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# The origin of inertia and the nature of inertial rest mass. An important issue for dark matter and dark energy.

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## Abstract

If equilibrium is to be a frame-independent condition, it is necessary for all non-electromagnetic forces to have precisely the same transformation law as that of the Lorentz-force. Therefore, gravity must be described by a gravitomagnetic theory just like the electromagnetic theory, with the gravitational mass as a Lorentz invariant. If acceleration is to be relative, we must accept the Principle of General Covariance and the Equivalence Principle. Using the above we prove that:

1. The external inertial forces, felt by an accelerating body, are inductive effects of the entire Universe and are real and not fictitious forces.
2. The internal inertial forces depend on the internal structure of the body, but in a free fall they are canceled due to the Principle of Equivalence and the body feels only the external inertial forces. That's why all bodies fall at the same rate in a gravitational field.
3. The inertial rest mass of a body depends on the distribution of the matter in the Universe and this seems very important for dark matter and dark energy.
4. The gravitational field affects the spacetime metric and a freely moving particle moves along a geodesic.
5. The Schwarzschild metric is a consequence of the new theory and therefore the new theory agrees with all past experiments and observations.

**Keywords:** gravitomagnetism , relative acceleration , Mach's principle , origin of inertia , nature of inertial rest mass , dark matter , dark energy

## 1 Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. Inertial forces appear in a non-inertial frame of reference. But what determines an inertial frame?

The first answer comes from Descartes and Newton, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the absolute space and the motion is absolute. The inertial forces, such as the centrifugal force, must arise from acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter.

The second answer comes from Leibniz, Berkeley and Mach and is known as Mach' principle, according to which, an inertial frame of reference is a frame that moves with constant velocity, with respect to the rest of the matter in the Universe, and the motion is relative. The inertial forces, such as the centrifugal

force, are more likely caused by acceleration, with respect to the fixed stars. This idea implies that the properties of space arise from the matter contained therein and are meaningless in empty space.

The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important when we consider subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle, is motion relative to other matter in the Universe, has never found its complete expression in a physical theory. The Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity, attempted to continue this relativization and interpret inertia considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2].

The Principle of Equivalence is an essential part of General relativity. But although the Principle of Equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3]. Moreover, as pointed out by Henri Poincare, if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force [4]. But this does not happen with gravity as it is described by General relativity.

It is the purpose of this paper to show the origin of inertia. To achieve this we will first accept that gravity is described by a gravitomagnetic theory with the gravitational mass as a Lorentz invariant, so that the gravitomagnetic force has precisely the same transformation law as that of the Lorentz-force. Second we will accept that all kinds of motion must be relative, and for this to be true we must accept the Principle of General Covariance and the Equivalence Principle. From the Equivalence Principle it follows that the gravitational field affects the spacetime metric and that a freely moving particle always moves along a geodesic. However, we will show that we can also find the motion of a freely moving body in a gravitational field using forces which is a great deal simpler. Using only the above, we can derive the Schwarzschild metric without the field equations of General relativity. Therefore, the new theory, which is full consistent with the Mach's principle, agrees with all past experiments and observations, but also goes further and reveals to us the origin of inertia and the nature of the inertial rest mass of a body, which seems very important for dark matter, dark energy, nuclear and particle physics.

## 2 Gravitomagnetic theory

In an inertial frame of reference  $S$ , let's have a system of two non spinning bodies with gravitational masses and positive electric charges, in a region of free space where there are no external forces. We suppose that the two bodies are at rest in the inertial frame  $S$ , under equilibrium conditions, i.e. the force of gravitational attraction balances that of electrostatic repulsion. But what is observed by another inertial frame of reference  $S'$ , moving with uniform velocity relative to the frame  $S$ ? Let's imagine that if the bodies collide, they will explode. It is impossible for one observer to see an explosion and for another to not see it. **Therefore, the equilibrium must be a frame-independent condition.**

In order for equilibrium to be a frame-independent condition, it is necessary the gravitational force to have precisely the same transformation law as that of the Lorentz-force. For this reason we accept that:

- **Gravity should be described by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory with the gravitational mass as a Lorentz invariant.**

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformations (for coordinates, velocities, potentials, forces) and the following series of remarks [5][6]:

1. The Coulomb potential at a distance  $r$  from a stationary point-charge  $q$  in vacuum is:  $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric point-charge produces a scalar potential  $\varphi_e$  and a vector potential  $\vec{A}_e$ , which together form a four-vector,  $A_e = \left(\frac{\varphi_e}{c}, \vec{A}_e\right)$
3. The potentials produced by a point-charge moving in any way, depend only upon the velocity and position at the retarded time.

where  $\epsilon_0$  is the vacuum permittivity. Of course we need to know how to get the Coulomb's law from the scalar potential. Therefore, if we want to obtain a gravitomagnetic theory, with equations that have the same mathematical form, as those of the electromagnetic theory, first we must accept that the gravitational mass is a Lorentz invariant and second that the same series of remarks must be met for gravity. We already have the first remark, that is, the gravitational potential at a distance  $r$  from a stationary gravitational point-mass  $m$  in vacuum is,  $\varphi_g = -G\frac{m}{r}$  where  $G$  is the gravitational constant, but this is only the one remark. Therefore, we need the other two, as well. We will obtain them with the following two principles:

**Principle 1**

**A gravitational point-mass produces a scalar potential  $\varphi_g$  and a vector potential  $\vec{A}_g$ , which together form a four-vector,  $A_g = \left(\frac{\varphi_g}{c}, \vec{A}_g\right)$**

**Principle 2**

**The potentials produced by a gravitational point-mass moving in any way, depend only upon the velocity and position at the retarded time.**

So, the potentials produced by a gravitational point-mass  $m$  moving with any velocity have the same mathematical form as the Lienard-Wiechert potentials for an electric point-charge moving with any velocity, but with a negative sign,

$$\varphi_g = -G \left[ \frac{m}{r - \vec{r}\vec{v}/c} \right] \quad \text{and} \quad \vec{A}_g = -\frac{G}{c^2} \left[ \frac{m\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_g \vec{v}] \quad (2.1)$$

where  $\vec{r}$  is the vector from the gravitational point-mass to the point where the potential is evaluated,  $c$  is the speed of light in vacuum and the quantities  $r$ ,  $\vec{r}$  and  $\vec{v}$  (the velocity of the point-mass) in the square bracket are to have their values at the retarded time. Starting from the potentials, in order to find the fields, we have the equations

$$\vec{E}_g = -\vec{\nabla}\varphi_g - \frac{\partial\vec{A}_g}{\partial t} \quad (2.2)$$

$$\vec{B}_g = \vec{\nabla} \times \vec{A}_g \quad (2.3)$$

When a gravitational mass  $m$  moves with velocity  $\vec{v}$  in the above fields, it feels the force,

$$\vec{F}_g = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (2.4)$$

where  $\vec{E}_g$  is the gravitational field and  $\vec{B}_g$  the gravitomagnetic field.

So, we have now a gravitomagnetic theory, with equations that have the same mathematical form as those of the electromagnetic theory. Therefore, we expect that there are gravitomagnetic waves that propagate in vacuum with the speed of light but with one important difference. An isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment. **However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity.** The reason is simple. The electric dipole moment can move around with respect to the center of mass but the gravitational dipole moment is identical in location with the center of mass, and due to the law of conservation of momentum, cannot accelerate or radiate [7].

### 3 General relativity of motion

**We will follow now, the fundamental idea of the relativity of all kinds of motion.** In accordance with this idea we can detect and measure the motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion. The rotational motion of a rigid body, as a whole, cannot be relative because different parts of the rotating body perceive different motion of the rest of the Universe. However, we can consider that a rotating rigid body consists of a large number of sufficiently small parts and the circular motion of each small part separately is relative. Therefore, following the fundamental idea, an observer inside a rocket which is accelerated, cannot distinguish whether the rocket is accelerated and the remainder of the Universe is at rest or whether the rocket is at rest and the remainder of the Universe is accelerated in the opposite direction. Acceleration, as well as velocity, is relative. So, we are looking for the principles that will ensure the fundamental idea of the relativity of all kinds of motion.

In order for all kinds of motion to be relative, the laws of physics should have the same mathematical form in all frames of reference since otherwise the difference in form could provide a criterion for judging the absolute motion. For this reason we accept the next principle:

#### Principle 3 - The Principle of General Covariance

**The laws of physics have the same mathematical form in all frames of reference. In inertial frames of reference the laws of physics reduce to simpler mathematical forms which agree with the laws of Special theory of relativity.**

The above principle is not enough to preserve the idea of the relativity of all kinds of motion. This arises from the fact that the expression of the equations of physics in a form which is independent of the frame of reference does not in general prevent a change in their numerical content when we change from one frame of reference to another [8].

**Let's make now a thought experiment, the lab frame experiment, to find out the next principle we need.** We suppose that we use a space station, which is far from any massive body, as a laboratory. We will call the local inertial frame where the space station is always at rest, the lab frame. **The lab frame, as a local inertial frame, is only expected to function over a small region of space.** An observer L is at rest in the lab frame. **We assume that the distribution of matter in the Universe is such that the gravitational field in the lab frame is zero.** This means that the gravitational scalar potential  $\varphi_g$ , of the entire Universe, has the same value everywhere in the lab frame, and so,

$$\vec{\nabla}\varphi_g = 0 \tag{3.1}$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitomagnetic vector potential due to one part of the mass-current, is canceled out by

the vector potential due to another part of the mass-current, owing to its symmetry. **Therefore, the gravitomagnetic vector potential  $\vec{A}_g$  from the entire Universe in the lab frame is zero,**

$$\vec{A}_g = 0 \quad (3.2)$$

This would also happen if all the bodies of the Universe were at rest, relative to the lab frame. So, we can say that the lab frame is at rest relative to the Universe, or at rest relative to the fixed stars.

We suppose now that in the region of the lab frame, an observer R is at rest in a uniformly accelerated reference frame  $R$ , that is, the observer R feels a constant force in its instantaneous rest frame. **The instantaneous rest frame of an accelerating body is the inertial frame of reference in which the body is instantaneously at rest.** Therefore, the observer R undergoes a constant proper acceleration, that is, a constant acceleration relative to its instantaneous rest frame. Since we accept that acceleration is relative, from the point of view of the observer R, the lab frame and the fixed stars are accelerated relative to him. Therefore, from equation (2.2) expressed in a form which is the same for all frames of reference, for the observer R an induced gravitational field appears in the uniformly accelerated frame  $R$ . So, from the point of view of the observer R, he is at rest in a frame with a gravitational field.

From the point of view of the observer L (lab frame), the observer R is accelerated while the lab frame is a local frame without gravity and inertial forces, an inertial frame. We assume that the gravitational forces between the accelerated reference frame  $R$  and the space station are negligible. **Therefore, the observer L using the Special theory of relativity is able to describe what physical effects are observed by the observer R in the uniformly accelerated frame  $R$  due to the acceleration relative to the lab frame.** This is the well known study of a uniformly accelerated reference frame, in Special relativity. So, some of the well known physical effects, that observed by the observer R due to the acceleration, **except of course of the inertial force**, are [9]:

1. Redshift or blueshift of a light ray moving parallel to the direction of the acceleration.
2. Varying coordinate speed of light; fixed local relative speed of light.
3. Spacetime is endowed with a metric.
4. Maximum proper time as the law of motion of freely moving bodies.
5. Horizons.

However, since we accept that acceleration is relative, the observer R is permitted to believe that he is at rest in a frame with a gravitational field while the lab frame is accelerated. If the induced gravitational field did not exist, the above physical effects would have to occur in the lab frame and not in the frame  $R$ . However, the gravitational field exists and the above physical effects occur in the frame  $R$  and not in the lab frame. The only explanation for the observer R, about what is happening to the frame  $R$ , is that the above physical effects that occur in the frame  $R$  are due to the gravitational field and not to the acceleration relative to the lab frame. **In order for this to happen, the gravitational field should be capable of causing all the above physical effects, with the same numerical values.** For this reason, we accept the next principle for an infinitesimal region of spacetime where we can assume that, the gravitational field of a spherical massive body and of the accelerated frame  $R$  is uniform:

#### Principle 4 - The Principle of Equivalence

**Physics in a non accelerating frame  $S$ , with a uniform gravitational field where all the released bodies fall with acceleration  $\vec{g}$ , is equivalent to physics in a frame without gravity but with translational acceleration  $\vec{a} = -\vec{g}$  and velocity zero with respect to the inertial frame in which the non accelerating frame  $S$  is at rest.**

According to the Principle of Equivalence all bodies fall at the same rate in a gravitational field. However, we will return to the above phenomenon later.

Since the observer R is permitted to believe that he is at rest in a gravitational field, for him, everything outside of the rocket, that is, the lab frame and the fixed stars, make free fall in the gravitational field. However, none of the above physical effects occur in the lab frame although it is accelerated in the gravitational field. **The only explanation for the observer R, about what is happening to the lab frame, is that the gravitational field must exactly cancel the acceleration of the lab frame so that, no sign of either acceleration or gravitation can be found by any physical means in the lab frame.** So, we have an alternative expression of the equivalence principle:

#### Principle 4 - The Principle of Equivalence

**Physics in a local frame freely falling in a gravitational field is equivalent to physics in an inertial frame without gravity.**

For this to happen, the total gravitational field inside the free-falling lab frame must be zero. Therefore, when a body makes a free fall in the gravitational field of a stationary huge body, its acceleration relative to the fixed stars is such that, the induced gravitational force that the body feels due to the acceleration relative to the fixed stars, which is the inertial force as we will show, is equal in magnitude to the gravitational force of the huge body but opposite in direction. For this reason we accept the next principle:

#### Principle 5 - The Law of motion

**The motion of a body is such that, in its proper frame, i.e. the frame where the body is always at rest, the total force on the body is always zero.**

According to the Law of motion, the force that accelerates a body and the inertial force that the body feels in its proper frame are equal in magnitude but opposite in direction.

Now the question is: for the observer R, do the fixed stars that make free fall emit radiation field? According to the Equivalence Principle, no sign of either acceleration or gravitation can be found by any physical means in the free-falling stars. Because the radiation field is a sign of acceleration, the observer R must not observe any radiation field from the free-falling stars. **In order for this to happen, for the observer R, the fields of the free falling stars must be like the fields of the same stars if they were always moving in a straight line with constant speed, and the reason is simple, the fields also make free fall in the gravitational field just like the stars.** So, the fields of the free falling stars are carried along convectively with the stars and thus, the potentials of the free falling stars for the observer R at the time  $t$ , must be exactly the same as the potentials of the same stars if they were always moving in a straight line with constant speed, equal to the instantaneous speed of the lab frame relative to the observer R at the time  $t$ . Therefore, we come to the conclusion that:

- **We can find the instantaneous potentials of the free falling stars for the observer R at the time  $t$ , simply with the Lorentz transformations, using as velocity the instantaneous velocity of the observer R relative to the lab frame at the time  $t$ .**

So, when the observer R is accelerated relative to the fixed stars, it is equivalent to say either that the observer R is accelerated while the fixed stars are stationary, or that the observer R is stationary in a gravitational field where all the stars and their fields make free fall. So, with the Principle of General Covariance and the Principle of Equivalence we have ensured the relativity of all kinds of motion. Acceleration, as well as velocity, is relative.

Moreover, as we have seen from the physical effects observed by observer R, the Equivalence Principle shows us that spacetime is endowed with a metric and the gravitational field affects the spacetime metric so that, the maximum proper time is the law of motion of a freely moving body in a gravitational field. The two above physical effects are so important that we will elevate them to physical principles:

### Principle 6 - The Principle of Spacetime Metric

**Spacetime is endowed with a metric. The spacetime interval between two events is:**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where  $g_{\mu\nu}$  is the metric tensor.

### Principle 7 - The Principle of Geodesic Motion or of Maximum Proper Time

**A freely moving body always moves along a geodesic:**

$$\delta \int ds = 0$$

Finally, we need to remember that according to the local-flatness theorem, the metric in the immediate neighborhood of a point P is, to a close approximation, the Minkowski spacetime metric and the laws of Special theory of relativity are valid there [10]. Therefore,

- **Everywhere locally the laws of Special theory of relativity are valid.**

So, in order for the equilibrium to be a frame-independent condition and the acceleration to be relative, we now have a new theory of gravity, which, however, has all the principles of Einstein's General theory of relativity except the equivalence of gravitational and inertial mass. With General relativity the field equations determine the curvature of spacetime, and the acceleration of a test particle is a consequence of this curvature. With the new theory, the gravitomagnetic theory determines the fields and the curvature of spacetime is a consequence of them. To obtain the equations of motion of a freely moving body in a gravitational field, one can either use the geodesic equation or one can use the relativistic mechanics properly modified due to the curvature of spacetime, that is, using forces as we will show later.

## 4 Inertia

### 4.1 Inertial forces

Let's now continue the lab frame experiment to find out the inertial forces. **We suppose that a test-body K of gravitational mass m, which is initially at rest in the lab frame, begins to accelerate making translational motion along the x-axis of the lab frame.** We have shown in the previous section, that we can find the potentials of the free falling stars for the observer R, simply with the Lorentz transformations. Therefore, when the instantaneous velocity of the test-body K is  $v$  in the positive x-direction as measured in the lab frame, the Lorentz transformations which give the gravitational scalar potential  $\varphi'_g$  and the gravitomagnetic vector potential  $\vec{A}'_g$  in the instantaneous local rest frame of the test-body K, in terms of the potentials  $\varphi_g$  and  $\vec{A}_g$  in the lab frame, are:

$$\begin{aligned} \varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}), & A'_{g-y} &= A_{g-y} \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g), & A'_{g-z} &= A_{g-z}, & \gamma(v) &= \left(\sqrt{1 - v^2/c^2}\right)^{-1} \end{aligned} \tag{4.1}$$



Let's recall that the gravitomagnetic vector potential  $\vec{A}_g$  from the entire Universe in the lab frame is zero. Therefore, the potentials in the instantaneous local rest frame of the test-body K in vector form are,

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.2)$$

$$\vec{A}'_g = -\frac{1}{c^2}\gamma(v)\varphi_g\vec{v} = -\frac{1}{c^2}\varphi'_g\vec{v} \quad (4.3)$$

As the test-body K accelerates, the potentials in its proper local frame change. **We now assume that the test-body K and its proper local frame are so small that we can consider that the spacetime is flat in the proper local frame and so we can apply there the laws of Special relativity.** So, according to eq. (2.2), due to the acceleration relative to the fixed stars the test-body K feels an induced gravitational field

$$\vec{E}'_g = -\vec{\nabla}'\varphi'_g - \frac{\partial\vec{A}'_g}{\partial t'} \quad (4.4)$$

where  $\partial t'$  is the time interval in the proper frame of the test-body K, that is, the proper time. **According to the clock postulate, the time interval in the proper frame of the test-body K, is equal to the time interval in its instantaneous rest frame.** The gravitomagnetic field in the proper local frame of the test-body K is zero, because the additional motion of the fixed stars relative to the test-body K is translational motion and so,

$$\vec{B}'_g = \vec{\nabla}' \times \vec{A}'_g = 0 \quad (4.5)$$

The proper local frame of the test-body K is so small that we can assume that the Lorentz factor  $\gamma(v)$  has the same value everywhere. The same happens with the scalar potential  $\varphi'_g$  because of equation (4.2) and therefore,

$$\vec{\nabla}'\varphi'_g = 0 \quad (4.6)$$

Hence, the induced gravitational field that the test-body K feels, becomes

$$\vec{E}'_g = -\frac{\partial\vec{A}'_g}{\partial t'} \quad (4.7)$$

Therefore an induced gravitational force will be exerted on the test-body K

$$\vec{F}'_g = m\vec{E}'_g = -\frac{\partial(m\vec{A}'_g)}{\partial t'} \quad (4.8)$$

If we assume now that the gravitational scalar potential  $\varphi_g$  is independent of time (that's why we call the stars, fixed stars) substituting for  $\vec{A}'_g$  from equation (4.3) into equation (4.8), we get

$$\vec{F}'_g = -\left(-\frac{1}{c^2}m\varphi_g\right)\frac{\partial[\gamma(v)\vec{v}]}{\partial t'} = -\left(-\frac{1}{c^2}m\varphi_g\right)\gamma^3(v)\frac{d\vec{v}}{dt'} \quad (4.9)$$

If we recall now that the gravitational scalar potential is negative, it is obvious from equation (4.9) that the induced gravitational force on the test body K resists changes in its velocity. **It is an inertial force!**

**We will call the inertial force which is given by equations (4.8), external gravitational inertial force  $\vec{F}'_{inert}$  because it is due to the acceleration with respect to the fixed stars. So,**

$$\vec{F}'_{inert} = \vec{F}'_g = -\frac{\partial(m\vec{A}'_g)}{\partial t'} \quad (4.10)$$

In addition to the external inertial force, there is also an internal inertial force. This is a well known effect which has the name radiation reaction [11] [12]. We do not know exactly the mechanism that causes it but we know that it exists. The picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. **We will call this self-force, internal inertial force, because it depends on the internal structure of the body.**

According to the Equivalence Principle, when a body makes free fall, the gravitational field exactly cancels the acceleration of the body so that, no sign of either acceleration or gravitation can be found by any physical means, inside the body. **Therefore, because the internal inertial force is a sign of acceleration, it should be canceled when the body makes free fall.** So, we come to the conclusion that:

- **When a body makes free fall in a gravitational field, the internal structure of the body plays no role and thus, only the external gravitational inertial force acts on the body.**

We can obtain now, some very important results for non-relativistic velocities, before moving on and considering the subject in the relativistic domain. So, for non-relativistic velocities, from equation (4.9), the external gravitational inertial force on the accelerating test-body K is

$$\vec{F}'_{inert} = - \left( -\frac{1}{c^2} m \varphi_g \right) \frac{d\vec{v}}{dt} = \left( -\frac{1}{c^2} m \varphi_g \right) (-\vec{a}) \quad (4.11)$$

where  $dt$  is the time interval in the lab frame and  $\vec{a}$  is the acceleration with respect to the lab frame. **Let's imagine now, that the test-body K is a body without internal structure and thus, when it is accelerated by a force  $\vec{F}$ , it does not feel any internal inertial force but only the external gravitational inertial force.** According to the Law of motion of principle 5, in the proper frame of the test-body K, the total force on the body is zero. Therefore, the force  $\vec{F}$  that must be exerted on the test body K to accelerate it with acceleration  $\vec{a}$ , is

$$\vec{F} = -\vec{F}'_{inert} = \left( -\frac{1}{c^2} m \varphi_g \right) \vec{a} = m_{in} \vec{a} \quad (4.12)$$

**The equation (4.12) is Newton's Second Law, for non-relativistic velocities, which obviously results from the Law of Motion. Therefore, the inertial mass  $m_{in}$  of the test-body K, for non-relativistic velocities, is**

$$m_{in} = \left( -\frac{1}{c^2} m \varphi_g \right) \quad (4.13)$$

We will call the inertial mass of the test-body K gravitational inertial mass and its momentum, gravitational momentum, because they are due to the gravitational potential of the rest of the Universe.

**So, the gravitational inertial mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential of the entire Universe.** It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. Therefore, the distant matter is of predominant importance, while local matter has only a very small effect on the gravitational inertial mass of a body.

Let's suppose now that a test-body of gravitational mass  $m$ , makes free fall in the gravitational field of a massive body which has spherically symmetric gravitational mass  $M$  with  $M \gg m$ , in a region of space where the gravitational scalar potential from the rest of the Universe is  $\varphi_g$ . **As we have shown, when a body makes a free fall in a gravitational field only the external gravitational inertial force acts on the body.** Therefore, for non-relativistic velocities, Newton's Law of Universal Gravitation and Newton's Second Law gives for the magnitude of the radial acceleration of the test-body,

$$G \frac{Mm}{r^2} = \left( -\frac{1}{c^2} m \varphi_g \right) a \quad (4.14)$$

where  $r$  is the distance of the test-body from the center of the massive body. **It is obvious that the gravitational mass  $m$  of the test-body is canceled in equation (4.14).**

- **Therefore, the acceleration of a free-falling body is independent of its gravitational mass and so, all bodies fall at the same rate in a gravitational field.**

This is a fundamental experimental result that was tested with great accuracy with the Eötvös experiment. We must emphasize that this was a free fall experiment. It's noteworthy that James Hartle writes for the Eötvös experiment [13]:

“The masses are free to move in the direction perpendicular to both the fiber and the rod. Gravity is the only force acting in this “twisting direction” along which the masses are effectively freely falling. Any difference between the acceleration of the two masses would cause the pendulum to twist.”

In Einstein's General relativity, the above experimental result is interpreted by accepting the equivalence of gravitational and inertial mass.

## 4.2 Relativistic momentum and the nature of the inertial rest mass

Let's continue the lab frame experiment, to find out the relativistic momentum and the nature of the rest mass of a body. **We suppose that in the lab frame a test-body  $K$  of gravitational mass  $m$ , electric charge  $q$  and no internal structure, which is initially at rest on the  $x$ -axis, begins to accelerate along the  $x$ -axis due to the electric field  $E_{e-x}$  of an electric point charge  $B$ , which is always stationary on the  $x$ -axis.** There are no other electric charges or currents in the vicinity.

Let's suppose that in the area of the lab frame, a local inertial frame  $S$  which has in common with the lab frame the  $x, x'$  axis, moves along the  $x$ -axis with uniform velocity  $V$  relative to the lab frame. We assume that when the test-body  $K$  has instantaneous velocity  $v$  relative to the lab frame, has instantaneous velocity  $w$  relative to the frame  $S$  and therefore, the frame  $S$  has instantaneous velocity  $(-w)$  relative to the instantaneous rest frame of the test-body  $K$ . **All the above velocities are along the  $x$ -axis and therefore parallel.**

Because the velocity of the test-body  $K$  is along a line passing through the charge  $B$ , i.e. the  $x$ -axis, no magnetic force is exerted on the test-body  $K$  in the lab frame and in the frame  $S$ . The same happens of course in its instantaneous rest frame, because it is at rest there.

**When the test-body  $K$  has instantaneous velocity  $v$  relative to the lab frame, from the point of view of the test-body  $K$ ,** the fixed stars are accelerated, along the  $x$ -axis and the gravitomagnetic vector potential from them changes and causes the external gravitational inertial force which, according to equation (4.10), is

$$F'_{inert-x} = -\frac{\partial (mA'_{g-x})}{\partial t'} \quad (4.15)$$

where  $\partial t'$  is the time interval in the proper frame of K. The gravitomagnetic vector potential  $A'_{g-x}$  in the proper frame of the test-body K, according to equations (4.2) and (4.3), is

$$A'_{g-x} = \gamma(v) \left( -\frac{1}{c^2} \varphi_g \right) v = -\frac{1}{c^2} \varphi'_g v \quad (4.16)$$

The accelerated test-body K feels in its proper frame the external gravitational inertial force and the electric force  $F'_x$  that accelerates it,

$$F'_x = qE'_{e-x} = q \left( -\frac{\partial \varphi'_e}{\partial x'} - \frac{\partial A'_{e-x}}{\partial t'} \right) \quad (4.17)$$

where  $\varphi'_e$  the electric scalar potential and  $A'_{e-x}$  the magnetic vector potential of the electric charge B.

**According to the Law of motion in the proper-frame of the test-body K the total force on it is zero. Therefore,**

$$F'_x = -F'_{inert-x} \quad (4.18)$$

**The electric force has the same value in the proper frame of the accelerated test-body K as it does in its instantaneous rest frame.** Hence, the electric force that accelerates the test body K in its instantaneous rest frame is

$$F'_x = qE'_{e-x} \quad (4.19)$$

and since the electric charge is a Lorentz invariant, the same electric force in the local frame S, is

$$F''_x = qE''_{e-x} = q \left( -\frac{\partial \varphi''_e}{\partial x''} - \frac{\partial A''_{e-x}}{\partial t''} \right) \quad (4.20)$$

where the double-prime indicates physical quantities in the frame S. It is well known from the Lorentz transformations that the longitudinal component of the electric field has the same value in the lab frame, in the frame S and in the instantaneous rest frame of the test-body K [14]. So,

$$E_{e-x} = E'_{e-x} = E''_{e-x} \quad (4.21)$$

Hence, the electric force that accelerate the test-body K has the same value in its instantaneous rest frame as it does in the frame S,

$$F'_x = F''_x \quad (4.22)$$

Therefore, the equation (4.18) becomes

$$F''_x = -F'_{inert-x} \quad (4.23)$$

**From the point of view of an observer at rest in the inertial frame S, the test-body K is accelerated by an electric force and its momentum changes.** But which momentum? In Lagrangian mechanics we have three momentums. The canonical gravitational momentum of the test-body K in the frame S, is the sum of its kinetic gravitational momentum and its potential gravitational momentum in the frame S,

$$\vec{P}''_g = \vec{p}''_g + m\vec{A}''_g \quad (4.24)$$

**We define the force  $\vec{F}$  that accelerates a body moving with velocity  $\vec{v}$ , as the total derivative of the canonical momentum  $\vec{P}$  of the body with respect to time:**

$$\vec{F} \equiv \frac{d\vec{P}}{dt} = \frac{\partial \vec{P}}{\partial t} + \frac{d\vec{r}}{dt} \cdot \vec{\nabla} \vec{P} = \frac{\partial \vec{P}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{P} \quad (4.25)$$

for a reason that will soon become apparent.

In the local frame S, where the test-body K is accelerated by the electric force  $F_x''$  along the x-axis, the total derivative results in the partial derivative with respect to time, for a reason that we will explain below. So, in the frame S,

$$F_x'' = \frac{\partial P_{g-x}''}{\partial t''} + \frac{\partial P_{g-x}''}{\partial x''} \frac{\partial x''}{\partial t''} = \frac{\partial P_{g-x}''}{\partial t''} \quad (4.26)$$

Therefore, using the equations (4.15) and (4.26), the equation (4.23):  $F_x'' = -F'_{inert-x}$ , becomes

$$\frac{\partial P_{g-x}''}{\partial t''} = \frac{\partial (mA'_{g-x})}{\partial t'} \quad (4.27)$$

where  $\partial t''$  is the time interval in the inertial frame S and  $\partial t'$  the time interval in the accelerated proper frame of the test-body K, i.e. the proper time. **According to the clock postulate, the time interval in the proper frame of the test-body K, is equal to the time interval in its instantaneous rest frame.** Therefore, from the Lorentz transformation, since the frame S has velocity  $(-w)$  relative to the instantaneous rest frame of the test-body K, we have,

$$t'' = \gamma(w) (t' + wx'/c^2) \quad (4.28)$$

From the above equation we get,

$$\frac{\partial t''}{\partial t'} = \gamma(w) \quad \text{and} \quad t' = \frac{1}{\gamma(w)} \left( t'' - \gamma(w) \frac{wx'}{c^2} \right) \quad (4.29)$$

**From the last two equations, using the chain rule we can find the Lorentz transformation of the differential operator  $\partial/\partial t'$  for constant  $x'$  (condition for the formula of time dilation)**

$$\frac{\partial}{\partial t'} = \frac{\partial t''}{\partial t'} \frac{\partial}{\partial t''} + \frac{\partial x'}{\partial t'} \frac{\partial}{\partial x'} = \frac{\partial t''}{\partial t'} \frac{\partial}{\partial t''} = \gamma(w) \frac{\partial}{\partial t''} \quad (4.30)$$

which is like the well known equation that relates the non-proper time interval  $dt''$  to the proper time interval  $dt'$ ,  $dt'' = \gamma(w)dt'$ , but for infinitesimal time intervals. The Lorentz factor  $\gamma(w)$  does not depend on time because the instantaneous rest frame of the test-body K moves with uniform velocity relative to the frame S. **Therefore, using the equation (4.30), the equation (4.27) becomes**

$$\frac{\partial P_{g-x}''}{\partial t''} = \frac{\partial (mA'_{g-x})}{\partial t'} = \gamma(w) \frac{\partial (mA'_{g-x})}{\partial t''} = \frac{\partial [\gamma(w)mA'_{g-x}]}{\partial t''} \quad (4.31)$$

**Integrating and substituting for  $A'_{g-x}$  from equation (4.16) into equation (4.31), we get the canonical gravitational momentum of the test-body K in the frame S**

$$P_{g-x}'' = \gamma(w)mA'_{g-x} = \gamma(w)\gamma(v) \left( -\frac{1}{c^2}m\varphi_g \right) v \quad (4.32)$$

where the constant of integration is set to zero, for a reason that will soon become apparent. **It is obvious that the momentum in equation (4.32) cannot be the kinetic gravitational momentum of the body K in the frame S.** Recognizing now that the canonical gravitational momentum of the body K may depend on position via  $\varphi_g$ , as well as time, and the position depend on time via  $v$ , we used the total derivative in the new definition of force. But in our case, since the gravitational scalar potential  $\varphi_g$  of the entire Universe has the same value everywhere in the local lab frame and also everywhere in the local frame S, the canonical gravitational momentum does not depend on position and thus the total derivative in equation (4.26) results in the partial derivative. But this may not always be the case.

**Let's find out now the kinetic gravitational momentum of the test-body K in the inertial frame S.** As we said before, when the instantaneous rest frame of the body K moves with velocity  $v$  relative to the lab frame, the frame S moves with velocity  $(-w)$  relative to the instantaneous rest frame of the test-body K. The above velocities are parallel along the x-axis and therefore, we have two successive Lorentz transformations along the x-axis. The coordinates  $(x', t')$  of the instantaneous rest frame of the test-body K, which moves with velocity  $v$  relative to the lab frame, are related to the coordinates  $(x, t)$  of the lab frame by the Lorentz transformations

$$x' = \gamma(v)(x - vt) \quad \text{and} \quad t' = \gamma(v)\left(t - \frac{vx}{c^2}\right) \quad (4.33)$$

The coordinates  $(x'', t'')$  of the inertial frame S, which moves with velocity  $(-w)$  relative to the instantaneous rest frame of the test-body K, are related to the coordinates  $(x', t')$  of the instantaneous rest frame of the test-body K by the Lorentz transformations

$$x'' = \gamma(w)(x' + wt') \quad \text{and} \quad t'' = \gamma(w)\left(t' + \frac{wx'}{c^2}\right) \quad (4.34)$$

If we now use the first Lorentz transformations as a definition of the prime variables and plug them into the second Lorentz transformations, we have

$$x'' = \gamma(w)\gamma(v)\left(x - vt + wt - \frac{wvx}{c^2}\right) \quad \text{and} \quad t'' = \gamma(w)\gamma(v)\left(t - \frac{vx}{c^2} + \frac{wx}{c^2} - \frac{wvt}{c^2}\right) \quad (4.35)$$

Collect terms

$$x'' = \gamma(w)\gamma(v)\left[x\left(1 - \frac{wv}{c^2}\right) - t(v - w)\right] \quad \text{and} \quad t'' = \gamma(w)\gamma(v)\left[t\left(1 - \frac{wv}{c^2}\right) - x\left(\frac{v}{c^2} - \frac{w}{c^2}\right)\right] \quad (4.36)$$

The coordinates  $(x'', t'')$  of the frame S, which moves at the same time with velocity  $V$  relative to the lab frame, are related directly to the coordinates  $(x, t)$  of the lab frame by the Lorentz transformations

$$x'' = \gamma(V)(x - Vt) \quad \text{and} \quad t'' = \gamma(V)\left(t - \frac{Vx}{c^2}\right) \quad (4.37)$$

In order for the equations (4.36) and (4.37) to be equivalent, the following equations must be valid

$$\gamma(w)\gamma(v)(v - w) = \gamma(V)V \quad \text{and} \quad \gamma(w)\gamma(v)\left[1 - \frac{vw}{c^2}\right] = \gamma(V) \quad (4.38)$$

If we divide the above two equations we get the relativistic parallel velocity-addition law,

$$V = \frac{v - w}{1 - (vw/c^2)} \quad (4.39)$$

Multiplying the first of the equations (4.38) by  $\left(-\frac{1}{c^2}m\varphi_g\right)$  we get

$$\gamma(w)\gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right)v - \gamma(w)\gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right)w = \gamma(V)\left(-\frac{1}{c^2}m\varphi_g\right)V \quad (4.40)$$

**By rearranging and comparing with equation (4.32), we get a relationship for the canonical gravitational momentum of the test-body K in the frame S**

$$P''_{g-x} = \gamma(w)\gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right)v = \gamma(w)\gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right)w + \gamma(V)\left(-\frac{1}{c^2}m\varphi_g\right)V \quad (4.41)$$

The potential gravitational momentum of the test-body **K** in the frame **S**, which moves at the same time with velocity  $V$  relative to the lab frame, according to eq. (4.3), is the last term of eq. (4.41),

$$mA''_{g-x} = \gamma(V) \left( -\frac{1}{c^2} m\varphi_g \right) V \quad (4.42)$$

Comparing the eq. (4.42), (4.41) and (4.24) we get the kinetic gravitational momentum of the test-body **K** in the frame **S**,

$$p''_{g-x} = \gamma(w)\gamma(v) \left( -\frac{1}{c^2} m\varphi_g \right) w = \gamma(w)m_{in-g0}w \quad (4.43)$$

where  $m_{in-g0}$  the gravitational inertial rest mass of the test-body **K** in the frame **S**,

$$m_{in-g0} = \gamma(v) \left( -\frac{1}{c^2} m\varphi_g \right) = -\frac{1}{c^2} m\varphi'_g \quad (4.44)$$

and  $\varphi'_g$  the gravitational scalar potential of the entire Universe, in the instantaneous rest frame of the test-body **K**, according to equation (4.2). **We'll return to the above equation (4.44) below!**

**From equations (4.20) and (4.25) we get the equation of motion of the test-body **K** in the frame **S**,**

$$\frac{dP''_{g-x}}{dt''} = \frac{d(p''_{g-x} + mA''_{g-x})}{dt''} = q \left( -\frac{\partial\varphi''_e}{\partial x''} - \frac{\partial A''_{e-x}}{\partial t''} \right) \quad (4.45)$$

which results from the Law of Motion. **In the local inertial frame **S****, since the scalar potential of the entire Universe has the same value everywhere, the momentum  $p''_{g-x}$  does not depend on position and the potential  $A''_{g-x}$  is constant. **So, in the frame **S**, eq. (4.45) result in Newton's Second Law.** Nothing changes in the previous study if the frame **S** is not an inertial frame (using instantaneous speeds). **So, eq. (4.45) applies in all frames of reference since it contains the inertial forces.**

**Let's find out now the nature of the gravitational inertia rest mass of the test-body **K**.** Let's suppose that we want to find the gravitational potential energy of the entire Universe relative to the test-body **K**, when it moves with velocity  $\vec{v}$  relative to the lab frame. The instantaneous sum of the gravitational four-potentials of all the bodies in the Universe is also a four-vector, the total gravitational four-potential:

$$A_g = \left( \frac{\varphi_g}{c}, \vec{A}_g \right) \quad (4.46)$$

and the four-velocity of the test-body **K** is:  $U = \gamma(v)(c, \vec{v})$ .

We know that the quantity we are looking for must depend on both  $U$  and  $A_g$ , and it is a scalar. The product  $mUA_g$  has physical dimensions of energy and it is a scalar, because the gravitational mass is a scalar, and the product of two four-vectors is a Lorentz invariant, i.e. a scalar. Evaluating the product  $mUA_g$  in the rest frame of the test-body **K**,

$$mUA_g = m(c, 0) \left( \frac{\varphi'_g}{c}, \vec{A}'_g \right) = m\varphi'_g \quad (4.47)$$

we obtain the gravitational potential energy of the entire Universe relative to the test-body **K**, which is the very thing we wanted and is a Lorentz invariant, i.e. all observers agree on its value at any instant of the test-body's history. Hence,

$$m_{in-g0} = -\frac{1}{c^2} mUA_g \quad (4.48)$$

**So, the gravitational inertial rest mass of a body without internal structure, is not an intrinsic property of the body but is proportional to the gravitational potential energy of the entire Universe relative to the body, and is a Lorentz invariant!**

Since the gravitational inertial rest mass of the test-body  $K$  is a Lorentz invariant, we can define its kinetic gravitational four-momentum in the usual way

$$P_g = m_{in-g0}U = (m_{in-g}c, \vec{p}_g) = (E_g/c, \vec{p}_g) \quad (4.49)$$

So, during a collision, if the kinetic gravitational four-momentum is conserved in one inertial frame, it is conserved in any inertial frame [15].

**In the Special Theory of relativity the inertial rest mass of a body should have the same numerical value when the body is at rest in all inertial frames. This is an experimental result.** However, from equation (4.44) we can see that the rest mass of the test-body  $K$  depends on its velocity relative to the lab frame and therefore, does not have the same value when it is at rest in different inertial frames, because different inertial frames move with different velocities relative to the lab frame. **Nevertheless, we will prove that because of the vast and expanding Universe, the rest mass of a body has practically the same value when it is at rest in all inertial frames.**

In order to prove the above, we will consider a simple no-relativistic model of the Universe [16]. The retarded gravitational scalar potential  $\varphi_{g-1}$  of the entire Universe in position 1 which is the position of the lab frame, is given from the integral over all the observable Universe

$$\varphi_{g-1} = -G \int \frac{[\rho]}{r_{12}} dV_2 \quad (4.50)$$

where the element of volume  $dV_2$  in position 2, is at a distance  $r_{12}$  from the lab frame and the density of matter  $[\rho]$  of the volume element  $dV_2$ , is the value at the retarded time  $(t - r_{12}/c)$ . According to Hubble's law, the galaxies are moving away with speeds proportional to their distance  $r_{12}$  from the lab frame

$$v = H_0 r_{12} \quad (4.51)$$

where  $H_0$  is the Hubble constant. We assume that matter receding with velocity greater than that of light makes no contribution to the scalar potential, so that the integral in equation is taken over the spherical volume of the Hubble radius  $c/H_0$  which is approximately 14.4 billion light years. **If we now assume that the density is uniform, not only in space but also in time, equation (4.50) gives**

$$\varphi_{g-1} = -2\pi G\rho(c/H_0)^2 \quad (4.52)$$

According to equation (4.52), the 99 per cent of the gravitational scalar potential arises from matter further away than  $10^9$  light-years. **If we now take into account that the very distant matter moves with relativistic velocities and the density of matter was much higher in the past,** we conclude that the 99,99 per cent of the gravitational scalar potential arises from matter further away than  $10^{10}$  light-years. Therefore, practically the 99,99 percent of the gravitational scalar potential arises from matter moving with speeds that are 99,99 percent of the speed of light. Thus, because of the relativistic velocity-addition law, practically, the velocity of the very distant matter relative to the test-body  $K$  is almost independent of the velocity of the test-body  $K$  relative to the lab frame. It is like the speed of light that is the same for all the observers. **Therefore, from equation (4.44) it follows that practically the gravitational inertial rest mass of a body has the same value when the body is at rest in all inertial frames.** So, the equations (4.44) and (4.43) becomes

$$m_{in-g0} \approx \left(-\frac{1}{c^2}m\varphi_g\right) \quad \text{and} \quad p''_{g-x} \approx \gamma(w) \left(-\frac{1}{c^2}m\varphi_g\right) w \quad (4.53)$$

If the frame  $S$  is at rest in the lab frame, the equation (4.43) has the product of two Lorentz factors  $\gamma(v)$ . The first Lorentz factor is caused by the motion of the test body  $K$  relative to the lab frame and the second is caused by the relativity of the time measurements and so, we come to equation (4.53) again.



Let us now consider, what happens if there are other electrical charges, with such a distribution and motion that the electric scalar potential  $\varphi_e$  in the lab frame is not zero but is the same everywhere, so that  $\vec{\nabla}\varphi_e = 0$  and the magnetic vector potential  $\vec{A}_e$  is zero. Since the equations of electromagnetism have the same mathematical form as the equations of gravitomagnetism, we will only state the important results. Let's suppose that the test-body K is accelerated in the lab frame along the x-axis just like in the previous study. So, when the instantaneous velocity of the test-body K relative to the frame S is  $w$  and relative to the lab frame is  $v$ , **the kinetic gravitoelectric momentum of the test-body K in the frame S, will be**

$$p''_{ge-x} = \gamma(w)\gamma(v) \left[ -\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] w = \gamma(w)m_{in-ge0}w \quad (4.54)$$

**and its gravitoelectric inertial rest mass  $m_{in-ge0}$  will be a Lorentz invariant.**

Let's suppose that there are  $n$  electric charges, each at a different distance  $r_i$  from the test-body K, as measured in the lab frame. For non relativistic velocities, according to equations (4.54) and (4.44), the gravitoelectric inertial rest mass of the the test-body K in the lab frame, is

$$m_{in-ge0} = m_{in-g0} - \frac{1}{c^2} \left( \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{qq_i}{r_i} \right) \quad (4.55)$$

**Finally, the Special theory of relativity will give us the rest mass of a body with internal structure, a composite body.** If we apply the conservation of the kinetic four-momentum in an inelastic collision where  $n$  free moving particles without internal structure, collide and create a composite body M which is at rest in the lab frame, the rest mass  $m_{in-0}$  of the composite body M in the lab frame is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + T/c^2 + E_{field}/c^2 \quad (4.56)$$

where  $m_{in-ge0i}$  is the gravitoelectric rest mass of each particle that makes up the composite body M,  $T$  is the kinetic energy of the relative motion of all the particles and  $E_{field}$  is the potential energy of the interaction of all the particles [17]. **The rest mass  $m_{in-0}$  of the composite body M, is also a Lorentz invariant as is well known from the Special theory of relativity [18].**

### 4.3 Zero rest mass

Let us now imagine, using classical physics, that we have an accelerating particle A of gravitational mass  $m$ , electric charge  $q$  and without internal structure, inside a thin spherical shell of radius  $R$  with charge  $q$  uniformly distributed on its surface. The electric scalar potential inside the spherical shell is given by the well known equation,  $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$ . The gravitoelectric rest mass of the particle A, according to equation (4.55), is

$$m_{in-ge0} = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R} \quad (4.57)$$

From equation (4.57) we can see that the gravitoelectric rest mass of the particle A becomes zero when the radius take the value  $R_{critical}$ . From equation (4.57) we get for  $R_{critical}$ ,

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{R_{critical}} \iff R_{critical} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{m_{in-g0}} \quad (4.58)$$

If we imagine that the particle A is like the proton  $R_{critical} \approx 1,53 \times 10^{-18}m$  and if is like the electron  $R_{critical} \approx 2,81 \times 10^{-15}m$ . **Therefore, the effect of the electric scalar potential on the inertial rest mass of a particle, becomes significant in the subatomic world !!!** And now the question is: What happens if the inertial rest mass can become negative? Does repulsive force become attractive?

## 5 Spacetime metric

The Equivalence Principle shows us that, spacetime is endowed with a metric and the gravitational field affects the spacetime metric. We will derive now the spacetime metric outside of a static, spherically symmetric body B of gravitational mass  $M$ . We will consider the freely motion of a test-body A, of gravitational mass  $m$  with  $m \ll M$ , in the radial direction of the gravitational field of the body B. We will follow a method based on a paper of F. Tangherlini [19][20][21].

The equation of motion of the test-body A, is obtained from the principle 7 of geodesic motion in section 3:

$$\delta \int ds = 0 \quad (5.1)$$

where, from the principle 6:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (5.2)$$

By well-known arguments we may bring the spacetime interval outside of a static, spherically symmetric body into the standard Schwarzschild form [22]:

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{11}(r) dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.3)$$

The scalar functions  $g_{00}$  and  $g_{11}$ , are functions only of the coordinate distance or radial coordinate  $r$ . For purely radial motion of the test-body A, the spacetime interval of equation (5.3), becomes

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{11}(r) dr^2 \quad (5.4)$$

The metric (5.3) should give us to infinity the Minkowski metric in spherical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.5)$$

Therefore, we must have the boundary conditions

$$\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1 \quad \lim_{r \rightarrow \infty} g_{11}(r) \rightarrow -1 \quad \lim_{r \rightarrow \infty} g_{00}(r)g_{11}(r) \rightarrow -1 \quad (5.6)$$

The equations of motion of the test-body A possess the well known first integral [23]:

$$g_{00} \frac{dt}{ds} = k_0 \quad (5.7)$$

Equation (5.7) states that the energy (per unit mass)  $k_0$  is a constant of the motion (the energy conservation law). We can have a second first integral, for the equation of motion in the radial direction, by dividing the equation (5.4) by  $ds^2$

$$1 = g_{00}c^2 \left( \frac{dt}{ds} \right)^2 + g_{11} \left( \frac{dr}{ds} \right)^2 \quad (5.8)$$

Equation (5.8) expresses the invariant relation between energy and momentum (per unit mass). Eliminating  $\frac{dt}{ds}$  from equation (5.8), using the first integral of equation (5.7), we get

$$1 = \frac{c^2 k_0^2}{g_{00}} + g_{11} \left( \frac{dr}{ds} \right)^2 \quad (5.9)$$

Since the metric tensor, in our case, is diagonal, one should have

$$g^{00} = \frac{1}{g_{00}} \quad \text{and} \quad g^{11} = \frac{1}{g_{11}} \quad (5.10)$$

Dividing the equation (5.9) by  $g_{11}$  we get

$$\frac{1}{g_{11}} = \frac{c^2 k_0^2}{g_{00} g_{11}} + \left( \frac{dr}{ds} \right)^2 \quad (5.11)$$

and using the equations (5.10) we get

$$g^{11} = c^2 k_0^2 (g^{00} g^{11}) + \left( \frac{dr}{ds} \right)^2 \quad (5.12)$$

The scalar functions  $g_{00}$  and  $g_{11}$ , are functions of only  $r$  and  $r$  is a function of only  $s$  since  $d\tau = ds/c$  where  $d\tau$  is the proper time. So, using the chain rule, we get

$$\frac{dg^{00}}{ds} = \frac{dg^{00}}{dr} \frac{dr}{ds} = g^{00}{}_{,r} \frac{dr}{ds} \quad \text{and} \quad \frac{dg^{11}}{ds} = \frac{dg^{11}}{dr} \frac{dr}{ds} = g^{11}{}_{,r} \frac{dr}{ds} \quad (5.13)$$

where a comma denotes ordinary differentiation. Using the relations (5.13) we differentiate the equation (5.12) in respect to  $s$  and we obtain

$$g^{11}{}_{,r} \frac{dr}{ds} = c^2 k_0^2 (g^{00} g^{11}){}_{,r} \frac{dr}{ds} + 2 \frac{dr}{ds} \frac{d^2 r}{ds^2} \quad (5.14)$$

Cancelling  $\frac{dr}{ds}$  we get

$$\frac{d^2 r}{ds^2} = -\frac{c^2 k_0^2}{2} (g^{00} g^{11}){}_{,r} + \frac{1}{2} g^{11}{}_{,r} \quad (5.15)$$

**Equation (5.15) is the Tangherlini equation for purely radial motion** [21]. Using the relation  $ds^2 = c^2 d\tau^2$ , we get the equation of motion of the test-body A in the form,

$$\frac{d^2 r}{d\tau^2} = -\frac{c^4 k_0^2}{2} (g^{00} g^{11}){}_{,r} + \frac{c^2}{2} g^{11}{}_{,r} \quad (5.16)$$

From equation (5.16) we can see that because of the  $k_0$  term, the radial acceleration depends on the energy which the test-body A had initially, i.e. the radial velocity with which the test-body A was launched. However, the electric force on a charge is strictly independent of the charge's velocity [24]. The same happens with the gravitational force on a gravitational mass, because we accept that both forces are described by equations with the same mathematical form. **Therefore, the proper acceleration of the test body A, in the radial direction of the gravitational field of the body B, is strictly independent of the radial velocity with which the test-body A was launched.** [25].

According to the principle of consistency, in the Newtonian limit, i.e. when the test-body A moves with non-relativistic velocity  $v \ll c$ , in a weak and static gravitational field, the equation (5.16) must reduce to the Newtonian equation of motion of the test-body A, which is

$$\frac{d^2 \rho}{dt^2} = -\frac{m}{m_{in-g0}} \frac{d\varphi_{gB}}{d\rho} = -\frac{m}{m_{in-g0}} \frac{d}{d\rho} \left( -\frac{GM}{\rho} \right) = -\frac{m}{m_{in-g0}} \frac{GM}{\rho^2} \quad (5.17)$$

where  $\varphi_{gB}$  is the gravitational scalar potential of the body B and  $\rho$  is the radial distance or 'physical' proper distance. We use the gravitational rest mass  $m_{in-g0}$  because the test-body A makes a free fall and thus feels only the external gravitational inertial force. In the Newtonian limit, the proper acceleration, the radial acceleration and the acceleration in equation (5.17), are all the same. So, in the Newtonian limit the radial acceleration of the test-body A is strictly independent of its velocity. **In order for this to happen, the following equation must be valid**

$$g^{00} g^{11} = const. \quad (5.18)$$

Because the scalar functions  $g_{00}(r)$  and  $g_{11}(r)$  are independent of the speed, the equation (5.18) must be valid also for relativistic velocities. Therefore, the radial acceleration of the test-body A is strictly independent of the radial velocity with which it was launched. Because of the boundary conditions (5.6), the equation (5.18) becomes

$$g^{00}g^{11} = -1 \quad (5.19)$$

Using the equations (5.10), we get

$$g_{11} = -\frac{1}{g_{00}} \quad (5.20)$$

Thus, the equation (5.3) for the spacetime interval outside of the body B, can be written in the form

$$ds^2 = g_{00}c^2dt^2 - g_{00}^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.21)$$

From equations (5.20) and (5.10), we get

$$g^{11} = -g_{00} \quad (5.22)$$

Using the equations (5.18) and (5.22), the equation of motion of the test-body A (5.16), becomes

$$\frac{d^2r}{d\tau^2} = -\frac{c^2}{2}g_{00,r} \quad (5.23)$$

So, in the Newtonian limit, where the time  $t$  is approximately equal to the proper time  $\tau$  :  $dt \approx d\tau$  and the coordinate distance  $r$  is approximately equal to the radial distance  $\rho$  :  $dr \approx d\rho$ , comparing the equations (5.17) and (5.23), we get

$$-\frac{c^2}{2} \frac{dg_{00}}{dr} = -\frac{m}{m_{in-g0}} \frac{d\varphi_{gB}}{dr} \iff \frac{dg_{00}}{dr} = \frac{2m}{c^2 m_{in-g0}} \frac{d\varphi_{gB}}{dr} \quad (5.24)$$

As we have shown, the local matter has almost no effect on the rest mass of the test-body A. **So, we accept that the gravitational rest mass of the test-body A, is constant during the motion.** Hence, by integration and using the boundary conditions (5.6), we obtain for  $g_{00}$  and  $g_{11}$  from eq. (5.24),

$$g_{00} = 1 + \frac{2m}{c^2 m_{in-g0}} \varphi_{gB} = 1 - \frac{2m}{c^2 m_{in-g0}} \frac{GM}{r} \quad (5.25)$$

Therefore, the equation (5.21) for the spacetime interval outside of the body B, becomes

$$ds^2 = \left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.26)$$

We must emphasize that, according to equation (4.44),

$$\frac{m}{m_{in-g0}} = -\frac{c^2}{\varphi_g} \quad (5.27)$$

Therefore, the spacetime interval outside of the body B, is independent of the gravitational mass of the test-body A. **In the International system of units the ratio of the gravitational mass to the gravitational rest mass of the test-body A, is equal to unity and so, the equation (5.26) becomes the Schwarzschild metric**

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.28)$$

**To obtain any orbital motion of the test-body A in the gravitational field of the body B, one can either follow the path of General relativity using the geodesic equation, or one can use the relativistic mechanics.** For the latter we will follow a method based on a paper of J. Wild [26]. If we use the relativistic mechanics, we have to remember that the acceleration of the test-body A is determined by the radial field of force diverging from the body B, and the gravitational mass of the body B is defined, in suitable units, as the flux of this field through any closed surface outside it (Gauss's law). Thus, in flat spacetime, the flux of the gravitational field  $E_{gB}$  of the body B through a spherical surface of radius  $\rho$  centered on the body B is equal to  $4\pi\rho^2 E_{gB}$ , but in curved spacetime the area of the same spherical surface is not equal to  $4\pi\rho^2$ . However, if we make a transformation of the coordinate  $\rho \rightarrow r$  so as to make the spherical surface always equal to  $4\pi r^2$ , then the magnitude of the field  $E_{gB}$  is correctly given by the equation  $E_{gB} = -GM/r^2$ . In addition, in relativistic mechanics we have to replace the time  $dt$  by the proper time  $d\tau = ds/c$ . By doing these, the equation (5.17) becomes

$$\frac{d^2r}{ds^2} = -\frac{m}{m_{in-g0}} \frac{GM}{c^2 r^2} \quad (5.29)$$

For motion of the test-body A in the  $\theta = \pi/2$  plane we have from equation (5.21)

$$g_{00}c^2 \left(\frac{dt}{ds}\right)^2 - g_{00}^{-1} \left(\frac{dr}{ds}\right)^2 - r^2 \left(\frac{d\phi}{ds}\right)^2 = 1 \quad (5.30)$$

or

$$\left(\frac{dr}{ds}\right)^2 + g_{00}r^2 \left(\frac{d\phi}{ds}\right)^2 - g_{00}^2 c^2 \left(\frac{dt}{ds}\right)^2 + 1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r} = 0 \quad (5.31)$$

With  $\mu = (g_{00}cdt/ds)^2$ , differentiation of equation (5.31) with respect to  $r$ , for purely radial motion ( $d\phi/ds = 0$ ), gives

$$2\frac{d^2r}{ds^2} + \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r^2} + \frac{d\mu}{dr} = 0 \quad (5.32)$$

Comparison with equation (5.29) shows that  $\mu = constant$ . Thus, using,  $\frac{m}{m_{in-g0}} = 1$  as in the International system of units, for easy comparison, equation (5.31) becomes

$$\left(\frac{dr}{ds}\right)^2 + \left(1 - \frac{2GM}{c^2 r}\right) r^2 \left(\frac{d\phi}{ds}\right)^2 - \frac{2GM}{c^2 r} = constant \quad (5.33)$$

The above equation is a well known equation of General relativity that incorporates the general relativistic analogues of energy conservation and angular momentum conservation and can also be obtained using the geodesic equation [27]. The equation (5.33) and the equation:  $r^2 \frac{d\phi}{ds} = constant$ , that gives the conservation of the angular momentum, define the motion of a planet and also (with  $ds = 0$ ) the gravitational deflection of a light ray. It is then a textbook procedure to show that they lead to the well-known formula for the precession of planetary orbits and the deflection of a light ray by the Sun [28].

## 6 Dark matter and dark energy

### 6.1 Dark matter

In the Universe there are planets, stars, galaxies, clusters of galaxies and so on. **If we now think that the gravitational scalar potential and the gravitomagnetic vector potential satisfy the wave equation and they travel with the speed of light in space, they must behave like the**

light that bends when traveling near a large gravitational mass. Therefore, we conclude that the gravitational potential of the entire Universe is more concentrated in places with higher densities of matter. Hence, because the gravitational inertial rest mass of a star depends on the gravitational scalar potential, it seems that the position where a star is located, affects the inertial mass of the star. In places with higher density of matter the inertial mass of a star will be greater than the inertial mass of an identical star, in a place with lower density of matter. **So, the inertial mass of a star near the center of a galaxy is greater than the inertial mass of an identical star at the edges of that galaxy.** Therefore, the stars at the edges of a rotating, spiral galaxy will have to move faster than Newtonian physics predicts. This seems to be very important for the problem of dark matter.

## 6.2 Dark energy

Let us now consider the light emitted by atoms on the surface of a static, spherically symmetric star of gravitational mass  $M$ . We assume that an atom of gravitational mass  $m$  and gravitational rest mass  $m_{in-g0}$ , which emits light, is at a distance  $r_{em}$  from the center of the star. From equation (5.26) arises the equation relating the proper time  $d\tau_{em}$  at the point of emission, with the proper time  $d\tau_{\infty}$  at infinity where is the point of observation [29]

$$d\tau_{\infty} = \frac{d\tau_{em}}{\sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}}} \quad (6.1)$$

So, the frequency of the light observed by an observer, at the point of observation, will be

$$f_{\infty} = f_{em} \sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}} \quad (6.2)$$

Equation (6.2) describes the redshift of light emitted by an atom in a gravitational field and received by an observer who is very far away, essentially at infinity. **As the Universe expands, the density of matter decreases and so, according to equation (4.50), the gravitational inertial mass of an atom decreases over time.** Therefore, as it emerges from equation (6.2), the light emitted by the atoms of two identical supernovas Ia at different times in the history of the Universe, will have different red shift. The atoms of a younger (most recent) supernova will have smaller gravitational rest mass than the atoms of an older supernova. **Hence, the light emitted by the atoms of a younger supernova Ia will have larger red shift than the light emitted by the atoms of an older supernova Ia.** This phenomenon has been observed, but the inability to explain it has led to the theory that the Universe expands in an accelerating way, because of dark energy.

## Conclusions

In this paper we have described a new theory which has all the hallmarks of Einstein's General theory of relativity, except the field equations and the equivalence of gravitational and inertial mass. The Schwarzschild metric is one of its consequences and therefore agrees with all previous experiments and observations. However, the new theory goes further and its other consequences are very important for dark matter, dark energy, nuclear and particle physics. Due to the similarity of gravitational and electric forces we have to check if the electric potential also affects the spacetime metric, by measuring the rate of decay of unstable particles inside a spherical shell with charge uniformly distributed on its surface so that the electric field inside to be zero while the electric potential is not zero. Finally, if the inertial mass can take any value, as the new theory shows, then a future technology can make space travel an easy affair !!!

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