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► To cite this version:

Konstantinos I. Tsarouchas. The origin of inertia and the nature of inertial rest mass. An important issue for dark matter and dark energy.. 2022. hal-01474982v8

HAL Id: hal-01474982

<https://hal.science/hal-01474982v8>

Preprint submitted on 6 Nov 2022 (v8), last revised 3 Jul 2023 (v10)

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The origin of inertia and the nature of inertial rest mass. An important issue for dark matter and dark energy.

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Abstract

We will show in this paper that gravity, in flat spacetime, must be described by a gravitomagnetic theory with the gravitational mass as Lorentz invariant and not equivalent to the inertial mass. We can then discover the origin of inertia and the nature of the inertial rest mass if we extend the principle of relativity to all kinds of motion by accepting the principle of General Covariance and the principle of Equivalence. Thus, we can prove that:

1. The external inertial forces, felt by an accelerating body, are inductive effects of the entire Universe while the internal inertial forces depend on the internal structure of the body. In a free fall the internal inertial forces are canceled due to the principle of Equivalence and the body feels only the external inertial forces. That's why all bodies fall at the same rate in a gravitational field.
2. The inertial rest mass of a body depends on the distribution of the matter in the Universe and this seems very important for dark matter and dark energy. The inertial mass of a charged particles in the subatomic world can take any value.
3. The gravitational forces affect the spacetime metric and the Schwarzschild metric is a consequence of the new theory.

Keywords: gravitomagnetism , relative acceleration , Mach's principle , origin of inertia , nature of inertial rest mass , dark matter , dark energy

1 Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. Inertial forces appear in a non-inertial frame of reference. But what determines an inertial frame?

The first answer comes from Descartes and Newton, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the absolute space and the motion is absolute. The inertial forces, such as the centrifugal force, must arise from acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter.

The second answer comes from Leibniz, Berkeley and Mach and is known as Mach' principle, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the rest of the matter in the Universe, and the motion is relative. The inertial forces, such as the centrifugal force, are more likely caused by acceleration, with respect to the fixed stars. This idea implies that the properties of space arise from the matter contained therein and are meaningless in empty space.

The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important when we consider subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle, is motion relative to other matter in the Universe, has never found its complete expression in a physical theory. The Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity, attempted to continue this relativization and interpret inertia considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2].

The principle of Equivalence is an essential part of General relativity. But although the principle of Equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3]. As Richard Feynman writes in Lectures on gravitation: "...perhaps if we consider alternative theories which do not seem a priori justified, and we calculate what things would be like if such a theory were true, we might all of sudden discover that's way it really is."

2 Gravitomagnetic theory

The first step in order to explain the inertial forces, is to accept that the gravitational forces must be described by a gravitomagnetic theory. Why should we do this? Let's do a thought experiment. Let us have a system of two non spinning bodies with gravitational masses and positive electric charges, in a region of free space where there are no external forces. We suppose that the two bodies are at rest in an inertial frame of reference S under equilibrium conditions, i.e. the force of gravitational attraction balances that of electrostatic repulsion. But what is observed by another inertial frame of reference S' , moving with uniform velocity relative to the frame S? Let us imagine that if the bodies collide, they will explode. It is impossible for one observer to see an explosion and for another to not see it. **Therefore, the equilibrium must be a frame-independent condition.**

In order for equilibrium to be a frame-independent condition, it is necessary that the gravitational forces, in flat spacetime, to have exactly the same transformation law as that of the Lorentz force. But this does not happen with gravity as it is described by the General theory of relativity. The linear approximation of the General Relativity, which is only valid for weak gravitational fields, gives us an equation analogous to the Lorentz force but the analogy is not perfect [4]. **For this reason we accept that gravity must be described, in flat spacetime, by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory with the gravitational mass as a Lorentz invariant.**

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformations (for coordinates, velocities, potentials, forces) and the following series of remarks [5][6]:

1. The Coulomb potential at a distance r from a stationary point-charge q in vacuum is: $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric point-charge produces a scalar potential φ_e and a vector potential \vec{A}_e , which together form a four-vector, $A_e = \left(\frac{\varphi_e}{c}, \vec{A}_e \right)$
3. The potentials produced by a point-charge moving in any way, depend only upon the velocity and position at the retarded time.

where ε_0 is the vacuum permittivity. Of course we need to know how to get the Coulomb's law from the scalar potential. **Therefore, if we want to obtain a gravitomagnetic theory, with equations that have the same mathematical form, as those of the electromagnetic theory, first we must accept that the gravitational mass is a Lorentz invariant and second that the same series of remarks must be met for gravity.** We already have the first remark, that is, the gravitational potential at a distance r from a stationary gravitational point-mass m in vacuum is, $\varphi_g = -G\frac{m}{r}$ where G is the gravitational constant, but this is only the one remark. Therefore, we need the other two, as well. We will obtain them with the following two principles:

Principle 1

A gravitational point-mass produces a scalar potential φ_g and a vector potential \vec{A}_g , which together form a four-vector, $A_g = \left(\frac{\varphi_g}{c}, \vec{A}_g\right)$

Principle 2

The potentials produced by a gravitational point-mass moving in any way, depend only upon the velocity and position at the retarded time.

So, the potentials produced by a gravitational point-mass m moving with any velocity have the same mathematical form as the Lienard-Wiechert potentials for an electric point-charge moving with any velocity, but with a negative sign,

$$\varphi_g = -G \left[\frac{m}{r - \vec{r}\vec{v}/c} \right] \quad \text{and} \quad \vec{A}_g = -\frac{G}{c^2} \left[\frac{m\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_g \vec{v}] \quad (2.1)$$

where \vec{r} is the vector from the gravitational point-mass to the point where the potential is evaluated, c is the speed of light in vacuum and the quantities r , \vec{r} and \vec{v} (the velocity of the point-mass) in the square bracket are to have their values at the retarded time. Starting from the potentials, in order to find the fields, we have the equations

$$\vec{E}_g = -\vec{\nabla}\varphi_g - \frac{\partial\vec{A}_g}{\partial t} \quad (2.2)$$

$$\vec{B}_g = \vec{\nabla} \times \vec{A}_g \quad (2.3)$$

When a gravitational mass m moves with velocity \vec{v} in the above fields, it feels the force,

$$\vec{F}_g = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (2.4)$$

where \vec{E}_g is the gravitational field and \vec{B}_g the gravitomagnetic field.

So we expect that there are gravitomagnetic radiations propagating in vacuum at the speed of light but with a significant difference compared to electromagnetic radiations. As I. Ciufolini and J. A. Wheeler write in "Gravitation and Inertia" [7], an isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment $d_e \equiv \sum_i q_i x^i$, that is, $L_{e,dipole} = \frac{2}{3} \ddot{d}_e^2 = \frac{2}{3} (\sum_i q_i \ddot{x}^i)^2$, where a dot denotes first time derivative and two dots second time derivative. **However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity.** The reason is simple. The electric dipole moment can move around with respect to the center of mass but the gravitational dipole moment $d_g \equiv \sum_i m_i x^i$ is identical in location with the center of mass, and due to the law of conservation of momentum, cannot accelerate or radiate $\dot{d}_g = \sum_i m_i \dot{x}^i = \sum_i p_i = 0$.

For the quantum field theory, the gravitomagnetic theory must be described in flat spacetime by a spin-1 gauge field, because it is just like electromagnetic theory. Let's see how this can happen. It is well known that the free Dirac Lagrangian density for a particle [8]

$$\mathcal{L} = i\hbar c \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} \bar{\Psi} \Psi \quad (2.5)$$

where m_{in-0} the inertial rest mass of the particle, is invariant under the transformation

$$\Psi \rightarrow e^{i\theta} \Psi \quad (2.6)$$

where θ is a real number. But the Lagrangian density is not invariant under the following transformations

$$\Psi \rightarrow e^{i\theta(x)} \Psi \quad (2.7)$$

where $\theta(x)$ is now function of x . Under the local phase transformation we get

$$\mathcal{L}' = \mathcal{L} - \hbar c (\partial_\mu \theta) \bar{\Psi} \gamma^\mu \Psi \quad (2.8)$$

If we define

$$\theta(x) = -\frac{q}{\hbar c} \lambda_e(x) - \frac{m}{\hbar c} \lambda_g(x) \quad (2.9)$$

where q the electric charge and m the gravitational mass of the particle, the equation (2.8) becomes

$$\mathcal{L}' = \mathcal{L} + [q(\partial_\mu \lambda_e) + m(\partial_\mu \lambda_g)] \bar{\Psi} \gamma^\mu \Psi \quad (2.10)$$

Now, if we demand that the complete Lagrangian must be invariant under local phase transformation, we are forced to add something to soak up the extra term

$$\mathcal{L} = [i\hbar c \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} \bar{\Psi} \Psi] - (\bar{\Psi} \gamma^\mu \Psi) [qA_e + mA_g] \quad (2.11)$$

where the vectors qA_e and mA_g are transformed under the local gauge transformation according to the rule

$$qA_e \rightarrow qA_e + q\partial_\mu \lambda_e \quad \text{and} \quad mA_g \rightarrow mA_g + m\partial_\mu \lambda_g \quad (2.12)$$

Thus, the complete Lagrangian becomes

$$\mathcal{L} = [i\hbar \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m_{in-0} \bar{\Psi} \Psi] - (\bar{\Psi} \gamma^\mu \Psi) (qA_e + mA_g) - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - \frac{1}{16\pi} G^{\mu\nu} G_{\mu\nu} \quad (2.13)$$

where

$$F^{\mu\nu} \equiv \partial^\mu A_e^\nu - \partial^\nu A_e^\mu \quad \text{and} \quad G^{\mu\nu} \equiv \partial^\mu A_g^\nu - \partial^\nu A_g^\mu \quad (2.14)$$

The complete Lagrangian (2.13) is now locally invariant by introducing the electromagnetic field A_e and the gravitomagnetic field A_g . Both fields must be mass-less, otherwise the invariance will be lost.

In order for like masses to attract and opposites to repel, the opposite of electric charges, we will accept that the gravitational mass is an imaginary number. We can do this because, as we will show below in equations (4.15) and (4.31), any measurable quantity, such as the inertial mass, the momentum and the kinetic energy, is the product of two gravitational masses and therefore is a real number. The gravitatonal mass will appear as a square in any Feynman diagram, via the coupling constant, because always two vertices are involved. **So, it will change the sign of the energy corresponding to this diagram so that likes attract and unlikes repel in gravitomagnetism.** However, for simplicity, in what follows the gravitational mass continues to be used as if it were a real number.

Now having a gravitomagnetic theory, just like the electromagnetic theory, we have the law of induction, given by equation (2.2). Therefore, if we can apply the laws of physics to a reference frame that is accelerating relative to the fixed stars, we expect an induced gravitational field to appear in that frame. This gravitational field may be the cause of inertial forces. However, in order to apply the laws of physics to an accelerating frame of reference, we need to extend the principle of relativity. For this reason we will follow the fundamental idea of the relativity of all kinds of motion.

3 General relativity of motion

According to Richard Tolman, the fundamental principles of General theory of relativity, i.e. the principle of General Covariance and the principle of Equivalence, may be regarded as based on the fundamental idea of the relativity of all kinds of motion [9]. In accordance with this idea we can detect and measure the motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion. Therefore, an observer inside an accelerated rocket cannot distinguish whether the rocket is accelerated and the remainder of the Universe is at rest or whether the rocket is at rest and the remainder of the Universe is accelerated in the opposite direction.

According to R. Tolman, in order to ensure the relativity of all kinds of motion we need the principle of General Covariance and the principle of Equivalence. We need the principle of General Covariance because the laws of physics should have the same mathematical form in all frames of reference since otherwise the difference in form could provide a criterion for judging the absolute motion. We need the principle of Equivalence, because the fact that the expression of the equations of physics in a form which is independent of the reference frame does not in general prevent a change in their numerical content when we change from one reference frame to another and it is only by relating such changes in numerical content to conceivable changes in gravitational field that we are able to eliminate criteria for absolute motion and to preserve the idea of the relativity of all kinds of motion. So, we accept the next principles:

Principle 3 - The principle of General Covariance

The laws of physics have the same mathematical form in all frames of reference. In inertial frames of reference the laws of physics reduce to simpler mathematical forms which agree with the laws of Special theory of relativity.

Principle 4 - The principle of Equivalence

Physics in a non accelerating frame S, with a uniform gravitational field where all the released bodies fall with acceleration \vec{g} , is equivalent to physics in a frame without gravity but with translational acceleration $\vec{a} = -\vec{g}$ and velocity zero with respect to the inertial frame in which the non accelerating frame S is at rest.

or,

Physics in a local frame freely falling in a gravitational field is equivalent to physics in an inertial frame without gravity.

Using the Special theory of relativity we are able to describe what physical effects are observed by an observer at rest in a uniformly accelerated frame of reference. **The most well-known of them, apart from the inertial forces, are [10]:**

1. Redshift or blueshift of a light ray moving parallel to the direction of the acceleration.
2. Varying coordinate speed of light; fixed local relative speed of light.
3. Spacetime is endowed with a metric.
4. Maximum proper time as the law of motion of freely moving bodies.

According to the principle of Equivalence the same effects must occur in a gravitational field . Therefore, the spacetime is endowed with a metric and the gravitational field affects the spacetime metric so that, the maximum proper time is the law of motion of a freely moving body in a gravitational field. The two above physical effects are so important that we will elevate them to physical principles:

Principle 5 - The Principle of Spacetime Metric

Spacetime is endowed with a metric. The spacetime interval between two events is: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ where $g_{\mu\nu}$ is the metric tensor.

Principle 6 - The Principle of Geodesic Motion or of Maximum Proper Time

A freely moving body always moves along a geodesic: $\delta \int ds = 0$

If the ratio of gravitational mass m to the inertial rest mass m_{in-0} of a freely moving particle K: $\frac{m}{m_{in-0}}$, is the same for all freely moving particles, the geodesic equation is

$$m_{in-0} \frac{d^2 x^\mu}{ds^2} + m \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} = 0 \quad (3.1)$$

where $\Gamma_{\nu\sigma}^\mu$ the Christoffel symbol. However, if the gravitational field is so weak that the spacetime is nearly flat, according to the gravitomagnetic theory, the equation of motion of the particle K is

$$m_{in-0} \frac{d^2 x'^\mu}{ds^2} = \frac{m}{c} G^\mu{}_\nu \frac{dx'^\nu}{ds} \quad (3.2)$$

where $G^\mu{}_\nu$ is the gravitomagnetic tensor. Therefore, if the gravitational field is so weak that the spacetime is nearly flat, the term: $m \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds}$ must reduce to the term: $-\frac{m}{c} G^\mu{}_\nu \frac{dx^\nu}{ds}$, that is

$$m \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} \rightarrow -\frac{m}{c} G^\mu{}_\nu \frac{dx^\nu}{ds} \quad (3.3)$$

According to C. Duarte, who examines this subject in the paper entitled “The classical geometrization of the electromagnetism” [11], for a weak gravitational field the metric tensor is

$$g_{\mu\nu} = \begin{pmatrix} 1 + 2\phi_g/c^2 & A_{g1}/c & A_{g2}/c & A_{g3}/c \\ A_{g1}/c & -1 & 0 & 0 \\ A_{g2}/c & 0 & -1 & 0 \\ A_{g3}/c & 0 & 0 & -1 \end{pmatrix}$$

So, the relation (3.3) shows us that the components of the metric tensor must depend on the components of the gravitational four-potential and thus the metric tensor will have as many degrees of freedom as the electromagnetic field. **Therefore, gravity can be described by a massless spin-1 gauge field.**

We will derive later, using only the principles 5 and 6 and the equation $\vec{E}_g = -\vec{\nabla}\varphi_g - \partial\vec{A}_g/\partial t$, the Schwarzschild metric but let us first discover the origin of inertia and the nature of the inertial rest mass which will justify all the above that we accepted in the present and also in the previous chapter.

According to the principle of Equivalence, in a local frame freely falling in a gravitational field, the gravitational field must exactly cancel the acceleration so that, no sign of either acceleration or gravitation can be found by any physical means in this local frame. In order for this to happen, the total gravitational field inside the free-falling frame must be zero. For this reason we accept the next principle:

Principle 7 - The Law of motion

The motion of a body is such that, in its proper frame, i.e. the frame where the body is always at rest, the total force on the body is always zero.

According to the Law of motion, the force that accelerates a body and the inertial force that the body feels in its proper frame are equal in magnitude but opposite in direction.

According to the principle of Equivalence, for an observer R accelerating relative to an inertial frame of reference S, the fixed stars make free fall in the gravitational field that he perceives. **Now the question is: for the accelerated observer R, do the fixed stars that make free fall emit radiation field?** According to the principle of Equivalence, no sign of either acceleration or gravitation can be found by any physical means in the free-falling stars. Because the radiation field is a sign of acceleration, the accelerated observer must not observe any radiation field from the free-falling stars. In order for this to happen, the fields of the free-falling stars must be like the fields of the same stars if they were always moving in a straight line with constant speed, and the reason is simple, the fields also make free fall in the gravitational field just like the stars. So, the instantaneous potentials of the free falling stars must be exactly the same as the potentials of the same stars if they were always moving in a straight line with constant speed, equal to the instantaneous speed of the inertial frame S relative to the accelerated observer R. **Therefore, we can find the instantaneous potentials of the fixed stars for the accelerating observer R at time t simply by the Lorentz transformations, using as the velocity the instantaneous velocity of the inertial frame S relative to the observer R at time t.** Having the potentials we can find the gravitational field perceived by the accelerated observer R.

According to the local-flatness theorem, the metric in the immediate neighborhood of a point P is, to a close approximation, the Minkowski spacetime metric and the laws of Special theory of relativity are valid there [12]. **So, everywhere locally the laws of Special theory of relativity are valid.** We now have all the tools we need to search for the origin of inertia.

4 Inertia

4.1 Inertial forces

Let's make now a thought experiment, the lab frame experiment. We suppose that we use a space station, which is far from any massive body, as a laboratory. We will call the local inertial frame where the space station is always at rest, the lab frame. The lab frame, as a local inertial frame, is only expected to function over a small region of space. We assume that the distribution of matter in the Universe is such that the gravitational field in the lab frame is zero. This means that the gravitational scalar potential φ_g , of the entire Universe, has the same value everywhere in the lab frame, and so,

$$\vec{\nabla}\varphi_g = 0 \quad (4.1)$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitomagnetic vector potential due to one part of the mass-current, is canceled out by the vector potential due to another part of the mass-current, owing to its symmetry. Therefore, the gravitomagnetic vector potential \vec{A}_g from the entire Universe in the lab frame is zero,

$$\vec{A}_g = 0 \quad (4.2)$$

We suppose that we have a rocket that is so small that we can always consider that the spacetime is flat on it and so, we can apply the laws of special relativity. The rocket, which is initially at rest in the lab frame, begins to accelerate making translatory motion along the x axis. **We have shown in the previous section, that we can find the potentials of the free falling stars for the accelerating observer R and therefore for the accelerating rocket, simply with the Lorentz transformations.** Therefore, when the instantaneous velocity of the rocket is v in the positive x -direction as measured in the lab frame, the Lorentz transformations which give the gravitational scalar potential

φ'_g and the gravitomagnetic vector potential \vec{A}'_g in the rocket, in terms of the potentials φ_g and \vec{A}_g in the lab frame, are:

$$\begin{aligned}\varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}), & A'_{g-y} &= A_{g-y} \\ A'_{g-x} &= \gamma(v)\left(A_{g-x} - \frac{v}{c^2}\varphi_g\right), & A'_{g-z} &= A_{g-z}, & \gamma(v) &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\end{aligned}\quad (4.3)$$

Therefore, using vector notation, the potentials inside the rocket are

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.4)$$

$$\vec{A}'_g = -\frac{1}{c^2}\gamma(v)\varphi_g\vec{v} = -\frac{1}{c^2}\varphi'_g\vec{v} \quad (4.5)$$

As the rocket accelerates, the potentials inside the rocket change. As we said, the rocket is so small that we can always consider that the spacetime is flat on it and so, we can apply the laws of special relativity. Hence, according to equation (2.2) an induced gravitational field appears now inside the rocket which is

$$\vec{E}'_g = -\vec{\nabla}'\varphi'_g - \frac{\partial\vec{A}'_g}{\partial t'} \quad (4.6)$$

where $\partial t'$ is the time interval, in the rocket frame.

The gravitomagnetic field, inside the rocket, is zero because all the fixed stars make translatory motion in respect to the rocket and so,

$$\vec{B}'_g = \vec{\nabla}' \times \vec{A}'_g = 0 \quad (4.7)$$

The $\gamma(v)$ factor is the same everywhere within the rocket. So, from the equation (4.4), the scalar potential φ'_g is always the same everywhere within the rocket and therefore,

$$\vec{\nabla}'\varphi'_g = 0 \quad (4.8)$$

Hence, the gravitational field within the rocket becomes

$$\vec{E}'_g = -\frac{\partial\vec{A}'_g}{\partial t'} \quad (4.9)$$

The vector potential is also the same everywhere within the rocket. **Therefore, an induced uniform gravitational field appears within the accelerated rocket, whereas in the lab frame there is no gravitational field.** If a test-body K with gravitational mass m, is at rest inside the rocket, will experience a gravitational force,

$$\vec{F}'_g = m\vec{E}'_g = -\frac{\partial\left(m\vec{A}'_g\right)}{\partial t'} \quad (4.10)$$

If we assume now that the gravitational scalar potential φ_g is independent of time (that's why we call the stars, fixed stars) substituting for \vec{A}'_g from equation (4.5) into equation (4.10), we get

$$\vec{F}'_g = -\left(-\frac{1}{c^2}m\varphi_g\right)\frac{\partial[\gamma(v)\vec{v}]}{\partial t'} = -\left(-\frac{1}{c^2}m\varphi_g\right)\gamma^3(v)\frac{d\vec{v}}{dt'} \quad (4.11)$$

If we recall now that the gravitational scalar potential is negative, it is obvious from equation (4.11) that the induced gravitational force on the test body K resists changes in its velocity. **It is an inertial force!**

We will call the inertial force which is given by equations (4.10), external gravitational inertial force \vec{F}'_{inert} because it is due to the acceleration with respect to the fixed stars. So,

$$\vec{F}'_{inert} = \vec{F}'_g = -\frac{\partial (m\vec{A}'_g)}{\partial t'} \quad (4.12)$$

So, inertial frame is a local frame that moves with uniform velocity relative to the lab frame or better, relative to the fixed stars or a local frame free-falling in a gravitational field.

In addition to the external inertial force, there is also an internal inertial force. This is a well known effect which has the name radiation reaction [13] [14]. We do not know exactly the mechanism that causes it but we know that it exists. The picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. **We will call this self-force, internal inertial force, because it depends on the internal structure of the body.**

According to the principle of Equivalence, when a body makes free fall, the gravitational field exactly cancels the acceleration of the body so that, no sign of either acceleration or gravitation can be found by any physical means, inside the body. Therefore, because the internal inertial force is a sign of acceleration, it should be canceled when the body makes free fall. So we conclude that: **When a body makes free fall in a gravitational field, the internal structure of the body plays no role and thus, only the external gravitational inertial force acts on the body.**

We can obtain some very important results for non-relativistic velocities, before moving on and considering the subject in the relativistic domain. So, for non-relativistic velocities, from equation (4.11), the external gravitational inertial force on the accelerating test-body K is

$$\vec{F}'_{inert} = -\left(-\frac{1}{c^2}m\varphi_g\right)\frac{d\vec{v}}{dt} = \left(-\frac{1}{c^2}m\varphi_g\right)(-\vec{a}) \quad (4.13)$$

where dt is the time interval in the lab frame and \vec{a} is the acceleration with respect to the lab frame. **Let's imagine now, that the test-body K is a body without internal structure and thus, when it is accelerated by a force \vec{F} , it does not feel any internal inertial force but only the external gravitational inertial force.** According to the Law of motion of principle 7, in the proper frame of the test-body K, the total force on the body is zero. Therefore, the force \vec{F} that accelerates the test-body K with acceleration \vec{a} , must be

$$\vec{F} = -\vec{F}'_{inert} = \left(-\frac{1}{c^2}m\varphi_g\right)\vec{a} = m_{in}\vec{a} \quad (4.14)$$

The equation (4.14) is Newton's Second Law, for non-relativistic velocities, which obviously results from the Law of Motion. Therefore, the inertial mass m_{in} of the test-body K, for non-relativistic velocities, is (this equation will change a bit for relativistic velocities so that the inertial mass is a Lorentz invariant)

$$m_{in} = \left(-\frac{1}{c^2}m\varphi_g\right) \quad (4.15)$$

We will call the inertial mass of the test-body K gravitational inertial mass and its momentum, gravitational momentum, because they are due to the gravitational potential of the rest of the Universe.

We must emphasize that the gravitational inertial mass of a body is just a part (a coefficient) of the inertial force and thus only appears when the body is accelerated. It

makes no sense when the body is moving uniformly. So, the gravitational inertial mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential of the entire Universe.

It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. Therefore, the distant matter is of predominant importance, while local matter has only a very small effect.

Let's suppose now that a test-body of gravitational mass m , makes free fall in the gravitational field of a massive body which has spherically symmetric gravitational mass M with $M \gg m$, in a region of space where the gravitational scalar potential from the rest of the Universe is φ_g . **As we have shown, when a body makes a free fall in a gravitational field only the external gravitational inertial force acts on the body.** Therefore, for non-relativistic velocities, Newton's Law of Universal Gravitation and Newton's Second Law gives for the magnitude of the radial acceleration of the test-body,

$$G \frac{Mm}{r^2} = \left(-\frac{1}{c^2} m \varphi_g \right) a \quad (4.16)$$

where r is the distance of the test-body from the center of the massive body. **It is obvious that the gravitational mass m of the test-body is canceled in equation (4.16). Therefore, the acceleration of a free-falling body is independent of its gravitational mass and so, all bodies fall at the same rate in a gravitational field.** This is a fundamental experimental result that was tested with great accuracy with the Eötvös experiment. In Einstein's General relativity, the above experimental result is interpreted by accepting the equivalence of gravitational and inertial mass.

4.2 Relativistic momentum and the nature of the inertial rest mass

Let's continue the lab frame experiment, to find out the momentum and the nature of the inertial rest mass of a body for relativistic velocities. Let's suppose that in the area of the lab frame, a local inertial frame S which has in common with the lab frame the x, x' axis, moves along the x -axis with uniform velocity V relative to the lab frame.

We suppose that a test-body K of gravitational mass m , electric charge q and no internal structure, which is initially at rest on the x -axis of the lab frame, begins to accelerate along the x -axis due to the electric field E_{e-x} of an electric point charge B , which is always stationary on the x -axis. There are no other electric charges or currents in the vicinity. Let's suppose that when the test-body K has instantaneous velocity v relative to the lab frame, has instantaneous velocity w relative to the frame S and therefore, the frame S has instantaneous velocity $(-w)$ relative to the instantaneous rest frame of the test-body K .

Because the velocity of the test-body K is along a line passing through the charge B , i.e. the x -axis, no magnetic force is exerted on the test-body K in the lab frame, in the frame S and its instantaneous rest frame.

When the test-body K is accelerated along the x -axis relative to the lab frame, from its point of view, the fixed stars are accelerated, along the x -axis and the gravitomagnetic vector potential from them changes and causes the external gravitational inertial force which, according to equation (4.12), is

$$F'_{inert-x} = -\frac{\partial (mA'_{g-x})}{\partial t'} \quad (4.17)$$

where $\partial t'$ is the time interval in the proper frame of the test-body K .

When the test-body K has instantaneous velocity v along the x-axis, relative to the lab frame, the gravitational scalar potential φ'_g and the gravitomagnetic vector potential A'_{g-x} in the proper frame of the test-body K, according to equations (4.4) and (4.5), are

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.18)$$

$$A'_{g-x} = \gamma(v) \left(-\frac{1}{c^2}\varphi_g \right) v = -\frac{1}{c^2}\varphi'_g v \quad (4.19)$$

The accelerated test-body K feels in its proper frame the external gravitational inertial force and the electric force F'_x that accelerates it, which is

$$F'_x = qE'_{e-x} \quad (4.20)$$

According to the Law of motion, in the proper-frame of the test-body K the total force on it is zero. Therefore,

$$F'_x = -F'_{inert-x} \Leftrightarrow F'_x = \frac{\partial (mA'_{g-x})}{\partial t'} \quad (4.21)$$

The electric force has the same value in the proper frame of the accelerated test-body K as in its instantaneous rest frame. Therefore, if the force accelerating the test body K in its instantaneous rest frame is defined as the partial derivative of its kinetic gravitational momentum with respect to time in that frame, we have

$$\frac{\partial \vec{p}'_{g-x}}{\partial t'} = \frac{\partial (mA'_{g-x})}{\partial t'} \quad (4.22)$$

We use the same time in both frames since according to the clock postulate, the time interval in the proper frame of the test-body K, is equal to the time interval in its instantaneous rest frame. By integration, we get

$$p'_{g-x} = mA'_{g-x} + C \quad (4.23)$$

where C is a constant. However, the kinetic gravitational momentum of the test-body K in its instantaneous rest frame is zero. **So, if this is not the kinetic momentum, what momentum is it?**

In Lagrangian mechanics we have three momentums. In the frame S for example, the canonical gravitational momentum \vec{P}''_g of the test-body K is the sum of its kinetic gravitational momentum and its potential gravitational momentum in the frame S,

$$\vec{P}''_g = \vec{p}'_g + m\vec{A}''_g \quad (4.24)$$

where the double-prime indicates physical quantities in the frame S. Hence, the momentum given by equation (4.23) must be the potential gravitational momentum of the test-body K in its instantaneous rest frame. **Therefore, we conclude that, the force that accelerate the test-body K, in its instantaneous rest frame, must be defined as the partial derivative of the canonical gravitational momentum of the test-body K with respect to time, in order to give us only the potential gravitational momentum. So, we must define the force that accelerates a body as**

$$\vec{F} \equiv \frac{\partial \vec{P}}{\partial t} \quad (4.25)$$

So, the potential gravitational momentum of the test-body K in its instantaneous rest frame is given from the equation (4.23). According to the Lorentz transformations (4.3), in the frame S, which has instantaneous velocity $(-w)$ relative to the instantaneous rest frame of the test-body K, its potential gravitational momentum is

$$m\vec{A}''_{g-x} = \gamma(w)m(A'_{g-x} + \frac{w}{c^2}\varphi'_g) \quad (4.26)$$

By substituting for A'_{g-x} and φ'_g from the equations (4.18) and (4.19) into equation (4.26), we get

$$m\vec{A}''_{g-x} = \gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) v + \gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) (-w) \quad (4.27)$$

By rearranging we get

$$\gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) v = \gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) (w) + m\vec{A}''_{g-x} \quad (4.28)$$

Comparing the equation (4.28) with the equation (4.24) we get the canonical gravitational momentum

$$P_{g-x}^{\vec{}} = \gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) v \quad (4.29)$$

and the kinetic gravitational momentum of the test-body K in the frame S,

$$p''_{g-x} = \gamma(w)\gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) w = \gamma(w)m_{in-g0}w \quad (4.30)$$

where m_{in-g0} the gravitational inertial rest mass of the test-body K,

$$m_{in-g0} = \gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) = -\frac{1}{c^2}m\varphi'_g \quad (4.31)$$

Let us now find out what is the gravitational inertial rest mass of the test-body K and whether it is a Lorentz invariant. Let's suppose that we want to find the gravitational potential energy of the entire Universe relative to the test-body K, when it moves with velocity \vec{v} relative to the lab frame. The instantaneous sum of the gravitational four-potentials of all the bodies in the Universe is also a four-vector, the total gravitational four-potential:

$$A_g = \left(\frac{\varphi_g}{c}, \vec{A}_g \right) \quad (4.32)$$

and the four-velocity of the test-body K is: $U = \gamma(v)(c, \vec{v})$.

We know that the quantity we are looking for must depend on both U and A_g , and it is a scalar. The product mUA_g has physical dimensions of energy and it is a scalar, because the gravitational mass is a scalar, and the product of two four-vectors is a Lorentz invariant, i.e. a scalar. Evaluating the product mUA_g in the rest frame of the test-body K,

$$mUA_g = m(c, o) \left(\frac{\varphi'_g}{c}, \vec{A}'_g \right) = m\varphi'_g \quad (4.33)$$

we obtain the gravitational potential energy of the entire Universe relative to the test-body K, which is the very thing we wanted and is a Lorentz invariant, i.e. all observers agree on its value at any instant of the test-body's history. Comparing the equation (4.33) with the equation (4.31) we get

$$m_{in-g0} = -\frac{1}{c^2}mUA_g \quad (4.34)$$

So, the gravitational inertial rest mass of a body without internal structure, is not an intrinsic property of the body but is proportional to the gravitational potential energy of the entire Universe relative to the body, and is a Lorentz invariant.

Since the gravitational inertial rest mass of the test-body K is a Lorentz invariant, we can define its kinetic gravitational four-momentum in the usual way

$$P_g = m_{in-g0}U = (m_{in-g}c, \vec{p}_g) = (E_g/c, \vec{p}_g) \quad (4.35)$$

So, during a collision, if the kinetic gravitational four-momentum is conserved in one inertial frame, it is conserved in any inertial frame [15].

In the Special Theory of relativity the inertial rest mass of a body should have the same numerical value when the body is at rest in all inertial frames. This is an experimental result. However, from equation (4.31) we can see that the rest mass of the test-body K depends on its velocity relative to the lab frame and therefore, does not have the same value when it is at rest in different inertial frames, because different inertial frames move with different velocities relative to the lab frame. **Nevertheless, we will prove that because of the vast and expanding Universe, the rest mass of a body has practically the same value when it is at rest in all inertial frames.**

In order to prove the above, we will consider a simple no-relativistic model of the Universe [16]. The retarded gravitational scalar potential φ_{g-1} of the entire Universe in position 1 which is the position of the lab frame, is given from the integral over all the observable Universe

$$\varphi_{g-1} = -G \int \frac{[\rho]}{r_{12}} dV_2 \quad (4.36)$$

where the element of volume dV_2 in position 2, is at a distance r_{12} from the lab frame and the density of matter $[\rho]$ of the volume element dV_2 , is the value at the retarded time $(t - r_{12}/c)$. According to Hubble's law, the galaxies are moving away with speeds proportional to their distance r_{12} from the lab frame

$$v = H_0 r_{12} \quad (4.37)$$

where H_0 is the Hubble constant. We assume that matter receding with velocity greater than that of light makes no contribution to the scalar potential, so that the integral in equation is taken over the spherical volume of the Hubble radius c/H_0 which is approximately 14.4 billion light years. **If we now assume that the density is uniform, not only in space but also in time, equation (4.36) gives**

$$\varphi_{g-1} = -2\pi G\rho(c/H_0)^2 \quad (4.38)$$

According to equation (4.38), the 99 per cent of the gravitational scalar potential arises from matter further away than 10^9 light-years. **If we now take into account that the very distant matter moves with relativistic velocities and the density of matter was much higher in the past,** we conclude that the 99,99 per cent of the gravitational scalar potential arises from matter further away than 10^{10} light-years. Therefore, practically the 99,99 percent of the gravitational scalar potential arises from matter moving with speeds that are 99,99 percent of the speed of light. Thus, because of the relativistic velocity-addition law, practically, the velocity of the very distant matter relative to the test-body K is almost independent of the velocity of the test-body K relative to the lab frame. It is like the speed of light that is the same for all the observers. **Therefore, from equation (4.31) it follows that practically the gravitational inertial rest mass of a body has the same value when the body is at rest in all inertial frames.** So, the equations (4.31) and (4.30) becomes

$$m_{in-g0} \approx \left(-\frac{1}{c^2}m\varphi_g\right) \quad \text{and} \quad p''_{g-x} \approx \gamma(w) \left(-\frac{1}{c^2}m\varphi_g\right) w \quad (4.39)$$

If the frame S is at rest in the lab frame, the equation (4.30) has the product of two Lorentz factors $\gamma(v)$. The first Lorentz factor is caused by the motion of the test body K relative to the lab frame and the second is caused by the relativity of the time measurements and so, we come to equation (4.39) again.

Let us now consider, what happens if there are other electrical charges, with such a distribution and motion that the electric scalar potential φ_e in the lab frame is not zero but is the same everywhere, so that $\vec{\nabla}\varphi_e = 0$ and the magnetic vector potential \vec{A}_e is zero. Since the equations of electromagnetism have the same mathematical form as the equations of gravitomagnetism, we will only state the important results. Let's suppose that the test-body K is accelerated in the lab frame along the x-axis just like in the previous study. So, when the instantaneous velocity of the test-body K relative to the frame S is w and relative to the lab frame is v , the kinetic gravitoelectric momentum of the test-body K in the frame S, will be

$$p''_{ge-x} = \gamma(w)\gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] w = \gamma(w)m_{in-ge0}w \quad (4.40)$$

and its gravitoelectric inertial rest mass m_{in-ge0} will be a Lorentz invariant.

Let's suppose that there are n electric charges, each at a different distance r_i from the test-body K, as measured in the lab frame. For non relativistic velocities, the gravitoelectric inertial rest mass of the the test-body K in the lab frame, is

$$m_{in-ge0} = m_{in-g0} - \frac{1}{c^2} \left(\frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{qq_i}{r_i} \right) \quad (4.41)$$

Finally, the Special theory of relativity will give us the rest mass of a body with internal structure, a composite body. If we apply the conservation of the kinetic four-momentum in an inelastic collision where n free moving particles without internal structure, collide and create a composite body M which is at rest in the lab frame, the rest mass m_{in-0} of the composite body M in the lab frame is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + T/c^2 + E_{field}/c^2 \quad (4.42)$$

where $m_{in-ge0i}$ is the gravitoelectric rest mass of each particle that makes up the composite body M, T is the kinetic energy of the relative motion of all the particles and E_{field} is the potential energy of the interaction of all the particles [17]. **The rest mass m_{in-0} of the composite body M, is also a Lorentz invariant as is well known from the Special theory of relativity [18].**

4.3 Zero rest mass

Let us now imagine, using classical physics, that we have an accelerating particle A of gravitational mass m, electric charge q and without internal structure, inside a thin spherical shell of radius R with charge q uniformly distributed on its surface. The electric scalar potential inside the spherical shell is given by the well known equation, $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$. The gravitoelectric rest mass of the particle A, according to equation (4.41), is

$$m_{in-ge0} = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R} \quad (4.43)$$

From equation (4.43) we can see that the gravitoelectric rest mass of the particle A becomes zero when the radius take the value $R_{critical}$. From equation (4.43) we get for $R_{critical}$,

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R_{critical}} \iff R_{critical} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{m_{in-g0}} \quad (4.44)$$

If the particle A is an electron $R_{critical} \approx 2,81 \times 10^{-15}m$ and if is a proton $R_{critical} \approx 1,53 \times 10^{-18}m$. Therefore, the effect of the electric scalar potential on the inertial rest mass of a particle, becomes significant in the subatomic world. This can be tested experimentally by measuring the inertial mass of moving electrons in a magnetic field if we put the whole device in a negatively charged box.

5 The Schwarzschild metric

We will derive now the spacetime metric outside of a static and spherically symmetric body B of gravitational mass M . We will consider the freely motion of a test-body A, of gravitational mass m with $m \ll M$, in the radial direction of the gravitational field of the body B. We will follow a method based on a paper of F. Tangherlini [19][20]. The equation of motion of the test-body A, is obtained from the principle 6 of geodesic motion:

$$\delta \int ds = 0 \quad (5.1)$$

where, from the principle 5:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (5.2)$$

By well-known arguments, we may bring the spacetime interval outside of a static and spherically symmetric body into the standard Schwarzschild form [21]:

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{11}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.3)$$

The scalar functions g_{00} and g_{11} , are functions only of the radial coordinate r . The metric (5.3) should give us to infinity the Minkowski metric in spherical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.4)$$

Therefore, we must have the boundary conditions

$$\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1 \quad \lim_{r \rightarrow \infty} g_{11}(r) \rightarrow -1 \quad \lim_{r \rightarrow \infty} g_{00}(r)g_{11}(r) \rightarrow -1 \quad (5.5)$$

For purely radial motion of the test-body A, the spacetime interval becomes

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{11}(r)dr^2 \quad (5.6)$$

From the principle of geodesic motion we have

$$\delta \int ds = 0 \Rightarrow \delta \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = 0 \Rightarrow \delta \int \sqrt{L} d\tau = 0 \quad (5.7)$$

The L may be termed a 'lagrangian'. So, the lagrangian is

$$L = g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \quad (5.8)$$

It is more convenient, however, instead of $\delta \int ds = 0$ to take $\delta \int ds^2 = 0$, which is, with L from eq. (5.8)

$$\delta \int L d\tau = 0 \quad (5.9)$$

For purely radial motion the lagrangian becomes

$$L = g_{00}c^2 \left(\frac{dt}{d\tau} \right)^2 + g_{11} \left(\frac{dr}{d\tau} \right)^2 \quad (5.10)$$

Using the calculus of variation, we obtain the Euler-Lagrange equations with $\dot{t} = dt/d\tau$ and $\dot{r} = dr/d\tau$

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}} \right) - \frac{\partial L}{\partial t} = 0, \quad \text{and} \quad \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0 \quad (5.11)$$

Since the Lagrangian does not depend on time, we have $\partial L/\partial t = 0$. Hence, from the first of the equations (5.11), arise that $\partial L/\partial \dot{t} = \text{const}$. Therefore, the term $\partial L/\partial \dot{t}$, is a conserved quantity. Performing the differentiation in the equation (5.10), using the fact that the metric function $g_{00}(r)$ doesn't depend on \dot{t} and the relation $ds^2 = c^2 d\tau^2$, where $d\tau$ is the proper time, we obtain

$$\frac{\partial L}{\partial \dot{t}} = 2g_{00}c^2 \frac{dt}{d\tau} = \text{const.} \Rightarrow g_{00} \frac{dt}{ds} = k_0 \quad (5.12)$$

Equation (5.12) is a first integral for the equation of motion of the test-body A and states that the energy (per unit mass) k_0 is a constant of the motion (the energy conservation law). We can have a second first integral, for the equation of motion in the radial direction, by dividing the equation (5.6) by ds^2

$$1 = g_{00}c^2 \left(\frac{dt}{ds}\right)^2 + g_{11} \left(\frac{dr}{ds}\right)^2 \quad (5.13)$$

Equation (5.13) expresses the invariant relation between energy and momentum (per unit mass). Eliminating dt/ds from equation (5.13), using the first integral of equation (5.12), we get

$$1 = \frac{c^2 k_0^2}{g_{00}} + g_{11} \left(\frac{dr}{ds}\right)^2 \quad (5.14)$$

Dividing the equation (5.14) by g_{11} we get

$$\frac{1}{g_{11}} = \frac{c^2 k_0^2}{g_{00}g_{11}} + \left(\frac{dr}{ds}\right)^2 \quad (5.15)$$

Since the metric tensor, in our case, is diagonal, one should have

$$g^{00} = \frac{1}{g_{00}} \quad \text{and} \quad g^{11} = \frac{1}{g_{11}} \quad (5.16)$$

So, the equation (5.15) becomes

$$g^{11} = c^2 k_0^2 (g^{00} g^{11}) + \left(\frac{dr}{ds}\right)^2 \quad (5.17)$$

The scalar functions g_{00} and g_{11} , are functions of only r and r is a function of only s . So, by differentiation of the equation (5.17) in respect to s , using the chain rule, we get

$$g^{11},r \frac{dr}{ds} = c^2 k_0^2 (g^{00} g^{11}),r \frac{dr}{ds} + 2 \frac{dr}{ds} \frac{d^2 r}{ds^2} \quad (5.18)$$

where a comma denotes ordinary differentiation. Cancelling $\frac{dr}{ds}$ we get

$$\frac{d^2 r}{ds^2} = -\frac{c^2 k_0^2}{2} (g^{00} g^{11}),r + \frac{1}{2} g^{11},r \quad (5.19)$$

Equation (5.19) is the Tangherlini equation for purely radial motion [19]. Using the relation $ds^2 = c^2 d\tau^2$, we get

$$\frac{d^2 r}{d\tau^2} = -\frac{c^4 k_0^2}{2} (g^{00} g^{11}),r + \frac{c^2}{2} g^{11},r \quad (5.20)$$

From equation (5.20) we can see that because of the k_0 term, the radial acceleration depends on the energy which the test-body A had initially, i.e. the radial velocity with which the test-body A was launched.

However, it is well known that the electric force on a charge is strictly independent of the charge's velocity [22]. Therefore, the same is true for the gravitational force on a gravitational mass. Hence, the proper acceleration of the test-body A, in the radial direction of the gravitational field of the body B, is strictly independent of the radial velocity with which the test-body A was launched [23]. For non-relativistic velocities, the proper acceleration and the radial acceleration are equal. **Therefore, for non-relativistic velocities, the radial acceleration of the test-body A is strictly independent of its velocity. For this to happen the following equation must hold**

$$g^{00}g^{11} = \text{const.} \quad (5.21)$$

Since the scalar functions $g_{00}(r)$ and $g_{11}(r)$ are velocity independent, equation (5.21) must also hold for relativistic velocities.

We must emphasize that only if gravity is described by a gravitomagnetic theory the proper acceleration of the test-body A is strictly independent of its velocity [24]. Using the boundary conditions (5.5), the equation (5.21) becomes

$$g^{00}g^{11} = -1 \quad (5.22)$$

and using the equations (5.16) we get

$$g_{11} = -\frac{1}{g_{00}} \quad (5.23)$$

Therefore, the equation (5.3) for the spacetime interval outside of the body B, can be written in the form

$$ds^2 = g_{00}c^2dt^2 - g_{00}^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.24)$$

From equations (5.23) and (5.16), we get

$$g^{11} = -g_{00} \quad (5.25)$$

Finally, using the equations (5.21) and (5.25), the Tangherlini equation becomes

$$\frac{d^2r}{d\tau^2} = -\frac{c^2}{2}g_{00,r} \quad (5.26)$$

According to the principle of consistency, when the test-body A moves with non-relativistic velocity $v \ll c$, in a weak and static gravitational field, equation (5.26) must be reduced to the equation

$$\frac{d^2r}{dt^2} = -\frac{m}{m_{in-g0}} \frac{d\varphi_{gB}}{dr} \quad (5.27)$$

where φ_{gB} is the gravitational scalar potential of the body B and m_{in-g0} the gravitational inertial rest mass of the test-body A. So, when $dt \approx d\tau$, comparing the equations (5.26) and (5.27), we get

$$\frac{dg_{00}}{dr} = \frac{2m}{c^2m_{in-g0}} \frac{d\varphi_{gB}}{dr} \quad (5.28)$$

The gravitational inertial rest mass of the test-body A, according to the way of deriving equation (4.31), does not depend on the body B that accelerate it. Hence, from equation (5.28) by integration and using the boundary conditions (5.5), we obtain

$$g_{00} = 1 - \frac{m}{c^2m_{in-g0}} \frac{2GM}{r} \quad (5.29)$$

Therefore, the equation (5.24) for the spacetime interval outside of the body B, becomes

$$ds^2 = \left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.30)$$

In the International System of Units, the ratio of the gravitational mass to the gravitational rest mass of the test body A in our region of space is equal to unity, and so equation (5.30) becomes the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.31)$$

So, the new theory is in agreement with all past experiments and observations. In another region in our Galaxy or in the Universe, the ratio of gravitational to inertial rest mass will not be equal to unity. There the phenomena will be the same qualitatively but not quantitatively.

6 Some ideas about the dark matter and the dark energy

6.1 Dark matter

From equation (4.31), it follows that the inertial mass of a star depends on the gravitational scalar potential of the entire Universe, i.e. the inertial mass of a star depends on the distribution of matter in the Universe. In the Universe there are planets, stars, galaxies, clusters of galaxies and so on. Therefore, the position where a star is located, affects the inertial mass of the star. In places with higher density of matter the inertial mass of a star will be greater than the inertial mass of an identical star, in a place with lower density of matter. Moreover, if we think that the gravitational scalar potential and vector potential satisfy the wave equation and travel at the speed of light through space, they must behave like the light that bends when traveling near a large gravitational mass. Therefore, the gravitational potential of the entire Universe is more concentrated in places with higher densities of matter. Hence, this is a second reason why in places with higher density of matter the inertial mass of a star will be greater than the inertial mass of an identical star, in a place with lower density of matter.

So, the inertial mass of a star near the center of a galaxy is greater than the inertial mass of an identical star at the edges of that galaxy. Therefore, stars at the edges of a rotating spiral galaxy, are moving faster than Newtonian physics predicts by assuming that the inertial mass is the same everywhere. This seems to be very important for the problem of dark matter.

6.2 Dark energy

Let us now consider the light emitted by atoms on the surface of a static, spherically symmetric star of gravitational mass M. We assume that an atom of gravitational mass m and gravitational rest mass m_{in-g0} , which emits light, is at a distance r_{em} from the center of the star. From equation (5.30) arises the equation relating the proper time $d\tau_{em}$ at the point of emission, with the proper time $d\tau_{\infty}$ at infinity where is the point of observation [25]

$$d\tau_{\infty} = \frac{d\tau_{em}}{\sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}}} \quad (6.1)$$

So, the frequency of the light observed by an observer, at the point of observation, will be

$$f_{\infty} = f_{em} \sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}} \quad (6.2)$$

Equation (6.2) describes the redshift of light emitted by an atom in a gravitational field and received by an observer who is very far away, essentially at infinity. **As the Universe expands, the density of matter decreases and therefore, the gravitational inertial mass of an atom decreases over time.** Therefore, as it emerges from equation (6.2), the light emitted by the atoms of two identical supernovas Ia at different times in the history of the Universe, will have different red shift. The atoms of a younger (most recent) supernova will have smaller gravitational rest mass than the atoms of an older supernova. **Hence, the light emitted by the atoms of a younger supernova Ia will have larger red shift than the light emitted by the atoms of an older supernova Ia.** This phenomenon has been observed, but the inability to explain it has led to the theory that the Universe expands in an accelerating way, because of dark energy.

Conclusions

In this paper we have described a new theory for gravity which has all the principles of Einstein's General theory of relativity, except the equivalence of gravitational and inertial mass. The Schwarzschild metric is one of its consequences and therefore agrees with all previous experiments and observations. However, the new theory goes further and explain the origin of inertia and the nature of the inertial rest mass which seems very important for dark matter, dark energy, nuclear and particle physics. **The new theory is fully consistent with Mach's principle.** Due to the similarity of gravitational and electric forces we need to check whether the electric forces also affects the spacetime metric. **Finally, if the inertial mass can take any value, as the new theory shows, then a future technology can make space travel an easy affair !!!**

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