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Relativistic Gravity and the Origin of Inertia and Inertial Mass: An important topic for the explanation of dark matter and dark energy

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Abstract

If equilibrium is to be a frame-independent condition, it is necessary the gravitational force to have precisely the same transformation law as that of the Lorentz-force. Therefore, gravity should be described by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory, with the gravitational mass as a Lorentz invariant. Using this gravitomagnetic theory, in order to ensure the relativity of all kinds of translatory motion, we accept the principle of covariance and the equivalence principle and thus we prove that,

1. The external inertial force, perceived by an accelerating body, is real gravitational force due to induction effects from the entire Universe.
2. The internal inertial force, depends on the body's internal structure, but in a free fall it is canceled because of the equivalence principle and the body experiences only the external inertial force. That is why all bodies, fall with the same acceleration.
3. The inertial mass of a body depends on the distribution of matter in the Universe and this seems very important for the explanation of dark matter and dark energy.
4. The gravitational field affects the spacetime metric and all freely moving bodies follow geodesic of the metric.
5. We can obtain the Schwarzschild metric and thus, the new theory is in agreement with all past experiments and observations.

Keywords: Gravitomagnetism , Mach' principle , origin of inertia , dark matter , dark energy.

1 Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. According to Newton, the inertial forces, such as the centrifugal force, must arise from acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter. According to Mach's principle, the inertial forces, such as the centrifugal force, are more likely caused by acceleration, with respect to the mass of the celestial bodies. This idea implies that the properties of space arise from the matter contained therein and are meaningless in an empty space. The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important for subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle, is motion relative to other matter in the Universe, has never found its complete expression in a physical theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity, attempted to interpret inertia considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2]. The principle of equivalence is an essential part of General relativity. But although the principle of equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3].

Finally, as pointed out by Henri Poincare, if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force [4]. But this does not happen with gravity as it is described by General relativity. In order for this to happen, gravity should be described by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory with the gravitational mass as a Lorentz invariant. Moreover, if we want to extend the principle of relativity to all kinds of motion, we must accept the principle of covariance and the equivalence principle. According to the equivalence principle the gravitational field should affect the spacetime metric and all the freely moving test bodies follow geodesic of the metric. All of the above, are enough to obtain the Schwarzschild metric, without the field equations of General relativity. So, the new theory will be in agreement with all past experiments and observations. The relativity of acceleration together with the gravitomagnetic theory will show us the origin of inertial forces and the origin of inertial mass. This is the main idea that we will describe in this paper.

2 Relativistic Gravity

In an inertial reference frame K , let's have a system of two non spinning bodies with gravitational masses and positive electric charges, in a region of free space where there are no external forces. We suppose that the two bodies are at rest in the inertial frame K , under equilibrium conditions, i.e. the force of gravitational attraction balances that of electrostatic repulsion. But what is observed by another inertial frame of reference K' , moving with uniform velocity relative to the frame K ? Let's imagine that if the bodies collide, they will explode. It is impossible for one observer to see an explosion and for another to not see it. So, the equilibrium must be a frame-independent condition. In order for this to happen, it is necessary the gravitational force to have precisely the same transformation law as that of the Lorentz-force. So, gravity should be described by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory with the gravitational mass as a Lorentz invariant.

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformations (for force, potential, velocity and co-ordinate) and the following series of remarks [5][6]:

1. The Coulomb potential for a stationary charge in vacuum is, $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric charge produces a scalar potential and a vector potential, which together form a four-vector, $A_e = \left(\frac{\varphi_e}{c}, \mathbf{A}_e\right)$
3. The potentials produced by a charge moving in any way, depend only upon the velocity and position at the retarded time.

Of course we need to know how to get the Coulomb's law from the scalar potential. So, if we want to obtain a gravitomagnetic theory, where its equations have the same mathematical form, as those

of the electromagnetic theory, the same series of remarks must be met for gravity. We already have the first remark, that is, the gravitational potential for a stationary gravitational mass m in vacuum is $\varphi_g = -\frac{1}{4\pi g_0} \frac{m}{r}$ where $g_0 = \frac{1}{4\pi G}$ and G is the Newton's universal gravitational constant, but this is only the one remark. So, we need the other two, as well. We will obtain them with the following two principles:

Principle 1

A gravitational mass produces a scalar potential and a vector potential, which together form a four-vector, $A_g = \left(\frac{\varphi_g}{c}, \mathbf{A}_g\right)$

From principle 1, follows that the gravitational mass is an invariant quantity. As we will show later, **the inertial mass and the gravitational mass are different entities but all the bodies released from the same point in a gravitational field, fall with the same acceleration.**

Principle 2

The potentials produced by a gravitational mass moving in any way, depend only upon the velocity and position at the retarded time.

So, the potentials produced by a gravitational point-mass m moving with any velocity have the same mathematical form as the Lienard-Wiechert potentials for an electric point-charge moving with any velocity, but with a negative sign,

$$\varphi_g = -\frac{1}{4\pi g_0} \left[\frac{m}{r - \mathbf{r}\mathbf{v}/c} \right] \quad (2.1)$$

$$\mathbf{A}_g = -\frac{1}{c^2} \frac{1}{4\pi g_0} \left[\frac{m\mathbf{v}}{r - \mathbf{r}\mathbf{v}/c} \right] = \frac{1}{c^2} [\varphi_g \mathbf{v}] \quad (2.2)$$

where \mathbf{r} is the vector from the gravitational point-mass to the point where the potential is evaluated, c is the speed of light in vacuum and all the quantities in the square bracket are to have their values at the retarded time. Starting from the potentials, in order to find the fields, we have the equations

$$\mathbf{E}_g = -\nabla \varphi_g - \frac{\partial \mathbf{A}_g}{\partial t} \quad (2.3)$$

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g \quad (2.4)$$

The force, that a gravitational mass m experiences, when it moves with velocity \mathbf{v} in the above fields is,

$$\mathbf{F}_g = m(\mathbf{E}_g + \mathbf{v} \times \mathbf{B}_g) \quad (2.5)$$

where \mathbf{E}_g is the gravitational field and \mathbf{B}_g the gravitomagnetic field.

So, we have now a gravitomagnetic theory, with equations that have the same mathematical form as those of the electromagnetic theory. Therefore, we expect that there are gravitomagnetic waves that propagate, in vacuum with the speed of light, and that they are described by equations which have the same mathematical form as the corresponding equations for electromagnetic waves, but with one important difference. An isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment. However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity. The reason is simple. The electric dipole moment can move around with respect to the center of mass but the gravitational dipole moment is identical in location with the center of mass, and due to the law of conservation of momentum, cannot accelerate or radiate [7].

3 General relativity of translatory motion

We will follow now, the fundamental idea of relativity of all kinds of translatory motion. In accordance with this idea we can detect and measure the translatory motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion. So, acceleration, as well as the velocity, is relative.

The rotational motion of a rigid body, as a whole, cannot be relative. Different parts of the rotating body perceive different motion of the other bodies. Nevertheless, we can consider that a rotational rigid body consists of an infinite number of point particles and the instantaneous velocity and acceleration of every point particle separately, is relative.

In order for all kinds of translatory motion to be relative, the laws of physics should have the same mathematical form when referred to different reference frames, which are in relative translatory motion, since otherwise the difference in form could provide a criterion for judging the absolute motion. So, first we accept the next principle,

Principle 3 - The principle of general relativity (The principle of covariance)

The laws of physics which are valid in an inertial reference frame, i.e. the laws of Special theory of relativity, have the same mathematical form in all reference frames which are in relative translatory motion.

However, it does not end with the above principle. We need one more principle to ensure the relativity of all kinds of translatory motion. This arises from the fact that the expression of the equations of physics in a form which is independent of the reference frame does not in general prevent a change in their numerical content when we change from one reference frame to another and it is only by relating such changes in numerical content to conceivable changes in gravitational field that we are able to eliminate criteria for absolute motion and to preserve the idea of the relativity of all kinds of translatory motion [8]. Let's make now a thought experiment, **the lab frame experiment**, to make the above argument clear.

Let's suppose that we have a space station, far from any massive body, that we use it as a laboratory. **We will call the local frame of the space station, lab frame. An observer L is at rest in the lab frame.** We assume that the distribution of matter in the Universe is such, that **the gravitational field in the lab frame is equal to zero.** This means that the gravitational scalar potential φ_g , from the entire Universe, has the same value everywhere in the lab frame, and so,

$$\nabla\varphi_g = 0 \quad (3.1)$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitational vector potential due to one part of the mass-current, is canceled out by the vector potential due to another part of the mass-current, owing to its symmetry. This means that the gravitational vector potential \mathbf{A}_g from the entire Universe is equal to zero, everywhere in the lab frame,

$$\mathbf{A}_g = 0 \quad (3.2)$$

This would also happen if all the bodies of the Universe were at rest, relative to the laboratory. So, we can say that the lab frame is at rest relative to the Universe, or at rest relative to the fixed stars.

Let's suppose that in the lab frame, a small rocket starts to accelerate making translatory motion, with constant proper acceleration, i.e. feels a constant force in its instantaneous rest frame. **We will call the local reference frame of the rocket, rocket frame. An observer R is at rest in the rocket frame.** According to the principle of general relativity we can apply the covariant laws of gravitomagnetism in the rocket frame. Because we accept that acceleration is relative, the fixed stars are

accelerating relative to the rocket frame, in the same way for all parts of the rocket frame. Therefore, according to the Faraday's law for gravitomagnetism, the covariant laws of gravitomagnetism will give us an induced uniform gravitational field in the rocket frame. We will consider this in detail, later.

So, the observer R is at rest in a local reference frame with a uniform gravitational field, while the observer L is at rest in a local reference frame without gravity, that is, an inertial reference frame. We assume that the gravitational forces between the rocket and the laboratory are negligible. The observer L, who is at rest in a local inertia frame, using the Special theory of relativity is able to describe what physical effects are observed by the observer R who is at rest in the accelerating rocket frame. This is the well known study of a uniformly accelerated rigid reference frame, in Special relativity. So, from the viewpoint of the observer L, the well known physical effects, which are observed by the observer R, are [9]:

1. Redshift or blueshift of a light ray which moves parallel to the direction of the acceleration.
2. Varying coordinate speed of light; fixed local relative speed of light.
3. Spacetime is endowed with a metric.
4. Maximum proper time as the law of motion of freely moving bodies.
5. Horizons.

Because we accept that acceleration is relative, from the viewpoint of the observer R, who is at rest in the rocket frame, it is the lab frame that makes accelerating motion, relative to the rocket frame. So, if the induced gravitational field did not exist, by symmetry, the rocket frame would be equivalent to the lab frame and the above physical effects would have to occur in the lab frame and not in the rocket frame, for the observer R. Nevertheless, the uniform gravitational field exists and the above physical effects occur in the rocket frame and not in the lab frame, for both observers. **In order for this to happen, the only way is to think that the induced uniform gravitational field, in the rocket frame, should be capable of causing all the above physical effects, with the same numerical values.** For this reason, we accept the next principle,

Principle 4 - The principle of equivalence

Physics in an accelerating local reference frame with uniform acceleration $a = -g$, in a region without gravity, is equivalent to physics in a non accelerating local reference frame with a uniform gravitational field, where all the released bodies fall with acceleration g .

From the viewpoint of the observer R, the lab frame makes a free fall in the gravitational field that he perceives but none of the above physical effects happen in the lab frame, although it is accelerating. **In order for this to happen, the only way is to think that the gravitational field must exactly cancel the acceleration of the lab frame so that, no sign of either acceleration or gravitation can be found by any physical means in the lab frame.** So, we have an alternative expression of the equivalence principle,

Principle 4 - The principle of equivalence

Physics in a local reference frame, freely falling in a gravitational field, is equivalent to physics in an inertial reference frame without gravity.

From the viewpoint of the observer R, the fixed stars and the lab frame, fall in the gravitational field that he perceives. Thus, according to the equivalence principle, no sign of either acceleration or gravitation can be found by any physical means in the fixed stars and in the lab frame because they are freely falling in a gravitational field. **Therefore, the observer R doesn't observe any radiation field from them, because radiation is a sign of acceleration or gravitation. In order for this to happen, the only way is to think that the fields make a free fall just like the stars.**

So, for the the observer R, the instantaneous potentials of the free falling stars are the same with the potentials that they would have if they were moving with uniform velocity, equal to the instantaneous relative velocity.

Therefore, we can find the instantaneous potentials of the free falling stars in the rocket frame, from the potentials of them in the lab frame, using just the Lorentz transformation with the instantaneous relative velocity.

Thus, when the rocket accelerates relative to the fixed stars it is equivalent to say either that

1. the rocket accelerates and thus radiates because of the acceleration, while the fixed stars are stationary and thus they do not radiate, or that
2. the rocket is stationary in a universal gravitational field where all the fixed stars make free fall, and thus they do not radiate, whereas the rocket which is at rest in the gravitational field radiates because of the gravitational field.

So, with the principle of general relativity and the principle of equivalence we have ensured the relativity of all kinds of translatory motion. Moreover, as we have seen, the equivalence principle shows us that spacetime is endowed with a metric and the gravitational field affects the spacetime metric so that, the maximum proper time is the law of motion of a freely moving body in a gravitational field. The two above physical effects are so important that we will elevate them to physical principles. So, we will accept the next two principles:

Principle 5 - The principle of spacetime metric

Spacetime is endowed with a metric. The spacetime interval between two events is:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where $g_{\mu\nu}$ is the metric tensor.

Principle 6 - The principle of geodesic motion or of maximum proper time

Freely moving test bodies, in a gravitational field, follow geodesic of the metric:

$$\delta \int ds = 0$$

The gravitational force affects the spacetime metric and so, we can say that gravity curves the spacetime. The spacetime of Special theory of relativity is the Minkowski spacetime which is a flat spacetime. While it is clear that flat and curved spaces are different entities, they are closely related. We are familiar from our experience with smoothly curved surfaces that any smoothly curved space can be approximated locally by a flat plane. This is the content of the local-flatness theorem. According to the local-flatness theorem, the metric in the immediate neighborhood of a point P is, to a close approximation, the Minkowski spacetime metric and the laws of Special relativity are valid there [10][11]. Therefore, **everywhere locally the laws of Special theory of relativity are valid.**

Let's follow now the principle of general relativity by applying the Newton's First Law, which is a law of Special relativity, **in the proper-frame of an accelerating body, i.e. the frame where the body is always at rest.** We assume that the the proper-frame is sufficiently small so that we can use the local-flatness theorem. Newton's First Law of motion states that a body, subject to no forces, remains at rest or continues to move in a straight line with constant speed. In its proper frame the body is at rest and so, according to the Newton's First Law the net force on it is zero. **So, we come to the conclusion that, in the proper-frame of any-body the total force on the body is equal to zero.**

So, in order for the acceleration to be relative, we have now all the principles of Einstein's General theory of relativity except the weak equivalence principle, i.e. the equality of the gravitational mass and the inertial mass. Now, having created all the tools we need, we can move on and consider what are the inertial forces and the inertial mass.

4 Inertia and inertial mass

Let's continue now, the lab frame experiment. We assume that the lab frame and the rocket frame have the three sets of axes parallel and common the x, x' axis. **The rocket, which is initially at rest** in the lab frame, begins to accelerate along the x axis. We assume that the rocket frame is sufficiently small, so that, according to the local-flatness theorem, we can consider that the spacetime is flat in the rocket frame, and so we can apply the laws of Special relativity. We have shown in the previous chapter, that **we can find the potentials of the fixed stars in the accelerated rocket frame, from their potentials in the lab frame, using the Lorentz transformations with the instantaneous relative velocity.**

The transformation laws which give the gravitational scalar potential φ'_g and the vector potential \mathbf{A}'_g in a moving frame S' , in terms of φ_g and \mathbf{A}_g in a stationary frame S , as measured at the same point, at the same time by people in the two frames, are

$$\begin{aligned}\varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}), & A'_{g-y} &= A_{g-y} \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g), & A'_{g-z} &= A_{g-z}, & \gamma(v) &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\end{aligned}\tag{4.1}$$

This assumes that the primed reference frame is moving with speed v in the positive x -direction, as measured in the unprimed reference frame. We consider now that the rocket frame is the moving frame and the lab frame is the stationary frame. When the instantaneous speed of the rocket frame is v in the positive x -direction, as measured in the lab frame, it is straightforward to find the instantaneous potentials φ'_g and \mathbf{A}'_g in the accelerated rocket frame, in terms of φ_g and \mathbf{A}_g in the stationary lab frame. Substituting for A_{g-x} from equation (3.2) into equation (4.1) and using vector notation with \mathbf{v} the velocity of the rocket in respect to the lab frame, the potentials in the rocket frame are [12][13],

$$\varphi'_g = \gamma(v)\varphi_g\tag{4.2}$$

$$\mathbf{A}'_g = -\frac{1}{c^2}\varphi'_g\mathbf{v} = -\frac{1}{c^2}\gamma(v)\varphi_g\mathbf{v}\tag{4.3}$$

The gravitational field in the rocket frame now, is given from the equation (2.3),

$$\mathbf{E}'_g = -\nabla'\varphi'_g - \frac{\partial\mathbf{A}'_g}{\partial t'}\tag{4.4}$$

where $\partial t'$ is the time interval, in the rocket frame. The gravitomagnetic field, in the rocket frame, is zero because all the fixed stars make translatory motion in respect to the rocket frame and so, $\nabla' \times \mathbf{A}'_g = 0$.

The $\gamma(v)$ factor is the same everywhere in the rocket frame. Hence, from the equation (4.2), the scalar potential φ'_g is always the same everywhere in the rocket frame and therefore,

$$\nabla' \varphi'_g = 0 \quad (4.5)$$

So, the gravitational field in the rocket frame becomes

$$\mathbf{E}'_g = -\frac{\partial \mathbf{A}'_g}{\partial t'} \quad (4.6)$$

Because the vector potential is also the same everywhere in the rocket frame, an induced uniform gravitational field appears in the accelerated rocket frame, whereas in the lab frame there is no gravitational field.

If a test-body K with gravitational mass m , is at rest in the rocket frame, will experience a gravitational force,

$$\mathbf{F}'_g = m\mathbf{E}'_g = -\frac{\partial (m\mathbf{A}'_g)}{\partial t'} \quad (4.7)$$

Substituting for \mathbf{A}'_g from equation (4.3) into equation (4.7), we obtain,

$$\mathbf{F}'_g = \frac{m}{c^2} \frac{\partial(\varphi'_g \mathbf{v})}{\partial t'} = \frac{m}{c^2} \left(\frac{\partial \varphi'_g}{\partial t'} \mathbf{v} + \varphi'_g \frac{\partial \mathbf{v}}{\partial t'} \right) \quad (4.8)$$

We can have now, some very important results for non relativistic velocities. So, for $\gamma(v) = 1$, $\varphi'_g = \varphi_g$, $\partial t' = \partial t$ and $\frac{\partial \varphi'_g}{\partial t'} = \frac{\partial \varphi_g}{\partial t} = 0$. Thus, the equation (4.8) becomes

$$\mathbf{F}'_g = \frac{m\varphi_g}{c^2} \frac{\partial \mathbf{v}}{\partial t} = \frac{1}{c^2} m\varphi_g \mathbf{a} = \left(-\frac{1}{c^2} m\varphi_g \right) (-\mathbf{a}) \quad (4.9)$$

where \mathbf{a} is the acceleration of the test-body K in respect to the lab frame. If we recall now that, the gravitational scalar potential is negative, it is obvious from the equation (4.9) that the induced gravitational force on the test-body K, is opposite in direction to the acceleration of the body and thus resists to any change of its speed. **It is an inertial force!**

We will call the force which is given from the equations (4.7), external gravitational inertial force because it is due to the acceleration in respect to the external bodies, i.e. in respect to the fixed stars. So, the external gravitational inertial force on the test-body K, for all speeds, is given by the equation

$$\mathbf{F}'_{ext-in-g} = \mathbf{F}'_g = -\frac{\partial (m\mathbf{A}'_g)}{\partial t'} \quad (4.10)$$

In addition to the external inertial force, there is also a self-force. Lorentz originally calculated the electromagnetic self-force using a spherical charge distribution [14]. The well known picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. **We will call this self-force, internal inertial force, because it depends on the internal structure of the body.**

According to the equivalence principle, when a body makes free fall in a gravitational field, the gravitational field exactly cancels the acceleration so that, no sign of either acceleration or gravitation can be found by any physical means on the body. **Therefore, because the internal inertial force is a sign of acceleration, it should be canceled when the body makes free fall.** So, we come to the conclusion that,

- **When a body makes a free fall in a gravitational field, it experiences only the external inertial force.**

Let's imagine now, that the test-body K is a body without internal structure and thus, when it is accelerated by a force \mathbf{F} in the lab frame, it does not experience any internal inertial force. The inertial force on the body is only the external gravitational inertial force. According to the Newton's First Law, as we restate it, in the proper-frame of the test-body K, the total force on the body, is zero. Therefore, for non relativistic velocities, from the equation (4.9), the force \mathbf{F} that accelerates the test-body K is

$$\mathbf{F} = -\mathbf{F}'_{ext-in-g} = -\mathbf{F}'_g = \left(-\frac{1}{c^2}m\varphi_g\right)\mathbf{a} = m_{in-g0}\mathbf{a} \quad (4.11)$$

With m_{in-g0} as inertial rest mass, the equation (4.11) is the Newton's Second Law for non relativistic velocities. We shall call the inertial rest mass of the test-body K, gravitational inertial rest mass m_{in-g0} and its momentum gravitational momentum \mathbf{p}_g because they are due to external gravitational forces. Hence, from the equation (4.11), the gravitational inertial rest mass m_{in-g0} of a body without internal structure, with gravitational mass m, is

$$m_{in-g0} = \left(-\frac{1}{c^2}m\varphi_g\right) \quad (4.12)$$

So, the gravitational inertial rest mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential from the entire Universe at the position of the body.

From equation (4.12) it follows that the gravitational inertial rest mass of a body is independent of the direction of its acceleration. This problem has the name "anisotropy of inertia" and was the subject of experimental investigation with negative results [15].

It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. With this consideration, the distant matter is of predominant importance, while local matter has only a small effect on the gravitational scalar potential.

If the Universe consists of n discrete gravitational masses, each at a different distance r_i from the the test-body K, as measured in the lab frame, the gravitational inertial rest mass of the the test-body B, in the lab frame, is

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} \quad (4.13)$$

Let's consider now if the gravitational inertial rest mass of the test-body K is Lorentz invariant. In the equation (4.13) the distances are measured in the lab frame and so they are proper distances and therefore they are Lorentz invariant. The gravitational masses are also Lorentz invariant **So, the gravitational inertial rest mass of a body, without internal structure, is an invariant quantity.**

In another area in the Universe, we can have a second lab frame with different gravitational scalar potential φ'_g and gravitational vector potential also equal to zero. The gravitational inertial rest mass of the test-body K, in this second lab frame, will be

$$m_{in-g0} = -\frac{1}{c^2}m\varphi'_g \quad (4.14)$$

So, the gravitational inertial rest mass of a body without internal structure is Lorentz invariant but it does not have the same value everywhere.

Having the relation for the gravitational inertial rest mass, the gravitational momentum of the test-body K, in the lab frame, is

$$\mathbf{p}_g = \gamma(v) \left(-\frac{1}{c^2}m\varphi_g \right) \mathbf{v} = m_{in-g} \mathbf{v} \quad (4.15)$$

where m_{in-g} the gravitational inertial mass of the test-body K, in the lab frame.

When the test-body K is at rest in the lab frame, its gravitational rest energy is

$$E_{g0} = m_{in-g0}c^2 = \left(-\frac{1}{c^2}m\varphi_g \right) c^2 = -m\varphi_g \quad (4.16)$$

and its gravitational potential rest energy is

$$U_{g0} = m\varphi_g \quad (4.17)$$

Thus, the total energy of the test-body K, is

$$E_{g0} + U_{g0} = -m\varphi_g + m\varphi_g = 0 \quad (4.18)$$

It's noteworthy that Richard Feynman writes in the "Lectures on Gravitation" [16]:

"Another spectacular coincidence relating the gravitational constant to the size of the universe comes in considering the total energy. The total gravitational energy of all the particles of the universe is something like GMM/R , where $R=Tc$, and T is the Hubble's time. [...] If now we compare this number to the total rest energy of the universe, Mc^2 , lo and behold, we get the amazing result that $GM^2/R = Mc^2$, so that the total energy of the universe is zero. [...] Why this should be so is one of the great mysteries and therefore one of the important question of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate?"

Let's suppose now that a body A with internal structure and gravitational mass m , makes free fall in the gravitational field of a body B with spherically symmetric gravitational mass M and $M \gg m$, in a place where the gravitational scalar potential is φ_g and the gravitational vector potential is zero. **In the free fall of a body in a gravitational field, as we have shown, the internal structure of the body does not play any role and the body experiences only the external inertial force.** So, for non relativistic velocities, the Universal Newton's Law of gravitation and the Newton's Second Law gives for the magnitude of the acceleration of the body A,

$$G \frac{Mm}{r^2} = \left(-\frac{1}{c^2}m\varphi_g \right) a \quad (4.19)$$

where r the body's A distance from the centre of the body B. It is obvious that the gravitational mass m is canceled in equation (4.19). **So, for non relativistic velocities, all the bodies released from the same point in a gravitational field, fall with the same acceleration.** This is a fundamental

experimental result that has been tested with enormous precision. In Einstein's General relativity, the above result is interpreted by accepting the equality of the gravitational mass and the inertial mass.

Let's consider now the case where there are electric charges around the lab frame with such a distribution that the total electric scalar potential φ_e in the lab frame is not zero but is the same everywhere so that, $\nabla\varphi_e = 0$ and the total electromagnetic vector potential \mathbf{A}_e of the electric charges in the lab frame, is zero. So, the electric field in the lab frame is zero. Let's have a test-body B without internal structure, with gravitational mass m and electric charge q which is accelerated in the lab frame. From the viewpoint of the test-body B all the other charges make a free fall in the gravitational field that it perceives and so, they do not radiate. We shall call the momentum \mathbf{p}_{ge} of the test-body B gravitoelectric momentum and its inertial mass m_{in-ge} gravitoelectric inertial mass because they are due to external gravitational and electric forces. Because the equations of electromagnetism have the same mathematical form as the equations of gravitomagnetism, the gravitoelectric momentum of the test-body B is

$$\mathbf{p}_{ge} = \gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \mathbf{v} \quad (4.20)$$

and the gravitoelectric inertial rest mass m_{in-ge0} of the test-body B, which is Lorentz invariant, is

$$m_{in-ge0} = \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \quad (4.21)$$

We assume now, that the Universe consists of n discrete gravitational masses and m discrete electric charges, each at a different distance r_i from the the test-body B, as measured in the lab frame. In this case, according to equations (4.21), the gravitoelectric inertial rest mass of the the test-body B, in the lab frame, is

$$m_{in-ge0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \sum_{i=1}^m \frac{qq_i}{r_i} \right) \quad (4.22)$$

Let's calculate now the order of magnitude of the distance between the test-body B and a charged particle with charge Q , in order for the effect of the electric scalar potential on the inertial rest mass of the test-body B to be significant. The gravitoelectric inertial mass of the test-body B according to the equation (4.22), is

$$m_{in-ge0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^N \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \right) = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad (4.23)$$

For like charges there is a distance where the inertial mass of the test-body B "becomes almost zero". Let's calculate approximately this distance. When this happens, equation (4.23) becomes

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \Rightarrow r = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{m_{in-g0}} \quad (4.24)$$

For $q = Q$ if the test-body B is a proton $r \approx 1,53 \times 10^{-18}m$ and if the test-body B is an electron $r \approx 2,81 \times 10^{-15}m$. **So, the effect of electric potential becomes significant in the subatomic world.**

Having finished with the gravitoelectric inertial rest mass of a body without internal structure, let's consider now the total inertial rest mass of a composite body M, a body with internal structure. We shall call it just inertial rest mass. The Special theory of relativity accepts the principle of conservation of four-momentum, i.e. the sum of the four-momentum of all the

particles going into a collision, is the same as the sum of the four- momentum of all those coming out. If we apply the conservation of four-momentum in a inelastic collision where n free moving point particles without internal structure, collide and create a composite body M which is at rest in a reference frame, the inertial rest mass m_{in-0} of the composite body M is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + \sum T/c^2 + E_{field}/c^2 \quad (4.25)$$

where $m_{in-ge0i}$ is the gravitoelectric inertial rest mass of each particle, without internal structure, that makes up the body M , T is the kinetic energy of the relative motion of the particles that make up the body M , and E_{field} is the potential energy of the interaction of the particles that make up the body M [17]. **The inertial rest mass m_{in-0} of the composite body M , is also Lorentz invariant as is well known from the Special theory of relativity [18].**

Therefore, if we have the gravitoelectric inertial rest mass of each point particle, without internal structure, using the principle of conservation of four-momentum, we end up with the relation that gives us the inertial rest mass of the body M with internal structure, which is Lorentz invariant.

5 Spacetime metric

The equivalence principle shows us that, spacetime is endowed with a metric and the gravitational forces affects the spacetime metric. So, we will find now the spacetime metric outside of a stationary and static body B with spherically symmetric distribution of gravitational mass M . We will consider the freely motion of a test-body A with gravitational mass m in the radial direction of the gravitational field of the body B . We suppose that $M \gg m$. We will follow a new method based on a paper of F. Tangherlini [19][20] [21].

From the principle of spacetime metric we have the spacetime interval between two events

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (5.1)$$

It is well documented that we can bring the spacetime interval outside of a stationary body with spherically symmetric distribution of gravitational mass, into the standard Schwarzschild form [22]

$$ds^2 = g_{00}c^2dt^2 + g_{11}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.2)$$

We assume now that,

1. The scalar functions g_{00} and g_{11} , of the metric tensor, are functions only of the distance from the centre of the body B .
2. The gravitational inertial rest mass m_{in-g0} , of the test body A , is constant during the radial motion (We use the gravitational inertial rest mass because the test body A makes a free fall and thus experiences only the external gravitational inertial force).
3. The metric (5.2) should give us to infinity the Minkowski metric in spherical coordinates

$$ds^2 = c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.3)$$

So we must have the boundary conditions

$$\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1 \quad \lim_{r \rightarrow \infty} g_{11}(r) \rightarrow -1 \quad \lim_{r \rightarrow \infty} g_{00}(r)g_{11}(r) \rightarrow -1 \quad (5.4)$$

For radial motion of the test-body A, the spacetime interval of equation (5.2), becomes

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{11}(r)dr^2 \quad (5.5)$$

From the principle of geodesic motion we have

$$\delta \int ds = 0 \Rightarrow \delta \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = 0 \Rightarrow \delta \int \sqrt{L} d\tau = 0 \quad (5.6)$$

where τ is the proper time. The L may be termed a 'lagrangian'. Using the relations

$$\dot{x}^\mu = \frac{dx^\mu}{d\tau} \quad \dot{x}^\nu = \frac{dx^\nu}{d\tau} \quad (5.7)$$

the lagrangian L becomes

$$L = g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \quad (5.8)$$

For radial motion the lagrangian becomes

$$L = g_{00}c^2 \left(\frac{dt}{d\tau} \right)^2 + g_{11} \left(\frac{dr}{d\tau} \right)^2 \quad (5.9)$$

From the lagrangian, using the calculus of variation, we obtain the Euler-Lagrange system of equations

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}} \right) - \frac{\partial L}{\partial t} = 0 \quad \text{and} \quad \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0 \quad (5.10)$$

where

$$\dot{t} = \frac{dt}{d\tau} \quad \text{and} \quad \dot{r} = \frac{dr}{d\tau}$$

Because the Lagrangian does not depend on time,

$$\frac{\partial L}{\partial t} = 0 \quad (5.11)$$

Therefore, from equations (5.10) we have

$$\frac{\partial L}{\partial \dot{t}} = \text{const.} \quad (5.12)$$

So, the term $\frac{\partial L}{\partial \dot{t}}$ is a conserved quantity. Performing the differentiation in the equation (5.9), using the fact that the metric function $g_{00}(r)$ doesn't depend on t and the relation $ds^2 = c^2 d\tau^2$, we obtain

$$\frac{\partial L}{\partial \dot{t}} = 2g_{00}c^2 \frac{dt}{d\tau} = \text{const.} \Rightarrow g_{00} \frac{dt}{ds} = k_0 \quad (5.13)$$

Equation (5.13) is a well known first integral for the equation of motion of the test-body A, in the radial direction and states that the body's energy k_0 (per unit mass) is a constant of the motion. We can have a second first integral for the equation of motion in the radial direction by dividing the equation (5.5) by the spacetime interval ds^2

$$1 = g_{00}c^2 \left(\frac{dt}{ds} \right)^2 + g_{11} \left(\frac{dr}{ds} \right)^2 \quad (5.14)$$

Equation (5.14) is the second first integral for the equation of motion of the test-body A, in the radial direction and states the invariant relation between energy and momentum (per unit mass). Eliminating $\frac{dt}{ds}$ from the equation (5.14), using the first integral from equation (5.13), we have

$$1 = \frac{c^2 k_0^2}{g_{00}} + g_{11} \left(\frac{dr}{ds} \right)^2 \quad (5.15)$$

We define a matrix $g^{\nu\sigma}$ as the inverse of $g_{\nu\sigma}$, that is, $g^{\nu\sigma} g_{\kappa\nu} = \delta_{\kappa}^{\sigma}$. Because the metric tensor is diagonal we have $g^{\nu\nu} g_{\nu\nu} = 1$ and so

$$g^{00} = \frac{1}{g_{00}} \quad \text{and} \quad g^{11} = \frac{1}{g_{11}} \quad (5.16)$$

Dividing the equation (5.15) by g_{11} and using the equations (5.16) we have

$$\frac{1}{g_{11}} = \frac{c^2 k_0^2}{g_{00} g_{11}} + \left(\frac{dr}{ds} \right)^2 \Rightarrow g^{11} = c^2 k_0^2 (g^{00} g^{11}) + \left(\frac{dr}{ds} \right)^2 \quad (5.17)$$

Because we assume that g_{00} and g_{11} are functions only of r

$$\frac{dg^{00}}{ds} = \frac{\partial g^{00}}{\partial r} \frac{dr}{ds} = g^{00},r \frac{dr}{ds} \quad \text{and} \quad \frac{dg^{11}}{ds} = \frac{\partial g^{11}}{\partial r} \frac{dr}{ds} = g^{11},r \frac{dr}{ds} \quad (5.18)$$

where a comma denotes ordinary differentiation. Using the relations (5.18) we differentiate the equation (5.17) in respect to s and we obtain the following equation

$$g^{11},r \frac{dr}{ds} = c^2 k_0^2 (g^{00} g^{11}),r \frac{dr}{ds} + 2 \frac{dr}{ds} \frac{d^2 r}{ds^2} \Rightarrow \frac{d^2 r}{ds^2} = -\frac{c^2 k_0^2}{2} (g^{00} g^{11}),r + \frac{1}{2} g^{11},r \quad (5.19)$$

Using the relation $ds^2 = c^2 d\tau^2$, we obtain the radial geodesic equation (5.19) in the form,

$$\frac{d^2 r}{d\tau^2} = -\frac{c^4 k_0^2}{2} (g^{00} g^{11}),r + \frac{c^2}{2} g^{11},r \quad (5.20)$$

In the equation (5.20) we can see that because of the k_0 term, the radial acceleration depends on the energy which the test-body A had initially, i.e. the radial velocity with which the test-body A was launched. However, it is well known that the electric force on a charge is strictly independent of the charge's velocity [23]. Therefore, the proper acceleration of a charge due to an electric force, for a motion of the charge along the line of the force, is strictly independent of the charge's velocity [24]. The same happens with the gravitational force, because we accept that both forces are described by equations with the same mathematical form. **Therefore, the radial acceleration of the test-body A must also be independent of the radial velocity with which it was launched.**

Thus, in order for the radial acceleration to be independent of the energy which the test-body A had initially, this must be valid:

$$(g^{00} g^{11}),r = 0 \Rightarrow g^{00} g^{11} = \text{const.} \quad (5.21)$$

Taking into consideration the boundary conditions (5.4) and the equation (5.16), we obtain

$$g^{00} g^{11} = -1 \Rightarrow g^{11} = -\frac{1}{g^{00}} \Rightarrow g_{11} = -\frac{1}{g_{00}} \quad (5.22)$$

So, the line element wich is given from the equation (5.2), can be written as

$$ds^2 = g_{00}(r) c^2 dt^2 - g_{00}(r)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.23)$$

From the equation (5.22), using the equation (5.16), we have

$$g^{00}g^{11} = -1 \Rightarrow g^{11} = -\frac{1}{g^{00}} \Rightarrow g^{11} = -g_{00} \quad (5.24)$$

So, using the relations (5.21) and (5.24), the radial geodesic equation of motion (5.20), becomes

$$\frac{d^2r}{d\tau^2} = -\frac{c^2}{2}g_{00,r} \quad (5.25)$$

The final step in our derivation is to use the principle of consistency and the Newtonian limit to relate the g_{00} component of the metric tensor to the gravitational potential of the spherically symmetric body B centred on the origin. The principle of consistency asserts that a new theory that aims to replace or supersede earlier theories should account for the successful predictions of those earlier theories. According to the principle of consistency, in the Newtonian limit, i.e. when the test-body A moves with nonrelativistic velocity $v \ll c$, in a weak and static gravitational field, the equation (5.25) reduces to the radial equation of motion of the test-body A in Minkowski spacetime, which is

$$m_{in-g0} \frac{d^2\mathbf{r}}{dt^2} = -m\nabla\varphi_{gB} \Rightarrow \frac{d^2\mathbf{r}}{dt^2} = -\frac{m}{m_{in-g0}}\nabla\varphi_{gB} \quad (5.26)$$

where m_{in-g0} is the gravitational inertial rest mass of the test-body A and φ_{gB} is the gravitational scalar potential of the body B. So, in the Newtonian limit, by taking into consideration that a comma denotes ordinary differentiation, from the equations (5.25) and (5.26), this must be valid:

$$-\frac{c^2}{2} \frac{\partial g_{00}}{\partial r} = -\frac{m}{m_{in-g0}} \frac{\partial \varphi_{gB}}{\partial r} \Rightarrow \frac{\partial g_{00}}{\partial r} = \frac{2m}{c^2 m_{in-g0}} \frac{\partial \varphi_{gB}}{\partial r} \quad (5.27)$$

Because we assume that the gravitational inertial rest mass m_{in-g0} of the test body A, is constant during the radial motion, using the boundary conditions (5.4) we obtain for g_{00} from the equation (5.27),

$$g_{00} = 1 + \frac{2m}{c^2 m_{in-g0}} \varphi_{gB} \quad (5.28)$$

and for g_{11} , because of the equation (5.22),

$$g_{11} = -\frac{1}{1 + \frac{2m}{c^2 m_{in-g0}} \varphi_{gB}} \quad (5.29)$$

So, the spacetime interval outside of the body B, from the equation (5.23), becomes

$$ds^2 = \left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{m}{m_{in-g0}} \frac{2GM}{c^2 r}\right)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.30)$$

In the International system of units the ratio of the gravitational mass to the gravitational inertial rest mass of the test-body A, is equal to unity and so, the equation (5.30) becomes the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5.31)$$

So, the new theory is in agreement with all the past experiments and observations. In another area in our Galaxy or in the Universe, the ratio of gravitational to inertial mass is not equal to unity. There, the phenomena will be the same qualitatively but not quantitatively.

6 Dark matter and dark energy

6.1 Dark matter

From equation (4.22), which is valid for non relativistic velocities, it follows that the inertial mass of a star depends on the gravitational scalar potential of the entire Universe, i.e. the inertial mass of a star depends on the distribution of matter in the Universe. From astronomical observation, we know that the distribution of matter in the Universe is highly inhomogeneous; there are planets, the sun, stars, galaxies, clusters of galaxies and so on. So, it seems that the position where a star is located, affects significantly the inertial mass of the star. In places with higher density of matter the inertial mass of a star will be greater than the inertial mass of the same star, in a place with lower density of matter. This phenomenon has been observed, but the inability to explain it has led to the theory that in the Universe most of the matter is dark matter. It is very likely that the equation (4.22) provides a solution to this problem.

6.2 Dark energy

Let's consider now the light emitted by an atom, with gravitational mass m and gravitational inertial rest mass m_{in-g0} , at a distance r_{em} from the centre of a star, which is stationary with spherically symmetric gravitational mass M . From equation (5.30) arises the equation relating the proper time $d\tau_{em}$ at the point of emission, with the proper time $d\tau_{\infty}$ at infinity where is the point of observation [25]

$$d\tau_{\infty} = \frac{d\tau_{em}}{\sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}}} \quad (6.1)$$

So, the frequency of light observed by an observer, in the point of observation, will be

$$f_{\infty} = f_{em} \sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-g0}}} \quad (6.2)$$

The equation (6.2) describes the red shift of spectral lines, which is emitted by an atom in a gravitational field and is received by a body, which is out of the gravitational field. This phenomenon is known as gravitational red shift.

As the Universe expands, according to the equation (4.22), the gravitational inertial rest mass of a star decreases. Thus, the gravitational inertial rest mass of an atom, which emits light, decreases over time. As it emerges from equation (6.2) the light emitted by two identical supernovas Ia which move at the same speed in respect to us, at different moments in the history of the Universe, will have different red shift. Because the atoms in a younger supernova have smaller gravitational inertial rest mass than the gravitational inertial rest mass of the atoms in an older supernova, the light emitted by a younger supernova Ia has greater red shift than the light emitted by an older supernova Ia. This phenomenon has been observed, but the inability to explain why the red shift of spectral lines is greater, has led to the theory that the Universe expands in an accelerating way, because of dark energy. It is very likely that the equations (4.22) and (6.2) provide a solution to this problem.

Conclusions

So, in order for the equilibrium to be a frame-independent condition and the acceleration to be relative, we have now a theory that describes gravity by equations which have the same mathematical form as

those of the electromagnetic theory but also has all the features of Einstein's General theory of relativity that have been confirmed experimentally. The field equations of gravitomagnetism determine the field of force, and the curvature of space is a consequence of this field. The mystery with the inertial and gravitational mass has been solved in a very logical way. All bodies can fall with the same acceleration in a gravitational field without the two masses having to be equivalent.

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