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The origin of inertia and the nature of the inertial rest mass.

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Abstract

In order to explain the origin of inertia and the nature of the inertial rest mass, we must first accept that gravity is described by a gravitomagnetic theory just like the electromagnetic theory with the gravitational mass as a Lorentz invariant, and secondly the fundamental idea of the relativity of all kinds of motion. By doing this we can prove that:

1. The external inertial forces, felt by an accelerating body, are inductive effects of the entire Universe while the internal inertial forces depend on the internal structure of the body. When a body moves freely in a gravitational field, its internal structure plays no role and the body feels only the external inertial forces. That is why all bodies fall at the same rate in a gravitational field.
2. The inertial rest mass of a body depends on the distribution of the matter in the Universe and this seems very important for dark matter and dark energy. The inertial mass of a charged particle depends on the distribution of other charges in its neighborhood and this effect turns out to be important in the subatomic world.
3. The gravitational field affects the geometry of space-time which is a Finsler-Randers space-time and all the freely moving bodies in a gravitational field follow geodesics of this space-time.

Keywords: gravitomagnetism , Mach's principle , origin of inertia , dark matter , dark energy

1 Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. Inertial forces appear in a non-inertial frame of reference. But what determines an inertial frame?

The first answer comes from Descartes and Newton, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the absolute space and the motion is absolute. The inertial forces, such as the centrifugal force, must arise from acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter.

The second answer comes from Leibniz, Berkeley and Mach and is known as Mach' principle, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the rest of the matter in the Universe, and the motion is relative. The inertial forces, such as the centrifugal force, are more likely caused by acceleration, with respect to the fixed stars. This idea implies that the properties of space arise from the matter contained therein and are meaningless in empty space.

The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its

vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important when we consider subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle, is motion relative to other matter in the Universe, has never found its complete expression in a physical theory. The Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity, attempted to continue this relativization and interpret inertia considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2].

The principle of Equivalence is an essential part of General relativity. But although the principle of Equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3].

In this paper, we will describe a theory that is consistent with Mach's principle and gives us a satisfactory solution for the inertial forces. We will describe gravity by a gravitomagnetic theory just like the electromagnetic theory. There is a well-established belief that we cannot have a spin-1 gauge field theory of gravity because one consequence of the spin-1 is that likes repel. However, the nature of the inertial rest mass allows us to solve this problem. R. Feynman writes in "Lectures on Gravitation" [4]: "...perhaps if we consider alternative theories which do not seem a priori justified, and we calculate what things would be like if such a theory were true, we might all of sudden discover that's way it really is."

2 Gravitomagnetic theory

The first step in order to explain the origin of inertia and the nature of the inertial rest mass, is to accept that gravity must be described by a gravitomagnetic theory just like the electromagnetic theory with the gravitational mass being a Lorentz invariant quantity and not equivalent to the inertial mass. However, we will prove that all bodies fall at the same rate in a gravitational field.

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformations (for coordinates, velocities, potentials, forces) and the following series of remarks [6] [7]:

1. The Coulomb potential at a distance r from a stationary point-charge q in vacuum is: $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric point-charge produces a scalar potential φ_e and a vector potential \vec{A}_e , which together form a four-vector, $A_e = \left(\frac{\varphi_e}{c}, \vec{A}_e \right)$
3. The potentials produced by a point-charge moving in any way, depend only upon the velocity and position at the retarded time.

where ϵ_0 is the vacuum permittivity and c the speed of light in vacuum. Of course we need to know how to get the Coulomb's law from the scalar potential.

Therefore, if we want to obtain a gravitomagnetic theory, with equations that have the same mathematical form, as those of the electromagnetic theory, first we must accept that the gravitational mass is a Lorentz invariant and second that the same series of remarks must be met for gravity. We already have the first remark, that is, the gravitational potential at a distance r from a stationary gravitational point-mass m in vacuum is,

$$\varphi_g = -G \frac{m}{r} \quad (2.1)$$

where G is the gravitational constant, but this is only the one remark. Therefore, we need the other two, as well. We will obtain them with the following two principles:

Principle 1

A gravitational point-mass produces a scalar potential φ_g and a vector potential \vec{A}_g , which together form a four-vector, $A_g = \left(\frac{\varphi_g}{c}, \vec{A}_g\right)$

Principle 2

The potentials produced by a gravitational point-mass moving in any way, depend only upon the velocity and position at the retarded time.

So, the potentials produced by a gravitational point-mass m moving with any velocity have the same mathematical form as the Lienard-Wiechert potentials for an electric point-charge moving with any velocity, but with a negative sign,

$$\varphi_g = -G \left[\frac{m}{r - \vec{r}\vec{v}/c} \right] \quad \text{and} \quad \vec{A}_g = -\frac{G}{c^2} \left[\frac{m\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_g \vec{v}] \quad (2.2)$$

where \vec{r} is the vector from the gravitational point-mass to the point where the potential is evaluated and the quantities r , \vec{r} and \vec{v} (the velocity of the point-mass) in the square bracket are to have their values at the retarded time. Starting from the potentials, in order to find the fields, we have the equations

$$\vec{E}_g = -\vec{\nabla}\varphi_g - \frac{\partial\vec{A}_g}{\partial t} \quad (2.3)$$

$$\vec{B}_g = \vec{\nabla} \times \vec{A}_g \quad (2.4)$$

When a gravitational mass m moves with velocity \vec{v} in the above fields, it feels the gravitomagnetic Lorentz force,

$$\vec{F}_g = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (2.5)$$

where \vec{E}_g is the gravitational field and \vec{B}_g the gravitomagnetic field.

So we expect that there are gravitomagnetic radiations propagating in vacuum at the speed of light but with a significant difference compared to electromagnetic radiations. It is well known that an isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment $d_e \equiv \sum_i q_i x^i$, that is,

$$L_{e,dipole} = \frac{2}{3} \ddot{d}_e^2 = \frac{2}{3} \left(\sum_i q_i \ddot{x}^i \right)^2 \quad (2.6)$$

where a dot denotes first time derivative and two dots second time derivative. However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity [8]. The reason is simple. The electric dipole moment can move around with respect to the center of the inertial mass but the gravitational dipole moment $d_g \equiv \sum_i m_i x^i$ is identical in location with the center off the inertial mass, and due to the law of conservation of momentum, cannot accelerate or radiate because

$$\dot{d}_g = \sum_i m_i \dot{x}^i = K \sum_i m_{in(i)} \dot{x}^i = K \sum_i p_i = 0 \quad (2.7)$$

where $m_{in(i)}$ the inertial mass and $m_i = K m_{in(i)}$ with K a constant which we will find later.

The gravitomagnetic theory must be described in flat spacetime by a spin-1 gauge field, because it is just like the electromagnetic theory. We will show that, the well-established belief that we cannot have a spin-1 field theory of gravity, is not the case.

The free Dirac Lagrangian of a particle is

$$\mathcal{L} = i\hbar c \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} c^2 \bar{\Psi} \Psi \quad (2.8)$$

where m_{in-0} the inertial rest mass of the particle, which we will prove later that is a scalar. It is well known that the free Dirac Lagrangian is invariant under the global gauge transformation [9]

$$\Psi \rightarrow e^{i\theta} \Psi \quad (2.9)$$

where θ is a real number. But the Lagrangian is not invariant under the local gauge transformations

$$\Psi \rightarrow e^{i\theta(x)} \Psi \quad (2.10)$$

where $\theta(x)$ is now function of x^i . Under the local gauge transformation we get

$$\mathcal{L}' = \mathcal{L} - \hbar c (\partial_\mu \theta) \bar{\Psi} \gamma^\mu \Psi \quad (2.11)$$

If we define

$$\theta(x) = -\frac{q}{\hbar c} \lambda_e(x) - \frac{m}{\hbar c} \lambda_g(x) \quad (2.12)$$

where q the electric charge and m the gravitational mass of the particle, the equation (2.11) becomes

$$\mathcal{L}' = \mathcal{L} + [q(\partial_\mu \lambda_e) + m(\partial_\mu \lambda_g)] \bar{\Psi} \gamma^\mu \Psi \quad (2.13)$$

Now, if we demand that the complete Lagrangian must be invariant under local gauge transformation, we are forced to add something to soak up the extra term

$$\mathcal{L} = [i\hbar c \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} c^2 \bar{\Psi} \Psi] - (\bar{\Psi} \gamma^\mu \Psi) [qA_e + mA_g] \quad (2.14)$$

where the vectors qA_e and mA_g are transformed under the local gauge transformation according to the rule

$$qA_e \rightarrow qA_e + q\partial_\mu \lambda_e \quad \text{and} \quad mA_g \rightarrow mA_g + m\partial_\mu \lambda_g \quad (2.15)$$

The full Lagrangian must include the free terms for the gauge fields. Thus, the full Lagrangian becomes

$$\mathcal{L} = [i\hbar c \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} c^2 \bar{\Psi} \Psi] - (\bar{\Psi} \gamma^\mu \Psi) (qA_e + mA_g) - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - \frac{1}{16\pi} G^{\mu\nu} G_{\mu\nu} \quad (2.16)$$

where

$$F^{\mu\nu} \equiv \partial^\mu A_e^\nu - \partial^\nu A_e^\mu \quad \text{and} \quad G^{\mu\nu} \equiv \partial^\mu A_g^\nu - \partial^\nu A_g^\mu \quad (2.17)$$

the electromagnetic and gravitomagnetic tensor respectively. The full Lagrangian is now locally gauge invariant, by introducing the electromagnetic field A_e and the gravitomagnetic field A_g . Both fields must be mass-less, otherwise the invariance will be lost.

However, as Richard Feynman writes in ‘‘Lectures on Gravitation’’ [10]: ‘‘A spin-1 theory would be essentially the same as electrodynamics. There is nothing to forbid the existence of two spin-1 fields, but gravity cant’be one of them, because one consequence of the spin-1 is that likes repel, and unlikes attract.’’ This well-established belief forces us to present the solution to the problem prematurely, before we discover the nature of the inertial rest mass. We will solve this problem by replacing the gravitational mass m with im (an imaginary number). We can do this because, as we will see, in the equations (4.24) and (4.25), the inertial rest mass is defined as the product of two gravitational masses. Therefore, every measurable quantity, such as the inertial mass and the kinetic energy, is expressed by a real number while the gravitational mass itself is not a measurable quantity. In any Feynman diagram describing the gravitational interaction, the gravitational mass will appear as a square via the coupling constant, because always two vertices are involved. Thus, by replacing the m with im , it will change the sign of the energy corresponding to this diagram so that likes attract in the gravitomagnetic theory. However, for simplicity, in what follows the gravitational mass continues to be expressed with m (a real number).

3 General relativity of motion

According to Richard Tolman, the fundamental principles of Einstein's General theory of relativity, i.e. the principle of General Covariance and the principle of Equivalence, may be regarded as based on the fundamental idea of the relativity of all kinds of motion [11]. We will now follow this fundamental idea. In accordance with this idea, we can detect and measure the motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion. The Special theory of relativity makes only a restricted use of this general idea, since it merely assumes the relativity of uniform translatory motion in a region of free space where gravitational effect can be neglected. Thus, the Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. In accordance with the fundamental idea of the relativity of all kinds of motion, an observer inside an accelerated rocket cannot distinguish whether the rocket is accelerated and the remainder of the Universe, matter and fields, is at rest or whether the rocket is at rest and the remainder of the Universe, matter and fields, is accelerated in the opposite direction.

In order to ensure the relativity of all kinds of motion the laws of physics should have the same mathematical form in all frames of reference since otherwise the difference in form could provide a criterion for judging the absolute motion. So, we accept the next principle:

Principle 3 - The principle of General Covariance

The laws of physics have the same mathematical form in all frames of reference.

In inertial frames of reference the laws of physics reduce to simpler mathematical forms which agree with the laws of Special theory of relativity. The fact that the expression of the equations of physics in a form which is independent of the motion of a reference frame relative to the fixed stars, does not in general prevent a change in their numerical content when we change from one reference frame to another. However, having a gravitomagnetic theory, just like the electromagnetic theory, we have the law of induction, given by equation (2.3). Therefore, we expect an induced gravitational field to appear in a reference frame that is accelerating relative to the fixed stars. We take for granted in this section the experimental fact that all bodies fall with the same acceleration in a gravitational field and therefore the ratio of gravitational mass to inertial rest mass for all freely moving bodies in a gravitational field is constant and can be considered, for the moment, equal with the unit. We will prove why this happens in the next section where we deal in detail with the induced gravitational field and the nature of the inertial rest mass. Thus, by relating the changes in numerical content, when we change from one reference frame to another, with changes in the induced gravitational field, we are able to eliminate the criteria for absolute motion and to preserve the idea of the relativity of all kinds of motion. Therefore, we accept the next principle:

Principle 4 - The principle of Equivalence

Physics in a non accelerating frame S, with a uniform gravitational field where all the released bodies fall with acceleration \vec{g} , is equivalent to physics in a local frame without gravity but with translational acceleration $\vec{a} = -\vec{g}$ and velocity zero with respect to the inertial frame in which the non accelerating frame S is at rest.

or,

Physics in a local frame freely falling in a gravitational field is equivalent to physics in an inertial frame without gravity.

Using the Special theory of relativity we are able to describe what physical effects are observed by an observer at rest in a uniformly accelerated frame of reference. The most well-known of them, apart from the inertial forces, are [12]:

1. Redshift or blueshift of a light ray moving parallel to the direction of the acceleration.
2. Varying coordinate speed of light; fixed local relative speed of light.
3. Space-time is endowed with a metric.
4. Maximum proper time as the law of motion of freely moving bodies.
5. Horizon

According to the principle of Equivalence the same effects must occur in a gravitational field. Therefore, the space-time is endowed with a metric and the gravitational field affects the space-time metric so that, the maximum proper time is the law of motion of a freely moving body in a gravitational field. The two above physical effects are so important that we will elevate them to physical principles:

Principle 5 - The Principle of Space-time Metric

The space-time interval between two neighbouring points events is:

$$ds^2 = g_{\mu\nu}(x^i, dx^i) dx^\mu dx^\nu$$

where $g_{\mu\nu}(x^i, dx^i)$ the metric tensor which depends not only on the position but also on the direction/velocity (Finsler geometry).

Principle 6 - The Principle of Geodesic Motion or of Maximum Proper Time

A freely moving body always moves along a geodesic: $\delta \int ds = 0$

Therefore, the fundamental idea of the relativity of all kinds of motion leads us to the conclusion that the gravitomagnetic field affects the geometry of space-time. However, the space-time now is not a pseudo-Riemannian space-time but a Finsler-Randers space-time [13].

Let us now see how the gravitomagnetic field affects the space-time. We will follow the Randers approach where the equation of motion of a test-body in a gravitomagnetic field results naturally as the geodesic of a Finsler-Randers space-time [14]. The lagrangian of a test-body of gravitational mass m and inertial rest mass m_{in-0} (which we will prove later that is a scalar) moving with four-velocity \dot{x}^μ in a gravitomagnetic field is

$$L = \sqrt{n_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu} + \frac{m}{m_{in-0}c^2} A_{g-\mu}\dot{x}^\mu \tag{3.1}$$

where $n_{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$ the Minkowski metric, $A_{g-\mu}$ the gravitational four-potential and $\dot{x}^\mu = dx/d\tau$ the four-velocity where $d\tau = \sqrt{n_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu}$. The first variation of the action corresponding to the langrangian (3.1) gives the Euler-Lagrange equations:

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right) - \frac{\partial L}{\partial x^\mu} = 0 \tag{3.2}$$

If we substitute the explicit form of the Lagrangian (3.1) in (3.2) we get the gravitomagnetic Lorentz equation of motion

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{m}{m_{in-0}c^2} G^{\mu\nu} \frac{dx_\nu}{d\tau} \tag{3.3}$$

where $G^{\mu\nu} \equiv \partial^\mu A_g^\nu - \partial^\nu A_g^\mu$, the gravitomagnetic-field tensor.

In order for the gravitomagnetic field to affect the space-time, we accept that the space-time is a Finsler-Randers space-time and we identify the metric function $F(x, \dot{x})$ of this space-time with the Lagrangian (3.1). So we get a Finsler-Randers space-time with metric function given by the principle [15]:

Principle 7 - The metric function of the Finsler-Randers spacetime is:

$$F(x, \dot{x}) = \sqrt{n_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu} + \frac{m}{m_{in-0}c^2}A_{g-\mu}\dot{x}^\mu$$

In this case the four velocity is $\dot{x}^\mu = dx/ds$, where ds is the Finsler-Randers proper time, because the measurable quantity in Finsler-Randers space-time is ds and not $d\tau$. In the absence of gravity, $ds = d\tau$. So, whenever exist a gravitomagnetic field in a region of space-time the space-time becomes Finslerian and the isotropy breaks. The metric function represent the distance ds between two neighbouring points represented by the coordinates x^i and $x^i + dx^i$

$$ds = F(x^i, dx^i) \quad (3.4)$$

Thus, using the principle of geodesic motion, the geodesic equation for this Finsler-Randers space-time is

$$\frac{d^2x^\mu}{ds^2} = \frac{m}{m_{in-0}c^2}G^{\mu\nu}\frac{dx_\nu}{ds} \quad (3.5)$$

which is the gravitomagnetic Lorentz equation of motion. As we said before, we assume that the ratio of gravitational mass to inertial rest mass, for all freely moving bodies in a gravitational field, is equal to unity. Therefore, the Lorentz equation of motion occurs physically from the geometry of the Finsler-Randers space-time.

According to P. Stavrinou [16], in a Finsler-Randers space-time every particle moving along a geodesic of the space-time satisfies the gravitomagnetic Lorentz equation. This is identified with the gravitomagnetic Lorentz equation of a particle moving in a curve of the gravitomagnetic field of the Minkowski space-time (not geodesic). Thus, we can describe the motion of a test-body in a gravitational field either by using forces in Minkowski space-time or by saying that the gravitomagnetic field curves the space-time and the test-body move along a geodesic.

The metric tensor of a Finsler-Randers space-time is given by the equation

$$g_{\mu\nu}(x^i, \dot{x}^i) = \frac{1}{2} \frac{\partial^2 F^2(x^i, \dot{x}^i)}{\partial \dot{x}^\mu \partial \dot{x}^\nu} \quad (3.6)$$

and it depends not only on the position but also on the velocity of the test-body. The metric function of the space-time may be given in terms of $g_{\mu\nu}$ as

$$F^2(x^i, \dot{x}^i) = g_{\mu\nu}(x^i, \dot{x}^i)\dot{x}^\mu\dot{x}^\nu \quad (3.7)$$

or, using the equation (3.4) $ds = F(x^i, dx^i)$, in terms of differentials

$$ds^2 = g_{\mu\nu}(x^i, dx^i)dx^\mu dx^\nu \quad (3.8)$$

According to P. Stavrinou [17], if the speed of the test-body is zero, $\dot{x}^\mu = (c, 0, 0, 0)$, we get from the metric tensor

$$g_{00} = 1 + \kappa^2\varphi_g^2 + 2\kappa\varphi_g \quad (3.9)$$

where $\kappa = \frac{m}{m_{in-0}c^2}$. Since the second term is very small compared to the third term, we get

$$g_{00} \approx 1 + 2\kappa\varphi_g = 1 + 2\frac{m}{m_{in-0}c^2}\varphi_g \quad (3.10)$$

Therefore, the line element in spherical coordinates, outside and at a distance r from the center of a static and stationary body B, with spherically symmetric distribution of gravitational mass M, becomes

$$ds^2 = \left(1 - \frac{2GMm}{rm_{in-0}c^2}\right)c^2 dt^2 - dr^2 \quad (3.11)$$

Equation (3.11) gives the effect of gravitational time dilation and the redshift of light emitted by an atom in a gravitational field for the static case.

4 Inertia

4.1 Gravitational inertial rest mass of a body with no internal structure

In accordance with the fundamental idea of the relativity of all kinds of motion, an observer inside an accelerated rocket cannot distinguish whether the rocket is accelerated and the fixed stars with their fields are at rest or whether the rocket is at rest and the fixed stars with their fields are accelerated in the opposite direction. The fields of the fixed stars are carried along convectively with the stars, just like the fields of the stars that moves with uniform velocity, and thus there is no radiation field of the fixed stars. Therefore, the instantaneous potentials of the fixed stars for an accelerating observer, is like the potentials of the same fixed stars if they were always moving in a straight line with constant speed. Therefore, we can find the instantaneous potentials of the fixed stars for an accelerating observer at some instant of time simply by the Lorentz transformations, using as velocity the instantaneous velocity of the fixed stars relative to the observer at the same instant of time.

Let's make now a thought experiment, the lab frame experiment. We suppose that we use a space station, which is far from any massive body, as a laboratory. We will call the local inertial frame where the space station is always at rest, the lab frame. The lab frame, as a local inertial frame, is only expected to function over a small region of space. We assume that the distribution of matter in the Universe is such that the gravitational field in the lab frame is zero. This means that the gravitational scalar potential φ_g , of the entire Universe, has the same value everywhere in the lab frame, and so,

$$\vec{\nabla}\varphi_g = 0 \quad (4.1)$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitomagnetic vector potential due to one part of the mass-current, is canceled out by the vector potential due to another part of the mass-current, owing to its symmetry. Therefore, the gravitomagnetic vector potential \vec{A}_g from the entire Universe in the lab frame is zero,

$$\vec{A}_g = 0 \quad (4.2)$$

This would also happen if all the bodies of the Universe were at rest, relative to the lab frame. So, we can say that the lab frame is at rest relative to the Universe, or at rest relative to the fixed stars.

We suppose that a point-particle, the test-body K, which is initially at rest in the lab frame, begins to accelerate making translatory motion along the x axis. We can find the potentials of the fixed stars as measured in the test-body K, from the potentials of the fixed stars as measured in the lab frame using the Lorentz transformations. Therefore, when the instantaneous velocity of the test-body K is v in the positive x -direction as measured in the lab frame, the Lorentz transformations which give the gravitational scalar potential φ'_g and the gravitomagnetic vector potential \vec{A}'_g in the test-body K, in terms of the potentials φ_g and \vec{A}_g in the lab frame, are:

$$\begin{aligned} \varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}), & A'_{g-y} &= A_{g-y} \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g), & A'_{g-z} &= A_{g-z}, & \gamma(v) &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned} \quad (4.3)$$

Therefore, using vector notation, the potentials in the test-body K are

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.4)$$

$$\vec{A}'_g = -\frac{1}{c^2}\gamma(v)\varphi_g\vec{v} = -\frac{1}{c^2}\varphi'_g\vec{v} \quad (4.5)$$

As the test-body K accelerates, the potentials change. Hence, according to equation (2.3) an induced gravitational field appears now in the test-body K which is

$$\vec{E}'_g = -\vec{\nabla}'\varphi'_g - \frac{\partial \vec{A}'_g}{\partial t'} \quad (4.6)$$

where $\partial t'$ is the time interval in the proper frame of the test-body K, i.e. the frame where the test-body K is always at rest. The gravitomagnetic field in the test-body K, is zero because all the fixed stars make translatory motion in respect to the test-body K and so,

$$\vec{B}'_g = \vec{\nabla}' \times \vec{A}'_g = 0 \quad (4.7)$$

Since the $\gamma(v)$ factor is the same everywhere in the proper frame of the test-body K, the scalar potential φ'_g is also the same everywhere and thus,

$$\vec{\nabla}'\varphi'_g = 0 \quad (4.8)$$

Therefore, the gravitational field in the test-body K becomes

$$\vec{E}'_g = -\frac{\partial \vec{A}'_g}{\partial t'} \quad (4.9)$$

If the test-body K has gravitational mass m, it will experience an induced gravitational force,

$$\vec{F}'_g = m\vec{E}'_g = -\frac{\partial (m\vec{A}'_g)}{\partial t'} \quad (4.10)$$

If we assume now that the gravitational scalar potential φ_g is independent of time (that's why we call the stars, fixed stars), substituting for \vec{A}'_g from equation (4.5) into equation (4.10), we get

$$\vec{F}'_g = -\left(-\frac{1}{c^2}m\varphi_g\right) \frac{\partial [\gamma(v)\vec{v}]}{\partial t'} = -\left(-\frac{1}{c^2}m\varphi_g\right) \gamma^3(v) \frac{d\vec{v}}{dt'} \quad (4.11)$$

If we recall now that the gravitational scalar potential is negative, it is obvious from equation (4.11) that the induced gravitational force on the test body K resists changes in its velocity. It is an inertial force!

We will call the inertial force which is given by equations (4.10), external gravitational inertial force \vec{F}'_{inert} because it is due to the acceleration with respect to the fixed stars. So,

$$\vec{F}'_{inert} = \vec{F}'_g = -\frac{\partial (m\vec{A}'_g)}{\partial t'} \quad (4.12)$$

Therefore, an inertial reference frame is a frame moving at a constant velocity relative to the fixed stars and an accelerating reference frame is a frame accelerating relative to the fixed stars. Any difference between an inertial and an accelerating frame, is only due to the above induced gravitational field. So an accelerating frame is just an inertial frame with an induced gravitational field.

In order for a body to move with acceleration relative to the fixed stars, an external force equal in magnitude but opposite in direction to the inertial force must be exerted on the body. Thus, the total force on a body in its proper frame is always zero, whether the body is moving with uniform velocity or is being accelerated relative to the fixed stars. So, we accept the Law of motion:

The motion of a body is such that, in its proper frame the total force on the body is always zero.

So, the force that accelerates a body and the inertial force that the body feels in its proper frame are equal in magnitude but opposite in direction. Therefore, inside a small free-falling elevator in a gravitational field, the total gravitational field is zero. That's why the free-falling elevator is an inertial frame and not because, as in Einstein's General relativity, a coordinate transformation vanishes the gravitational field.

In addition to the external inertial force, there is also an internal inertial force. This is a well known effect which has the name radiation reaction [18] [19]. We do not know exactly the mechanism that causes it but we know that it exists. The picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. We will call this self-force, internal inertial force, because it depends on the internal structure of the body.

The above effect, i.e. the self-force, is due to the induced gravitational field because of the acceleration relative to the fixed stars. However, according to the Law of motion, when a body makes free fall in a gravitational field, the total gravitational field in its proper frame is zero and therefore, the self-force is zero. So we conclude that:

When a body makes free fall in a gravitational field, the internal structure of the body plays no role and thus, only the external gravitational inertial force acts on the body.

We can obtain some very important and useful results using non-relativistic velocities. So, for non-relativistic velocities, from equation (4.10), the external gravitational inertial force on the accelerating test-body K is

$$\vec{F}'_{inert} = - \left(-\frac{1}{c^2} m \varphi_g \right) \frac{d\vec{v}}{dt} = \left(-\frac{1}{c^2} m \varphi_g \right) (-\vec{a}) \quad (4.13)$$

where dt is the time interval in the lab frame and \vec{a} is the acceleration with respect to the lab frame. Let's imagine now, that the test-body K is a body without internal structure and thus, when it is accelerated by a force \vec{F} , it does not feel any internal inertial force but only the external gravitational inertial force. According to the Law of motion, in the proper frame of the test-body K, the total force on the body is zero. Therefore, the force \vec{F} that accelerates the test-body K with acceleration \vec{a} , must be

$$\vec{F} = -\vec{F}'_{inert} = \left(-\frac{1}{c^2} m \varphi_g \right) \vec{a} = m_{in-g0} \vec{a} \quad (4.14)$$

The equation (4.14) is Newton's Second Law, for non-relativistic velocities, which obviously results from the Law of Motion. Therefore, the inertial rest mass m_{in-g0} of the test-body K is

$$m_{in-g0} = \left(-\frac{1}{c^2} m \varphi_g \right) \quad (4.15)$$

We will call the inertial rest mass m_{in-g0} of the test-body K gravitational inertial rest mass and its momentum \vec{p}_g , gravitational momentum, because they are due to the gravitational potential of the rest of the Universe. We must emphasize that the gravitational inertial rest mass of a body is just a part (a coefficient) of the inertial force and thus only appears when the body is accelerated. It makes no sense when the body is moving uniformly. So, the gravitational inertial rest mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential of the entire Universe.

For non relativistic velocities of the test-body K, its gravitational energy is: $E_g = m_{in-g0}c^2 = -m\varphi_g$ and its gravitational potential energy is: $U_g = m\varphi_g$. Therefore, its total gravitational energy is zero. Since the Universe consists of bodies such as the test body K (the internal structure does not concern us here) the total gravitational energy of the Universe is zero! It's noteworthy that Richard Feynman writes in "Lectures on Gravitation" [20]:

"Another spectacular coincidence relating the gravitational constant to the size of the universe comes in considering the total energy. The total gravitational energy of all the particles of the universe is something like GMM/R , where $R=Tc$, and T is the Hubble's time. [...] If now we compare this number to the total rest energy of the universe, Mc^2 , lo and behold, we get the amazing result that $GM^2/R = Mc^2$, so that the total energy of the universe is zero. Actually, we don't know the density nor that radius well enough to claim equality, but the fact that these two numbers should be of the same magnitude is a truly amazing coincidence. [...] Why this should be so is one of the great mysteries and therefore one of the important question of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate?"

It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential, and consequently also the inertial rest mass. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. Therefore, the distant matter is of predominant importance, while local matter has only a very small effect. Thus, it is difficult to observe any difference in the gravitational inertial rest mass with local experiments.

Let's suppose now that a test body of gravitational mass m , with internal structure, i.e. a composite body, is free-falling in the gravitational field of a large-mass body which has spherically symmetric gravitational mass M with $M \gg m$, in the region of the lab frame where the gravitational scalar potential from the rest of the Universe is φ_g . As we have shown, when a body makes a free fall in a gravitational field, only the external gravitational inertial force acts on the body. Therefore, for non-relativistic velocities, Newton's Law of Universal Gravitation and Newton's Second Law gives for the magnitude of the radial acceleration of the body

$$G\frac{Mm}{r^2} = \left(-\frac{1}{c^2}m\varphi_g\right) a \quad (4.16)$$

where r is the distance of the test-body from the center of the large-mass body. It is obvious that the gravitational mass m of the test body is canceled in equation (4.16). Therefore, the acceleration of a free-falling body is independent of its gravitational mass and thus, all bodies fall at the same rate in a gravitational field. This is a fundamental experimental result that was tested with great accuracy with the Eötvös experiment. In Einstein's General relativity, the above experimental result is interpreted by accepting the equivalence of gravitational mass and inertial rest mass.

Let us now return to relativistic physics to study the gravitational momentum of the test-body K of gravitational mass m with no internal structure. It is well known from the Special theory of relativity that if we wish to salvage Newton's law of momentum conservation, we must define the gravitational momentum \vec{p}_g of the test-body K in an inertial frame of reference S, where the test-body K moves with velocity \vec{u} , as follows

$$\vec{p}_g = \frac{m_{in-g0}}{\sqrt{1 - u^2/c^2}} \vec{u} = \gamma(u)m_{in-g0}\vec{u} \quad (4.17)$$

where the gravitational inertial rest mass m_{in-g0} of the test-body K must be a Lorentz invariant, i.e. all observers agree on its value at any instant of test-body's history.

As we showed earlier, for non relativistic velocities, the gravitational inertial rest mass of the test-body K is the negative of its gravitational potential energy with the rest of the Universe. Let us now find

the equation that describe the gravitational inertial rest mass for relativistic velocities. This means that we want to find the gravitational potential energy of the entire Universe relative to the test-body K, when it moves with velocity \vec{v} relative to the lab frame. The instantaneous sum of the gravitational four-potentials of all the bodies in the Universe, at a certain point, is also a four-vector, the total gravitational four-potential:

$$A_g = \left(\frac{\varphi_g}{c}, \vec{A}_g \right) \quad (4.18)$$

and the four-velocity of the test-body K is: $U = \gamma(v)(c, \vec{v})$.

We know that the quantity we are looking for must depend on both U and A_g , and it is a scalar. The product mUA_g has physical dimensions of energy and it is a scalar, because the gravitational mass is a scalar, and the product of two four-vectors is a Lorentz invariant, i.e. a scalar. Evaluating the product mUA_g in the rest frame of the test-body K, where the gravitational scalar potential from the entire Universe according to equation (4.4) is φ'_g , we get

$$mUA_g = m(c, o) \left(\frac{\varphi'_g}{c}, \vec{A}'_g \right) = m\varphi'_g \quad (4.19)$$

Thus we obtain the gravitational potential energy of the entire Universe relative to the test-body K, which is the very thing we wanted and is a Lorentz invariant. So, for relativistic velocities, the gravitational inertial rest mass of the test-body K is

$$m_{in-g0} = -\frac{1}{c^2}mUA_g - \frac{1}{c^2}m\varphi'_g \quad (4.20)$$

Substituting for φ'_g from equation (4.4) into equation (4.20) we get

$$m_{in-g0} = -\frac{1}{c^2}m\varphi'_g = \gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) \quad (4.21)$$

So, the gravitational inertial rest mass of a body without internal structure, is not an intrinsic property of the body but is proportional to the gravitational potential energy of the entire Universe relative to the body, and is a Lorentz invariant. The gravitational inertial rest mass of a body depends on its velocity relative to the fixed stars.

4.2 Gravitoelectric inertial rest mass of a body with no internal structure

Let us now consider, what happens if there are other electrical charges, in the neighborhood of the lab frame, with such a distribution and motion that the electric scalar potential φ_e in the lab frame is not zero but is the same everywhere, so that $\vec{\nabla}\varphi_e = 0$ and the magnetic vector potential \vec{A}_e is zero. We assume that a test-body K of gravitational mass m and electric charge q , with no internal structure, is accelerated in the lab frame. Since the equations of electromagnetism have the same mathematical form as the equations of gravitomagnetism, the test-body K will experience an induced gravitational and electric inertial force. In this case we will call the inertial rest mass of the test-body K, gravitoelectric inertial rest mass m_{in-ge0} . Using the same argument we used earlier for the gravitational inertial rest mass, we conclude that the gravitoelectric inertial rest mass is given by the equation

$$m_{in-ge0} = \gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \quad (4.22)$$

and is a Lorentz invariant quantity. So, when the instantaneous velocity of the test-body K relative to an inertial frame S is \vec{u} and relative to the lab frame is \vec{v} , the gravitoelectric momentum of the test-body K in the frame S, will be

$$p_{ge} = \gamma(u)\gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \vec{u} = \gamma(u)m_{in-ge0}\vec{u} \quad (4.23)$$

We now assume that the Universe consists of n distinct gravitational masses and m distinct electric charges, moving at non-relativistic velocities relative to the lab frame. If the test-body K moves with no relativistic velocity relative to the lab frame, its gravitoelectric inertial rest mass is

$$m_{in-ge0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \sum_{i=1}^m \frac{qq_i}{r_i} \right) \quad (4.24)$$

where $G = 1/4\pi g_0$ and the distance r from the test-body K , is measured in the lab frame. Therefore, the gravitoelectric inertial rest mass of a body depends on the distribution of the matter in the Universe.

4.3 Inertial rest mass of a composite body

So far we have only considered the test-body K with no internal structure. The Special theory of relativity will now give us the inertial rest mass of a body with internal structure, a composite body. If we apply the conservation of the four-momentum in an inelastic collision where n free moving particles without internal structure, collide and create a composite body M , the inertial rest mass m_{in-0} of the composite body M is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + T/c^2 + E_{field}/c^2 \quad (4.25)$$

where $m_{in-ge0i}$ the gravitoelectric inertial rest mass of each particle that makes up the composite body M , T is the kinetic energy of the relative motion of all the particles and E_{field} the potential energy of the interaction of all the particles [21]. The inertial rest mass of the composite body M , is also a Lorentz invariant as is well known from the Special theory of relativity [22].

From equations (4.24) and (4.25) we can see that if we replace the gravitational mass m with im (an imaginary number), the inertial rest mass which is a measurable quantity, is expressed by a real number, as we mentioned earlier in section 2.

4.4 Zero gravitoelectric inertial rest mass

Let us now imagine, using non relativistic physics, that we have an accelerating particle A of gravitational mass m , electric charge q and without internal structure, inside a thin spherical shell of radius R with charge q uniformly distributed on its surface. The electric scalar potential inside the spherical shell is the same everywhere and it is given by the well known equation, $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$. The gravitoelectric inertial rest mass of the particle A , according to equation (4.24), is

$$m_{in-ge0} = \frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R} = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R} \quad (4.26)$$

From equation (4.26) we can see that the gravitoelectric rest mass of the particle A can be zero if the radius takes the value $R_{critical}$ which is

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R_{critical}} \iff R_{critical} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{m_{in-g0}} \quad (4.27)$$

If the particle A is an electron which considered as a particle without internal structure, the charge q in equation (4.27) is the charge of the electron and for the $R_{critical}$ we get $R_{critical} \approx 2,81 \times 10^{-15}m$. Therefore, the effect of the electric scalar potential on the inertial rest mass of a charged particle, becomes significant in the subatomic world !!! This can be tested experimentally by measuring the inertial mass of moving electrons in a magnetic field if we put the whole device in a negatively charged spherical shell.

5 Some ideas for dark matter and dark energy

5.1 Dark matter

From equations (4.24) and (4.25), it follows that the inertial rest mass of a star depends on the gravitational scalar potential of the entire Universe, i.e. the inertial rest mass of a star depends on the distribution of matter in the Universe. In the Universe there are planets, stars, galaxies, clusters of galaxies and so on. Therefore, the position where a star is located, affects the inertial rest mass of the star. In places with higher density of matter the inertial rest mass of a star will be greater than the inertial rest mass of an identical star, in a place with lower density of matter.

Moreover, since the gravitational scalar potential and the gravitational vector potential satisfy the wave equation and travel at the speed of light in space, they must behave like the light that bends when traveling near a large gravitational mass. Therefore, the gravitational potentials of the entire Universe are more concentrated in places with higher densities of matter. This is a second reason why in places with higher density of matter the inertial rest mass of a star will be greater than the inertial rest mass of an identical star, in a place with lower density of matter.

So, the inertial rest mass of a star near the center of a galaxy is greater than the inertial rest mass of an identical star at the edges of that galaxy. Therefore, stars at the edges of a rotating spiral galaxy, are moving faster than Newtonian physics predicts by assuming that the inertial rest mass is the same everywhere. This phenomenon has been observed, but the inability to explain it has led to the theory of dark matter. It is very likely that the above ideas provide a solution to this problem.

5.2 Dark energy

Let us now consider the light emitted by an atom on the surface of a static, spherically symmetric star of gravitational mass M . We assume that an atom of gravitational mass m and inertial rest mass m_{in-0} , which emits light, is at a distance r_{em} from the center of the star. From equation (3.11) arises the equation relating the frequency f_{em} of the light at the point of emission, with the frequency f_{∞} of the light at infinity where is the point of observation [23]

$$f_{\infty} = f_{em} \sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-0}}} \quad (5.1)$$

Equation (5.1) describes the redshift of light emitted by an atom in a gravitational field and received by an observer who is very far away, essentially at infinity.

As the Universe expands, the density of matter decreases and therefore, the inertial rest mass of an atom decreases over time. Therefore, as it emerges from equation (5.1), the light emitted by the atoms of two identical supernovas Ia at different times in the history of the Universe, will have different red shift. The atoms of a younger (most recent) supernova will have smaller inertial rest mass than the atoms of an older supernova. Hence, the light emitted by the atoms of a younger supernova Ia will have larger red shift than the light emitted by the atoms of an older supernova Ia. This phenomenon has been observed, but the inability to explain it has led to the theory that the Universe expands in an accelerating way, because of dark energy. It is very likely that the above idea provides a solution to this problem.

Conclusions

In this paper we have shown that we can have a spin-1 gauge field theory of gravity that affects the space-time metric and also explains the origin of inertia and the nature of the inertial rest mass. The new theory is fully consistent with Mach's principle, and its equation for the inertial rest mass seems very

important for dark matter, dark energy, nuclear and particle physics. Due to the similarity of gravitational and electric forces we need to check whether the electric forces also affects the space-time metric. We can measure, if any, the frequency shift of a laser beam in a region of strong electric field. The new theory show us that the motion of a tiny body is affected by the entire Universe, and a tiny body gives us information about the entire Universe. Therefore, deserve to be told how Dennis Sciama ended an article on inertia [24]: “If atomic properties are in fact so determined, we shall again be faced with the dual situation: Distant matter influencing local phenomena and local phenomena giving us information about distant matter. The scientist would then be able to claim that his imagination had out-stripped the poet’s. For he would see the world not in a “grain of sand” but in an atom”

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