M_{p2}
Stage nozzle expansion ratio	ϵ_1, ϵ_2
Stage grain relative length	RL_1, RL_2
Stage combustion depth	W_1, W_2

The uncertain variables taken into account are the 1^{st} stage combustion regression rate coefficient $\mathcal{N}(3.99, 0.05)$ in $cm/s/MPa^{0.3}$ and the 2^{nd} stage dry mass error $\mathcal{N}(0, 50)$ in *kg*. The uncertainty on the combustion model through the combustion regression rate results in uncertainty on the 1^{st} stage thrust. The mission has to ensure that the payload reaches at least an altitude of 300km (with a probability of failure of 3×10^{-2}). CMA-ES optimization algorithm [30] is used at the subsystem-level for MHOU. The same feasible baseline is considered as initialization for the three methods. The baseline corresponds to the deterministic optimal solution of the two stage sounding rocket problem (Fig. 32) found by a deterministic MDF approach. However, this solution is not robust to the presence of uncertainty. Indeed, the deterministic optimal solution does not succeed to reach with a probability of failure lower than 3×10^{-2} an altitude of 300km, the failure rate is around 70%.

Results

MHOU (6.68t) and IDF-PCE (6.88t) presents better characteristics in terms of quality of objective function than MDF (7.07t) for a fixed discipline evaluation budget (Fig.31). MDF, IDF-PCE and MHOU solutions satisfy the constraints especially the apogee altitude of 300km as illustrated in Fig. 33 for MDF and MHOU. Only 2.9% of the trajectories do not reach the required apogee altitude. MHOU ensures interdisciplinary coupling satisfaction for the feedback couplings as illustrated by the comparison of the couplings found respectively by the coupled approach and the decoupled approach for the optimal solution found by MHOU. The same coupling satisfaction are found for IDF-PCE. The separation altitude and velocity distributions for the optimal MHOU found solution are similar by using MDA or MHOU (Fig. 35-38). Moreover, the interdisciplinary coupling error for the separation altitude and velocity are represented in Fig. 39 and 40. The coupling error is always lower than 2% and concentrated around 0%-0.5%. The design space dimension for the system-level is increased from 10 for MDF to 13 for MHOU, however it enables multi-level optimization where each stage subsystem handles its local design variables. For IDF-PCE, the dimension of the system-level design space is 22. Thanks to the two levels of optimization, MHOU allows to converge to a better optimum than IDF-PCE in this test case while enabling decoupled design strategy and autonomy to each engineering team working on each stage.

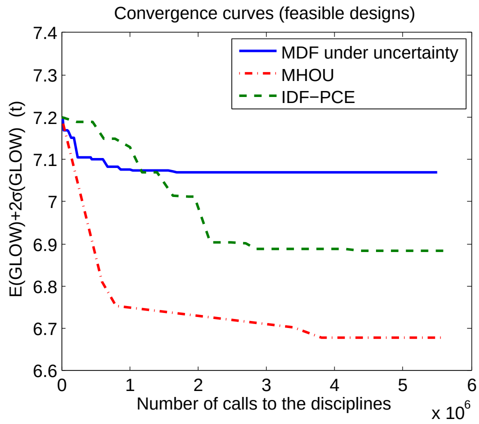


Figure 31: Convergence curves with the points satisfying the constraints

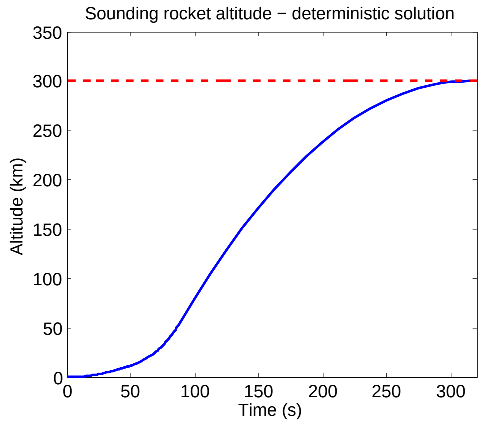


Figure 32: Optimal sounding rocket altitude without uncertainty

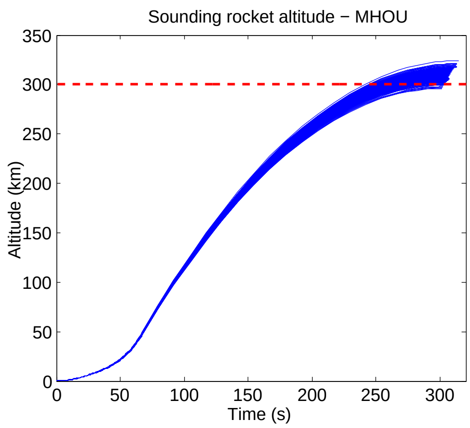


Figure 33: Optimal sounding rocket altitude

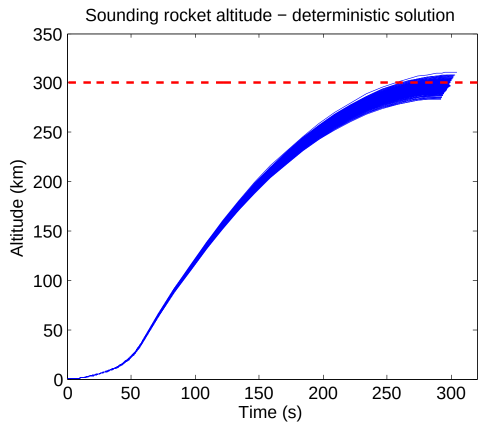


Figure 34: Deterministic optimal sounding rocket altitude in the presence of uncertainty

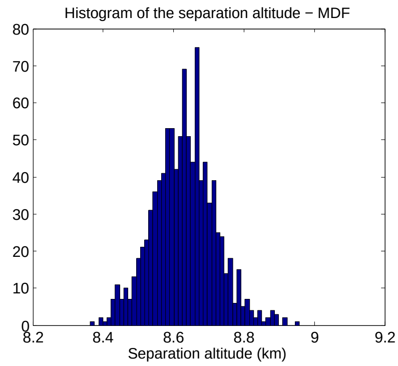


Figure 35: Distribution of the separation altitude for the optimal MHOU solution - by MDA

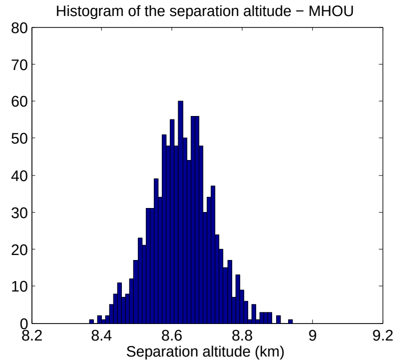


Figure 36: Distribution of the separation altitude for the optimal MHOU solution

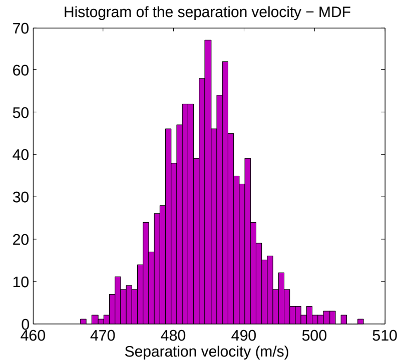


Figure 37: Distribution of the separation velocity for the optimal MHOU solution - by MDA

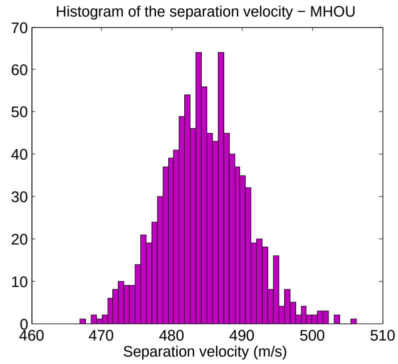


Figure 38: Distribution of the separation velocity for the optimal MHOU solution

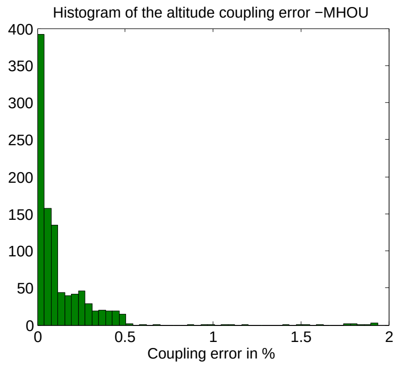


Figure 39: Distribution of the altitude coupling error MHOU

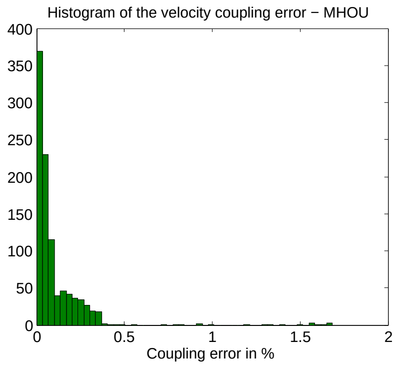


Figure 40: Distribution of the velocity coupling error MHOU

4 Conclusion

This chapter describes several methods to handle multidisciplinary and uncertainty aspects in the context of aerospace vehicle design process. Such processes are complex and present some specificities (*e.g.* predominance of trajectory for LVD) that stress the need to adapt existing MDO methods to exploit these latter and improve the problem solving efficiency. A dedicated hierarchical MDO method (SWORD) has been described in this chapter and compared to classical MDF on a three-stage-to-orbit launch vehicle design problem. The results of this comparison shown that such a dedicated MDO method allows to obtain a better optimum with respect MDF with less computation time. In addition to the multidisciplinary aspects, considering both epistemic and aleatory uncertainties in the design process is primordial in order to assess the designed vehicle performance and to ensure its reliability. For that purpose, a method allowing to bridge the gap between classical deterministic optimization and full probabilistic optimization has been described. The proposed bi-level optimization approach optimizes the safety factors at the upper-level and perform a full design / redesign process at the lower-level, providing for the designer with a set of optimal design rules satisfying the reliability requirements without having overly conservative designs. In the third part of this chapter, one single-level (IDF-PCE) and one multi-level (MHOU) formulations have been proposed in order to solve MDO problems in the presence of uncertainty. These methods allow to ensure the interdisciplinary functional coupling satisfaction for all the realizations of the uncertain variables. These approaches have been compared to MDF on two LVD problems and allow to obtain a better optimum with a less computational cost than classical MDF.

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